

CHAPTER 4

MODELLING MANUFACTURING SYSTEMS FOR CONTROL: A VALIDATION STUDY

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This contribution deals with the modelling of manufacturing systems for control. First the concept of effective process times is introduced as a means to arrive at relatively simple discrete event models of manufacturing systems based on measured data. Secondly, a control framework is presented. Thirdly, a validation study is presented which shows that the currently available PDE-models for describing manufacturing systems need further improvement. Finally some criteria are specified which a PDE-model should at least meet in order to be considered valid.

4.1. Introduction

The dynamics of manufacturing systems has been a subject of study for several decades^{9,12}. Over the last years, manufacturing systems have become more and more complex. A good understanding of the dynamics of manufacturing systems has therefore become even more important.

A living cell can also be considered as a tiny manufacturing system which produces certain parts via a system of “protein machines” (enzyme molecules). Parts produced by one “machine” then move to other “machines” to be processed. For a better understanding of this cell-dynamics, experiences from studying the dynamics of manufacturing systems might be helpful, and vice versa.

The goal of this contribution is to introduce the “outsider” to recent developments in the modelling and control of manufacturing systems and to provide some references that can be used as starting points for the interested

reader. Since no familiarity with manufacturing systems is assumed, in Section 4.2 first some terminology and basic properties of manufacturing systems are introduced.

A commonly accepted approach for modelling the dynamics of manufacturing systems is by means of discrete event models, in which each product and each individual production step is modelled in great detail. In Section 4.3, the concept of effective process times is introduced as a means to arrive at relatively simple discrete event models of manufacturing systems. When building a discrete event model, one usually tries to include all possible disturbances due to machine failures, availability of operators and tools, maintenance, breaks, etc. Instead of using this white-box approach, a workstation is considered to be a black-box, whose input-output behaviour can be determined from real manufacturing data.

Using effective process times as a means to arrive at a relatively simple discrete event model of a manufacturing system is only the first step in our framework to control a manufacturing system. This framework is presented in Section 4.4. A second important ingredient of this control framework is the accurate approximation of the discrete event model's dynamics by a continuous model. Recently, a new class of continuous models has been proposed to capture the dynamics of manufacturing systems. This new class of models (PDE-models) is introduced in Section 4.5.

In Section 4.6 a summary is given of PDE-models that have been proposed in literature so far. The dynamic behaviour resulting from these models is compared with the dynamic behaviour that results from discrete event simulation. Unfortunately, none of the presented models describes the dynamics satisfactorily. Since this validation study calls for improved models, Section 4.7 concludes this chapter with a list of elementary properties that valid models should satisfy.

4.2. Preliminaries

We first need to introduce a few basic quantities and the main principles for manufacturing system analysis. The items produced by a manufacturing system are called *lots*. Also the words product and job are commonly used. The total number of lots in a manufacturing system is called *wip* (work-in-process) w . To characterise the behaviour of a manufacturing system two important measures are being used. The first measure is the *throughput* δ , i.e., the number of lots per time-unit that leaves the manufacturing system. The second measure is the *flow time* φ , i.e., the time from release of a lot

in the system until the finished lot leaves the system. Instead of flow time the words cycle time and throughput time are also commonly used.

Ideally, a manufacturing system should both have a high throughput and a low flow time or low wip. Unfortunately, these goals can not both be met simultaneously. These two goals are conflicting, as can be seen from Fig. 4.1.

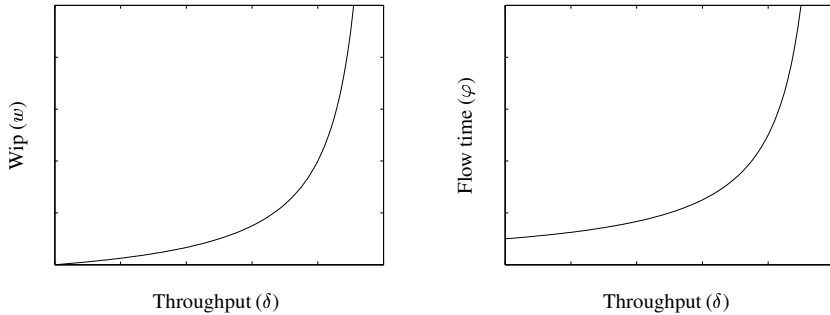


Fig. 4.1. Basic relations between basic quantities for manufacturing systems.

On the one hand, if we want to have a high throughput, we need to make sure that machines are always busy. Since from time to time disturbances like machine failures happen, we should make sure that we have buffers between two consecutive machines to make sure that the second machine can still continue if the first machine fails (or vice versa). As a result, for a high throughput we need to have many lots in the manufacturing system, i.e., we have a high wip. Therefore, if a new lot starts in the system it has a large flow time, since all lots that are currently in the system need to be completed first.

On the other hand, the least possible flow time can be achieved if a lot arrives at a completely empty system and never has to wait before processing at any machine takes place. As a result, for that system we have a small wip level, but also most of the time machines are not processing, yielding a small throughput.

When trying to control manufacturing systems, we need to make a trade-off between throughput and flow time, so the nonlinear (steady state) relations depicted in Fig. 4.1 need to be incorporated in any reasonable model of manufacturing systems.

approximation models of discrete event models. As these continuous approximation models should be valid, some validation studies are presented.

4.3. Effective Process Times (EPT's)

For the processing of a lot at a machine, many steps may be required. For example, it could be that an operator needs to get the lot from a storage device, setup a specific tool that is required for processing the lot, put the lot on an available machine, start a specific program for processing the lot, wait until this processing has finished (meanwhile doing something else), inspect the lot to determine if all went well, possibly perform some additional processing (e.g., rework), remove the lot from the machine and put it on another storage device and transport it to the next machine. At all of these steps something might go wrong: the operator might not be available, after setting up the machine the operator finds out that the required recipe can not be run on this machine, the machine might fail during processing, no storage device is available anymore so the machine can not be unloaded and is blocked, etc.

