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## Strategic allocation of cyclically arriving container vessels to inter-related terminals

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### **Abstract**

We consider a port consisting of a cluster of inter-related terminals, where container vessels arrive cyclically. The problem is to strategically assign a terminal and a time interval of berthing to each of the vessels in the cycle. Restricting properties are terminal quay lengths and quay crane capacity. Conflicting objectives are i) minimizing the number of required quay cranes, ii) minimizing the amount of inter-terminal traffic and iii) minimizing the total weighted deviation from desired berthing intervals. We formulate both a straightforward and an alternative mixed integer linear program to model this system. Results show that the alternative model is much faster solvable and enables to optimize real-life problems within a couple of hours.

# 1 Introduction

In 1960 people started using containers for international conveyance of sea freight for the first time. Since then the containerization has grown rapidly. Nowadays, deep-sea vessels can carry about 15 thousand TEU's (Twenty feet Equivalent Units) and mega container ports are processing up to 15 million TEU's a year. In order to cope with these tremendous amounts of cargo, port operators have to develop efficient logistics systems. Descriptions and classifications as well as solution methods for the main logistics processes in container ports are given in [1], [2] and [3]. These studies determine the so-called berth allocation problem (BAP) as one of the key issues in a container port. We explicitly distinguish between i) the single-terminal BAP, which is concerned with the allocation of a set of vessels to one terminal and ii) the multi-terminal BAP, which is concerned with the allocation of a set of vessels to a cluster of inter-related terminals.

In the last two decades intensive research has been conducted on the single-terminal BAP. The single-terminal BAP consists of two interrelated assignment problems: assign i) a berthing position at the terminal and ii) a time interval of berthing to each vessel. The problem can be represented in a two-dimensional space, where each vessel is a small rectangle, whose dimensions are the vessel's length and handling time. These small rectangles have to be placed within a large rectangle, with dimensions quay length and considered time horizon, such that the small rectangles are not overlapping and satisfy some additional, case-related constraints. A vessel's handling time depends on both the amount of containers to be loaded and unloaded and the number of quay cranes assigned to the vessel. The objective of the single-terminal BAP is usually to minimize the total weighted handling time.

In general, either a static or dynamic, single-terminal BAP is considered. The static case assumes all vessels are in the port before the berths become available. This turns the problem into an assignment problem and makes it solvable in polynomial time with the Hungarian method [4]. This method assigns jobs to machines by sequentially computing shortest paths until each job is assigned to a machine.

In the dynamic case vessels arrive while work is in progress. In addition, a discrete and a continuous, single-terminal BAP is discussed. In the discrete case, the terminal is divided into a finite set of segments. A vessel can only berth at one (or more) of these segments, which means that the length of the vessels is not (or partly) taken into consideration. The problem can then be modeled as a parallel machine scheduling problem [5], where each vessel is a job and the berth is the machine. In the continuous case, vessels can berth anywhere along the terminal. The discrete as well as the continuous case have been proven to be NP-hard [6].

[7] consider the static, single-terminal BAP and the quay crane assignment within one terminal simultaneously by using a two-phase solution procedure. The first phase determines the scheduling and allocation of the vessels as well as the number of quay cranes assigned to each vessel. They assume a vessel's handling time to be inversely proportional to the number of quay cranes assigned to that vessel. In the second phase, a detailed schedule for each quay crane is constructed.

[8] investigate the dynamic, single-terminal BAP in a container port and develop a heuristic procedure dependent on the relaxation of the static, single-terminal BAP. They show that the dynamic case is easy to solve as long as it is closely related to the static one, which means that most of the vessels are already in the port before the berths become available. [9] propose a heuristic for the dynamic, single-terminal BAP and compare results of the discrete and continuous case.

[10] solve different variations of the discrete, single-terminal BAP with the First Fit Decreas-

ing Heuristic. Their results show that the heuristic is effective in obtaining near-optimal solutions. A nonlinear integer program and a genetic algorithm for a different perspective on the single-terminal BAP are proposed in [11]. They consider berth segments where, dependent on their lengths, at most two vessels can berth simultaneously. Furthermore, additional constraints relative to the water depths at the different berths are included. [12] study the discrete, single-terminal BAP, which subsumes as particular cases the single-terminal BAP with priorities, the dynamic, single-terminal BAP and the static single-terminal BAP. In addition both handling time as well as handling costs are dependent on the berth position. A variable neighborhood search heuristic is presented and shown to outperform three other heuristics.

[13] transforms the continuous, single-terminal BAP into a two-dimensional packing problem and formulates it as a graph-theoretical problem. Furthermore, a heuristic is proposed to solve the model efficiently. [14] use a simulated annealing method to solve the continuous, single-terminal BAP and show that the results are similar to the optimal solution. A sequence pair based simulated annealing method is used by [15] to efficiently solve the rectangle packing problem, which is similar to the continuous, single-terminal BAP. [16] propose a stochastic beam search for the berth allocation problem and show that it outperforms both the state-of-the-art simulated annealing meta-heuristic of [17] and the traditional deterministic beam search. In [18], mega-ports with indented berths are modeled and subsequently genetic algorithms are applied to solve the model. Results show that while the indented berths serve mega-vessels faster, the total lead time of all vessels at ports with indented berths is larger than the one at conventional ports.

[19] present models and heuristics for both the discrete and continuous, single-terminal BAP. In [14] and [7] each berthing point is penalized dependent on the deviation from a pre-determined optimal berthing point. However, [19] assume that a vessel's handling time depends on the distance between the berth position of that vessel and the destined stack of its containers.

To our knowledge and as stated in ([20]), hardly any research has been conducted towards the multi-terminal BAP. So far, studies present models and algorithms, which only solve a version of the single-terminal BAP. However, present ports often consist of a cluster of terminals (see Figure 1), where inter-terminal container traffic is established by trucks. Since most ports face a significant amount of transshipment traffic, the allocations of vessels to the different terminals become interdependent and cannot be considered separately anymore.

