ADAPTIVE AND FILTERED VISUAL SERVOING OF PLANAR ROBOTS

Erjen Lefeber [*] **Rafael Kelly** [**] **Romeo Ortega** [***]
Henk Nijmeijer [*,****]

[*] *Faculty of Mathematical Sciences, Department of Systems Signals and Control, P.O. Box 217, 7500 AE Enschede, The Netherlands,*
{A.A.J.Lefeber,H.Nijmeijer}@math.utwente.nl
[**] *División de Física Aplicada, CICESE, Apdo. Postal 2615, Adm. 1, Ensenada, B.C. 22800, Mexico,*
rkelly@cicese.mx
[***] *Laboratoire des Signaux et Systèmes, LSS/CNRS/Supélec, Plateau de Moulon, 91192 Gif Sur Yvette Cedex, France,*
Romeo.Ortega@lss.supelec.fr
[****] *Faculty of Mechanical Engineering Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, The Netherlands*

Abstract: In this paper we address the visual servoing of planar robot manipulators with a fixed–camera configuration. The control goal is to place the robot end-effector over a desired static target by using a vision system equiped with a fixed camera to 'see' the robot end-effector and target. To achieve this goal we introduce a class of visual servo state feedback controllers and output (position) feedback controllers provided the camera orientation is known. For the case of unknown camera orientation a class of adaptive visual servo controllers is presented. All three classes contain controllers that meet input constraints.

Keywords: Visual servoing, robotics, stability

1. INTRODUCTION

External sensors such as visual systems enlarge the potential applications of actual robot manipulators evolving in unstructured environments. Although this fact has been recognized decades ago, it is until recent years that its effectiveness has reached the real world applications thanks to the technological improvement in cameras and dedicated hardware for image processing (Hashimoto, 1993; Hutchinson *et al.*, 1996).

This paper deals with a fixed camera configuration for visual servoing of robot manipulators. Most previous research has been started with the optics of the kinematic control where the robot velocity control (in joint or Cartesian space) is assumed to be computed in advance, and therefore the robot dynamics can be neglected (Allen *et al.*, 1993; Castaño and Hutchinson, 1994; Chaumette *et al.*, 1991; Espiau, 1993; Feddema *et al.*, 1991; Hager *et al.*, 1995; Nelson *et al.*, 1996; Mitsuda *et al.*, 1996). This approach is an example of a mechanical control system in which a kinematic model is used for control design, that is, the velocity of the system is assumed to be a direct input which can be manipulated. In physical systems, however, actuators exert forces or torques. This control philosophy is certainly effective for slow robot motion but its application is of a limited value when high speed motions are demanded.

We focus the visual servoing problem from an automatic control point of view by considering the full robot nonlinear dynamics with the applied torques as the control actions, and a rigorous stability analysis is given for an appropriate (adaptive) set point controller. Also, we are interested in simple control schemes avoiding the common procedures of camera calibration, inversion of the robot Jacobian and computation of the inverse kinematics. Previous efforts in this subject have been reported in (Coste-Manière *et al.*, 1995; Kelly, 1996; Kelly *et al.*, 1996; Lei and Ghosh, 1993; Miyazaki and Masutani, 1990).

The main contributions of our work are extensions of the results in (Kelly, 1996) to the cases where velocity measurements are not available and the camera orientation parameter is unknown. The first problem is solved invoking the (by now) standard "dirty derivative" solution. However, the later problem involves a nonlinearly parametrized adaptive system, –a situation which is essentially unexplored in the field– hence special analysis and synthesis tools have to be developed for its solution. Furthermore, we provide a simple common framework to design standard proportional or saturated controllers.

The organisation of this paper is as follows. Section 2 contains the problem formulation, preliminaries and notation. In section 3 we introduce a class of visual servo controllers which includes the controllers reported in (Coste-Manière *et al.*, 1995; Kelly, 1996; Kelly *et al.*, 1996). In section 4 we derive a class of adaptive visual servo controllers in case the camera orientation is unknown. In section 5 we present a class of visual servo controllers in case we have no velocity measurements available. Section 6 contains our concluding remarks.

