EXPONENTIAL TRACKING CONTROL OF A MOBILE CAR USING A CASCADED APPROACH

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Abstract: In this paper we address the problem of designing simple global tracking controllers for a kinematic model of a mobile robot and a simple dynamic model of a mobile robot. For this we use a cascaded systems approach, resulting into linear controllers that yield global $\mathcal{K}$-exponential stability of the closed loop system.

Keywords: mobile robot, trajectory tracking, persistency of excitation, global $\mathcal{K}$-exponential stabilization.

1. INTRODUCTION

In recent years the control of nonholonomic dynamic systems has received considerable attention, in particular the stabilization problem. One of the reasons for this is that no smooth stabilizing state-feedback control law exists for these systems, since Brockett's necessary condition for smooth stabilization is not met (Brockett, 1983). For an overview we refer to the survey paper (Kolmanovsky and McClamroch, 1995) and references cited therein. In contrast to the stabilization problem, the tracking control problem for nonholonomic control systems has received little attention. In (Fierro and Lewis, 1995; Kanayama *et al.*, 1990; Micaelli and Samson, 1993; Murray *et al.*, 1992; Oelen and van Amerongen, 1994) tracking control schemes have been proposed based on linearization of the corresponding error model. In (Canudas de Wit *et al.*, 1996; Rui and McClamroch, 1995) the feedback design issue was addressed via a dynamic feedback linearization approach. All these papers solve the local tracking problem for some classes of nonholomic systems. The only global tracking results that we are aware of are (Samson and Ait-Abderrahim, 1991; Gusmak and Makarov, 1993; Jiang and Nijmeijer, 1997*b*; Jiang *et al.*, 1998).

Quite recently, the results in (Jiang and Nijmeijer, 1997*b*) have been extended to arbitrary chained form nonholonomic systems (Jiang and Nijmeijer, 1997*a*). The proposed backstepping-based recursive design turned out to be useful for simplified dynamic models of such chained form systems, see (Jiang and Nijmeijer, 1997*b*; Jiang and Nijmeijer, 1997*a*). However, it is clear that the technique used in (Jiang and Nijmeijer, 1997*b*) (and (Jiang and Nijmeijer, 1997*a*)) does not

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exploit the physical structure behind the model, and then the controllers may become quite complicated and computationally demanding when computed in original coordinates.
The purpose of this paper is to show that the nonlinear controllers proposed in (Jiang and Nijmeijer, 1997*b*) can be simplified into *linear* controllers for both the kinematic model and an 'integrated' simplified dynamic model of the mobile robot. Our approach is based on cascaded systems. As a result, instead of exponential stability for small initial errors as in (Jiang and Nijmeijer, 1997*b*), the controllers proposed here yield global $\mathcal{K}$-exponential stability (cf. (Sørdalen and Egeland, 1995)) of the closed loop system.
The organisation of the paper is as follows. Section 2 contains some definitions, preliminary results, the model of the mobile car, the tracking error dynamics and the problem under consideration. In Section 3 we derive linear feedback controllers that solve the global tracking problem. Section 4 shows how to extend our results for a dynamic extension of the mobile robot. Section 5 contains the conclusions.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 *Preliminaries*

To start with, we recall some basic concepts (see e.g. (Khalil, 1996; Vidyasagar, 1993)).

Definition 1. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to *class* $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

Definition 2. A continuous function $\beta : [0, a) \times [0, a) \to [0, \infty)$ is said to belong to *class* $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

Consider the system

$$\dot{x} = f(t, x) \qquad f(t, 0) = 0 \quad \forall t \geq 0 \tag{1}$$

where $f(t, x)$ is piecewise continuous in t and locally Lipschitz in x.

Definition 3. The system (1) is *uniformly stable* if for each $\epsilon > 0$ there is $\delta = \delta(\epsilon) > 0$, independent of t_0, such that

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| < \epsilon, \qquad \forall t \geq t_0 \geq 0.$$

Definition 4. The system (1) is *globally uniformly asymptotically stable (GUAS)* if it is uniformly stable and globally attractive, that is, there exists a class $\mathcal{KL}$ function $\beta(\cdot, \cdot)$ such that for all initial state $x(t_0)$:

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0), \quad \forall t \geq t_0 \geq 0$$

