

Observer Based Kinematic Tracking Controllers for a Unicycle-type Mobile Robot

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Abstract

In this paper the problem of global trajectory tracking for the kinematic model of a unicycle-type mobile robot is considered. It is assumed that some of the tracking error coordinates are not measurable. Using a cascaded systems approach we develop full order and reduced order observers, and introduce an observer based controller resulting in \mathcal{K} -exponential convergence of the tracking error. Simulations are provided to illustrate the results.

1 Introduction

In recent years the stabilization problem of non-holonomic systems has received considerable attention. One of the reasons for this is that no smooth time-invariant stabilizing state-feedback control law exists for these systems, since Brockett's necessary condition for smooth stabilization is not met [1]. For an overview we refer to the survey paper [2] and references cited therein. Although the stabilization problem for wheeled mobile robots is now well understood, the tracking problem has received less attention. As a matter of fact, it is not clear that the current stabilization methodologies can be extended easily to tracking problems. In [3, 4, 5, 6, 7] a linearization-based tracking control scheme was derived. The idea of input-output linearization was used in [8]. In [9] the trajectory stabilization problem was dealt with by means of a flatness approach. A systematic exposition of the feedback linearization technique for wheeled mobile robot was presented in [10]. All these papers solve the local tracking problem. The first global tracking control law that we are aware of was proposed in [11]. Another global tracking result was derived in [12] using integrator backstepping. Global tracking results yielding exponential convergence were presented in [13, 14] under a persistence of excitation assumption on the reference.

In this paper we elaborate on the results of [13] by considering the global state-tracking control problem for a unicycle-type wheeled robot under *output-*

feedback. It is allowed that not all position components of the tracking error can be measured. We solve this problem by means of *linear* dynamic controllers that yield global \mathcal{K} -exponential stability (cf. [15]), which is a *uniform* kind of stability and implies a certain robustness against disturbances. We arrive at our results by constructing both full- and reduced-order observers. The stability analysis is based on results of cascaded systems. A somewhat complementary problem of motion planning with measurements of the position coordinates has been solved in [16]. A fuzzy PD controller using look-up tables for mobile robots is given in [17].

The organization of the paper is as follows. Section 2 presents definitions, preliminary results, and the problem under consideration. In Section 3 we obtain a cascade structure in the tracking-error dynamics. In Section 4 we solve the global tracking problem by constructing a full order observer. In Section 5 we present a reduced order observer. The behavior of both controllers has been illustrated by means of computer simulations in Section 6. Section 7 concludes the paper.

2 Preliminaries and Problem Formulation

In order to make this paper self-contained we recall some standard concepts of stability theory [18].

2.1 Preliminaries

Definition 2.1. A continuous function $\alpha : [0, a) \rightarrow [0, \infty)$ is said to belong to **class** \mathcal{K} ($\alpha \in \mathcal{K}$) if it is strictly increasing and $\alpha(0) = 0$.

Definition 2.2. A continuous function $\beta : [0, a) \times [0, \infty) \rightarrow [0, \infty)$ is said to belong to **class** \mathcal{KL} ($\beta \in \mathcal{KL}$) if for each fixed s the mapping $\beta(r, s)$ belongs to class \mathcal{K} with respect to r and if for each fixed r the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \rightarrow 0$ as $s \rightarrow \infty$.

Definition 2.3. The equilibrium point $x = 0$ of a non-autonomous system $\dot{x} = f(t, x)$ is said to be

• **globally uniformly asymptotically stable (GUAS)** if a function $\beta \in \mathcal{KL}$ exists such that for all $(t_0, x(t_0)) \in \mathbb{R}_+ \times \mathbb{R}^n$, $t \geq t_0$, $\|x(t_0)\| \leq r$

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0), \quad (1)$$

• **globally uniformly exponentially stable (GUES)** if it is GUAS and (1) is satisfied with

$$\beta(r, s) = kre^{-\gamma s} \quad k > 0, \gamma > 0.$$

A notion that is equivalent to having both global uniform asymptotic stability and local uniform exponential stability (GUAS+LUES) is the following.