2. PROBLEM FORMULATION, PRELIMINARIES AND NOTATION

2.1 *Robot dynamics*

In the absence of friction or other disturbances, the dynamics of a serial 2–link rigid robot manipulator can be written as (see e.g. (Ortega and Spong, 1989; Spong and Vidyasagar, 1989)):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \qquad (1)$$

where q is the 2×1 vector of joint displacements, τ is the 2×1 vector of applied joint torques, $M(q)$ is the 2×2 symmetric positive definite manipulator inertia matrix, $C(q,\dot{q})\dot{q}$ is the 2×1 vector of centripetal and Coriolis torques, and $g(q)$ is the 2×1 vector of gravitational torques. Two important properties of the robot dynamic model are the following:

Property 1. (see e.g. (Ortega and Spong, 1989; Spong and Vidyasagar, 1989)) The time derivative of the inertia matrix, and the centripetal and Coriolis matrix satisfy:

$$\dot{q}^T \left[\frac{1}{2}\dot{M}(q) - C(q,\dot{q}) \right] \dot{q} = 0; \quad \forall\, q, \dot{q} \in I\!R^2. \qquad (2)$$

Property 2. (see e.g. (Craig, 1988)). The gravitational torque vector $g(q)$ is bounded for all $q \in I\!R^2$. This means there exist finite constants $k_i \geq 0$ such that

$$\max_{q \in I\!R^2} ||g_i(q)|| \leq k_i \quad i = 1, 2$$

where $g_i(q)$ stands for the elements of $g(q)$.

For the purposes of this paper we consider a planar two degrees of freedom robot arm. For convenience we define a Cartesian reference frame anywhere in the robot base.

2.2 *Output equation*

We consider a fixed CDD camera whose optical axis is perpendicular to the plane where the robot tip evolves. The orientation of the camera with respect to the robot frame is denoted by θ.

The image acquired by the camera supplies a two–dimensional array of brightness values from a three–dimensional scene. This image may undergo various types of computer processing to enhance image properties and extract image features. In this paper we assume that the image features are the projection into the 2D image plane of 3D points in the scene space.

The output variable $y \in I\!R^2$ is defined as the position (in pixels) of the robot tip in the image. The mapping from the joint positions q to the output y involves a rigid body transformation, a perspective projection and a linear transformation (Feddema *et al.*, 1991; Hutchinson *et al.*, 1996). The corresponding output equation has the form (Kelly, 1996)

$$y = ae^{-J\theta}[k(q) - \vartheta_1] + \vartheta_2 \qquad (3)$$

where $a > 0$ and ϑ_1, ϑ_2 denote intrinsic camera parameters (scale factors, focal length, center offset), $k : I\!R^2 \to I\!R^2$ stands for the robot direct kinematics, and

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

The direct kinematics yields $\dot{k} = J(q)\dot{q}$, where $J(q) \in I\!R^{2\times 2}$ is the analytic robot Jacobian. An important property of this Jacobian is the following (see e.g. (Spong and Vidyasagar, 1989)):

Property 3. The Jacobian is bounded for all $q \in I\!R^2$, i.e. there exists a finite constant J_M such that

$$||J(q)|| \leq J_M \quad \forall q \in I\!R^2$$

2.3 *Problem formulation*

Consider the robotic system (1) together with the output equation (3), where the camera orientation θ is known, but the intrinsic camera parameters a, ϑ_1 and ϑ_2 are unknown. Suppose that together with the position y of the robot tip in the image also measurements of the joint positions q and velocities $\dot{q}$ are available. Let $y_d \in I\!R^2$ be a desired constant position for the robot tip in the image plane. This corresponds to the image of a point target which is assumed to be located strictly inside the robot workspace. Then the control problem can be stated as to design a control law for the actuator torques τ such that the robot tip reaches, in the image supplied on the screen, the target point placed anywhere in the robot workspace. In other words:

$$\lim_{t\to\infty} y(t) = y_d$$

Later in this paper the assumption that the camera orientation θ is known will be relaxed, as well as the assumption that measurements of the joint velocities $\dot{q}$ are available.