Definition 5. The system (1) is *globally exponentially stable (GES)* if there exist $k > 0$ and $\gamma > 0$ such that for any initial state

$$\|x(t)\| \leq \|x(t_0)\| k \exp[-\gamma(t - t_0)].$$

A slightly weaker notion of exponential stability is the following (cf. (Sørdalen and Egeland, 1995))

Definition 6. We call the system (1) *globally* $\mathcal{K}$*-exponentially stable* if there exist $\gamma > 0$ and a class $\mathcal{K}$ function $k(\cdot)$ such that

$$\|x(t)\| \leq k(\|x(t_0)\|) \exp[-\gamma(t - t_0)] \tag{2}$$

2.2 *Cascaded systems*

Consider the system

$$\begin{cases} \dot{x} = f_1(t, x) + g(t, x, y)y \\ \dot{y} = f_2(t, y) \end{cases} \tag{3}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $f_1(t, x)$ is continuously differentiable in (t, x) and $f_2(t, y)$, $g(t, x, y)$ are continuous in their arguments, and locally Lipschitz in y and (x, y) respectively.
We can view the system (3) as the system

$$\Sigma_1 : \dot{x} = f_1(t, x)$$

that is perturbed by the output of the system

$$\Sigma_2 : \dot{y} = f_2(t, y).$$

For the cascaded system (3) we have:

Theorem 7. (see (Panteley and Loría, 1998)). The cascaded system (3) is GUAS if the following three assumptions hold:

- *assumption on* Σ_1*:* the system $\dot{x} = f_1(t, x)$ is GUAS and there exists a continuously differentiable function $V(t, x) : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}$ that satisfies
$$W(x) \leq V(t, x),$$
$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \cdot f_1(t, x) \leq 0, \forall \|x\| \geq \eta,$$
$$\left\| \frac{\partial V}{\partial x} \right\| \|x\| \leq cV(t, x), \quad \forall \|x\| \geq \eta,$$
where $W(x)$ is a positive definite proper function and $c > 0$ and $\eta > 0$ are constants,
- *assumption on the interconnection:* the function $g(t, x, y)$ satisfies for all $t \geq t_0$:
$$\|g(t, x, y)\| \leq \theta_1(\|y\|) + \theta_2(\|y\|)\|x\|,$$
where $\theta_1, \theta_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ are continuous functions,
- *assumption on* Σ_2*:* the system $\dot{y} = f_2(t, y)$ is GUAS and for all $t_0 \geq 0$:
$$\int_{t_0}^{\infty} \|y(t, t_0, y(t_0))\| dt \leq \kappa(\|y(t_0)\|),$$
where the function $\kappa(\cdot)$ is a class $\mathcal{K}$ function,

Lemma 8. If in addition to the assumptions in Theorem 7 both $\dot{x} = f_1(t, x)$ and $\dot{y} = f_2(t, y)$ are globally $\mathcal{K}$-exponentially stable, then the cascaded system (3) is globally $\mathcal{K}$-exponentially stable.

PROOF. Evidently the bound (2) is satisfied for $y(t)$, so it suffices to prove the same bound for $x(t)$.
Since all conditions of Theorem 7 are satisfied, system (3) is GUAS and $z = col(x, y)$ satisfies the bound $\|z(t, t_0, z_0)\| \le \beta(\|z_0\|, t - t_0)$, where $\beta(\cdot)$ is a class KL function. Then, for all initial conditions $z_0 \le r$ the function $g(t, x, y)$ can be upperbounded as $\|g(t, x, y)\| \le c_g$, where $c_g = c_g(r) > 0$ is a constant. Now consider the subsystem

$$\dot{x} = f_1(t, x) + g(t, x, y)y \tag{4}$$

By assumption $\dot{x} = f_1(t, x)$ and $\dot{y} = f_2(t, y)$ are globally K-exponentially stable, hence using converse Lyapunov theory, cf. (Khalil, 1996), on that ball there exist Lyapunov functions $V_1(t, x)$ and $V_2(t, y)$ such that

$$\alpha_1\|x\|^2 \le V_1 \le \alpha_2\|x\|^2$$
$$\dot{V}_1 = \frac{\partial V_1}{\partial x} f_1(t, x) \le -\alpha_3\|x\|^2$$
$$\|\frac{\partial V_1}{\partial x}\| \le \alpha_4\|x\|$$

and

$$\beta_1\|y\|^2 \le V_2 \le \beta_2\|y\|^2$$
$$\dot{V}_2 = \frac{\partial V_2}{\partial y} f_2(t, y) \le -\beta_3\|y\|^2$$
$$\|\frac{\partial V_2}{\partial y}\| \le \beta_4\|y\|$$