It is impossible to measure all sources of variability that might occur in a manufacturing system. One could incorporate some of these sources in a discrete event model. The number of operators and tools can be modelled explicitly and it is common practice to collect data on mean times to failure and mean times to repair of machines. Also schedules for (preventive) maintenance can be incorporated explicitly in a discrete event model. Nevertheless, still not all sources of variability are included. This is clearly illustrated in Fig. 4.3, obtained from¹³. The left graph contains actual realisations of flow times of lots leaving a real manufacturing system, whereas the right graph contains the results of a detailed deterministic simulation model and the graph in the middle contains the results of a similar model including stochasticity. It turns out that in reality flow times are much higher and much more irregular than simulation predicts. So, even if one tries hard to capture all variability present in a manufacturing system, still the outcome predicted by the model is far from reality.

Hopp and Spearman¹² use the term *effective process time* (EPT) as the time seen by lots from a logistical point of view. In order to determine this effective process time, Hopp and Spearman assume that the contribution of the individual sources of variability is known.

A similar description is given by Sattler¹⁸ who defines the effective process time as all flow time except waiting for another lot. It includes waiting

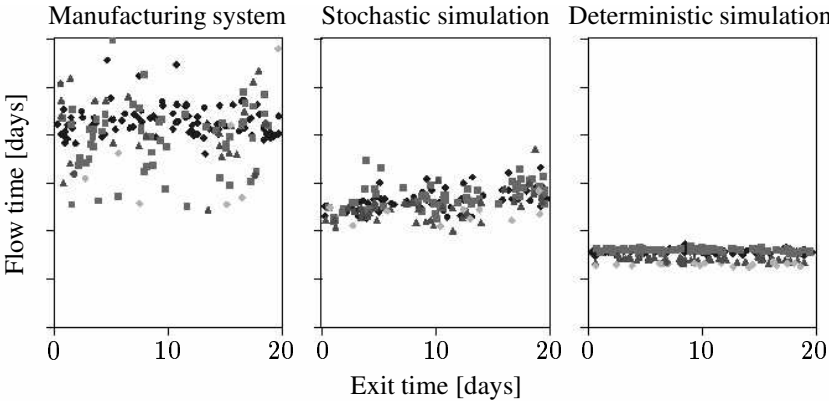


Fig. 4.3. A comparison.

for machine down time and operator availability and a variety of other activities. Sattler¹⁸ noticed that her definition of effective process time is difficult to measure.

Instead of taking the bottom-up view of Hopp and Spearman, a top-down approach can also be taken, as shown by Jacobs et al.¹³, where algorithms have been introduced that enable determination of effective process time realisations from a list of events. For these algorithms, the basic idea of the effective process time to include time losses was used as a starting point.

Consider the Gantt chart of Fig. 4.4. At $t = 0$ the first lot arrives at

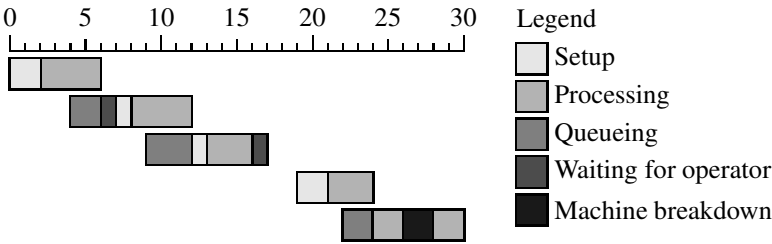


Fig. 4.4. Gantt chart of 5 lots at a workstation.

the workstation. After a setup, the processing of the lot starts at $t = 2$ and is completed at $t = 6$. At $t = 4$ the second lot arrives at the workstation.

At $t = 6$ this lot could have been started, but apparently there was no operator available, so only at $t = 7$ the setup for this lot starts. Eventually, at $t = 8$ the processing of the lot starts and is completed at $t = 12$. The fifth lot arrives at the workstation at $t = 22$, processing starts at $t = 24$, but at $t = 26$ the machine breaks down. It takes until $t = 28$ before the machine has been repaired and the processing of the fifth lot continues. The processing of the fifth lot is completed at $t = 30$.

If we take the point of view of a lot, what does a lot see from a logistical point of view? The first lot arrives at an empty system at $t = 0$ and departs from this system at $t = 6$. From the point of view of this lot, its processing took 6 time-units. The second lot arrives at a non-empty system at $t = 4$. Clearly, this lot needs to wait. However, at $t = 6$, if we would forget about the second lot, the system becomes empty again. So from $t = 6$ on there is no need for the second lot to wait. At $t = 12$ the second lot leaves the system, so from the point of view of this lot, its processing took from $t = 6$ till $t = 12$; the lot does not know whether waiting for an operator and a setup is part of its processing. Similarly, the third lot sees no need for waiting after $t = 12$ and leaves the system at $t = 17$, so it assumes to have been processed from $t = 12$ till $t = 17$. Following this reasoning, the resulting effective process times for lots are as depicted in Fig. 4.5.

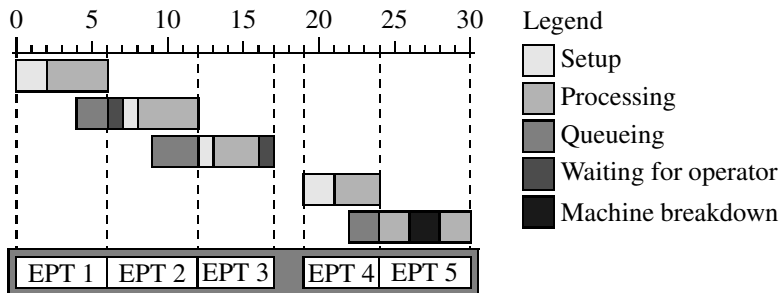


Fig. 4.5. EPT realisations of 5 lots at a workstation.

Notice that only arrival and departure events of lots to a workstation are needed for determining the effective process times. Furthermore, none of the contributing disturbances needs to be measured.