To be able to solve the problem formulated above we make the following assumptions:

Assumption 4. **(Problem solvability)** There exists a constant (unknown) vector $q_d \in I\!R^2$ such that

$$y_d = ae^{-J\theta}[k(q_d) - \vartheta_1] + \vartheta_2$$

Assumption 5. **(Nonsingularity at the desired configuration)** For the (unknown) vector $q_d \in I\!R^2$ it holds true that

$$\det\{J(q_d)\} \neq 0.$$

Corollary 6. There exists a neighborhood around q_d for which $\det\{J(q)\} \neq 0$ (by smoothness of the Jacobian).

It is worth noticing that in case y_d corresponds to the image of a point target located strictly inside the robot workspace, then Assumptions 4 and 5 are trivially satisfied. Also, under Assumptions 4 and 5 we have that $q = q_d \in I\!R^2$ is an isolated solution of

$$y_d = ae^{-J\theta}[k(q) - \vartheta_1] + \vartheta_2 \tag{4}$$

i.e. there exists a neighborhood around q_d for which $q = q_d$ is the only solution of (4).

2.4 *Notation*

Throughout we use the following notation.

Definition 7. Let F^n denote the class of continuous functions $f : I\!R^n \to I\!R^n$ for which there exists a positive definite $F : I\!R^n \to I\!R$ such that

$$f(x) = f(x_1, \ldots, x_n) = \begin{bmatrix} \dfrac{\partial F}{\partial x_1}(x_1, \ldots, x_n) \\ \vdots \\ \dfrac{\partial F}{\partial x_n}(x_1, \ldots, x_n) \end{bmatrix} \tag{5}$$

and for which $x^T f(x)$ is a positive definite function.

Definition 8. Let B^n denote the class of $f \in F^n$ that are bounded, i.e the class of $f \in F^n$ for which there exists a constant $f_M \in I\!R$ such that $||f(x)|| \leq f_M$ for all $x \in I\!R^n$.

An important property of $f \in F^n$ is the following:

Property 9. Let $f \in F^n$. Then $f(x) = 0$ if and only if $x = 0$.

In general it is not easy to verify whether a given $f : I\!R^n \to I\!R^n$ can be written as the gradient of a radially unbounded $F : I\!R^n \to I\!R$. However, a necessary condition for continuously differentiable f is that its Jacobian $\frac{\partial f}{\partial x}$ is symmetric.

It is easy to see that elements of F^n are the functions

$$f(x) = K_1[f_1(x_1), \ldots, f_n(x_n)]^T$$

and

$$f(x) = K_2 x$$

where $K_1 = K_1^T$ is a $n \times n$ diagonal positive definite matrix, $K_2 = K_2^T$ is a $n \times n$ (not necessarily diagonal) positive definite matrix, and f_i are continuous nondecreasing functions satisfying $f_i(0) = 0$ and $f_i'(0) > 0$ $(i = 1, \ldots, n)$. By choosing $f_i(x) = \tanh(\lambda_i x)$, $f_i(x) = \mathrm{sat}(\lambda_i x)$ or $f_i(x) = \frac{x}{\lambda_i + |x|}$ $(\lambda_i > 0)$ we obtain elements of B^n, whereas $f(x) = K_2 x$ is an element of F^n but not of B^n.

Throughout we denote for $f \in F^n$ by $F(x)$ the associated function of which f is the gradient (cf. (5)).

Furthermore, we define

$$\tilde{q} = q - q_d \text{ and } \tilde{y} = y - y_d.$$

Since y is measurable and y_d is given,

$$\tilde{y} = ae^{-J\theta}(k(q) - k(q_d))$$

can be measured too. However, since q_d is unknown, $\tilde{q}$ is *not* available for measurement.