Definition 2.4 ([15, Definition 2]). The equilibrium point $x = 0$ of a non-autonomous system $\dot{x} = f(t, x)$ is said to be **globally \mathcal{K} -exponentially stable** if a function $\kappa \in \mathcal{K}$ and a constant $\gamma > 0$ exist such that for all $(t_0, x(t_0)) \in \mathbb{R}_+ \times \mathbb{R}^n$ we have

$$\|x(t)\| \leq \kappa(\|x(t_0)\|)e^{-\gamma(t-t_0)} \quad \forall t \geq t_0 \geq 0.$$

Definition 2.5. A continuous function $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}$ is said to be **persistently exciting (PE)** if constants $\epsilon_1, \epsilon_2, \delta > 0$ exist such that for all $t \geq 0$

$$\epsilon_1 \leq \int_t^{t+\delta} \phi^2(\tau) d\tau \leq \epsilon_2.$$

Theorem 2.6 ([19]). Consider the system

$$\dot{x} = \begin{bmatrix} -c_1 & -c_2\phi(t) & d_1 & d_2\phi(t) \\ \phi(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & -l_2\phi(t) \\ 0 & 0 & \phi(t) & -l_1 \end{bmatrix} x. \quad (2)$$

When $\phi(t)$ is PE, $c_1 > 0$, $c_2 > 0$, $l_1 > 0$, $l_2 > 0$, then the system (2) is GUES.

Theorem 2.7 ([20, Theorem 3.4.6 (v)]). The system $\dot{x} = A(t)x$ is GUES if and only if it is GUAS.

2.2 Cascaded Systems

Consider a system $\dot{z} = f(t, z)$ that can be written as

$$\begin{aligned} \dot{z}_1 &= f_1(t, z_1) + g(t, z_1, z_2)z_2, \\ \dot{z}_2 &= f_2(t, z_2), \end{aligned} \quad (3)$$

where $z_1 \in \mathbb{R}^n$, $z_2 \in \mathbb{R}^m$, $(z_1, z_2) = (0, 0)$ is an equilibrium point of (3), $f_1(t, z_1)$ is continuously differentiable in (t, z_1) and $f_2(t, z_2)$, $g(t, z_1, z_2)$ are continuous in their arguments, and locally Lipschitz in z_2 and (z_1, z_2) , respectively.

Assumption 2.8. Assume that continuous functions $k_1 : \mathbb{R}_+ \rightarrow \mathbb{R}$ and $k_2 : \mathbb{R}_+ \rightarrow \mathbb{R}$ exist such that

$$\|g(t, z_1, z_2)\| \leq k_1(\|z_2\|) + k_2(\|z_2\|)\|z_1\|. \quad (4)$$

Then we can formulate the following corollary from a result presented in [21] (see also [13]).

Corollary 2.9. Assume that the subsystem $\dot{z}_1 = f_1(t, z_1)$ of (3) is GUES, the subsystem $\dot{z}_2 = f_2(t, z_2)$ is globally \mathcal{K} -exponentially stable and $g(t, z_1, z_2)$ satisfies (4). Then the cascaded system (3) is globally \mathcal{K} -exponentially stable.

2.3 Problem Formulation

A kinematic model of the unicycle-type mobile robot is given by the following equations

$$\dot{x} = v \cos \theta, \quad \dot{y} = v \sin \theta, \quad \dot{\theta} = \omega.$$

Coordinates (x, y, θ) are shown in figure 1. The forward velocity v and the angular velocity ω are considered as inputs.

Consider the problem of tracking a reference trajectory $(x_r, y_r, \theta_r, v_r, \omega_r)$ generated by the system

$$\dot{x}_r = v_r \cos \theta_r, \quad \dot{y}_r = v_r \sin \theta_r, \quad \dot{\theta}_r = \omega_r,$$

where v_r and ω_r are continuous functions of time.

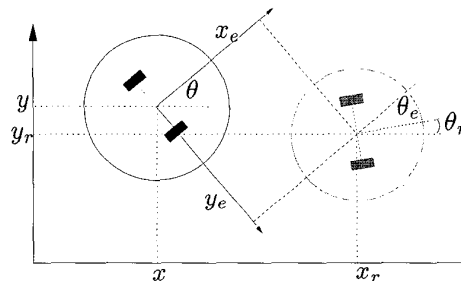


Fig. 1. The unicycle coordinates (x, y, θ) , reference position (x_r, y_r, θ_r) and the new error coordinates (x_e, y_e, θ_e) .