Hence, in order to derive an optimal schedule, it is necessary to incorporate the complex of interacting terminals in one model: this is called the multi-terminal BAP. This model should then also take into account the amount of inbound and outbound containers and their corresponding destinations. Inbound containers of an arriving vessel for instance could be partly destined for the hinterland and partly for another vessel. Hence, allocation of the two involved vessels to different terminals implies inter-terminal traffic and thus additional costs. However, due to other objectives and constraints this may still be the best or only solution.

## 2 Contributions and Outline

In this paper we consider the multi-terminal BAP. Although we guarantee that terminal quay lengths as well as quay crane capacities are never exceeded, the exact berth position and the exact quay crane allocation within a terminal are still to be determined at a tactical or operational level. This implies that we simultaneously consider a number of inter-dependent, one-dimensional packing problems, which allow capacitated parallel processing. Applying this cut (see Figure (2)), we are able to construct an accurate timetable for real-life instances

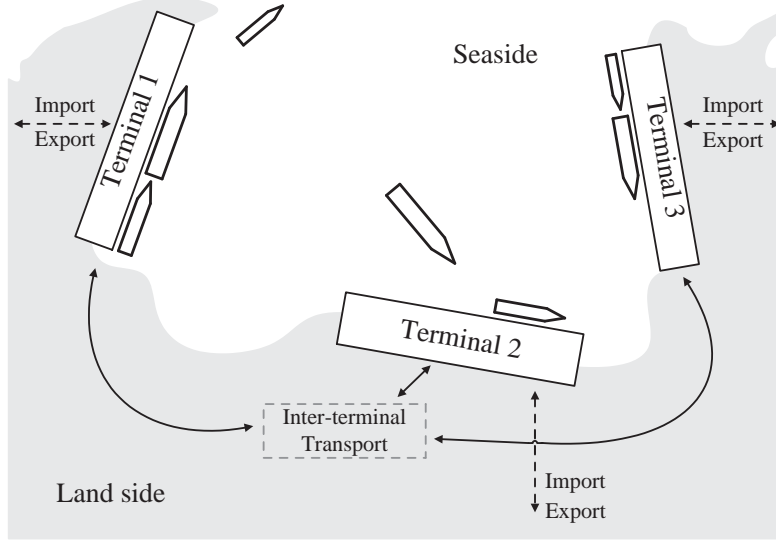


Figure 1: A cluster of inter-related terminals.

rather fast. Results show that, when real-life data is used, feasible position and quay crane allocations exist.

Previous studies after the BAP consider a set of vessels within a certain time horizon. The corresponding objective in these researches often reduces to fitting all vessels within a time horizon and minimizing the total weighted handling time for all vessels. However, in practice most vessels run a regular service on their ports, for instance once a week, which makes the system cyclic. Vessels can arrive at the end of the considered time period (cycle) and leave at the beginning of this time period (next cycle). Relating this to the packing problem implies that rectangles (vessels) can be cut into two pieces, where one piece is placed at the end of the time horizon and the other piece at the beginning, which has not yet been studied for the BAP.

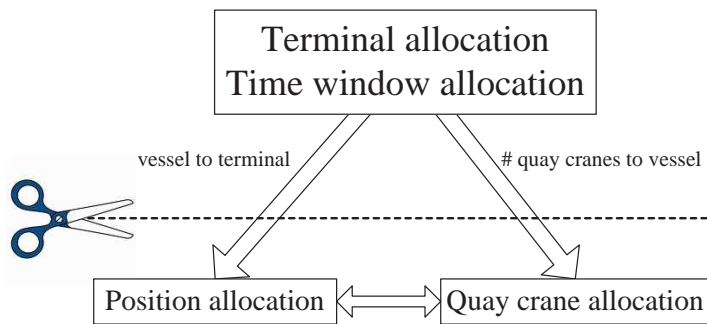


Figure 2: Schematic illustration of the chosen cut.

The contributions of this paper are the following:

- We address the multi-terminal BAP and allocate vessels to a certain terminal for a certain time interval.

- The model in this paper takes the cyclic nature of the system into consideration.
- An alternative approach, introduced here, is much faster solvable than the straightforward approach. Using the alternative approach, we are able to construct accurate allocations for real-life problems within a couple of hours.

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The paper is structured as follows: in Section 3, the problem considered is formally phrased. Then, we introduce a straightforward mixed integer linear program to solve this problem. In addition, we propose an alternative mixed integer linear program. We compare the performance and discuss feasibility aspects of both approaches in Section 4. Finally, in Section 5, we draw conclusions and make recommendations for future research.

## 3 Mathematical Models

In this section, we first describe the problem considered in detail. Next, we propose and discuss two mathematical formulations to solve the problem. The first approach is straightforward in the sense that the way of modeling the berthing of vessels is common. The second model contains a different way of approaching the modeling problem. The way of modeling the cyclic property of the system is similar for both approaches.

### 3.1 System Description

For all of this paper the following holds, unless stated differently:  $t \in \{1, 2, \dots, T\}$ , the cluster of terminals,  $v \in \{1, 2, \dots, V\}$ , the set of vessels,  $z \in \{0, 1, 2, \dots, V\}$ , the set of container destinations. Furthermore, we assume vessels to call cyclically, where each vessel in the set arrives exactly once each cycle. In general, the cycle length is in the order of a week for such a container operation. We consider discrete time  $k$  and unless stated differently,  $k \in \{1, 2, \dots, K\}$ , is the set of discrete time slots within the cycle.

In the cluster of terminals, the set of container vessels has to be unloaded and loaded. Vessel  $v$  imports a pre-determined number of inbound containers  $I_{vz} \in \mathbb{N}$  with destination(s)  $z$ , where  $v \neq z$ . In this context,  $z = 0$  means that containers are destined for the hinterland, whereas  $z = 1, 2, \dots, V$  means that containers are destined for vessels  $v = 1, 2, \dots, V$  respectively. Besides import containers brought in by vessels, a certain amount of containers  $H_v$  with destination  $v$  is imported from the hinterland by trucks and trains during the cycle. These containers are distributed among the different terminals dependent on their destinations. Furthermore, each vessel  $v$  exports a number of outbound containers  $O_v \in \mathbb{N}$ . Container transport between the cluster of terminals is established by trucks.