We conclude this section by noticing that since q_d is fixed, $\dot{\tilde{y}} = ae^{-J\theta}J(q)\dot{q}$ and therefore

$$\dot{F}(\tilde{y}) = a\dot{q}^T J(q)^T e^{J\theta} f(\tilde{y}).$$

3. A CLASS OF STABLE VISUAL SERVO CONTROLLERS

In this section we introduce a class of visual servo controllers which includes those reported in (Coste-Manière *et al.*, 1995; Kelly, 1996; Kelly *et al.*, 1996). Assuming that the camera orientation θ is known, and the full state $(q, \dot{q})$ is measured, these controllers ensure local regulation. This is formally stated in the next

Proposition 10. Consider the system (1) in closed-loop with the control law

$$\tau = g(q) - f_1(\dot{q}) - J(q)^T e^{J\theta} f_2(\tilde{y}) \tag{6}$$

where $f_1, f_2 \in F^2$. Under Assumptions 4–5 we have

$$\lim_{t\to\infty} \tilde{y}(t) = \lim_{t\to\infty} \dot{q}(t) = 0$$

provided the initial conditions $\dot{q}(0)$ and $\tilde{y}(0)$ are sufficiently small.

PROOF. Using the control law (6) results in the closed-loop dynamics

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + f_1(\dot{q}) + J(q)^T e^{J\theta} f_2(\tilde{y}) = 0 \tag{7}$$

According to Assumptions 4–5 this equation has an isolated equilibrium at $[q^T \;\; \dot{q}^T]^T = [q_d^T \;\; 0^T]^T$.

Consider the Lyapunov function candidate

$$V(\tilde{q},\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{a}F_2(\tilde{y})$$

which is a (locally) positive definite function.
Along the closed-loop dynamics (7) its time-derivative becomes, using Property 1:

$$\begin{aligned}\dot{V}(\tilde{q},\dot{q}) &= -\dot{q}^T f_1(\dot{q}) - \dot{q}^T J(q)^T e^{J\theta} f_2(\tilde{y}) + \\ &\quad + \dot{q}^T J(q)^T e^{J\theta} f_2(\tilde{y}) \\ &= -\dot{q}^T f_1(\dot{q}) \le 0\end{aligned}$$

which is negative semidefinite in the state $(\tilde{q},\dot{q})$. Using LaSalle's theorem and Corollary 6, for any initial condition in a small neighborhood of the equilibrium we have

$$\lim_{t\to\infty} \dot{q}(t) = \lim_{t\to\infty} f_2(\tilde{y}(t)) = 0$$

so we can conclude using Property 9:

$$\lim_{t\to\infty} \tilde{y}(t) = 0. \qquad \square$$

Consider the system (1) where we deal with the input constraints

$$|\tau_i(t)| \le \tau_{i,max} \quad i = 1, 2. \tag{8}$$

Then we can derive the following

Corollary 11. If $\tau_{i,max} > k_i$, where k_i has been defined in Property 2, then there exist $f_1,\ f_2 \in B^2$ such that the controller (6) meets (8) and in closed-loop with the system (1) yields

$$\lim_{t\to\infty} \tilde{y}(t) = 0$$

provided the initial conditions are sufficiently small.

4. ADAPTIVE VISUAL SERVOING

In this section we consider the case in which, in contrast with the previous section, also the camera orientation θ is unknown. Still assuming that the full state $(q,\dot{q})$ is available for measurement we introduce a class of adaptive controllers that ensure local regulation:

Proposition 12. Consider the system (1) in closed-loop with the control law

$$\tau = \begin{cases} g(q) - f_1(\dot{q}) - J(q)^T e^{J\hat{\theta}} f_2(\tilde{y}) \\ \qquad \text{if } \dot{q}^T J(q)^T e^{J\hat{\theta}} f_2(\tilde{y}) \ge 0 \\ g(q) - f_1(\dot{q}) + J(q)^T e^{J\hat{\theta}} f_2(\tilde{y}) \\ \qquad \text{if } \dot{q}^T J(q)^T e^{J\hat{\theta}} f_2(\tilde{y}) < 0 \end{cases} \tag{9}$$

where $f_1,\ f_2 \in F^2$. We update the parameter $\hat{\theta}$ as

$$\dot{\hat{\theta}} = \gamma \dot{q}^T J(q)^T J e^{J\hat{\theta}} f_2(\tilde{y}) \tag{10}$$

where $\gamma > 0$ is a constant. Under Assumptions 4–5 we have, if we define $\tilde{\theta} = \hat{\theta} - \theta$:

$$\lim_{t\to\infty} \tilde{y}(t) = \lim_{t\to\infty} \dot{q}(t) = \lim_{t\to\infty} \dot{\tilde{\theta}}(t) = 0$$

provided the initial conditions $\dot{q}(0)$, $\tilde{y}(0)$ and $\tilde{\theta}(0)$ are sufficiently small.