Taking the derivative of $V_1(t, x)$ with respect to (4) we obtain

$$\begin{aligned}\dot{V}_1 &\le -\alpha_3\|x\|^2 + \alpha_4\|g(t, x, y)\|\;\|x\|\;\|y\| \\ &\le -\alpha_3\|x\|^2 + \alpha_4 c_g\|x\|\;\|y\| \\ &\le -\frac{\alpha_3}{2}\|x\|^2 + \frac{\alpha_4^2 c_g^2}{2\alpha_3}\|y\|^2\end{aligned}$$

For the overall system consider the Lyapunov function

$$V(t, x, y) := V_1(t, x) + \delta V_2(t, y)$$

where $\delta = \delta(r) = \frac{\alpha_4^2 c_g^2(r)}{2\alpha_3}$. It is easy to see that the derivative of V along the solutions of (3) satisfies $\dot{V} \le -\gamma V$ with

$$\gamma = \frac{1}{2}\min\{\frac{\alpha_3}{\alpha_1}, \frac{\beta_3}{\beta_1}\} \tag{5}$$

Using the bounds on $V_1(t, x)$ from the last inequality we conclude that

$$\|x(t, t_0, x_0, y_0)\|^2 \le \frac{1}{\alpha_1} V(t_0, x_0, y_0) e^{-\gamma(t-t_0)}$$

hence for $x(t)$ the bound (2) is satisfied with γ defined in (5) and $k = \max\{\alpha_2, \beta\delta\}$. □

2.3 *A result from Model Reference Adaptive Control*

Lemma 9. (cf. e.g. (Khalil, 1996; Sastry and Bodson, 1989)). Consider the system

$$\begin{bmatrix} \dot{e} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} A_m & b_m w^T(t) \\ -\gamma w(t) c_m^T & 0 \end{bmatrix} \begin{bmatrix} e \\ \phi \end{bmatrix} \tag{6}$$

where $e \in I\!R^n$, $\phi \in I\!R^m$, $\gamma > 0$. Assume that $M(s) \stackrel{\Delta}{=} c_m^T \cdot$ $\cdot (sI - A_m)^{-1} b_m$ is a strictly positive real transfer function, i.e. $\mathrm{Re}[M(i\omega)] > 0$ for all $\omega \in I\!R$. Then $\phi(t)$ is bounded and

$$\lim_{t\to\infty} e(t) = 0.$$

If in addition $\omega(t)$ and $\dot{\omega}(t)$ are bounded for all $t \ge t_0$, and there are positive constants δ and k such that

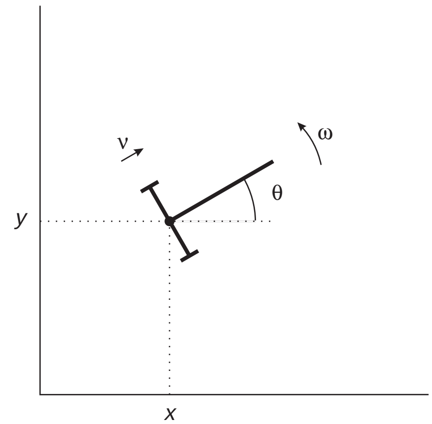

Fig. 1. The mobile car

$$\int_t^{t+\delta} \omega(\tau)\omega^T(\tau) d\tau \ge kI, \qquad \forall t \ge t_0 \tag{7}$$

then the system (6) is GES.

Remark 10. Note that in the model reference adaptive control problem the bound on $\omega(t)$ usually depends on the initial state $(e(0), \phi(0))^T$. Therefore, in general only global K-exponential stability can be claimed for the model reference adaptive control problem. However, when bounds on $\omega(t)$ are known apriori, GES can be claimed instead.
The condition (7) is known as the persistence-of-excitation condition.

