Output Feedback Tracking of Nonholonomic Systems in Chained Form

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Abstract

In this paper we study the tracking problem for the class of nonholonomic systems in chained form. In particular, with as outputs the first and last state component of the chained form, we suggest a solution for the output tracking problem by combining a time-varying state feedback controller with an observer for the chained form system. For the stability analysis of the "certainty equivalence type" of controller we use a cascaded systems approach. The resulting closed loop system is globally $\mathcal{K}$-exponentially stable.

1 Introduction

In recent years a lot of interest has been devoted to (mainly) stabilization and tracking of nonholonomic dynamic systems, see e. g. [1, 6, 8, 15, 17]. One of the reasons for the attention is the lack of a continuous static state-feedback control since Brockett's necessary condition for smooth stabilization is not met, see [3]. The proposed solutions to this problem follow mainly two routes, discontinuous and/or time-varying control. For a good overview, see the survey paper [10] and the references therein.

It is well known that the kinematic model of several nonholonomic systems can be transformed into a *chained form system*. The global tracking problem for chained form systems has recently been addressed in [6, 7, 8, 14, 17, 20]. In this paper we consider the output tracking problem for chained form systems. Our results are based on the construction of a time varying state-feedback controller in combination with an observer. However, the stability analysis and design are based on results for (time-varying) cascaded systems [18]. In the design we divide the chained form into a cascade of two sub-systems which we can stabilize independently of each other, and furthermore the same cascade results also apply for the controller-observer combination. Regarding the latter part, similar ideas were recently presented for the combination of high-gain controllers and high-gain observer for a class of triangular nonlinear systems [2], see also [12].

The organization of the paper is as follows. Section 2 contains some definitions, preliminary results and the problem formulation. Section 3 addresses the tracking problem based on time-varying state feedback and in section 4 we design an exponentially convergent observer for the chained system. In section 5 we combine the control law from section 3 with the observer from section 4 in a "certainty equivalence" sense. This yields a globally $\mathcal{K}$-exponentially stable closed loop system under the condition of persistently exciting reference trajectories. Finally, section 6 concludes the paper.

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2 Preliminaries, problem formulation

2.1 Stability

To start with, we recall some basic concepts (see e.g. [9, 23]).

Definition 2.1 *A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to* class K *if it is strictly increasing and $\alpha(0) = 0$.*

Definition 2.2 *A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to* class KL *if, for each fixed s, the mapping $\beta(r, s)$ belongs to class K with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.*

Consider the system

$$\dot{x} = f(t, x), \qquad f(t, 0) = 0 \quad \forall t \geq 0 \tag{1}$$

where $f(t, x)$ is piecewise continuous in t and locally Lipschitz in x.

Definition 2.3 *The system (1) is* uniformly stable *if for each $\epsilon > 0$ there is $\delta = \delta(\epsilon) > 0$, independent of t_0, such that*

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| < \epsilon, \quad \forall t \geq t_0 \geq 0.$$

Definition 2.4 *The system (1) is* globally uniformly asymptotically stable (GUAS) *if it is uniformly stable and globally attractive, that is, there exists a class KL function $\beta(\cdot, \cdot)$ such that for all initial state $x(t_0)$:*

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0), \quad \forall t \geq t_0 \geq 0$$

Definition 2.5 *The system (1) is* globally exponentially stable (GES) *if there exist $k > 0$ and $\gamma > 0$ such that for any initial state*

$$\|x(t)\| \leq \|x(t_0)\| k \exp[-\gamma(t - t_0)].$$

A slightly weaker notion of exponential stability is the following

Definition 2.6 ((cf. [21])) *We call the system (1)* globally K-exponentially stable *if there exist $\gamma > 0$ and a class K function $k(\cdot)$ such that*

$$\|x(t)\| \leq k(\|x(t_0)\|) \exp[-\gamma(t - t_0)] \tag{2}$$

2.2 Cascaded systems

Consider the system

$$\begin{cases} \dot{z}_1 &= f_1(t, z_1) + g(t, z_1, z_2) z_2 \\ \dot{z}_2 &= f_2(t, z_2) \end{cases} \tag{3}$$

where $z_1 \in I\!R^n$, $z_2 \in I\!R^m$, $f_1(t, z_1)$ is continuously differentiable in (t, z_1) and $f_2(t, z_2)$, $g(t, z_1, z_2)$ are continuous in their arguments, and locally Lipschitz in z_2 and (z_1, z_2) respectively.