PROOF. Using the control law (9) together with the parameter update law (10) results in the closed-loop dynamics

$$\begin{aligned} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + f_1(\dot{q}) &= \pm J(q)^T e^{J\hat{\theta}} f_2(\tilde{y}) \\ \dot{\tilde{\theta}} &= \gamma \dot{q}^T J(q)^T J e^{J\hat{\theta}} f_2(\tilde{y}) \end{aligned} \tag{11}$$

where the '$\pm$' reads as a '$+$' if $\dot{q}^T J(q)^T e^{J\hat{\theta}} f_2(\tilde{y}) \ge 0$ and as a '$-$' if $\dot{q}^T J(q)^T e^{J\hat{\theta}} f_2(\tilde{y}) < 0$.
Consider the Lyapunov function candidate

$$V(\tilde{q},\dot{q},\tilde{\theta}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{a}F_2(\tilde{y}) + \frac{1}{\gamma}(1-\cos\tilde{\theta}) \tag{12}$$

which is a (locally) positive definite function.
Along the closed-loop dynamics (11) its time-derivative becomes, using Property 1:

$$\begin{aligned}\dot{V}(\tilde{q},\dot{q},\tilde{\theta}) &= -\dot{q}^T f_1(\dot{q}) - \left|\dot{q}^T J(q)^T e^{J\hat{\theta}} f_2(\tilde{y})\right| + \\ &\quad + \dot{q}^T J(q)^T e^{J\theta} f_2(\tilde{y}) + \\ &\quad + \sin\tilde{\theta}\dot{q}^T J(q)^T J e^{J\hat{\theta}} f_2(\tilde{y}) \\ &= -\dot{q}^T f_1(\dot{q}) - \left|\dot{q}^T J(q)^T e^{J\hat{\theta}} f_2(\tilde{y})\right| + \\ &\quad + \dot{q}^T J(q)^T (e^{-J\tilde{\theta}} + \sin\tilde{\theta} J) e^{J\hat{\theta}} f_2(\tilde{y}) \\ &= -\dot{q}^T f_1(\dot{q}) - \left|\dot{q}^T J(q)^T e^{J\hat{\theta}} f_2(\tilde{y})\right| + \\ &\quad + \cos\tilde{\theta}\dot{q}^T J(q)^T e^{J\hat{\theta}} f_2(\tilde{y}) \\ &\le -\dot{q}^T f_1(\dot{q})\end{aligned}$$

which is negative semidefinite in the state $(\tilde{q},\dot{q},\tilde{\theta})$.
According to LaSalle's theorem, the closed-loop system tends to the largest invariant set of points $(\tilde{q},\dot{q},\tilde{\theta})$ for which $\dot{V} = 0$. From $0 = \dot{V} \le -\dot{q}^T f_1(\dot{q}) \le 0$ it follows that necessarily $\dot{q} = 0$. Then from the closed-loop dynamics (11) we know $\dot{\tilde{\theta}} = 0$ and using Corollary 6 also $f_2(\tilde{y}) = 0$. Therefore LaSalle's theorem gives us for any initial condition in a small neighborhood of the origin

$$\lim_{t\to\infty} \dot{\tilde{\theta}}(t) = \lim_{t\to\infty} \dot{q}(t) = \lim_{t\to\infty} \tilde{y}(t) = 0$$

$\square$

Remark 13. The switching nature of the controller (9) leads to chattering, which is undesirable. Using a suitably smoothed control law might be a way to overcome the chattering.