2.4 *Problem-formulation*

A kinematic model of a wheeled mobile robot with two degrees of freedom is given by the following equations

$$\begin{aligned}\dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \omega\end{aligned} \tag{8}$$

where the forward velocity v and the angular velocity ω are considered as inputs, (x, y) is the center of the rear axis of the vehicle, and θ is the angle between heading direction and x-axis (see Figure 1).
Consider the problem of tracking a reference robot as done in (Kanayama *et al.*, 1990):

$$\begin{aligned}\dot{x}_r &= v_r\cos\theta_r \\ \dot{y}_r &= v_r\sin\theta_r \\ \dot{\theta}_r &= \omega_r.\end{aligned}$$

Following (Kanayama *et al.*, 1990) we define the error coordinates (cf. Figure 2)

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}$$

One can verify that in these coordinates the error dynamics become

$$\begin{aligned}\dot{x}_e &= \omega y_e - v + v_r\cos\theta_e \\ \dot{y}_e &= -\omega x_e + v_r\sin\theta_e \\ \dot{\theta}_e &= \omega_r - \omega\end{aligned} \tag{9}$$

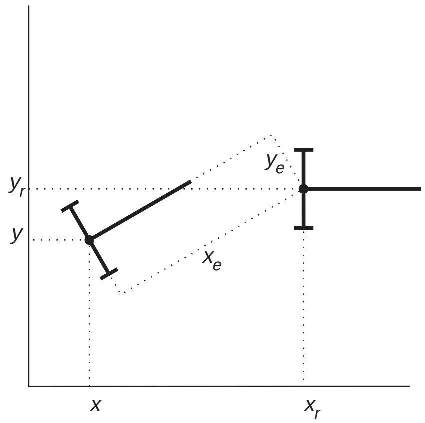

Fig. 2. The error dynamics

Our aim is to find appropriate velocity control laws v and ω of the form

$$\begin{aligned} v &= v(t, x_e, y_e, \theta_e) \\ \omega &= \omega(t, x_e, y_e, \theta_e) \end{aligned} \tag{10}$$

such that the closed-loop trajectories of (9,10) are globally $\mathcal{K}$-exponentially stable.

3. CONTROLLER DESIGN

As mentioned in the introduction, our goal is to find simple global tracking controllers for the system (9). The approach used in (Jiang and Nijmeijer, 1997*b*) is based on the integrator backstepping idea (Koditschek, 1987; Byrnes and Isidori, 1989; Tsinias, 1989; Krstić *et al.*, 1995) which consists of searching a stabilizing function for a subsystem of (9), assuming the remaining variables to be controls. Then new variables are defined, describing the difference between this desired dynamics and the real dynamics. Subsequently a stabilizing controller for this 'new system' is looked for.

This approach has the advantage that it can lead to globally stabilizing controllers. A disadvantage, however, is that they may cancel or compensate for high order nonlinearities yielding unnecessarily complicated control laws. The main reason for this is that the stability of a 'new system' is studied using a Lyapunov function expressed in 'new coordinates'. A result of this is that the controller also is expressed in these 'new coordinates'. When written in the original coordinates usually complex expressions are obtained.

To arrive at simple controllers our approach is different. We find inspiration in the recently developed studies on cascaded systems (Panteley and Loría, 1998; Janković *et al.*, 1996; Mazenc and Praly, 1996; Ortega, 1991; Sontag, 1989). Our main goal is then to subdivide the tracking control problem into two simpler and 'independent' problems: for instance, positioning and orientation. More precisely, we search for a subsystem of the form $\dot{y} = f_2(t, y)$ that is asymptotically stable. In the remaining dynamics we then can replace the appearance of this y by 0, leading to the system $\dot{x} = f_1(t, x)$. If this system is asymptotically stable we might be able to conclude asymptotic stability of the overall system.

Consider the error dynamics (9):

$$\dot{x}_e = \omega y_e - v + v_r \cos\theta_e \tag{11}$$

$$\dot{y}_e = -\omega x_e + v_r \sin\theta_e \tag{12}$$

$$\dot{\theta}_e = \omega_r - \omega \tag{13}$$

We can easily stabilize the mobile car's orientation change rate, that is the linear equation (13), by using the linear controller

$$\omega = \omega_r + c_1\theta_e \tag{14}$$

which yields GES for θ_e, provided $c_1 > 0$.

If we now set θ_e equal to 0 in (11,12) we obtain

$$\begin{aligned} \dot{x}_e &= \omega_r y_e - v + v_r \\ \dot{y}_e &= -\omega_r x_e \end{aligned} \tag{15}$$

where we used (14).