Following [3] we express the error coordinates in the moving frame in the form

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix},$$

and compute the error dynamics as

$$\begin{cases} \dot{x}_e = \omega y_e - v + v_r \cos \theta_e \\ \dot{y}_e = -\omega x_e + v_r \sin \theta_e \\ \dot{\theta}_e = \omega_r - \omega. \end{cases} \quad (5)$$

In certain pursuit navigation problems, particularly when one of position coordinates x_e, y_e differs substantially from the other, the measurements of the smaller coordinate cannot be accomplished accurately. Specifically, we assume that we are unable to measure the forward-error x_e , so only values of

y_e and θ_e are available. The case of unmeasured y_e can be addressed analogously. In the former case the available output is

$$y = [y_e \ \theta_e]^T. \quad (6)$$

Then the dynamic output-feedback state-tracking control problem can be formulated as

Find appropriate velocity control laws v and ω of the form

$$v = v(t, y_e, \theta_e, z), \quad \omega = \omega(t, y_e, \theta_e, z), \quad (7)$$

where z is generated from the observer

$$\dot{z} = f(t, y_e, \theta_e, z), \quad (8)$$

such that the closed-loop error system of (5, 7, 8) is globally \mathcal{K} -exponentially stable.

3 Important Observation

Assume that we apply the control law

$$\omega = \omega_r + c_1 \theta_e. \quad (9)$$

In this case, in combination with the error dynamics (5), we obtain the cascaded structure

$$\underbrace{\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} 0 & \omega_r \\ -\omega_r & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \end{bmatrix} + \begin{bmatrix} v_r - v \\ 0 \end{bmatrix}}_{\dot{z}_1 = f_1(t, z_1)} + \underbrace{\begin{bmatrix} c_1 y_e + v_r \frac{\cos \theta_e - 1}{\theta_e} \\ -c_1 x_e + v_r \frac{\sin \theta_e}{\theta_e} \end{bmatrix} \theta_e}_{g(t, z_1, z_2) z_2},$$

$$\underbrace{\dot{\theta}_e = -c_3 \theta_e}_{\dot{z}_2 = f_2(t, z_2)}$$

Assume that v_r is bounded. This being so, it is clear that $g(t, z_1, z_2)$ satisfies Assumption 2.8. Furthermore, the system $\dot{z}_2 = f_2(t, z_2)$ is GUES. As soon as we are able to find a control law for v such that the system

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} 0 & \omega_r \\ -\omega_r & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \end{bmatrix} + \begin{bmatrix} v_r - v \\ 0 \end{bmatrix} \quad (10)$$

is rendered GUES, we can conclude global \mathcal{K} -exponential stability of the resulting overall closed-loop system by means of Corollary 2.9.

This is precisely the approach used in [13], where

$$v = v_r + c_2 x_e, \quad c_2 > 0$$

was used for rendering (10) GUES, assuming ω_r persistently exciting. Notice that for any control law

$$v = v_r + c_2 x_e - c_3 \omega_r y_e, \quad c_2 > 0, c_3 > -1 \quad (11)$$

the same result can be established (cf. Theorem 2.6).

However, this approach can not only be used for studying the state-feedback problem, but also for studying the output-feedback problem. As θ_e is still available for measurement, the output-feedback problem can be reduced in a similar way to the problem of finding an output feedback for v such that the system

$$\begin{cases} \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} 0 & \omega_r \\ -\omega_r & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \end{bmatrix} + \begin{bmatrix} v_r - v \\ 0 \end{bmatrix} \\ y_1 = y_e \end{cases} \quad (12)$$

is rendered GUES. In that case we can again conclude global \mathcal{K} -exponential stability of the resulting overall closed-loop system by means of Corollary 2.9.

4 Full-order Observer

As explained in the previous section, the main problem we are interested in is stabilizing the system (12). From standard linear systems theory we know that this can be established by means of the dynamic output-feedback control law

$$v = v_r + c_2 \hat{x}_e - c_3 \omega_r \hat{y}_e, \quad (13a)$$

$$\begin{bmatrix} \dot{\hat{x}}_e \\ \dot{\hat{y}}_e \end{bmatrix} = \begin{bmatrix} 0 & \omega_r \\ -\omega_r & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_e \\ \hat{y}_e \end{bmatrix} + \begin{bmatrix} v_r - v \\ 0 \end{bmatrix} + \begin{bmatrix} -l_2 \omega_r \\ l_1 \end{bmatrix} (y_1 - \hat{y}_1), \quad (13b)$$

$$\hat{y}_1 = [0 \ 1] \begin{bmatrix} \hat{x}_e \\ \hat{y}_e \end{bmatrix}, \quad (13c)$$

with $c_2 > 0$, $c_3 > -1$, $l_1 > 0$, and $l_2 > -1$. Using this control law, the resulting closed-loop system (12, 13) is GUES, provided that ω_r is persistently exciting (cf. Theorem 2.6). By a direct application of Corollary 2.9 we obtain the following