In highly automated manufacturing systems, arrival and departure events of lots are being registered, so for these manufacturing systems, effective process time realisations can be determined rather easily. Next, these

EPT realisations can be used in a relatively simple discrete event model of the manufacturing system. This discrete event model only contains the architecture of the manufacturing system, buffers and machines. The process times of these machines are samples from their EPT-distribution as measured from real manufacturing data. There is no need for incorporating machine failures, operators, etc., as this is all included in the EPT-distributions. Furthermore, the algorithms as provided in¹³ are utilisation independent. That is, data collected at a certain throughput rate is also valid for different throughput rates. Also, machines with the same EPT-distribution can be added to a workstation. This makes it possible to study how the manufacturing systems responds in case a new machine is added, or all kinds of other what-if-scenario's. Finally, since EPT-realizations characterise operational time variability, they can be used for performance measuring. For more on this issue, the interested reader is referred to^{4,13}. What is most important in the current setting, is that EPT's can be determined from real manufacturing data and yield relatively simple discrete event models of the manufacturing system under consideration. These relatively simple discrete event models serve as a starting point for controlling manufacturing systems.

4.4. Control Framework

In the previous section, the concept of effective process times has been introduced as a means to arrive at relatively simple discrete event models of a manufacturing system, using measurements from the real manufacturing system under consideration. This would be the first step in the control framework. The idea is to develop a controller for the derived discrete event model. Once this controller yields good performance for the discrete event model, the controller can be applied to the real manufacturing system.

Even though control theory exists for controlling discrete event systems, unfortunately none of it is appropriate for controlling real-life discrete event models of manufacturing systems. This is mainly due to the large number of states a manufacturing system can be in. Therefore, a different approach is needed.

If we concentrate on mass production, the distinction between lots is not really necessary and lots can be viewed in a more continuous way. Instead of the discrete event model we might consider an approximation model. This would be the second step in the control framework. Next, we can use standard control theory for deriving a controller for the approximation model.

These first three steps in the control framework are illustrated in Fig. 4.6.

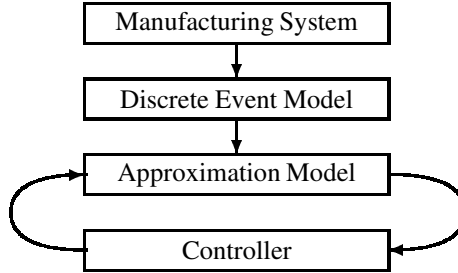


Fig. 4.6. Control framework (first three steps).

To make the second and third step more clear, a possible approximation model is presented in the next subsection, followed by a possible controller design based on this model. The final steps of the control framework conclude this section.

4.4.1. *Approximation Model*

Consider the manufacturing line in Fig. 4.7 which consists of two machines in series. Let $u_0(k)$ denote the number of lots that arrive at the system dur-

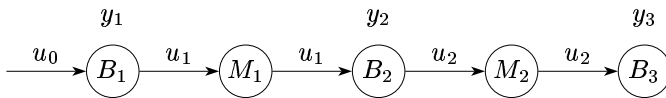


Fig. 4.7. A manufacturing line.

ing shift k , let $u_i(k)$ denote the number of lots which machine M_i produces during shift k , let $x_i(k) = y_i(k)$ denote the number of lots in buffer B_i at the beginning of shift k ($i \in \{1, 2\}$), and let $x_3(k) = y_3(k)$ denote the number of lots produced by the manufacturing system during shift k . Assume that machines M_1 and M_2 have a maximum capacity of respectively μ_1 and

μ_2 lots per shift. Then we obtain the following approximation model:

$$x(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} u(k), \quad (1a)$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} u(k), \quad (1b)$$

where $u = [u_0, u_1, u_2]^\top$ and $y = [y_1, y_2, y_3]^\top$. System (1) is a controllable linear system of the form

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k) + Du(k) \end{aligned}$$

as extensively studied in control theory (which explains the introduction of both x and y when deriving (1)). Therefore, many standard techniques from control theory can be used for deriving a controller for system (1).

4.4.2. Model Predictive Control (MPC)

For the continuous approximation model as derived in the previous section, we also have constraints. To be more precise, we have capacity constraints on the input u , as well as constraints on the state x and output y (the buffer contents should remain positive). These constraints can be expressed by means of the following equations:

$$\begin{aligned} 0 \leq u_1(k) \leq \mu_1, \quad x_1(k) \geq 0, \quad y_1(k) \geq 0, \\ 0 \leq u_2(k) \leq \mu_2, \quad x_2(k) \geq 0, \quad y_2(k) \geq 0, \\ x_3(k) \geq 0, \quad y_3(k) \geq 0. \end{aligned} \quad (2)$$

A standard control approach for controlling system (1) when having to deal with the constraints is Model Predictive Control (MPC). When using MPC, it is common practice to define a reference output $y_r(k)$ that the system (1) should track.

Assume that the buffer contents at the end of shift $k-1$ have just been measured, i.e., $y(k)$. Since $y(k) = x(k)$, the current state of the system is known. So using this measurement, for each possible plan of future inputs $u(k), u(k+1), \dots, u(k+p-1)$, by means of model (1) the resulting future outputs $y(k+1), y(k+2), \dots, y(k+p)$ can be determined, usually denoted as $y(k+1|k), \dots, y(k+p|k)$ to illustrate that these are predictions, while currently being at time-instant k . Next, costs can be associated with each

possible plan of future inputs. Usually, these costs consist of both a penalty for not being exactly at the desired reference output, and a penalty for the control effort used. In this way almost naturally an optimisation problem arises, as the expected costs should be minimised over all possible plans for future inputs. A typical optimisation problem using this approach would be:

$$\min_{u(k), \dots, u(k+p-1)} \sum_{i=1}^p [y(k+i|k) - y_r(k+i)]^\top Q [y(k+i|k) - y_r(k+i)] + u(k+i-1)^\top R u(k+i-1) \quad (3a)$$

subject to

$$0 \leq u_j(k+i-1) \leq \mu_j, \quad j \in \{1, 2\}, \quad y(k+i|k) \geq 0 \quad (i = 1, \dots, p). \quad (3b)$$

As all $y(k+i|k)$ are affine functions of $u(k+i-1)$, this optimisation problem is a quadratic program which can be solved easily. From the resulting optimal solution $u^*(k), \dots, u^*(k+p-1)$, only $u^*(k)$ is used as the production targets for the next period. Since disturbances might occur, this optimisation procedure is repeated each shift, resulting in a receding horizon scheme.

The mentioned scheme is one of the simplest versions of MPC. The interested reader is referred to literature, e.g., ^{5,7,17} for more information about this control strategy.