Concerning the positioning of the cart, if we choose the linear controller

$$v = v_r + c_2 x_e \tag{16}$$

where $c_2 > 0$, we obtain for the closed-loop system (15,16):

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} -c_2 & \omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \end{bmatrix} \tag{17}$$

which under some conditions on $\omega_r(t)$, see Section 2.3, is asymptotically stable. The following proposition makes this result rigorous.

Proposition 11. Consider the system (9) in closed-loop with the controller

$$\begin{aligned} v &= v_r + c_2 x_e \\ \omega &= \omega_r + c_1\theta_e \end{aligned} \tag{18}$$

where $c_1 > 0$, $c_2 > 0$. If $\omega_r(t)$, $\dot{\omega}_r(t)$, and $v_r(t)$ are bounded and there exist δ and k such that

$$\int_t^{t+\delta} \omega_r(\tau)^2 d\tau \geq k \qquad \forall t \geq t_0$$

then the closed-loop system (9,18) is globally $\mathcal{K}$-exponentially stable.

PROOF. Observing that $\sin\theta_e = \theta_e \int_0^1 \cos(s\theta_e)ds$ and $1 - \cos\theta_e = \theta_e \int_0^1 \sin(s\theta_e)ds$ we can write the closed-loop system (9,18) as

$$\begin{aligned} \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} &= \begin{bmatrix} -c_2 & \omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \end{bmatrix} + \begin{bmatrix} v_r \int_0^1 \sin(s\theta_e)ds + c_1 y_e \\ v_r \int_0^1 \cos(s\theta_e)ds - c_1 x_e \end{bmatrix} \theta_e \\ \dot{\theta}_e &= -c_1\theta_e \end{aligned} \tag{19}$$

which is of the form (3), where $x = (x_e, y_e)^T$, $y = \theta_e$, $f_2(t, y) = -c_1\theta_e$, $f_1(t, x) = \begin{bmatrix} -c_2 & \omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \end{bmatrix}$ and $g(t, x, y) = \begin{bmatrix} v_r \int_0^1 \sin(s\theta_e)ds + c_1 y_e \\ v_r \int_0^1 \cos(s\theta_e)ds - c_1 x_e \end{bmatrix}$.

To be able to apply Theorem 7 we need to verify the three assumptions:

- assumption on Σ_1: Due to the assumptions on $\omega_r(t)$ we have from Lemma 9 that $\dot{x} = f_1(t, x)$ is GES and therefore GUAS. From converse Lyapunov theory (see e.g. (Khalil, 1996) or the proof of Lemma 8) the existence of a suitable V is guaranteed.

- assumption on connecting term: Since $|v_r(t)| \leq v_r^{max}$ for all $t \geq 0$ we have:

$$\|g(t,x,y)\| \leq v_r^{max}\sqrt{2} + c_1\|x\|.$$

- assumption on Σ_2: Follows from GES of (13,14).

Therefore, we can conclude GUAS from Theorem 7. Since both Σ_1 and Σ_2 are GES, Lemma 8 gives the desired result. □

Remark 12. It is interesting to notice the link between the tracking condition that the reference trajectory should not converge to a point (or straight line) and the well known persistance-of-excitation condition in adaptive control theory. More precisely, we could think of (17) as a controlled system with state x_e, parameter estimation error y_e and as regressor, the *reference* trajectory ω_r.

Remark 13. It is important to remark that the cascaded decomposition used above is not unique. One may find other ways to subdivide the original system, for which different control laws may be found.

4. A SIMPLIFIED DYNAMIC MODEL

In this section we consider the dynamic extension of (9) as studied in (Jiang and Nijmeijer, 1997*b*):

$$\begin{aligned} \dot{x}_e &= \omega y_e - v + v_r\cos\theta_e \\ \dot{y}_e &= -\omega x_e + v_r \sin\theta_e \\ \dot{\theta}_e &= \omega_r - \omega \\ \dot{v} &= u_1 \\ \dot{\omega} &= u_2 \end{aligned} \tag{20}$$

where u_1 and u_2 are regarded as torques or generalized force variables for the two-degree-of-freedom mobile robot.

Our aim is to find a control law $u = (u_1, u_2)^T$ of the form

$$\begin{aligned} u_1 &= u_1(t, x_e, y_e, \theta_e, v, \omega) \\ u_2 &= u_2(t, x_e, y_e, \theta_e, v, \omega) \end{aligned} \tag{21}$$

such that the closed-loop trajectories of (20,21) are globally $\mathcal{K}$-exponentially stable.