Proposition 4.1. *Consider the tracking error dynamics (5) with output (6) in closed loop with the control laws (9, 13). Assume ω_r persistently exciting (PE) and v_r bounded. Then the resulting closed-loop system is globally \mathcal{K} -exponentially stable.*

5 Reduced-order Observer

Notice that (13b, 13c) is a full order observer for the system (12), i.e., even though we can measure y_e we also have generated an estimate for y_e . It is also possible to use a *reduced order* observer, i.e., to reconstruct only the unknown signal x_e .

In order to find a reduced observer for the system (12) we try to estimate some linear combination of the measured and the unknown signals. To be precise, we define a new variable z as

$$z = x_e - b y_1,$$

where b is a time-function still to be determined in order to guarantee asymptotic stability of the reduced order observer. Differentiating z with respect to time along the dynamics (12) yields

$$\begin{aligned}\dot{z} &= \omega_r y_e + (v_r - v) - \dot{b} y_e + b \omega_r x_e \\ &= b \omega_r (x_e - b y_e) + b^2 \omega_r y_e + \omega_r y_e + (v_r - v) - \dot{b} y_e \\ &= b \omega_r z + (b^2 \omega_r + \omega_r - \dot{b}) y_e + (v_r - v).\end{aligned}$$

On defining the reduced order observer dynamics as

$$\dot{\hat{z}} = b \omega_r \hat{z} + (b^2 \omega_r + \omega_r - \dot{b}) y_e + (v_r - v)$$

we obtain for the observation-error $\tilde{z} = z - \hat{z}$

$$\dot{\tilde{z}} = b \omega_r \tilde{z}. \quad (14)$$

Solutions of (14) satisfy

$$\tilde{z}(t) = \tilde{z}(t_0) e^{\int_{t_0}^t b(\tau) \omega_r(\tau) d\tau}.$$

If we now take $b = -l \omega_r$ with l a positive constant and we furthermore assume that ω_r is PE, we have the existence of $\epsilon_1 > 0$, $\epsilon_2 > 0$, and $\delta > 0$ such that

$$\frac{\epsilon_1}{\delta} (t - t_0) < \int_{t_0}^t \omega_r^2(\tau) d\tau < \frac{\epsilon_2}{\delta} (t - t_0)$$

which enables us to conclude that (14) is GUES.

We can combine this reduced observer with the controller (11):

Proposition 5.1. *Consider the tracking error dynamics (5) with output (6) in closed-loop with the control law*

$$\omega = \omega_r + c_1 \theta_e, \quad c_1 > 0, \quad (15a)$$

$$v = v_r + c_2 \hat{x}_e - c_3 \omega_r y_e, \quad c_2 > 0, c_3 > -1, \quad (15b)$$

where \hat{x}_e is generated by the reduced order observer

$$\hat{x}_e = \hat{z} - l \omega_r y_e, \quad l > 0, \quad (16a)$$

$$\dot{\hat{z}} = -l \omega_r^2 \hat{z} + (l^2 \omega_r^3 + \omega_r + l \dot{\omega}_r) y_e + (v_r - v). \quad (16b)$$

If v_r is bounded and ω_r is persistently exciting (PE), then the closed-loop system (5, 15, 16) is globally K -exponentially stable.