Having illustrated the second and third step of the control framework, the final steps of the framework can be introduced.

4.4.3. Control Framework (revisited)

At the beginning of this section the first steps of the control framework have been explained, cf. Fig. 4.6. Using effective process times a relatively simple discrete event model of a manufacturing system can be derived based on measurements from the real manufacturing system. Next, an approximation model of the discrete event model can be derived. Subsequently, by means of standard control theory a controller for this approximation model can be derived.

When the derived controller behaves as desired, as a fourth step this controller could be connected to the discrete event model. This can not be done straightforwardly, since the derived controller is not a discrete event controller. The control actions still need to be transformed into events. It might very well be that the optimal control action would be to produce

2.75 lots during the next shift. One still needs to decide how many jobs to really start (2 or 3), and also when to start them. This is the left conversion block in Fig. 4.8. From this figure, it can also be seen that a conversion is

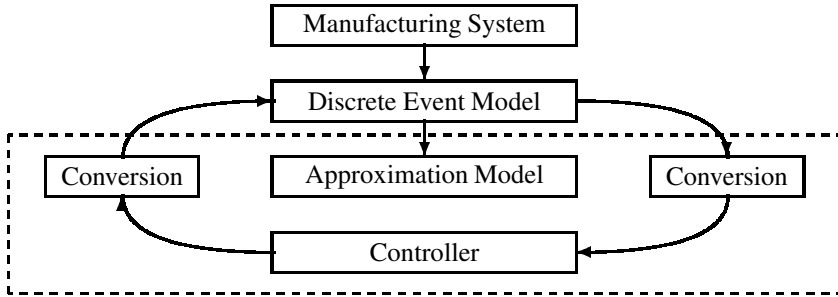


Fig. 4.8. Control framework (fourth step).

needed from discrete event model to controller. In the example treated in this section, it was decided to sample the discrete event model once every shift. Other strategies might be followed. For example, if at the beginning of a shift a machine breaks down it might not be such a good idea to wait until the end of the shift before setting new production targets. Designing proper conversion blocks would be the fourth step in the control framework.

After the fourth step, i.e., properly designing the two conversion blocks, a suitable discrete event controller for the discrete event model is obtained, as illustrated in Fig. 4.8 (dashed).

Eventually, as a fifth and final step, the designed controller can be disconnected from the discrete event model, and attached to the manufacturing system.

In the presented control framework two crucial steps can be distinguished. First, the discrete event model should be a good enough approximation of the real manufacturing system. For that reason, once a discrete event model of a manufacturing system has been made, the model needs to be validated. If results as shown in Fig. 4.3 are obtained the model needs further improvement. Second, the approximation model should be a good enough approximation of the discrete event model, or actually: of the discrete event model and the conversion blocks, since that is the system that needs to be controlled by the continuous controller. For that reason, ap-

proximation models of (discrete event models of) manufacturing systems also need to be validated. In the remainder of this chapter, some validation studies of approximation models are presented.

4.5. Modelling Manufacturing Systems

In the previous section, a control framework for controlling a manufacturing system has been presented. Similar ideas can be applied to the problem of controlling a network of interacting manufacturing systems. An illustrative example of a semiconductor manufacturing supply chain is given in Fig. 4.9.

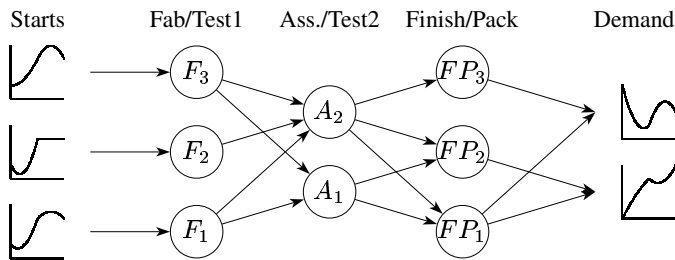


Fig. 4.9. A small semiconductor manufacturing supply chain.

In this figure, F_1 , F_2 , and F_3 denote wafer fabs, in which wafers are being produced, containing hundreds to thousands of integrated circuits (ICs) on its surface. Due to, among others, the large number of process steps, the re-entrant nature of the process flow, and the advanced process technologies, the fabrication of wafers is a complex manufacturing process. A typical flow time for a wafer fab is in the order of two months. That is, once a bare silicon wafer enters the manufacturing system, it typically takes about two months for the wafer to be completed.

Finished wafers are moved to an Assembly/Test facility, where individual chips are cut out of the wafer and each separated IC is assembled. Typical flow times for the manufacturing systems A_1 and A_2 are in the order of ten days. Finally, the chips are packaged in FP_1 , FP_2 , FP_3 , and can be shipped to customers. This takes in the order of five days.

The control of this supply chain is one of the problems the semiconductor industry currently faces. The fact that flow times are large and nonlinearly dependent on the load (cf. Fig. 4.1) is one of the most difficult aspects in

this problem. Notice that, even though the flow time of a wafer fab is in the order of two months, the raw process time of a wafer is less than two weeks. That is, if a wafer enters an empty wafer fab it would take less than two weeks for the wafer to be completed. This illustrates that the nonlinear relations between wip, throughput and flow time should be present in any approximation model.

Consider the manufacturing line depicted in Fig. 4.7 and assume that we start with an initially empty system and then turn on the machines, i.e., assume that $x(0) = 0$ and $u(k) = [\lambda, \lambda, \lambda]^\top$, ($0 < \lambda \leq \min(\mu_1, \mu_2)$, $k = 1, 2, 3, \dots$), model (1) predicts that products immediately start leaving the manufacturing system. Furthermore, according to model (1) for each feasible throughput any wip-level can be used. In particular also a wip-level of 0. As this example illustrates, these fluid models do not take into account the nonlinear relations between wip, throughput and flow time. As a result, these models can not be used as a valid approximation model.

Models like (1) have been used a lot in literature. Examples of these models are the flow model as initiated by Kimemia and Gershwin¹⁴ for modelling failure-prone manufacturing systems, the fluid models or fluid queues as proposed by queueing theorists¹⁰, or the stochastic fluid model as introduced by Cassandras et al.⁸

Recently, a new class of models for manufacturing systems has been introduced^{2,3,16}. In these models, the flow of products through a manufacturing system is modelled in a similar way as the flow of cars on a highway. Not only is the number of lots assumed to be continuous, also the position of a lot in the manufacturing system is assumed to vary continuously.