Terminal  $t$  has a restricted quay length  $L_t \in \mathbb{R}^+$  and a number of quay cranes  $N_t \in \mathbb{N}$ . Once berthing, vessel  $v$  requires a certain amount of quay meters  $l_v$ . In addition, this length  $l_v$  determines the maximum number of quay cranes  $S_v \in \mathbb{N}$  processing vessel  $v$  and the efficiency  $\eta_v \in [0, 1]$  of the quay cranes on vessel  $v$ . In practice, quay cranes with different processing rates are present in the terminals. We do not take the specific allocation of quay cranes to vessels into account, but we use the average processing rate  $\bar{\lambda}_t \in \mathbb{N}$  of all quay cranes in terminal  $t$ . Then the handling time of vessel  $v$  in terminal  $t$  depends on i) the mean processing rate  $\bar{\lambda}_t$  in terminal  $t$ , ii) the efficiency  $\eta_v$  of quay cranes operating vessel  $v$ , iii) the number of quay cranes processing vessel  $v$  and iv) the number of inbound and outbound

containers  $I_{vz}$  and  $O_v$  of vessel  $v$ . We assume the processing time of vessel  $v$  to be inversely proportional to the first three of these items and proportional to the latter. Furthermore, the number of quay cranes processing vessel  $v$  may change from one time slot to another. After the unloading and before the loading, containers can temporarily be stored in the yard of terminal  $t$  up to the yard's capacity  $W_t$ . The time it takes to transport containers from terminal  $p$  to terminal  $r$  is defined as  $\Delta_{pr} \in \mathbb{N}$ . Furthermore, we assume that the total number of time slots vessel  $v$  is actually berthing, is less than the number of time slots  $K$  in the cycle. In addition, we assume that vessels arrive at the beginning of a time slot and depart at the end of a time slot.

Our goal is to minimize the total costs of the system, which consist of three conflicting elements: first of all, costs are associated with the number of quay cranes that have to be installed in the terminals in order to satisfy the proposed schedule. We define  $c_t$  to be the average costs of a quay crane in terminal  $t$ . Second of all, a fixed amount of money  $c_{pr}$  has to be paid for each container that is transported from terminal  $p$  to terminal  $r$ . Finally, the terminal operator has some contractual agreements with each of the vessel lines with respect to the berthing interval of the corresponding vessels. We define  $A_v \in \mathbb{N}$  to be the preferred arrival time of vessel  $v$  in the port and  $D_v \in \mathbb{N}$  to be the preferred departure time of vessel  $v$ . We have to remark that, in our definition, this means that vessel  $v$  prefers to depart at the end of time slot  $[D_v - 2, D_v - 1]$ . If in the constructed strategic allocation, the actual berth time of vessel  $v$  deviates from its preferred arrival time  $A_v$ , fines have to be paid: we assign a factor of penalty costs  $c_v^a$  if vessel  $v$  berths later than the agreed arrival time  $A_v$ . Moreover, we assign factors of penalty costs  $c_v^e$  and  $c_v^d$  if in the constructed allocation vessel  $v$  departs earlier and later than its preferred departure time  $D_v$ , respectively. If the constructed service window of a certain vessel deviates from the preferred service window, negotiations between the terminal operator and the corresponding vessel line have to lead to the eventual allocation.

Parameter	Definition
$T$	Number of terminals in the cluster
$V$	Number of vessels in the set
$K$	Number of discrete time slots within the cycle
$L_t$	Quay length [m]
$l_v$	Quay length required for vessel [m]
$I_{vz}$	# inbound containers to be unloaded from vessel $v$ with destination $z$ and $v \neq z$
$O_v$	# outbound containers to be loaded onto vessel $v$
$H_v$	# containers with destination $v$ arriving from the hinterland during the cycle
$A_v$	Preferred arrival time of vessel $v$
$D_v$	Preferred departure time of vessel $v$
$E_v$	Parameter to distinguish between the cases $A_v < D_v$ and $D_v \geq A_v$
$N_t$	# quay cranes available in terminal $t$
$S_v$	Maximum # quay cranes, which can process vessel $v$
$\bar{\lambda}_t$	Mean processing rate of quay cranes in terminal $t$ [containers/time slot]
$\eta_v$	Vessel efficiency with respect to quay crane rate [-]
$\Delta_{pr}$	# time slots needed to transport containers from terminal $p$ to $r$
$W_t$	# containers that can be stored in terminal $t$
$c_v^a$	Factor of penalty costs for vessel $v$ for arriving too late $[\frac{\text{euro/container}}{\text{time slot}}]$
$c_v^e$	Factor of penalty costs for vessel $v$ for departing too early $[\frac{\text{euro/container}}{\text{time slot}}]$
$c_v^d$	Factor of penalty costs for vessel $v$ for departing too late $[\frac{\text{euro/container}}{\text{time slot}}]$
$c_{pr}$	Factor of transportation costs from terminal $p$ to $r$ [euro/ container]
$c_t$	Factor of costs for required equipment in terminal $t$ [euro/ quay crane]