We can view the system (3) as the system

$$\Sigma_1 : \dot{z}_1 = f_1(t, z_1)$$

that is perturbed by the output of the system

$$\Sigma_2 : \dot{z}_2 = f_2(t, z_2).$$

From the proof presented in [18] one can conclude:

Theorem 2.7 (based on [18]) *The cascaded system (3) is GUAS if the following three assumptions hold:*

- assumption on Σ_1: *the system $\dot{z}_1 = f_1(t, z_1)$ is GUAS and there exists a continuously differentiable function $V(t, z_1) : I\!R_+ \times I\!R^n \to I\!R$ that satisfies*

$$\begin{aligned} W(z_1) &\leq V(t, z_1), \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial z_1} \cdot f_1(t, z_1) &\leq 0, \quad \forall \|z_1\| \geq \eta, \\ \left\| \frac{\partial V}{\partial z_1} \right\| \|z_1\| &\leq c V(t, z_1), \quad \forall \|z_1\| \geq \eta, \end{aligned}$$

  where $W(z_1)$ is a positive definite proper function and $c > 0$ and $\eta > 0$ are constants,

- assumption on the interconnection: *the function $g(t, z_1, z_2)$ satisfies for all $t \geq t_0$:*

$$\|g(t, z_1, z_2)\| \leq \theta_1(\|z_2\|) + \theta_2(\|z_2\|)\|z_1\|,$$

  where $\theta_1, \theta_2 : I\!R_+ \to I\!R_+$ are continuous functions,

- assumption on Σ_2: *the system $\dot{z}_2 = f_2(t, z_2)$ is GUAS and for all $t_0 \geq 0$:*

$$\int_{t_0}^{\infty} \|z_2(t_0, t, z_2(t_0))\| dt \leq \kappa(\|z_2(t_0)\|),$$

  where the function $\kappa(\cdot)$ is a class K function,

Lemma 2.8 (see [17]) *If in addition to the assumptions in Theorem 2.7 both $\dot{z}_1 = f_1(t, z_1)$ and $\dot{z}_2 = f_2(t, z_2)$ are globally K-exponentially stable, then the cascaded system (3) is globally K-exponentially stable.*

2.3 A stability result

Theorem 2.9 *Consider the linear time-varying SISO system*

$$\begin{aligned} \dot{z} &= \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 \\ f(t) & \ddots & & & \vdots \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & f(t) & 0 \end{bmatrix} z + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} v \\ y &= [0 \ldots 0\, 1] z \end{aligned} \tag{4}$$

where $z \in I\!R^m$ ($m \geq 2$), $y, v \in I\!R$ and $f(t)$ is a locally bounded function.

Define

$$\omega(\sigma, t) = \int_{\sigma}^{t} f(\tau) d\tau. \tag{5}$$

Assume that there exist $\delta, \epsilon_1, \epsilon_2 > 0$ such that for all $t \geq t_0$:

$$\epsilon_1 \leq \int_{t}^{t+\delta} \omega(\sigma, t)^2 d\sigma \quad \text{and} \quad \int_{t}^{t+\delta} \omega(\sigma, t)^{2m-2} d\sigma \leq \epsilon_2. \tag{6}$$

Then there exist continuous $K(t)$ and $L(t)$ such that the controller-observer combination

$$\begin{aligned} u &= -K(t)\hat{x} \quad (7)\\ \dot{\hat{x}} &= A(t)\hat{x} + Bu + L(t)(y - C\hat{x}) \quad (8) \end{aligned}$$

yields global exponential stability of the closed-loop system.