Let $t \in \mathbb{R}^+$ denote the time and let $x \in [0, 1]$ denote the position of a lot in the manufacturing line (the degree of completion). The behaviour of lots flowing through the manufacturing line can be described by three variables that vary with time and position: *flow* $u(x, t)$, measured in unit lots per unit time, *density* $\rho(x, t)$, measured in unit lots per degree of completion, and *speed* $v(x, t)$, measured in degree of completion per unit time. First, we observe that flow is the product of density and speed:

$$u(x, t) = \rho(x, t)v(x, t). \quad (4)$$

Second, assuming no scrap, the number of products between any two “locations” x_1 and x_2 ($x_1 < x_2$) needs to be conserved at any time t , i.e., the change in the number of products between x_1 and x_2 is equal to the flow

entering at x_1 minus the flow leaving at x_2 :

$$\frac{\partial}{\partial t} \int_{x_1}^{x_2} \rho(x, t) \, dx = u(x_1, t) - u(x_2, t),$$

or in differential form:

$$\frac{\partial \rho}{\partial t}(x, t) + \frac{\partial u}{\partial x}(x, t) = 0. \quad (5)$$

Relations (4) and (5) are basic relations that any model must satisfy. As we have three variables of interest, (at least) a third relation is needed. Several choices can be made for this third (or more) relation(s), as the next section and¹ make clear.

As far as we know, the PDE-models as just described are the only ones that are solvable in limited time, describe the dynamics of a manufacturing system and incorporate both throughput and flow time. Flow or fluid models, like the one presented in (1), do not incorporate the nonlinear relation between throughput and flow time. Discrete event models do incorporate the nonlinear relation between throughput and flow time, but simulating these models takes a lot of time, making their on-line use computationally infeasible. A third class of models are queueing models like in^{6,19}. They provide many insights in steady state behaviour of manufacturing lines, but the dynamics of manufacturing lines is rarely addressed.

Even though discrete event simulation is computationally intensive and queueing theory is mainly concerned with steady state, results from these models can be used for validating PDE-models. A minimal requirement for a valid PDE-model would be that its steady state behaviour is in accordance with results from queueing theory. Also the dynamics of a PDE-model should be in accordance with the dynamics obtained from discrete event simulation. These checks are discussed in the next section, where queueing theory and discrete event simulation are used to validate PDE-models.

4.6. Validation of PDE-Models

In the previous section we introduced PDE-models as a way to model manufacturing systems. We only mentioned the basic ingredients (4), (5), and the need for a third relation. Also, we mentioned that results from queueing theory and from discrete event simulation can be used for validating PDE-models.

In this section we present validation studies for five PDE-models for manufacturing systems, using queueing theory for deriving the proper steady state and discrete event simulation for validating the dynamics.

4.6.1. *Manufacturing Systems*

When we consider the supply chain in Fig. 4.9, two typical manufacturing systems can be considered. On the one hand we have the factories F_1 , F_2 , and F_3 , which have a re-entrant nature, on the other hand we have the factories A_1 , A_2 , FP_1 , FP_2 , and FP_3 , which have the nature of a line of workstations. Therefore, we define two manufacturing systems:

Manufacturing System 1 A line consisting of 15 identical workstations (see Fig. 4.10). Lots visit the workstations according to the fol-

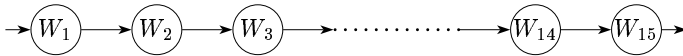


Fig. 4.10. An ordinary manufacturing line.

lowing recipe: 1-2-3-4-5-6-7-8-9-10-11-12-13-14-15. This is an “ordinary” manufacturing line, cf. A_1 , A_2 , FP_1 , FP_2 , and FP_3 .

Manufacturing System 2 A line consisting of five identical workstations. Lots visit the workstations according to the following recipe:

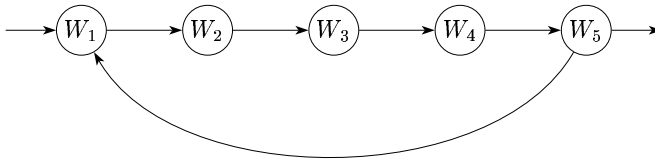


Fig. 4.11. A re-entrant manufacturing line.

1-2-3-4-5-1-2-3-4-5-1-2-3-4-5 (see Fig. 4.11). Since each lot re-enters the system twice, this is a re-entrant manufacturing line, cf. F_1 , F_2 , and F_3 .

We assume that each workstation consists of an infinite buffer, which operates under a FIFO policy (First In First Out), and a single machine whose effective process times are drawn from an exponential distribution with mean 1. If we furthermore assume that lots arrive at the manufacturing system according to a Poisson process with an arrival rate λ , we can derive the following steady state properties by means of queueing theory:

- For Manufacturing System 1 (Fig. 4.10), the mean number of lots equals $\frac{\lambda}{1-\lambda}$ in each workstation, resulting in a mean number of $\frac{15\lambda}{1-\lambda}$ lots in the system. Furthermore, the mean flow time of lots for Manufacturing System 1 is $\frac{15}{1-\lambda}$.
- For Manufacturing System 2 (Fig. 4.11), the mean number of lots equals $\frac{3\lambda}{1-3\lambda}$ in each workstation, resulting in a mean number of $\frac{15\lambda}{1-3\lambda}$ lots in the system. Furthermore, the mean flow time of lots for Manufacturing System 2 is $\frac{15}{1-3\lambda}$.

4.6.2. PDE-Models

In the validation studies we consider the following five models that have been proposed in literature.

Model 1: Single queue I² Relations (4), (5) together with

$$v(x, t) = \frac{\mu}{1 + \int_0^1 \rho(s, t) ds}, \quad (6)$$

where $\mu > 0$ is a constant representing the processing rate of the workstation.