Table 1: Model parameters



With respect to the cyclic property of the considered system, we have two additional remarks. First, we require conservation with respect to the arrival and departure of containers:

$$\sum_{i=1}^V I_{iv} + H_v = O_v \quad \forall v \quad (I)$$

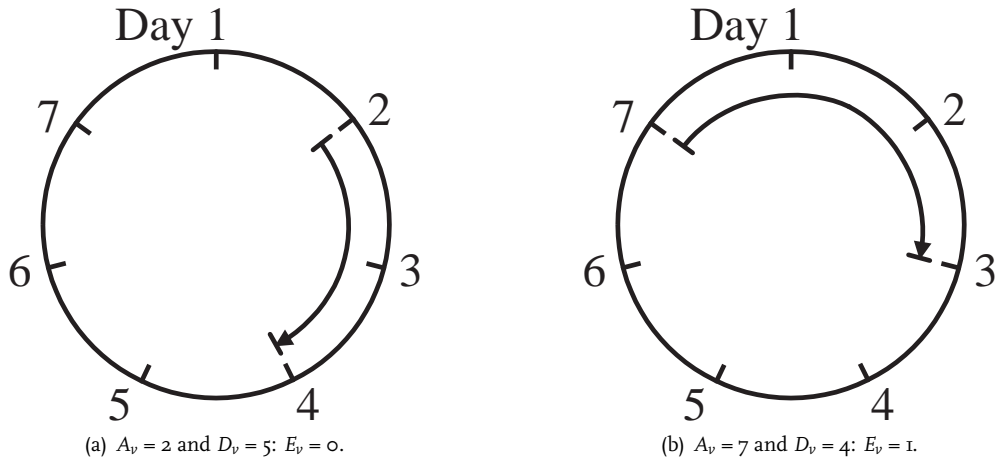


Figure 3: Possible arrivals and departures of vessel  $v$  in terminal  $t$  in a cyclic system with  $K = 7$ .

Second, we notice that both  $A_v \geq D_v$  and  $A_v < D_v$  are possible, since we model cyclically arriving container vessels (see also Figure (3)). Therefore, we introduce an auxiliary parameter  $E_v$ , which explicitly distinguishes between both cases:

$$E_v = \begin{cases} 1 & \text{if } A_v \geq D_v, \\ 0 & \text{if } A_v < D_v. \end{cases} \quad \forall v$$

In the next section it becomes clear why we need this parameter. The sets and parameters discussed above are conveniently arranged in Table 1.

### 3.2 Straightforward MILP

#### Binary variables

$$a_{tv}(k) = \begin{cases} 1 & \text{if in terminal } t \text{ vessel } v \text{ berths during time slot } [k-1, k], \\ 0 & \text{otherwise.} \end{cases}$$

$$d_{tv}(k) = \begin{cases} 1 & \text{if in terminal } t \text{ vessel } v \text{ departs during time slot } [k-2, k-1], \\ 0 & \text{otherwise.} \end{cases}$$

### Auxiliary binary variables

$$\begin{aligned}
 b_{tv}(k) &= \begin{cases} 1 & \text{if in terminal } t \text{ vessel } v \text{ is berthing during time slot } \langle k-1, k \rangle, \\ 0 & \text{otherwise.} \end{cases} \\
 e_{tv} &= \begin{cases} 1 & \text{if } a_{tv}(k_a) = 1 \text{ and } d_{tv}(k_d) = 1 \text{ and } k_a > k_d, \\ 1 & \text{if in terminal } t \text{ vessel } v \text{ is continuously berthing,} \\ 0 & \text{otherwise.} \end{cases} \\
 e_v^a &= \begin{cases} 1 & \text{if } a_{tv}(k_a) = 1 \text{ and } k_a < A_v, \\ 0 & \text{if } a_{tv}(k_a) = 1 \text{ and } k_a \geq A_v. \end{cases}
 \end{aligned}$$

### Continuous variables

$$\begin{aligned}
 m_{tv}(k) &= \text{Amount of quay meters consumed in terminal } t \text{ by vessel } v \text{ during} \\
 &\quad \text{time slot } \langle k-1, k \rangle \text{ [m]} \\
 q_{tv}(k) &= \text{Amount of quay processing vessel } v \text{ in terminal } t \text{ during time slot } \langle k-1, k \rangle \\
 h_{tv}(k) &= \text{Amount of containers from hinterland transported into terminal } t \\
 &\quad \text{with destination } v \text{ during time slot } \langle k-1, k \rangle \text{ [containers/ time slot]} \\
 f_{prv}(k) &= \text{Amount of containers transported from terminal } p \text{ to terminal } r \text{ with} \\
 &\quad \text{destination } v \text{ during time slot } \langle k-1, k \rangle \text{ [containers/ time slot], } p \neq r \\
 w_{tv}(k) &= \text{WIP in terminal } t \text{ with destination } v \text{ at time } k \\
 n_t &= \text{Number of quay cranes required in terminal } t
 \end{aligned}$$

### Integer variables

$$\begin{aligned}
 \Delta_v^a &= \text{Number of time slots vessel } v \text{ berths too late} \\
 \Delta_v^c &= \text{Number of time slots vessel } v \text{ departs too early} \\
 \Delta_v^d &= \text{Number of time slots vessel } v \text{ departs too late}
 \end{aligned}$$

### Constraints

Vessel  $v$  can only arrive once each cycle and only at one terminal:

$$\sum_{t=1}^T \sum_{k=1}^K a_{tv}(k) = 1 \quad \forall v \quad (2)$$

Furthermore, vessel  $v$  can only depart once each cycle from one terminal, where for vessel  $v$  the departure terminal is equal to the arrival terminal:

$$\sum_{k=1}^K a_{tv}(k) - \sum_{k=1}^K d_{tv}(k) = 0 \quad \forall t, v \quad (3)$$

Iff vessel  $v$  arrives at and departs from terminal  $t$ , terminal  $t$  is occupied by vessel  $v$  only between its arrival and departure time. We introduce the auxiliary binary variable  $e_{tv}$  to distinguish between the cases as given in the definition of  $e_{tv}$ , resulting from the cyclic nature of the system. The principle used here is similar to the one for the auxiliary parameter  $E_v$  (see also Figures 3a and 3b).

$$b_{tv}(k) - e_{tv} = \sum_{k'=k+1}^K (d_{tv}(k') - a_{tv}(k')) \quad \forall t, v, k \quad (4)$$