Proof. We can view the closed-loop system (5, 15, 16) as a cascaded system, i.e., a system of the form (3), where

$$\begin{aligned}z_1 &= [x_e \quad y_e \quad x_e - \hat{x}_e]^T, \quad z_2 = \theta_e, \\ f_1(t, z_1) &= \begin{bmatrix} -c_2 & (c_3 + 1)\omega_r & c_2 \\ -\omega_r & 0 & 0 \\ 0 & 0 & -l\omega_r^2 \end{bmatrix} z_1,\end{aligned}$$

$$f_2(t, z_2) = -c_2 z_2,$$

$$g(t, z_1, z_2) = \begin{bmatrix} c_1 y_e + v_r \frac{\cos \theta_e - 1}{\theta_e} \\ -c_1 x_e + v_r \frac{\sin \theta_e}{\theta_e} \\ c_1 y_e + v_r \frac{\cos \theta_e - 1}{\theta_e} + l \omega_r \left(v_r \frac{\sin \theta_e}{\theta_e} - c_1 x_e \right) \end{bmatrix}.$$

To be able to apply Corollary 2.9 we need to verify global uniform exponential stability (GUES) of the system $\dot{z}_1 = f_1(t, z_1)$, which can also be expressed as

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \underbrace{\begin{bmatrix} -c_2 & (c_3 + 1)\omega_r \\ -\omega_r & 0 \end{bmatrix}}_{\tilde{f}_1(t, \tilde{z}_1)} \begin{bmatrix} x_e \\ y_e \end{bmatrix} + \underbrace{\begin{bmatrix} c_2 \\ 0 \end{bmatrix}}_{\tilde{g}(t, \tilde{z}_1, \tilde{z}_2)} \tilde{z}_2, \quad (17a)$$

$$\dot{\tilde{z}}_2 = -l \omega_r^2 \tilde{z}_2. \quad (17b)$$

Solutions of the subsystem (17b) are given by

$$\tilde{z}_2(t) = \tilde{z}_2(t_0) e^{-l \int_{t_0}^t \omega_r^2(\tau) d\tau}.$$

Since ω_r is PE, the subsystem (17b) is GUES. Furthermore, the term $\tilde{g}(t, \tilde{z}_1, \tilde{z}_2)$ is bounded and the system $\dot{\tilde{z}}_1 = \tilde{f}_1(t, \tilde{z}_1)$ is GUES. From Corollary 2.9 we can conclude that the system $\dot{z}_1 = f_1(t, z_1)$ is GUAS. Since it is a linear time-varying system Theorem 2.7 enables us to conclude that $\dot{z}_1 = f_1(t, z_1)$ is GUES. Since also the system $\dot{z}_2 = f_2(t, z_2)$ is GUES and boundedness of both v_r and ω_r (cf: Definition 2.5) guarantees that the condition on $g(t, z_1, z_2)$ is met, Corollary 2.9 yields the desired result. \square

6 Simulations

With the purpose of illustrating the output-feedback state-tracking controllers derived in this paper, a number of simulations have been done. The simulations were carried out using MATHEMATICA. We considered the problem of tracking a circle with constant velocity, i.e., a reference trajectory that is given by $v_r = 1$, $\omega_r = 1$, where as in [12] we took for the initial error $(x_e(0), y_e(0), \theta_e(0)) = (-0.5, 0.5, 1)$. For comparison reasons we first simulated the state-feedback controller (9, 11) using the gains

$$c_1 = 5.9460, \quad c_2 = 1.3522, \quad c_3 = -0.4142. \quad (18)$$

We arrived at these gains by minimizing for the system (10) the costs

$$\int_0^\infty x_e^2(\tau) + y_e^2(\tau) + (v_r(\tau) - v(\tau))^2 d\tau$$

and making the θ_e -dynamics (which enters as a perturbation to this system) five times as fast. The resulting performance is depicted in figure 2.

For studying the behavior of the full-order observer we simulated the full-order output-feedback

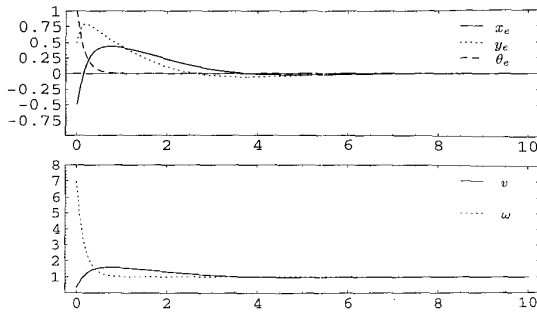


Fig. 2. Tracking errors and inputs for static state-feedback controller (9, 11) with controller gains (18).

controller (9, 13) with the controller gains (18), where we used the observer gains

$$l_1 = 6.6710, \quad l_2 = 34.3546. \quad (19)$$

We arrived at these gains by taking five times the eigenvalues of the corresponding linear closed-loop system (10). As an initial state-estimate we took $(\hat{x}_e(0), \hat{y}_e(0)) = (0, 0)$. The resulting performance is depicted in figure 3. When we compare the behavior