Proof A complete proof can be found in [11]. The main idea is as follows. First it is shown that the system (4) is uniformly controllable and uniformly observable if and only if (6) is satisfied. Then the result follows from standard linear systems theory (cf [19]). ∎

In general it takes some involving calculations to arrive at these gains $K(t)$ and $L(t)$. However, under an extra assumption we can arrive at rather simple expressions.

Theorem 2.10 *Consider again the system (4) which is such that the condition (6) is satisfied. If in addition $|f(t)|$ can be upper bounded by a polynomial in t, then we have that the controller-observer combination (7,8) yields global $\mathcal{K}$-exponential stability of the closed-loop system in case we use*

$$\begin{aligned} K(t) &= [k_1(t_0,t)\dots k_m(t_0,t)]\\ L(t) &= [l_m(t_0,t)\dots l_1(t_0,t)] \end{aligned}$$

where

$$\begin{aligned} k_i(t_0,t) &= (-1)^i l_i(t_0,t)\\ l_i(t_0,t) &= \sum_{j=0}^{m-i} \frac{\omega(t_0,t)^{2j+i-1}}{(j+i-1)!\,j!} \qquad i = 1,\dots,m \end{aligned}$$

with ω as defined in (5).

Proof A complete proof can be found in [11]. The proof is mainly based on the results of [4]. ∎

2.4 Problem formulation

The class of chained form nonholonomic systems we study in this paper is given by the following equations

$$\begin{aligned} \dot{x}_1 &= u_1\\ \dot{x}_2 &= u_2\\ \dot{x}_3 &= x_2 u_1 \quad (9)\\ &\vdots\\ \dot{x}_n &= x_{n-1}u_1 \end{aligned}$$

where $x = (x_1,\dots,x_n)$ is the state, u_1 and u_2 are control inputs.

Consider the problem of tracking a reference chained form system:

$$\begin{aligned} \dot{x}_{1,d} &= u_{1,d}\\ \dot{x}_{2,d} &= u_{2,d}\\ \dot{x}_{3,d} &= x_{2,d}u_{1,d}\\ &\vdots\\ \dot{x}_{n,d} &= x_{n-1,d}u_{1,d} \end{aligned}$$

When we define the tracking error $x_e = x - x_d$ we obtain as tracking error dynamics

$$\begin{aligned} \dot{x}_{1,e} &= u_1 - u_{1,d}\\ \dot{x}_{2,e} &= u_2 - u_{2,d}\\ \dot{x}_{3,e} &= x_2u_1 - x_{2,d}u_{1,d} \quad (10)\\ &\vdots\\ \dot{x}_{n,e} &= x_{n-1}u_1 - x_{n-1,d}u_{1,d} \end{aligned}$$

The state-feedback tracking control problem then can be formulated as to find appropriate control laws u_1 and u_2 of the form

$$u_1 = u_1(t, x, x_d, u_d) \quad \text{and} \quad u_2 = u_2(t, x, x_d, u_d) \qquad (11)$$

such that the closed-loop trajectories of (10,11) are globally $\mathcal{K}$-exponentially stable.

Assuming the output is given by

$$y = \begin{bmatrix} x_1 \\ x_n \end{bmatrix} \qquad (12)$$

we can formulate the output-feedback tracking problem as to find appropriate control laws u_1 and u_2 of the form

$$u_1 = u_1(t, y, x_d, u_d) \quad \text{and} \quad u_2 = u_2(t, y, x_d, u_d) \qquad (13)$$

such that the closed-loop trajectories of (10,13) are globally $\mathcal{K}$-exponentially stable.

3 The state feedback problem

The approach we use to solve our problem is based on the recently developed studies on cascaded systems [5, 13, 16, 18, 22], and that of Theorem 2.7 in particular.

We search for a subsystem which, with a stabilizing control law, can be written in the form $\dot{z}_2 = f_2(t, z_2)$ that is asymptotically stable. In the remaining dynamics we then can replace the appearance of this z_2 by 0, leading to the system $\dot{z}_1 = f_1(t, z_1)$. If this system is asymptotically stable we might be able to conclude asymptotic stability of the overall system using Theorem 2.7.