As in the previous section we can derive the following

Corollary 14. If $\tau_{i,max} > k_i$, where k_i has been defined in Property 2, then there exist $f_1,\ f_2 \in B^2$ such that the controller (9) meets (8) and in closed-loop with the system (1) yields

$$\lim_{t\to\infty} \tilde{y}(t) = \lim_{t\to\infty} \dot{q}(t) = \lim_{t\to\infty} \dot{\tilde{\theta}}(t) = 0$$

provided the initial conditions are sufficiently small.

Remark 15. For the system (1) it is well known (Ortega and Spong, 1989) that there exist a reparametrization of all unknown system parameters into a parameter vector $\Theta \in I\!R^p$ that enters linearly in the system dynamics (1). Therefore the following holds:

$$M(q,\Theta)\ddot{q} + C(q,\dot{q},\Theta)\dot{q} + g(q,\Theta) =$$
$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + g_0(q) + Y(q,\dot{q},\dot{q},\ddot{q})\Theta$$

We can cope with those unknown system parameters in the 'standard' way by adding $Y(q,\dot{q},\dot{q},\ddot{q})\hat{\Theta}$ to the control law (and replacing $g(q)$ with $g_0(q)$), where $\hat{\Theta}$ is updated according to

$$\dot{\hat{\Theta}} = -\Gamma Y^T(q,\dot{q},\dot{q},\ddot{q})\dot{q}$$

where $\Gamma = \Gamma^T > 0$ is a positive definite matrix. To prove asymptotic stability as in Proposition 12, we only add $\frac{1}{2}\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta}$ to the Lyapunov function (12), where we defined $\tilde{\Theta} = \hat{\Theta} - \Theta$.

5. FILTERED VISUAL SERVOING

In this section we consider the case in which, in contrast with section 3, no measurements of the joint velocities $\dot{q}$ are available. Assuming that the camera orientation θ is known and only measurements of the joint positions q are available we introduce a class of controllers and filters that ensure local regulation. This is formally stated in the next

Proposition 16. Consider the system (1) in closed-loop with the control law

$$\tau = g(q) - J(q)^T e^{J\theta} f_1(z) - J(q)^T e^{J\theta} f_2(\tilde{y}) \quad (13)$$

where $f_1,\ f_2 \in \mathcal{F}^2$, and z is generated from the filter

$$\begin{aligned} z &= \tilde{y} - w \\ \dot{w} &= \tilde{y} - w \end{aligned} \quad (14)$$

Under Assumptions 4–5 we have

$$\lim_{t\to\infty} w(t) = \lim_{t\to\infty} z(t) = \lim_{t\to\infty} \dot{q}(t) = \lim_{t\to\infty} \tilde{y}(t) = 0$$

provided the initial conditions $w(0)$, $\dot{q}(0)$, and $\tilde{y}(0)$ are sufficiently small.

PROOF. Using the control law (13) together with the filter (14) results in the closed-loop dynamics

$$\begin{aligned} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + J(q)^T e^{J\theta}[f_1(z) + f_2(\tilde{y})] &= 0 \\ \dot{z} = ae^{-J\theta}J(q)\dot{q} - z & \end{aligned} \quad (15)$$

Consider the Lyapunov function candidate

$$V(\tilde{q},\dot{q},z) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{a}F_2(\tilde{y}) + \frac{1}{a}F_1(z) \quad (16)$$

which is a (locally) positive definite function.

Along the closed-loop dynamics (15) its time-derivative becomes:

$$\begin{aligned} \dot{V}(\tilde{q},\dot{q},z) &= -\dot{q}^T J(q)^T e^{J\theta} f_1(z) - \dot{q}^T J(q)^T e^{J\theta} f_2(\tilde{y}) + \\ &\quad + \dot{q}^T J(q)^T e^{J\theta} f_2(\tilde{y}) + \frac{1}{a}\dot{z}^T f_1(z) \\ &= -\dot{q}^T J(q)^T e^{J\theta} f_1(z) + \dot{q}^T J(q)^T e^{J\theta} f_1(z) - \\ &\quad - \frac{1}{a} z^T f_1(z) \\ &= -\frac{1}{a} z^T f_1(z) \le 0 \end{aligned}$$

which is negative semidefinite in the state $(\tilde{q},\dot{q},z)$. Using LaSalle's theorem and Corollary 6, for any initial condition in a small neighborhood of the equilibrium we can conclude

$$\lim_{t\to\infty} w(t) = \lim_{t\to\infty} z(t) = \lim_{t\to\infty} \dot{q}(t) = \lim_{t\to\infty} \tilde{y}(t) = 0.$$