Model 2: Single queue II² Relations (4), (5) together with

$$\frac{\partial \rho v}{\partial t}(x, t) + \frac{\partial \rho v^2}{\partial x}(x, t) = 0, \quad (7)$$

and the additional boundary condition

$$\rho v^2(0, t) = \frac{\mu \cdot \rho v(0, t)}{1 + \int_0^1 \rho(s, t) ds}, \quad (8)$$

where $\mu > 0$ again denotes the processing rate of the workstation.

Model 3: Re-entrant I³ Relations (4), (5) together with

$$v(x, t) = v_0 \left(1 - \frac{1}{L_{\max}} \int_0^1 \rho(s, t) ds \right), \quad (9)$$

where $v_0 > 0$ is a constant representing the maximal speed that can be achieved (i.e., $1/v_0$ denotes the theoretical minimal flow time),

and $L_{\max} > 0$ is a constant representing the maximal number of lots that is allowed in the manufacturing system.

Model 4: Re-entrant II³ Relations (4), (5) together with (7), and the additional boundary condition

$$\rho v^2(0, t) = v_0 \left(1 - \frac{1}{L_{\max}} \int_0^1 \rho(s, t) \, ds \right) \rho v(0, t), \quad (10)$$

where v_0 and L_{\max} are the same as in (9).

Model 5: m identical machines¹⁶ Relations (4), (5) together with

$$v(x, t) = \frac{\mu}{m + \rho(x, t)}, \quad (11)$$

where $m > 0$ denotes the number of machines, and $\mu > 0$ denotes the processing rate of each workstation.

All five models have the boundary condition

$$\rho v(0, t) = \lambda(t),$$

where $\lambda(t)$ denotes the inflow to the manufacturing system in unit lots per unit time.

Recently, other PDE-models than models 1–5 have been proposed, cf. ¹ (an other chapter in this book). These models have not been incorporated in these validation studies.

4.6.3. Validation Study

In the previous subsections we introduced Manufacturing Systems 1 and 2, as well as the PDE-models used in the validation studies.

Manufacturing System 1 is an ordinary manufacturing line and does not have a re-entrant nature. As PDE-models 3 and 4 have been specifically designed for re-entrant manufacturing systems, they have not been used in the validation studies for Manufacturing System 1.

As mentioned, according to queueing theory the mean number of lots in Manufacturing System 1 is $\frac{15\lambda}{1-\lambda}$ in case we have a mean arrival rate of λ . Translated into PDE-terms we have in equilibrium

$$\rho(x, t) = \frac{15\lambda}{1-\lambda}, \quad v(x, t) = \frac{1-\lambda}{15}. \quad (12)$$

From (12) we obtain, by eliminating λ , that in steady state

$$v(x, t) = \frac{1}{15 + \rho(x, t)} = \frac{1}{15 + \int_0^1 \rho(s, t) \, ds}. \quad (13)$$

From (13) we obtain that the models 1, 2, and 5 are valid in steady state, provided that in (6) and (8) we replace the denominator $1 + \int_0^1 \rho(s, t) ds$ with $15 + \int_0^1 \rho(s, t) ds$, which is consistent with the results in². In² a single queue is assumed. If, instead, we assume a line of 15 workstations the mentioned modification of (6) and (8) results.

For Manufacturing System 2, the mean number of lots in the system equals $\frac{15\lambda}{1-3\lambda}$ with a mean flow time of $\frac{15}{1-3\lambda}$. Translated into PDE-terms we have in equilibrium

$$\rho(x, t) = \frac{15\lambda}{1-3\lambda}, \quad v(x, t) = \frac{1-3\lambda}{15}. \quad (14)$$

From (14) we obtain, by eliminating λ , that in steady state

$$v(x, t) = \frac{1}{15 + 3\rho(x, t)} = \frac{1}{15} \left(1 - \frac{\int_0^1 \rho(s, t) ds}{\frac{5}{1-3\lambda}} \right). \quad (15)$$

When we compare (15) with (9) and (10), we notice that in order for models 3 and 4 to be valid in steady state, we need $L_{\max} = \frac{5}{1-3\lambda}$, where λ denotes the steady state arrival rate. Since L_{\max} depends on λ , the re-entrant models 3 and 4 are not likely to be “globally” valid for re-entrant manufacturing systems, i.e., valid for an arbitrary arrival rate λ . In the best case they are valid “locally” around a certain λ . On the other hand, any manufacturing system can contain only a finite number of lots, arguing the validity of a queueing model with infinite buffers.

From (15) we obtain that the models 1, 2, and 5 are valid in steady state, provided that in (6) and (8) we replace the denominator $1 + \int_0^1 \rho(s, t) ds$ with $15 + 3 \int_0^1 \rho(s, t) ds$, and in (11) we replace the denominator with $15 + 3\rho(x, t)$. The former can be argued to be a suitable model for a re-entrant manufacturing line (a homogeneous velocity over the line is used), whereas the latter is not a proper model for a re-entrant manufacturing line. It would have been better to replace it with $15 + \rho(x, t) + \rho(\langle x + \frac{1}{3} \rangle_1, t) + \rho(\langle x + \frac{2}{3} \rangle_1, t)$, where $\langle \alpha \rangle_1$ denotes α modulo 1, i.e., all digits behind the decimal separator.

Next, we can use discrete event models of System 1 and System 2 to study the dynamics of the proposed PDE-models. For this we used the specification language χ^{11} . Starting with an initially empty system, we performed experiments where lots arrive according to a Poisson process with a mean arrival rate λ . During an experiment we collected at the times $t = 1, 2, 3, \dots$ the number of lots in each workstation as well as the number of lots that has been completed by the system. In order to guarantee

a 99% confidence interval with a relative width of less than 0.01 for *each* measurement, experiments have been repeated 1.000.000 times. We averaged all data, resulting in the average number of lots in each workstation, as well as the number of lots that has been completed by the system, at each time-instant. This we did for both Manufacturing System 1 and Manufacturing System 2, where we chose the arrival rate such that the steady state utilisation of the workstations was respectively 25%, 50%, 75%, 90%, and 95% (so $\lambda = 0.25, 0.5, 0.75, 0.90, 0.95$ for Manufacturing System 1 and $\lambda = 0.08333, 0.16667, 0.25, 0.3, 0.31667$ for Manufacturing System 2). These experiments provide more data than can be presented in this chapter. The interested reader is referred to¹⁵ for more results. Here we present some general findings.