Consider the tracking error dynamics (10). We can stabilize the $x_{1,e}$ dynamics by using the linear controller

$$u_1 = u_{1,d} - c_1 x_{1,e} \qquad (14)$$

which yields GES for $x_{1,e}$, provided $c_1 > 0$.

If we now set $x_{1,e}$ equal to 0 in (10) we obtain

$$\begin{aligned} \dot{x}_{2,e} &= u_2 - u_{2,d}\\ \dot{x}_{3,e} &= x_{2,e}u_{1,d} \quad (15)\\ &\vdots\\ \dot{x}_{n,e} &= x_{n-1,e}u_{1,d} \end{aligned}$$

where we used (14).

Notice that the system (15) can be seen as an $n-1$ dimensional linear time-varying system of the form (4) with $f(t) = u_{1,d}(t)$, $x = [x_{2,e}, x_{3,e}, \dots, x_{n,e}]^T$, and $u = u_2 - u_{2,d}$. From Theorem 2.9 we know the conditions for finding a controller that makes the closed-loop system GES. From Theorem 2.10 we have an explicit expression under some more restrictive conditions.

This observation leads to the following

Proposition 3.1 *Consider the system (10) in closed-loop with the controller*

$$
\begin{aligned}
u_1 &= u_{1,d} - c_1 x_{1,e} \\
u_2 &= u_{2,d} - c_2 [k_2(t_0, t), \dots, k_n(t_0, t)] \begin{bmatrix} x_{2,e} \\ \vdots \\ x_{n,e} \end{bmatrix}
\end{aligned} \tag{16}
$$

where $c_1, c_2 > 0$ *and*

$$k_i(t_0, t) = (-1)^i \sum_{j=0}^{n-i} \frac{\omega(t_0, t)^{2j+i-2}}{(j+i-2)!\, j!} \quad i = 2, \dots, n$$

with ω *defined as*

$$\omega(\sigma, t) = \int_\sigma^t u_{1,d}(\tau) d\tau = x_{1,d}(t) - x_{1,d}(\sigma). \tag{17}$$

If

- *there exist* $\delta, \epsilon_1, \epsilon_2 > 0$ *such that (6) is satisfied (where* $m = n - 1$*) with* ω *as defined in (17),*
- $|u_{1,d}(t)|$ *can be upper bounded by a polynomial function of* t*, and*
- $x_{2,d}, \dots, x_{n-1,d}$ *are bounded*

then the closed-loop system (10,16) is globally $\mathcal{K}$*-exponentially stable.*

Proof We can see the closed-loop system (10,16) as a system of the form (3) where

$$
\begin{aligned}
z_2 &= x_{1,e} \\
z_1 &= [x_{2,e}, \dots, x_{n,e}]^T \\
f_1(t, z_1) &= (A(t) - c_2 B K(t)) z_1 \\
f_2(t, z_2) &= -c_1 z_2 \\
g(t, z_1, z_2) &= -c_1 [0, x_2, x_3, \dots, x_{n-1}]^T
\end{aligned}
$$

with

$$
A(t) = \begin{bmatrix} 0 & \dots & \dots & \dots & 0 \\ u_{1,d}(t) & \ddots & & & \vdots \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & u_{1,d}(t) & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

To be able to apply Theorem 2.7 we need to verify the three assumptions:

- assumption on Σ_1: Due to the assumptions on $u_{1,d}(t)$ we have from Theorem 2.10 that $\dot{x} = f_1(t, x)$ is globally $\mathcal{K}$-exponentially stable and therefore GUAS. In [11] the existence of a suitable V is guaranteed.
- assumption on connecting term: Since $x_{2,d}, \dots, x_{n-1,d}$ are bounded, we have

$$
\begin{aligned}
\|g(t, z_1, z_2)\| &\le c_1 \left(\| \begin{bmatrix} 0 \\ x_{2,d} \\ \vdots \\ x_{n-1,d} \end{bmatrix} \| + \| \begin{bmatrix} 0 \\ x_{2,e} \\ \vdots \\ x_{n-1,e} \end{bmatrix} \| \right) \\
&\le c_1 M + c_1 \|z_1\|
\end{aligned}
$$

- assumption on Σ_2: Follows from GES of $\dot{z}_2 = -c_1 z_2$.