Furthermore, vessel  $v$  can only berth at one terminal:

$$\sum_{t=1}^T b_{tv}(k) \leq 1 \quad \forall v, k \quad (5)$$

The sum of lengths of all vessels berthing at terminal  $t$  during time slot  $\langle k-1, k \rangle$  should be less than or equal to the total quay length in terminal  $t$ :

$$\sum_{v=1}^V l_v \cdot b_{tv}(k) \leq L_t \quad \forall t, k \quad (6)$$

Vessel  $v$  can be operated by up to  $S_v$  quay cranes in terminal  $t$  during time slot  $\langle k-1, k \rangle$ , iff this vessel is berthing in terminal  $t$  during time slot  $\langle k-1, k \rangle$ :

$$q_{tv}(k) \leq S_v \cdot b_{tv}(k) \quad \forall t, v, k \quad (7)$$

Vessel  $v$  has to be fully processed during the cycle:

$$\sum_{t=1}^T \sum_{k=1}^K \eta_v \bar{\lambda}_t \cdot q_{tv}(k) = \sum_{z=0}^Z I_{vz} + O_v \quad \forall v \quad (8)$$

We want to minimize the maximum number of quay cranes in terminal  $t$  ever required during the cycle. Therefore, we introduce an auxiliary variable  $n_t$ , which is a soft upper bound on the number of quay cranes in terminal  $t$ . This variable  $n_t$  is present in the objective function:

$$\sum_{v=1}^V q_{tv}(k) \leq n_t \quad \forall t, k \quad (9)$$

The maximum number of quay cranes ever required in terminal  $t$  during the cycle cannot be larger than the number of quay cranes actually available in terminal  $t$ :

$$n_t \leq N_t \quad \forall t \quad (10)$$

The sum over the cycle's time slots of the number of containers with destination  $v$ , transported from the hinterland into the different terminals, should be equal to the total number of containers with destination  $v$  arriving from the hinterland during the cycle.

$$\sum_{k=1}^K \sum_{t=1}^T h_{tv}(k) = H_v \quad \forall v \quad (11)$$

Since the system is cyclic, the storage level in the terminals and the inter-terminal transport during time slot  $\langle k-1, k \rangle$  should equal the storage level in the terminals and the inter-terminal transport during time slot  $\langle k-1+\alpha K, k+\alpha K \rangle$ , where  $\alpha \in \mathbb{N}$ :

$$w_{tv}(k) = w_{tv}(k + \alpha K) \quad \forall t, v, k \quad (12)$$

and

$$f_{prv}(k) = f_{prv}(k + \alpha K) \quad \forall p, r, v, k \quad (13)$$

We assume that inbound containers with destination  $o$  ("hinterland") are transported into the hinterland directly after they arrive in the terminal and are not counted as stack.

The amount of containers in terminal  $t$  with destination  $v$  during time slot  $\langle k-1, k \rangle$  is equal to the amount of containers in terminal  $t$  with destination  $v$  during time slot  $\langle k-2, k-1 \rangle$  plus all incoming flows (inbound containers from vessels, containers from other terminals and containers from the hinterland) minus all outgoing flows (outbound containers to vessels and containers to other terminals). We assume that loading and unloading of containers from vessel  $v$  with different destinations is divided proportionally among the time slots vessel

$v$  is actually berthing. To this, we first define the constants  $\beta_{vz} = \frac{I_{vz}}{\sum_{z=0}^Z I_{vz} + O_v}$  and  $\gamma_v = \frac{O_v}{\sum_{z=0}^Z I_{vz} + O_v}$ , and derive appropriate constraints:

$$w_{tv}(k) = w_{tv}(k-1) + \sum_{i=1}^V \beta_{iv} \eta_i \bar{\lambda}_t \cdot q_{ti}(k) - \gamma_v \eta_v \bar{\lambda}_t \cdot q_{tv}(k) + h_{tv}(k) + \sum_{r=1}^T f_{rtv}(k - \Delta_{pr}) - \sum_{r=1}^T f_{trv}(k) \quad \forall t, v, k \quad (I4)$$

If we start with bringing containers into the yard during time slot  $\langle k-1, k \rangle$ , the following constraint has to be satisfied:

$$\sum_{v=1}^V (w_{tv}(k-1) + \sum_{i=1}^V \beta_{iv} \eta_i \bar{\lambda}_t \cdot q_{ti}(k) + h_{tv}(k) + \sum_{r=1}^T f_{rtv}(k - \Delta_{pr})) \leq W_t \quad \forall t, k \quad (I5)$$

If we start with taking away containers from the yard during time slot  $\langle k-1, k \rangle$ , the following constraint has to be satisfied:

$$w_{tv}(k-1) - \gamma_v \eta_v \bar{\lambda}_t \cdot q_{tv}(k) - \sum_{r=1}^T f_{trv}(k) \geq 0 \quad \forall t, v, k \quad (I6)$$

Whatever order is applied during the cycle, (I5) and (I6) guarantee that never too much and never too less (negative amount of) containers are in the yard.

We use the additional integer variables  $\Delta_v^a$ ,  $\Delta_v^c$  and  $\Delta_v^d$  to model the number of time slots vessel  $v$  berths too late, departs too early and departs too late, respectively. We assume vessel  $v$  arrives in the port at time  $A_v$ , which means that the actual berth time can only take place exactly at or later than  $A_v$ . We assume that the costs  $\Delta_v^a$  for late arrival of vessel  $v$  depend linearly on the difference between the preferred arrival time  $A_v$  and the actual arrival time. Due to the cyclic nature of the system, both  $a_{tv}(k_a) = 1$ , where  $k_a < A_v$  and  $k_a \geq A_v$  are possible. Hence, a jump in the cost function occurs at  $A_v$  as depicted in Figure (4).

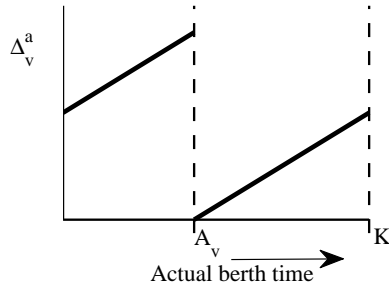


Figure 4: Costs for berthing too late.