□

Remark 17. The filter (14) can, similar to the one presented in (Lefeber and Nijmeijer, 1997), be seen as a simple representative of a whole class of possible filters. For instance if $f_1 \in \mathcal{F}^2$ satisfies the property that also $\Lambda f \in \mathcal{F}^2$, where Λ is an arbitrary positive definite matrix, then it can easily be seen that instead of (14) also the filter

$$\begin{aligned} z &= \Lambda_1\tilde{y} - \Lambda_2 w \\ \dot{w} &= \Lambda_3(\Lambda_2\tilde{y} - \Lambda_2 w) \end{aligned} \quad (17)$$

can be used (replace in (16) $F_1(\tilde{y})$ with the $F(\tilde{y})$ associated with $\Lambda_1^{-1} f_1(\tilde{y})$ to obtain

$$\dot{V} = -\frac{1}{a} z^T \Lambda_3\Lambda_2\Lambda_1^{-1} f_1(z) = z^T \tilde{f}_1(z)$$

with $\tilde{f}_1(z) \in \mathcal{F}^2$). The filter (17) is similar to the ones presented in (Ailon and Ortega, 1993; Berghuis and Nijmeijer, 1993). Also the more general class of linear filters presented in (Arimoto *et al.*, 1994; Kelly and Santibañez, 1996) can similarly be seen as a special case of (14). Also a wide variety of nonlinear filters can be rewritten as (14).

In general one can say that the filter (14) is a representative of a whole class of controllers that takes its simple form due to a well chosen change of coordinates.

To obtain other possible filters, just apply a suitable change of coordinates in z and w (suitable in the sense that $\dot{V}$ remains negative definite). As far as the proof is concerned, one possibly has to replace $F_1(\tilde{y})$ in (16) with a different F, as we have seen in deriving (17), sometimes resulting in a different expression for $f_1(z)$ in (13).

As in the previous sections we can derive the following

Corollary 18. If $\tau_{i,max} > k_i$, where k_i has been defined in Property 2, then there exist $f_1,\ f_2 \in \mathcal{B}^2$ such that the controller (13) meets (8) and in closed-loop with the system (1) yields

$$\lim_{t\to\infty} w(t) = \lim_{t\to\infty} z(t) = \lim_{t\to\infty} \dot{q}(t) = \lim_{t\to\infty} \tilde{y}(t) = 0$$

provided the initial conditions are sufficiently small.

6. CONCLUDING REMARKS

In this paper we addressed the visual servoing of planar robot manipulators under a fixed camera configuration. In case the camera orientation is known, we introduced a class of visual servo controllers for both the state feedback and output feedback case (position measurements). In case of unknown camera orientation a class of adaptive controllers has been presented. The results include controllers that satisfy input constraints.

ACKNOWLEDGEMENTS

This work was carried out while the first author was visiting R. Ortega in the frame of the Capital and Mobility European network on "Nonlinear and Adaptive Control: Towards a design Methodology for Physical Systems".

7. REFERENCES

Ailon, A. and R. Ortega (1993). An observer-based controller for robot manipulators with flexible joints. *Systems and Control Letters* **21**, 329–335.

Allen, P. K., A. Timcenko, B. Yoshimi and P. Mitchelman (1993). Automated tracking and grasping of a moving object with a robotic hand-eye system. *IEEE Transactions on Robotics and Automation* **9**(2), 152–165.

Arimoto, S., V. Parra-Vega and T. Naniwa (1994). A class of linear velocity observers for nonlinear mechanical systems. In: *Proceedings of the 1st Asian Control Conference*. Vol. 2. Tokyo, Japan. pp. 633–636.