Therefore, we can conclude GUAS from Theorem 2.7. Since both Σ_1 and Σ_2 are GES, Lemma 2.8 gives the desired result. ■

Remark 3.2 *Notice that for this result we used the controller proposed in Theorem 2.10. It is clear that by using the controller proposed in Theorem 2.9 a similar result can be achieved. As a consequence we can drop the assumption that* $|u_{1,d}(t)|$ *can be upper bounded by a polynomial function of* t*. The price we have to pay is that the expressions for* $K(t)$ *and* $L(t)$ *are more difficult to calcultate (since it involves determining the state-transition matrix* $\Phi(t, t_0)$*).*

Remark 3.3 *Notice that since*

$$u_1(t) = u_{1,d}(t) - c_1 x_{1,e}(t_0) \exp(-c_1(t - t_0))$$

the conditions on $u_{1,d}(t)$ *are satisfied if and only if the conditions on* $u_1(t)$ *are satisfied.*

Therefore, we can also see the closed-loop system (10,16) as a system of the form (3) where

$$
\begin{aligned}
z_2 &= x_{1,e} \\
z_1 &= [x_{2,e}, \dots, x_{n,e}]^T \\
f_1(t, z_1) &= (A(t) - c_2 B K(t)) z_1 \\
f_2(t, z_2) &= -c_1 z_2 \\
g(t, z_1, z_2) &= -c_1 [0, x_{2,d}, x_{3,d}, \dots, x_{n-1,d}]^T
\end{aligned}
$$

with

$$
A(t) = \begin{bmatrix} 0 & \dots & \dots & \dots & 0 \\ u_1(t) & \ddots & & & \vdots \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & u_1(t) & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

So when we replace the controller (16) with the one given by Theorem 2.9 we can copy the proof.

However, since the connecting term $g(t, z_1, z_2)$ *now can be bounded by a constant, we can claim not only* $\mathcal{K}$*-exponential stability, but even GES. More details can be found in [11].*

4 An observer

The observability property for chained form systems was considered in [1], in which a (local) observer was proposed in case $u_1(t) = -c_1 x_1(t)$. In this section we propose a globally exponentially stable observer for the chained system under an observability condition which is related to the persistence of excitation of the reference trajectory.

Proposition 4.1 *Consider the chained-form system (9) with output (12). Assume that*

- *There exist $\delta, \epsilon_1, \epsilon_2 > 0$ such that (6) is satisfied where we define*

$$\omega(\sigma, t) = \int_\sigma^t u_1(\tau) d\tau = x_1(t) - x_1(\sigma) \qquad (18)$$

- *$|u_1(t)|$ can be bounded by a polynomial function of t.*

Then the observer

$$\begin{aligned} \hat{x}_1 &= x_1 \\ \begin{bmatrix} \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \vdots \\ \dot{\hat{x}}_n \end{bmatrix} &= \begin{bmatrix} u_2 \\ \hat{x}_2 u_1 \\ \vdots \\ \hat{x}_{n-1} u_1 \end{bmatrix} - c_3 \begin{bmatrix} l_n(t_0, t) \\ l_{n-1}(t_0, t) \\ \vdots \\ l_2(t_0, t) \end{bmatrix} \tilde{x}_n \end{aligned}$$

where $\tilde{x} = \hat{x} - x$, $c_3 > 0$ is a constant and

$$l_i(t_0, t) = \sum_{j=0}^{n-i} \frac{[x_1(t) - x_1(t_0)]^{2j+i-2}}{(j+i-2)!\, j!} \qquad i = 2, \ldots, n$$

guarantees that the state observation error $\tilde{x}$ converges to zero exponentially.