To model this jump, we introduce the auxiliary binary variable  $e_v^a$  in an additional constraint:

$$\Delta_v^a = \sum_{t=1}^T \sum_{k=1}^K \left( - (A_v - k) \cdot a_{tv}(k) \right) + K \cdot e_v^a \quad \forall v \quad (I7)$$

where

$$K \cdot e_v^a \geq \sum_{t=1}^T \sum_{k=1}^K (A_v - k) \cdot a_{tv}(k) \quad \forall v \quad (I8)$$

With respect to departing too early or too late one can distinguish  $4! = 24$  permutations (of  $a_v$ ,  $A_v$ ,  $d_v$  and  $D_v$ ). It turns out that with help of the introduced auxiliary binary variables

$e_v^a$  and  $e_{tv}$ , and the auxiliary parameter  $E_v$  as defined in Section 3.1, we are able to construct appropriate constraints for  $\Delta_v^c$  and  $\Delta_v^d$  to satisfy each of the 24 cases:

$$\Delta_v^c \geq \sum_{t=1}^T \sum_{k=1}^K \left( (D_v - k) \cdot d_{tv}(k) \right) - K \cdot e_v^a + K \cdot E_v - \sum_{t=1}^T K \cdot e_{tv} \quad \forall v \quad (19)$$

where

$$\Delta_v^c \geq 0 \quad \forall v \quad (20)$$

and

$$\Delta_v^d \geq \sum_{t=1}^T \sum_{k=1}^K - \left( (D_v - k) \cdot d_{tv}(k) \right) + K \cdot e_v^a - K \cdot E_v + \sum_{t=1}^T K \cdot e_{tv} \quad \forall v \quad (21)$$

where

$$\Delta_v^d \geq 0 \quad \forall v \quad (22)$$

Finally, some of the continuous variables have to be lower-bounded:

$$q_{tv}(k) \geq 0 \quad (23)$$

$$h_{tv}(k) \geq 0 \quad (24)$$

$$f_{prz}(k) \geq 0 \quad (25)$$

$$w_{tz}(k) \geq 0 \quad (26)$$

### Objective function

Linear penalty costs are assigned when vessel  $v$  berths later than its arrival time and/ or when vessel  $v$  departs too early or too late ( $c_v^a$ ,  $c_v^c$ ,  $c_v^d$  respectively). Furthermore, a linear unit penalty cost is assigned when containers are transported from one terminal to another ( $c_{pr}$ ). Finally, linear costs are assigned to the number of required quay cranes in terminal  $t$  ( $c_t$ ). The decision variables are represented in a vector  $u(k) = [a_{tv}(k), d_{tv}(k), h_{tv}(k), q_{tv}(k), f_{prv}(k)]^T$  and the objective function is formulated:

$$\min_{u(1), \dots, u(K)} \sum_{v=1}^V \left( c_v^a \Delta_v^a + c_v^c \Delta_v^c + c_v^d \Delta_v^d \right) + \sum_{k=1}^K \sum_{p=1}^T \sum_{r=1}^T \sum_{z=1}^Z c_{pr} f_{prz}(k) + \sum_{t=1}^T c_t n_t \quad (27)$$

Remark: In the solution of this MILP it could be that an arbitrary amount of containers is stored in a certain terminal during the entire cycle. This could be prevented by assigning a small cost for each stored container.

### 3.3 Alternative MILP

In the previous section, we introduced a straightforward approach of modeling the problem. The cyclic nature of the system is taken into account by (4), (12) and (13), and (17) through (22). In this straightforward formulation, we used binary variables  $a_{tv}(k)$  and  $d_{tv}(k)$ , which indicate whether or not a vessel berths at or departs from terminal  $t$  during time slot  $[k-1, k]$ . In the alternative approach we split these variables into the integer variables  $a_v$  and  $d_v$  and the binary variable  $x_{tv}$ . Here,  $a_v$  and  $d_v$  denote the time slots vessel  $v$  berths and departs, respectively. In our definition  $d_v$  means that the processing of vessel  $v$  ends at the end of time slot  $[k-2, k-1]$ . Additionally,  $x_{tv}$  denotes the terminal in which vessel  $v$  berths. Consequently, some of the constraints of the straightforward approach have to be adapted and even some new constraints have to be introduced to describe the same problem. In the end, however, the alternative way of modeling uses only a fraction  $\frac{T+K+2}{3TK+T+1}$  of the number of binary variables in the straightforward way of modeling.

### Continuous variables

- $a_v$  = Actual berth time slot of vessel  $v$  (start of processing vessel  $v$ )  
 $d_v$  = Actual departure time slot of vessel  $v$  (processing of vessel  $v$  ends at the end of time slot  $\langle k-2, k-1 \rangle$ )

The rest of the continuous variables are equivalent to the continuous variables as described in Section 3.2.

### Binary variables

$$x_{tv} = \begin{cases} 1 & \text{if in terminal } t \text{ vessel } v \text{ berths,} \\ 0 & \text{otherwise.} \end{cases}$$

### Auxiliary binary variables

$$\begin{aligned}
 b_v(k) &= \begin{cases} 1 & \text{if vessel } v \text{ is berthing during time slot } \langle k-1, k \rangle, \\ 0 & \text{otherwise.} \end{cases} \\
 e_v &= \begin{cases} 1 & \text{if } a_v > d_v, \\ 0 & \text{if } a_v < d_v, \\ 1 & \text{if } a_v = d_v \text{ and vessel } v \text{ is continuously berthing,} \\ 0 & \text{if } a_v = d_v \text{ and vessel } v \text{ does not berth at all.} \end{cases} \\
 e_v^a &= \begin{cases} 1 & \text{if } a_v < A_v, \\ 0 & \text{if } a_v \geq A_v. \end{cases}
 \end{aligned}$$