Berghuis, H. and H. Nijmeijer (1993). Global regulation of robots using only position measurement. *Systems and Control Letters* **21**, 289–293.

Bishop, B. E. and M. Spong (1997). Adaptive calibration and control of 2D monocular visual servo system. In: *Proceedings of the IFAC Symposium on Robot Control (SYROCO)*. Nantes, France. pp. 531–537.

Castaño, A. and S. Hutchinson (1994). Visual compliance: Task-directed visual servo control. *IEEE Transactions on Robotics and Automation* **10**(3), 334–342.

Chaumette, F., P. Rives and B. Espiau (1991). Positioning of a robot with respect to an object, tracking it and estimating its velocity by visual servoing. In: *Proceedings of the 1991 IEEE Conference on Robotics and Automation*. Sacramenta, CA, USA. pp. 2248–2253.

Coste-Manière, E., P. Couvignou and P. K. Khosla (1995). Visual servoing in the task-function framework: A contour following task. *Journal of Intelligent and Robotic Systems* **12**, 1–12.

Craig, J. J. (1988). *Adaptive control of mechanical manipulators*. Addison-Wesley Pub.

Espiau, B. (1993). Effect of camera calibration errors on visual servoing in robotics. In: *Proceedings of Experimental Robotics III: The 3rd International Symposium*. Kyoto, Japan. pp. 182–192.

Feddema, J. T., C. S. G. Lee and O. R. Mitchell (1991). Weighted selection of image features for resolved rate visual feedback control. *IEEE Transactions on Robotics and Automation* **7**(1), 31–47.

Hager, G. D., W. C. Chang and A. S. Morse (1995). Robot hand-eye coordination based on stereo vision. *IEEE Control Systems Magazine* **15**(1), 30–39.

Hashimoto, K. (1993). *Visual servoing: Real-time control of robot manipulators based on visual sensory feedback*. Vol. 7. World Scientific Publishing Co.. Singapore.

Hutchinson, S., G. D. Hager and P. I. Corke (1996). A tutorial on visual servo control. *IEEE Transactions on Robotics and Automation* **12**(5), 651–670.

Kelly, R. (1996). Robust asymptotically stable visual servoing of planar robots. *IEEE Transactions on Robotics and Automation* **12**(5), 759–766.

Kelly, R. and V. Santibañez (1996). A class of global regulators by using output feedback for elastic joint robots. In: *Proceedings of the 13th Triennial World Congress*. San Francisco, California, USA. pp. 37–42.

Kelly, R., P. Shirkey and M. W. Spong (1996). Fixed camera visual servoing of planar robots. In: *Proc. IEEE Int. Conf. on Robotics and Automation*. Vol. 3. Minneapolis, MN, USA. pp. 2643–2649.

Lefeber, A. A. J. and H. Nijmeijer (1997). Bounded global tracking of rigid robot manipulators using composite controllers. Memorandum 1404. University of Twente. Faculty of Applied Mathematics, Enschede, The Netherlands.

Lei, M. and B. K. Ghosh (1993). Visually guided robotic tracking and grasping of a moving object with a robotic hand-eye system. In: *Proceedings of the 32nd Conference on Decision and Control*. San Antonio, TX, USA. pp. 1604–1609.

Mitsuda, T., N. Maru, K. Fujikawa and F. Miyazaki (1996). Visual servoing based on the use of binocular visual space. In: *Proceedings IROS 96*. pp. 1104–1111.

Miyazaki, F. and Y. Masutani (1990). Robustness of sensory feedback control based on imperfect jacobian. In: *Robotics Research: The Fifth International Symposium*. MIT Press. Cambridge, MA, USA. pp. 201–208.

Nelson, B., N. P. Papanikolopoulos and P. K. Khosla (1996). Robotic visual servoing and robotic assembly tasks. *IEEE Robotics and Automation Magazine* **3**(2), 23–31.

Ortega, R. and M. W. Spong (1989). Adaptive motion control of rigid robots: a tutorial. *Automatica* **25**(6), 877–888.

Spong, M. W. and M. Vidyasagar (1989). *Robot Dynamics and Control*. Wiley.