The first results we present are for Manufacturing System 1 with an arrival rate of $\lambda = 0.25$. Fig. 4.12 presents the evolution of the total number of lots in the system as a function of time. The solid line describes

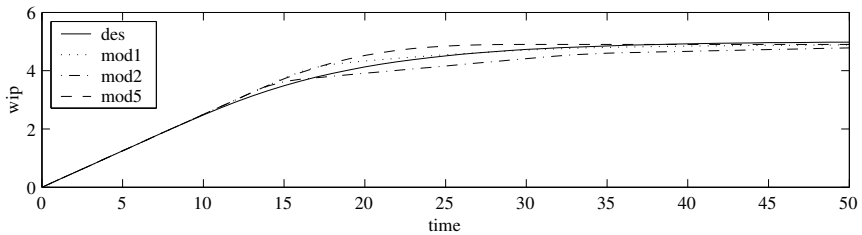


Fig. 4.12. Number of lots in Manufacturing System 1 for utilisation of 25%.

the (averaged) result of the discrete event simulations. The dotted line, the dash-dotted line, and the dashed line describe the result according to Model 1, Model 2, and Model 5 respectively. In Fig. 4.12 we see that initially the total number of lots in the line linearly increases. This is due to the fact that lots are only entering the system and it takes a while before lots start coming out. Also, we see that all models predict that in steady state five lots are in the system. This is as expected. When we closely look at Fig. 4.12 we see that around $t = 10$ the graph of the discrete event simulation bends off from the PDE-graphs, from which we can conclude that the moment at which the first lot leaves the system is overestimated by the PDE-models. That is, according to the discrete event simulation this should happen earlier. Also, we see that after $t = 40$ all three PDE-models

underestimate the number of products in the system. Therefore, all PDE-models predict that the system is later in steady state than according to the discrete event simulation.

The differences in behaviour become more clear when we consider the development of the density over time. This can be made most clear by means of a movie, for which the reader is referred to¹⁵. In Fig. 4.13 the most important parts of the behaviour are captured. The figure presents respectively $\rho(0, t)$, $\rho(0.5, t)$ and $\rho(1, t)$, again for the discrete event model, Model 1, Model 2, and Model 5. For the discrete event system we assume the density to be piecewise constant at intervals of width $\frac{1}{15}$. When looking

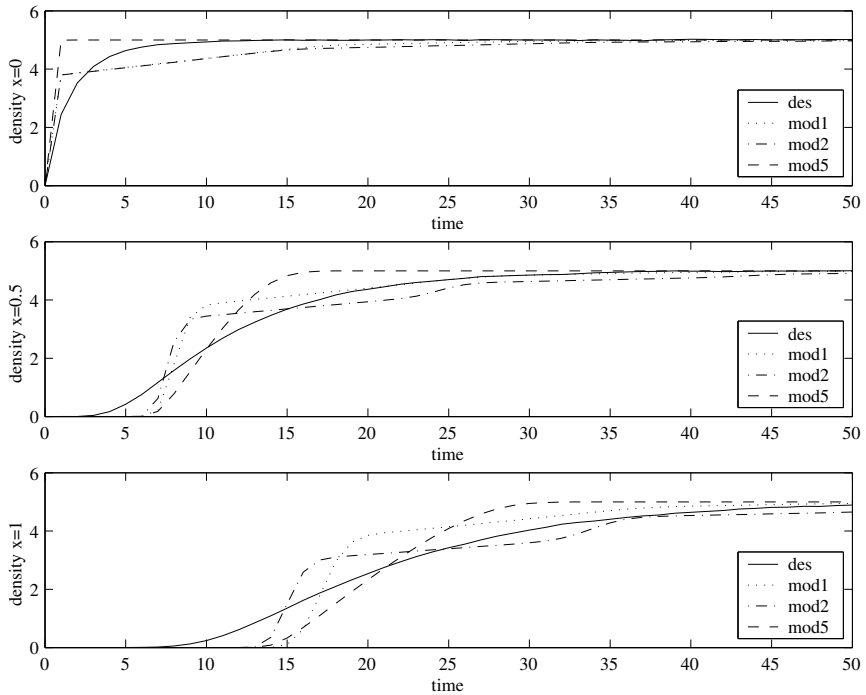


Fig. 4.13. Densities at $x = 0$, $x = 0.5$ and $x = 1$ for utilisation of 25%.

at the first graph, we see that the behaviour of Model 1 and Model 2 almost coincide. All three models predict a quicker raise of the density than the discrete event model predicts. If we look at the graph of $\rho(0.5, t)$ we see that initially the PDE-models underestimate the growth of the density, around

$t = 7$ the PDE-models show a strong increase in the density, resulting in an over-estimation of the density. Similar behaviour can be observed for $\rho(1, t)$.

The second results we present are for Manufacturing System 2 with an arrival rate of $\lambda = 0.08333$. Fig. 4.14 presents the evolution of the total number of lots in the system as a function of time. In addition to the

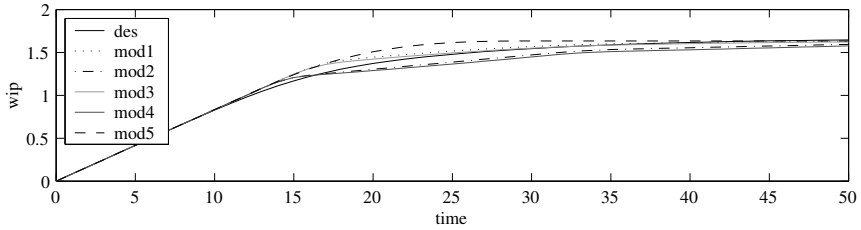


Fig. 4.14. Number of lots in Manufacturing System 2 for utilisation of 25%.

lines from the previous two figures, the light grey and dark grey solid line represent the output of Model 3 and Model 4 respectively.

For the case of Manufacturing System 2 we can make similar remarks as for Manufacturing System 1. Furthermore, a close resemblance between Model 1 and Model 3 can be noticed, as well as a close resemblance between Model 2 and Model 4.