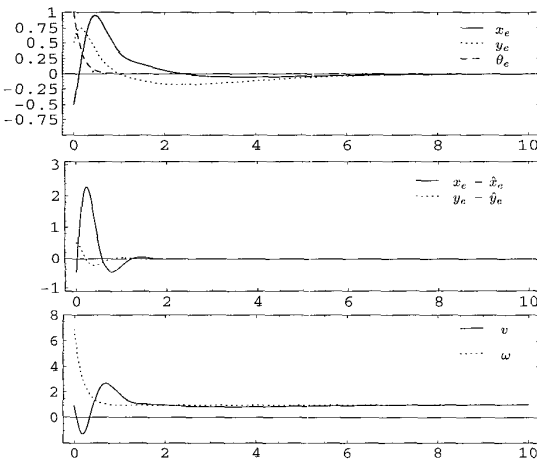


Fig. 3. Tracking errors and inputs for full-order output-feedback controller (9, 13) with controller and observer gains (18, 19).

with that of the state-feedback controller, as depicted in figure 2, we obtain a comparable tracking error performance.

For studying the behavior of the reduced-order observer, we simulated the reduced-order output-feedback controller (15, 16) with the controller gains (18), where we used the observer gain

$$l = 24.7461 \quad (20)$$

which guarantees that the error dynamics of the reduced-order observer (15, 16) converges as quickly

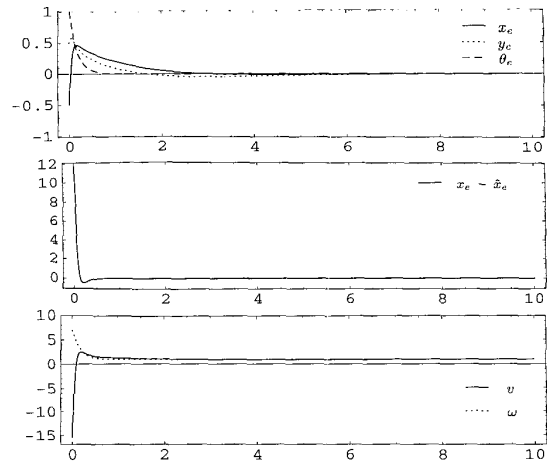


Fig. 4. Tracking errors and inputs for reduced-order output-feedback controller (15, 16) with controller- and observer-gains (18, 20).

as that of the full-order observer (13). The results are depicted in figure 4. The convergence of the tracking error dynamics is again comparable to that of the previous two simulations.

7 Concluding Remarks

By means of the output-feedback we have solved the state-tracking control problem for the unicycle-type mobile robot with restricted measurability of tracking error components. We arrived at our results by constructing both full- and reduced-order observers. Using the theory of cascaded systems, we were able to show global \mathcal{K} -exponential stability of the resulting tracking error dynamics. This is a uniform kind of stability which guarantees a certain robustness against disturbances. The theoretical results have been confirmed by simulations. We believe that the approach involved in our tracking controller design may be applicable to mobile robot navigation problems with restricted knowledge of the scene.

All our results assume the angular velocity of the reference to be persistently exciting. For this reason, our controllers are not capable of tracking e.g., straight lines. For the state-feedback problem there are two possible ways to overcome this problem. One way is to keep the control law for the forward velocity v , but to redesign the control law for the angular velocity ω by means of backstepping (starting from a virtual control $\theta_e = 0$). In that case we need that either v_r or ω_r should not tend to zero. A second way to overcome the PE-problem is to use the concept of $u\delta$ -PE as presented in [22]. Using this concept, GUAS of the resulting closed-loop system can be shown, where both the stabilization and tracking problem is solved by means of the same controller.

As this are two ways to overcome the PE-problem for the state-feedback case, it is worth investigating if these modifications also apply to the output-feedback case.

8 Acknowledgments

The work of the second and the third author was supported by the Polish State Committee for Scientific research under the research project "Modelling, motion planning, and control of mobile manipulators".