Proof The system (9) together with the observer (8) leads to $\tilde{x}_1 = 0$ and

$$\begin{bmatrix} \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \\ \vdots \\ \dot{\tilde{x}}_n \end{bmatrix} = \begin{bmatrix} 0 & \ldots & \ldots & 0 & -c_3 l_n(t_0, t) \\ u_1(t) & \ddots & & \vdots & \vdots \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \ldots & 0 & u_1(t) & -c_3 l_2(t_0, t) \end{bmatrix} \begin{bmatrix} \tilde{x}_2 \\ \tilde{x}_3 \\ \vdots \\ \tilde{x}_n \end{bmatrix}$$

which according to Theorem 2.10 is globally $\mathcal{K}$-exponentially stable. ■

Remark 4.2 *Notice that by using Theorem 2.9 we can achieve GES in a similar way at the expense of a more complicated $L(t)$.*

5 The output feedback problem

In section 3 we derived a state-feedback controller for tracking a desired trajectory, whereas in section 4 we derived an observer for a system in chained form. We can also combine these two results in a "certainty equivalence" sense:

Proposition 5.1 *Consider the system (9) with output (12) in closed-loop with the controller*

$$\begin{aligned} u_1 &= u_{1,d} - c_1 x_{1,e} \\ u_2 &= u_{2,d} - c_2 [k_2(t_0, t), \ldots, k_n(t_0, t)] \begin{bmatrix} \hat{x}_{2,e} \\ \vdots \\ \hat{x}_{n,e} \end{bmatrix} \end{aligned} \qquad (19)$$

where $c_1, c_2 > 0$ and $[\hat{x}_{2,e} \ldots \hat{x}_{n,e}]^T$ is generated from the observer

$$\begin{bmatrix} \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \vdots \\ \dot{\hat{x}}_n \end{bmatrix} = \begin{bmatrix} u_2 - u_{2,d} \\ \hat{x}_{2,e} u_{1,d} \\ \vdots \\ \hat{x}_{n,e} u_{1,d} \end{bmatrix} - c_3 \begin{bmatrix} l_n(t_0, t) \\ l_{n-1}(t_0, t) \\ \vdots \\ l_2(t_0, t) \end{bmatrix} (\hat{x}_n - x_n) \qquad (20)$$

where $c_3 > 0$ is a constant and

$$\begin{aligned} k_i(t_0, t) &= (-1)^i l_i(t_0, t) \\ l_i(t_0, t) &= \sum_{j=0}^{n-i} \frac{[x_{1,d}(t) - x_{1,d}(t_0)]^{2j+i-2}}{(j+i-2)!\, j!} \qquad i = 2, \ldots, n \end{aligned}$$

If

- *there exist $\delta, \epsilon_1, \epsilon_2 > 0$ such that (6) is satisfied with $\omega(\sigma, t) = x_{1,d}(t) - x_{1,d}(\sigma)$*
- *$|u_{1,d}(t)|$ can be upper bounded by a polynomial function of t, and*
- *$x_{2,d}(t), \ldots, x_{n-1,d}(t)$ are bounded*

then the closed-loop system (10,19,20) is globally $\mathcal{K}$-exponentially stable.

Proof Follows immediately from Theorems 2.10 and 2.7 along the lines of the proofs of Proposition 3.1. ■

Remark 5.2 *Similar to Remarks 3.2 and 3.3 we can claim even GES instead of $\mathcal{K}$-exponential stability. Also, it is clear that a similar result can even be achieved when we drop the assumption that $|u_{1,d}(t)|$ can be upper bounded by a polynomial. The only price we have to pay are more complex $K(t)$ and $L(t)$.*

6 Conclusions

In this paper we considered the output tracking problem for nonholonomic systems in chained form, by combining a time-varying state feedback controller with an observer for the chained form in a "certainty equivalence" way. The stability of the closed loop system is showed by results from time-varying cascaded systems. Under a condition of persistence of excitation, we have shown globally $\mathcal{K}$-exponential stability of the closed loop system.

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