### Constraints

Vessel  $v$  berths at only one terminal  $t$ :

$$\sum_{t=1}^T x_{tv} = 1 \quad \forall v \quad (28)$$

The arrival and departure times ( $a_v$  and  $d_v$  respectively) of vessel  $v$  are within the cycle:

$$1 \leq a_v \leq K \quad \forall v \quad (29)$$

and

$$1 \leq d_v \leq K \quad \forall v \quad (30)$$

Vessel  $v$  berths between its arrival and departure time,  $a_v$  and  $d_v$  respectively. We need generic constraints, which relate  $a_v$  and  $d_v$  to  $b_v(k)$  as well as  $b_v(k)$  to  $a_v$  and  $d_v$  for the cases where  $a_v < d_v$ ,  $a_v = d_v$  and  $a_v > d_v$ . To incorporate the latter case, which follows from the cyclic property of the system, we introduce an auxiliary binary variable  $e_v$ :

$$\sum_{k=1}^K (b_v(k) - e_v) = d_v - a_v \quad \forall v \quad (31)$$

and

$$1 - a_v \leq k \cdot (b_v(k) - e_v) \leq d_v - 1 \quad \forall v, k \quad (32)$$

and

$$d_v - K \leq (K - k) \cdot (b_v(k) - e_v) \leq K - a_v \quad \forall v, k \quad (33)$$

Vessel  $v$  requires an amount of quay meters  $l_v$  at a terminal  $t$  during time slot  $\langle k-1, k \rangle$ , iff the vessel is actually berthing during time slot  $\langle k-1, k \rangle$  at terminal  $t$ .

$$m_{tv}(k) \leq l_v \cdot x_{tv} \quad \forall t, v, k \quad (34)$$

and

$$\sum_{t=1}^T m_{tv}(k) = l_v \cdot b_v(k) \quad \forall v, k \quad (35)$$

Furthermore, the sum of lengths of all vessels berthing at terminal  $t$  during time slot  $\langle k-1, k \rangle$  should be less than or equal to the total quay length of terminal  $t$ :

$$\sum_{v=1}^V m_{tv}(k) \leq L_t \quad \forall t, k \quad (36)$$

Vessel  $v$  can only be operated in terminal  $t$  iff vessel  $v$  is berthing in terminal  $t$ . Furthermore, a maximum number of quay cranes  $S_v$  can be assigned to vessel  $v$ :

$$q_{tv}(k) \leq S_v \cdot x_{tv} \quad \forall t, v, k \quad (37)$$

and

$$q_{tv}(k) \leq S_v \cdot b_v(k) \quad \forall t, v, k \quad (38)$$

Next, constraints (8) through (16) from the straightforward MILP are also valid in this formulation. Furthermore, we have to slightly adapt constraints (17) through (22):

$$\Delta_v^a = -(A_v - a_v) + K \cdot e_v^a \quad \forall v \quad (39)$$

where

$$K \cdot e_v^a \geq A_v - a_v \quad \forall v \quad (40)$$

and

$$\Delta_v^c \geq (D_v - d_v) - K \cdot e_v^a + K \cdot E_v - K \cdot e_v \quad \forall v \quad (41)$$

where

$$\Delta_v^c \geq 0 \quad \forall v \quad (42)$$

and

$$\Delta_v^d \geq -((D_v - d_v) - K \cdot e_v^a + K \cdot E_v - K \cdot e_v) \quad \forall v \quad (43)$$

where

$$\Delta_v^d \geq 0 \quad \forall v \quad (44)$$

## Objective function

The decision variables are represented in a vector  $u(k) = [x_{tv}, a_v, d_v, h_{tv}(k), q_{tv}(k), f_{prz}(k)]^T$  and the objective function is equivalent to the one in (27).

## 4 Results

In the previous chapter, two approaches have been formulated, which model the described system. As a next step, both approaches are coded in Matlab and solved using CPLEX. Results for a large set of randomly generated instances suggest that the models encompass the same solution, but significantly differ in CPU time. In this section, we statistically compare the CPU times of both models dependent on the number of vessels  $V$  in the set. Results suggest that the alternative approach convincingly outperforms the straight-forward approach. Next, we investigate the CPU time of the alternative approach dependent on the number of time slots  $K$  in the considered cycle. Finally, we suggest that from generated terminal and time window allocations of realistic problems, we are able to construct feasible and satisfactory i) two-dimensional packing solutions and ii) quay crane allocations.

#### 4.1 Performance Analysis

In this section, we first compare the CPU time of both approaches dependent on the number of vessels in the set. We consider a problem with a cluster of three terminals  $T = 3$  and a one-week-cycle, where each time slot has a width of one day, hence  $K = 7$ . The rest of the parameters, as described in Section 3.1, are randomly generated. The parameter set is used as input data and, together with the model, fed into CPLEX. The mixed integer optimization is terminated as soon as it has found a feasible integer solution proven to be within 5% of optimal. For each value of  $V \in \{6, 7, 8, \dots, 40\}$ , we randomly (within reasonable bounds following from practice) generate 60 parameter sets and solve both approaches for these instances. Furthermore, for each of these optimizations, we monitor the time CPLEX's CPU time. Figure 5a shows the corresponding geometric means and 95% confidence intervals of the CPU time dependent on  $V$  for both approaches. Due to relatively large CPU times when using the straightforward approach,  $V$  is varied between six and twenty only in this case. The relative difference between the optimums of both approaches for each parameter set are found to be within the 5% optimality gap, which suggests that indeed both models describe the same problem.

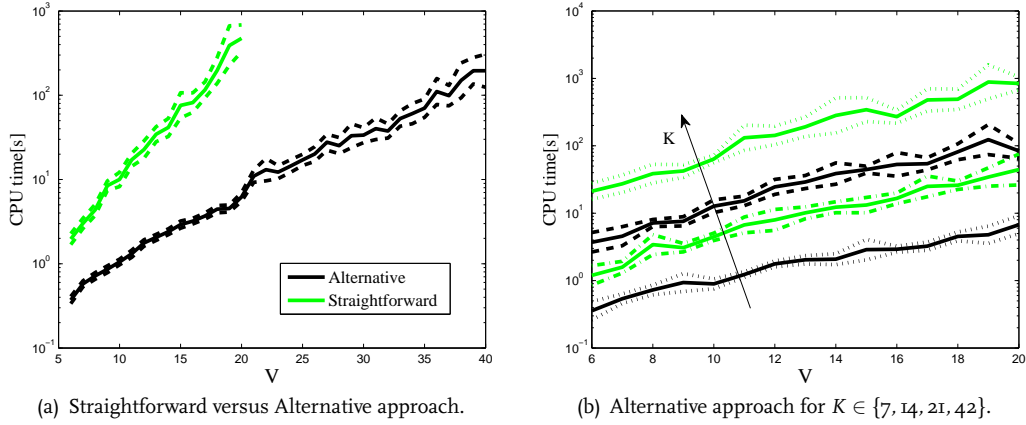


Figure 5: CPU time analysis.