To solve this problem we first define

$$\begin{aligned} v_e &= v - v_r \\ \omega_e &= \omega - \omega_r \end{aligned}$$

which leads to

$$\begin{bmatrix} \dot{x}_e \\ \dot{v}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} 0 & -1 & \omega_r(t) \\ 0 & 0 & 0 \\ -\omega_r(t) & 0 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ v_e \\ y_e \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} (u_1 - \dot{v}_r) + $$

$$+ \begin{bmatrix} v_r\int_0^1 \sin(s\theta_e)ds & y_e \\ 0 & 0 \\ v_r\int_0^1 \cos(s\theta_e)ds & -x_e \end{bmatrix} \begin{bmatrix} \theta_e \\ \omega_e \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} \dot{\theta}_e \\ \dot{\omega}_e \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_e \\ \omega_e \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u_2 - \dot{\omega}_r) \tag{23}$$

in which we again recognize a cascaded structure similar to the one in the previous section. We only need to find u_1 and u_2 such that the systems

$$\begin{bmatrix} \dot{x}_e \\ \dot{v}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} 0 & -1 & \omega_r(t) \\ 0 & 0 & 0 \\ -\omega_r(t) & 0 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ v_e \\ y_e \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} (u_1 - \dot{v}_r)$$

and

$$\begin{bmatrix} \dot{\theta}_e \\ \dot{\omega}_e \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_e \\ \omega_e \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u_2 - \dot{\omega}_r)$$

are globally $\mathcal{K}$-exponentially stable. In light of the previous section, that is not too difficult.

Proposition 14. Consider the system (20) in closed-loop with the controller

$$\begin{aligned} u_1 &= \dot{v}_r + c_3 x_e - c_4 v_e \\ u_2 &= \dot{\omega}_r + c_5\theta_e - c_6\omega_e \end{aligned} \tag{24}$$

where $c_3 > 0$, $c_4 > 0$, $c_5 > 0$, $c_6 > 0$. If $\omega_r(t)$, $\dot{\omega}_r(t)$ and $v_r(t)$ are bounded and there exist δ and k such that

$$\int_t^{t+\delta} \omega_r(\tau)^2 d\tau \geq k \qquad \forall t \geq t_0$$

then the closed-loop system (20,24) is globally $\mathcal{K}$-exponentially stable.

PROOF. The closed-loop system (20,24) can be written as

$$\begin{bmatrix} \dot{x}_e \\ \dot{v}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} 0 & -1 & \omega_r(t) \\ c_3 & -c_4 & 0 \\ -\omega_r(t) & 0 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ v_e \\ y_e \end{bmatrix} + $$

$$+ \begin{bmatrix} v_r\int_0^1 \sin(s\theta_e)ds & y_e \\ 0 & 0 \\ v_r\int_0^1 \cos(s\theta_e)ds & -x_e \end{bmatrix} \begin{bmatrix} \theta_e \\ \omega_e \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_e \\ \dot{\omega}_e \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ c_5 & -c_6 \end{bmatrix} \begin{bmatrix} \theta_e \\ \omega_e \end{bmatrix}$$

which is of the form (3). We again have to verify the three assumptions of Theorem 7:

- assumption on Σ_1: From Lemma 9 we have that Σ_1 is GES and therefore GUAS, where we used the assumptions on $\omega_r(t)$. The existence of a suitable V again follows from converse Lyapunov theory.
- assumption on connecting term: Since $|v_r(t)| \leq v_r^{max}$ for all $t \geq 0$ we have:

$$\|g(t,x,y)\| \leq v_r^{max}\sqrt{2} + \|x\|.$$

- assumption on Σ_2: Follows from GES of Σ_2.

Therefore we can conclude GUAS from Theorem 7. Since both Σ_1 and Σ_2 are GES, Lemma 8 gives the desired result. □

5. CONCLUSIONS

In this paper we addressed the problem of designing simple global tracking controllers for both a kinematic and a simple dynamic model of a mobile robot. We divided the tracking control problem into two simpler and 'independent' problems. Using cascaded systems theory we proved that it is possible to design *linear* controllers for both subsystems that yield global tracking.

An interesting further remark is the link between the persistance-of-exictation condition and the non-vanishing condition on the reference trajectory. It is in belief that a deeper understanding of this relationship might lead to interesting conclusions on both domains adaptive control and nonholonomic systems theory.

6. ACKOWLEDGEMENTS

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