9 References

- [1] Brockett, R.W., "Asymptotic stability and feedback stabilization," in *Differential Geometric Control Theory*, Brockett, R.W., Millman, R.S., and Sussmann, H.J., Eds., Boston, MA, USA, Birkhäuser, 1983, pp. 181–191.
- [2] Kolmanovsky, I., and McClamroch, N.H., "Developments in nonholonomic control problems," *IEEE Control Systems Magazine*, vol. 16, no. 6, pp. 20–36, 1995.
- [3] Kanayama, Y., et al., "A stable tracking control method for an autonomous mobile robot," in *Proceedings of the 1990 IEEE International Conference on Robotics and Automation*, pp. 384–389, 1990.
- [4] Murray, R.M., et al., "Stabilization and tracking for nonholonomic control systems using time-varying state feedback," in *IFAC Nonlinear control systems design*, Fliess, M., Ed., Bordeaux, 1992, pp. 109–114.
- [5] Micaelli, A., and Samson, C., "Trajectory tracking for unicycle-type and two-steering-wheels mobile robots," *Rapport de recherche no. 2097*, INRIA, Sophia-Antipolis, France, 1993.
- [6] Walsh, G., et al., "Stabilization of trajectories for systems with nonholonomic constraints," *IEEE Transactions on Automatic Control*, vol. 39, no. 1, pp. 216–222, 1994.
- [7] Fierro, R., and Lewis, F.L., "Control of a nonholonomic mobile robot: backstepping kinematics into dynamics," in *Proceedings of the 34th Conference on Decision and Control*, pp. 3805–3810, New Orleans, LA, USA, 1995.
- [8] Oelen, W., and van Amerongen, J., "Robust tracking control of two-degrees-of-freedom mobile robots," *Control Engineering Practice*, vol. 2, pp. 333–340, 1994.
- [9] Fliess, M., et al., "Design of trajectory stabilizing feedback for driftless flat systems," in *Proceedings of the 3rd European Control Conference*, pp. 1882–1887, Rome, 1995.
- [10] Canudas de Wit, C., et al., *Theory of Robot Control*, New York, Springer-Verlag, 1996.
- [11] Samson, C., and Ait-Abderrahim, K., "Feedback control of a nonholonomic wheeled cart in cartesian space," in *Proceedings of the 1991 IEEE International Conference on Robotics and Automation*, pp. 1136–1141, Sacramento, USA, 1991.
- [12] Jiang, Z.-P., and Nijmeijer, H., "Tracking control of mobile robots: a case study in backstepping," *Automatica*, vol. 33, no. 7, pp. 1393–1399, 1997.
- [13] Panteley, E., et al., "Exponential tracking control of a mobile car using a cascaded approach," in *Proceedings of the IFAC Workshop on Motion Control*, pp. 221–226, Grenoble, France, 1998.
- [14] Dixon, W.E., et al., "Global exponential tracking control of a mobile robot system via a PE condition," in *Proceedings of the 38th Conference on Decision and Control*, pp. 4822–4827, Phoenix, Arizona, USA, 1999.
- [15] Sørtdalen, O.J., and Egeland, O., "Exponential stabilization of nonholonomic chained systems," *IEEE Transactions on Automatic Control*, vol. 40, no. 1, pp. 35–49, 1995.
- [16] Guillaume, D., and Rouchon, P., "Observation and control of a simplified car," in *Proceedings of the IFAC Workshop on Motion Control*, pp. 63–67, Grenoble, France, 1998.
- [17] Ulyanov, V.S., et al., "Intelligent robust control of a robotic unicycle based on a new physical measure for mechanical controllability," *Advanced Robotics*, vol. 12, no. 4, pp. 455–482, 1998.
- [18] Khalil, H.K., *Nonlinear Systems*, Upper Saddle River, USA, Prentice-Hall, 2nd edition, 1996.
- [19] Lefeber, E., et al., "Linear controllers for exponential tracking of systems in chained form," *International Journal of Robust and Nonlinear Control*, vol. 10, no. 4, pp. 243–264, 2000.
- [20] Ioannou, P.A., and Sun, J., *Robust Adaptive Control*, Upper Saddle River, USA, Prentice-Hall, 1996.
- [21] Panteley, E., and Loría, A., "On global uniform asymptotic stability of nonlinear time-varying systems in cascade," *Systems and Control Letters*, vol. 33, no. 2, pp. 131–138, 1998.
- [22] Loría, A., et al., "A new notion of persistency-of-excitation for UGAS on NLTV systems: Application to stabilisation of nonholonomic systems," in *Proceedings of the 5th European Control Conference*, Paper no. 500, Karlsruhe, Germany, 1999.