From Figure 5a it is obvious that the alternative approach significantly outperforms the straightforward approach: First of all, the mean CPU time of the alternative approach is significantly shorter than the mean runtime of the straightforward approach for each point in the considered interval. Moreover, the fraction between the CPU time of the straightforward approach and the CPU time of the alternative approach increases exponentially with the number of vessels in the set. Furthermore, the confidence interval of the straightforward approach starts diverging at a smaller  $V$  value than the confidence interval of the alternative approach. We now assume that the order of diverging remains the same for larger  $V$  values. A real-life instance, where the number of vessels in the set is forty, then could CPLEX take weeks or even months if the straightforward approach is applied and only a couple of minutes if the alternative approach is applied. Therefore, in the remainder of this paper, only the alternative approach is used.

The final goal is to consider a real-life case and to improve its current allocation by applying the alternative approach. In order to generate a more and more detailed schedule, the width of a time slot should be decreased, at the expense of larger CPU times. Nowadays, planners of such container operations construct a schedule composed of time slots with 2 or 4 hours width. Therefore, we are interested in the CPU time dependent on the number of time slots



$K$  in the cycle. Again, we consider a port with three terminals  $T = 3$  and a one-week-cycle. Additionally, for each value of  $V \in \{6, 7, 8, \dots, 20\}$ , we generate 10 parameter sets and solve the model for  $K \in \{7, 14, 21, 42\}$ , sequentially. The geometric means and the 95% confidence intervals are shown in Figure 5b. As expected, the CPU time increases as  $K$  increases. Furthermore, the four graphs for the geometric means exhibit approximately the same slope, which suggests that, independent of  $K$  the CPU time grows with the same exponential rate as  $V$  increases. Additional experiments show that a problem of real-life size can be solved within about 6000 seconds for the case with 42 time slots (for the one-week-cycle, the width of a time slot is then 4 hours). This suggests that with the alternative approach a rather accurate allocation can be constructed for a real-life problem within a couple of hours.

## 4.2 Feasibility

Although the proposed method allocates a terminal, a number of quay cranes and a time interval of berthing to a vessel, the actual position within that terminal as well as the actual quay cranes processing that vessel are not specifically generated (see also Figure 2). The omission of these allocations allows to solve the model relatively fast, at the expense of possible infeasibility of the found solution on an operational level. If the terminals would continuously be utilized against their quay lengths capacities, this could lead to a situation where (6) is fulfilled, however a feasible two-dimensional packing solution does not exist. In practice however, ports require a significant utilization margin to compensate for disturbances (e.g. late arrivals and departures) on an operational level. Results show, when representative data from the port of Antwerp is used, that both a feasible position and quay crane allocation exist. Since in this paper, we focus on an allocation at a strategic level, an arbitrary feasible solution on an operational level is satisfactory at this point in time. A recently finished study considers optimization of the position and quay crane allocation at a second level. In a future study one could incorporate the position and quay crane allocation in the top-level optimization.

# 5 Conclusions, Recent Progress and Future Work

In this paper, we considered a cluster of inter-acting terminals, at which a set of container vessels arrive cyclically. By abstracting from position and quay crane allocation, we were able to strategically allocate a terminal and a time window to each of the vessels in the cycle. Moreover, we showed that an alternative MILP formulation decreases the CPU time of a straightforward MILP formulation several orders of magnitude for real-life instances. In fact, real-life problems can be solved within a couple of hours when the alternative approach is used. Results, using actual data from the port of Antwerp, showed that feasible position and quay crane allocations exist.

We make the following remarks with respect to recent progress and future work:

- The alternative approach has been used to solve a case study in the port of Antwerp. The cost reduction by adopting the resulting allocation looks promising to say the least. Results will be discussed in a subsequent paper.
- Once the strategic allocation has been generated, we have to verify whether the solution is feasible on an operational level. For problem sizes as faced by the port of Antwerp, we were able to manually construct a two-dimensional position and quay crane allocation. From a strategic point of view, being the abstraction level considered in this paper, an arbitrary feasible schedule on an operational level suffices. A recently finished study

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addresses both the position and the quay crane allocation problems resulting for the cut as proposed in this paper. Results will be discussed in a subsequent paper.

- To cope with stochastic arrivals of vessels in the port, a terminal operator provides a so-called arrival window rather than a single arrival time: iff a vessel arrives within its arrival window, the operator has to guarantee a maximal process time. If the vessel arrives out of its window, the operator is not bound to a maximal process time. We have developed a model, which takes these agreements into consideration and constructs an allocation with minimally required crane capacity in the worst case arrival scenario. Results will be discussed in a subsequent paper.
- In addition to the previous item, we have currently developed an online model predictive control algorithm, that reallocates the strategic allocation under disturbances (e.g. early/late vessels and crane breakdowns) to minimize the deviations from the reference allocation. Results will be discussed in a subsequent paper.
- In this paper, we assumed all vessels to arrive exactly once during the cycle, implying that all vessels have the same period. However, in practice it can happen that some vessels have deviating cycle lengths. An expansion of the model, which incorporates this phenomenon, is a recommendation for a future study.
- In today's ports, inter-terminal transport is not only established by trucks, but also by barges. In the current approach we model the resource utilization of the barges by reducing the quay lengths by 200 meters and occupying one quay crane in each terminal during the entire cycle. It is interesting to investigate the trade-off between the amount of inter-terminal transport by trucks and barges and therefore worth-while expanding the current model with the actual loading and unloading of barges.

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