4.7. Concluding Remarks

For controlling a complex network of interacting machines, models are needed that not only describe the dynamics of the network well, but are also suitable for applying control theory to.

In this chapter we illustrated that when building discrete event models of manufacturing systems, a workstation together with all its possible disturbances might be considered as a black box. Instead of using white box modelling and trying to capture all possible disturbances well (as is often done), one can also focus on accurately capturing the input-output behaviour. For this, the concept of effective process times (EPT's) can be used. Using this approach it is possible to arrive at valid discrete event models of manufacturing systems, using real manufacturing data.

Though EPT's can be used to arrive at valid discrete event models of manufacturing systems, these discrete event models can not be used for

deriving suitable controllers. Therefore, a control framework has been presented which makes use of an approximation model of the discrete event model. PDE-models have been mentioned as possible approximation models that are solvable in limited time, describe the dynamics of a manufacturing system and incorporate both throughput and flow time. However, the question remains: are these PDE-models valid models of manufacturing systems?

The presented validation studies showed that more accurate PDE-models are needed. Recently, new models have been proposed (cf. ¹) which might do a better job. Nevertheless, most of the recently developed PDE-models fail at least at one of the following elementary tests:

- Given a fixed set of model parameters, the correct steady state wip is achieved for arbitrary constant influx λ . Models 3 and 4 fail this test.
- For a manufacturing line, as depicted in Fig. 4.10, lots at the end of the line are not influenced by lots in the beginning of the line. In particular, assume the system is in a certain steady state and suddenly the influx decreases. This is not immediately noticed in the outflux. Models 1 and 2 fail this test.
- For a manufacturing system the steady state wip distribution is not only determined by the influx (arrival rate of lots), but also by its variance.

In the validation studies as presented, Poisson arrivals have been assumed, yielding a homogeneous wip distribution over the line (cf. (12) and (14)). However, if we would have used constant inter arrival times the wip distribution would not be homogeneous, but as depicted in Fig. 4.15. In case the inter arrival times would have

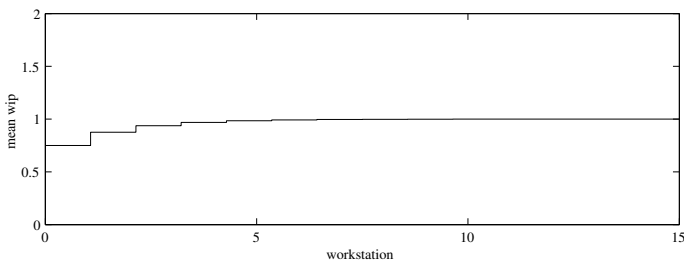


Fig. 4.15. Manufacturing System 1 with deterministic arrivals.

had a higher variance, the wip distribution as depicted in Fig. 4.16 might arise. For the same influx different steady state wip levels

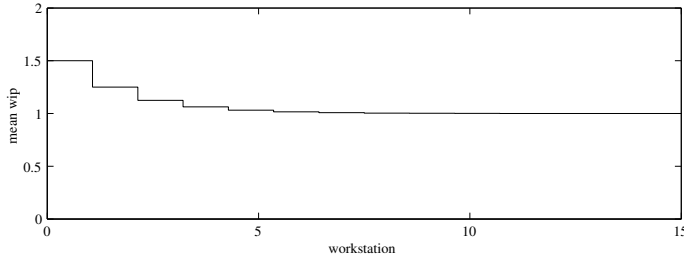


Fig. 4.16. Manufacturing System 1 with highly irregular arrivals.

results, depending on the variance of the influx. The higher the variance, the higher the steady state wip level.

Since the variance of the influx is not a system property, for a given set of model parameters, different steady state wip profiles should be achievable for different variances of the influx.

Models 1, 3, and 5 fail this test. With the current boundary conditions Models 2 and 4 fail this test too.

- In a manufacturing system, lots do not flow back wards. Assume that the first and the last machine of a manufacturing system fail. In that case both the influx and the outflux are zero. Furthermore, assume that initially the beginning of the line is empty (say $\rho(x, 0) = 0$ for $0 \leq x \leq \frac{1}{2}$), but that the end of the line contains some lots. Assume that both the influx and the outflux remain zero. This should also hold for a valid PDE-model.

Models 1–5 pass this test. Nevertheless, this is a test that models which do pass the previous test should also pass.

A final remark deals with the effects of correlation between influx and outflux. Assume that a to be developed PDE-model passes all of the above mentioned tests. If the presented control framework is used to derive a control strategy for the manufacturing system(s) under consideration, typically a feedback results. That is, the current state of the system determines what new influxes will be. This introduces correlations between the influx and the outflux.

To illustrate this, consider a manufacturing system consisting of only one

workstation with an infinite buffer. Assume that process times are drawn from an exponential distribution with mean 1 and assume arrivals according to a Poisson process with a mean arrival rate of $\lambda = 0.5$. Using queueing theory the mean number of lots in this manufacturing system equals 1 and the mean flow time equals 2. Furthermore, the outflux is also a Poisson process with a mean departure rate of $\lambda = 0.5$. We also know from queueing theory that a Poisson arrival process with a mean arrival rate of $\lambda = 1$ results in an unstable system.

Now we apply feedback to this system. The policy we use is to keep the total number of products in this system equal to 1. For the resulting closed-loop system, the mean wip number of lots in the system also equals 1. However, the mean flow time becomes 1 and a mean throughput of 1 results. So the arrival and departure processes both are Poisson processes with a mean rate of 1, something which was unfeasible for the system without feedback.

The system itself has not changed. The only thing that changed was that in the former case the influx and outflux were uncorrelated, whereas in the latter case they were correlated. This example illustrates that apparently the possibility of correlation between influx and outflux should be incorporated in the models too.

The goal of this contribution was to introduce the “outsider” to recent developments in the modelling and control of manufacturing systems and to provide some references that can be used as starting points for the reader who has become more interested. Firstly, we presented the concept of effective processing times. Instead of trying to model what is going on exactly, we try to capture only the input-output behaviour as good as possible using real manufacturing data. Secondly, we presented a framework for controlling manufacturing systems. Thirdly, we showed that the currently available PDE-models that try to capture the dynamics of manufacturing systems at a macroscopic level need further improvement.

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