# GLOBALLY BOUNDED TRACKING CONTROLLERS FOR RIGID ROBOT SYSTEMS

**A.A.J. Lefeber, H. Nijmeijer**

*Department of Applied Mathematics*
*University of Twente*
*P.O. Box 217*
*7500 AE Enschede*
*The Netherlands*
*Fax : +31 53 4340733*
*e-mail : {A.A.J.Lefeber,H.Nijmeijer}@math.utwente.nl*

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## Abstract

In this note we consider tracking control of rigid robot systems under input constraints. Using the combination of two controllers we are able to obtain globally asymptotically stable closed-loop error-dynamics while keeping the applied actuator torques within in advance specified bounds. This idea is also exploited successfully in case only position measurements are available, or when parameter uncertainties call for an adaptive control scheme.

## 1 Introduction

In recent years there has been a strong interest in the development of controllers for the regulation or tracking of rigid robot manipulators. Starting with the computed torque method several controllers have been designed, which today also exploit the physical nature of the robot system. Furthermore, several other aspects have been incorporated in modern robot controllers, as for instance, the construction of adaptive controllers in case parameter uncertainties are present in the manipulator model, or the development of controller-observer combinations when velocity measurements are not available for control, see e.g. [1, 7] as well as references therein.

In the last few years some interest has arisen in the from a practical perspective important question of designing tracking controllers which respect actuator constraints. In particular, for position control a bounded controller was developed in [4], see also [3] for an alternative bounded PD-like controller.

Sofar the tracking problem under input constraints has only partially been solved in that a semi-global tracking controller or controller-observer has been derived, see [1] and [6].

The purpose of the present contribution is to develop a globally bounded tracking controller for rigid robot systems. Our control scheme essentially combines a bounded regulation controller with a local asymptotically stable tracking controller, and in essence contains the earlier mentioned results on bounded regulation. The idea of using a combination of two controllers can be exploited in various cases including the situation where only position measurements are available or when an adaptation mechanism is required.

The organisation of this note is as follows. Section 1 contains the problem formulation, preliminaries and notation. In section 2 the key idea for the construction of a globally bounded controller is explained. In respectively sections 3, 4 and 5 the controller design with full state measurements, controller design with only position measurements and an adaptive controller design are considered in detail.

## 2 Problem formulation, preliminaries and notation

We consider a rigid robot manipulator

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where $q$ is the $n \times 1$ vector of joint displacements, $\tau$ is the $n \times 1$ vector of applied torques, $M(q)$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n \times 1$ vector of centripetal and Coriolis torques, and $G(q)$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $P(q)$, which without loss of generality, is assumed to have an absolute minimum at $q = 0$. We assume that the links are connected with revolute joints.

Some properties of this system are [9]:

- The matrix $M(q)$ is symmetric and positive definite for all $q \in I\!R^n$.
- The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, that is
$$x^T(\dot{M}(q) - 2C(q,\dot{q}))x = 0 \qquad \forall x \in I\!R^n$$
- The matrix $C$ satisfies
$$C(q,x)y = C(q,y)x$$
- The matrices $M(q)$, $C(q,\dot{q})$ and $G(q)$ are bounded with respect to $q$, i.e:
$$\begin{array}{ll} 0 < M_m \le ||M(q)|| \le M_M & \forall q \in I\!R^n \\ ||C(q,x)|| \le C_M||x|| & \forall q,x \in I\!R^n \\ ||G(q)|| \le G_M & \forall q \in I\!R^n \end{array}$$

Let $\mathcal{C}^k$ denote the set of $k$ times continuously differentiable functions.

We consider the problem of controlling the system (1) towards any desired trajectory $q_d(t) \in \mathcal{C}^2$, satisfying
$$||q_d(t)|| \le B_0, \quad ||\dot{q}_d(t)|| \le B_1, \quad ||\ddot{q}_d(t)|| \le B_2 \qquad (2)$$
under input limitations
$$||\tau(t)|| \le \tau_{max} \qquad \forall t \ge 0$$
where $||\cdot||$ is some norm, e.g. $||\cdot||_\infty$.

A continuous function $f : I\!R^+ \to I\!R^+$ is said to be a class $\mathcal{K}$ function if

- $f(x)$ is strictly increasing, and
- $f(0) = 0$.

Let $\mathcal{F}$ denote the set of strictly increasing $\mathcal{C}^1$ functions $f : I\!R \to I\!R$ satisfying $f(0) = 0$ and $f'(x) > 0$ for all $x \in I\!R$. Some properties of this set, which can easily be verified, are:

- Let $f, g \in \mathcal{F}$. Then $\alpha f(\beta x) + \gamma g(\delta x) \in \mathcal{F}$, provided that $\alpha\beta > 0$ and $\gamma\delta > 0$.
- Let $f, g \in \mathcal{F}$. Then $(fg)(x) \equiv f(x)g(x)$ is positive definite.

Let $\mathcal{B}$ denote the subset of $f \in \mathcal{F}$ for which $f$ is bounded. For instance $f(x) = \tanh(x)$ is an element of both $\mathcal{B}$ and $\mathcal{F}$, and $f(x) = x$ is an element of $\mathcal{F}$, but not of $\mathcal{B}$.

Throughout we use the following notations for $x \in I\!R^n$:

- $f(x) \stackrel{\Delta}{=} [f_1(x_1), \ldots, f_n(x_n)]^T$ where $f \in \mathcal{F}$ means $f_1, \ldots, f_n \in \mathcal{F}$,
- $\sqrt{x} \stackrel{\Delta}{=} \left[\sqrt{x_1}, \ldots, \sqrt{x_n}\right]^T \qquad (x_i \ge 0)$
- $F(x) \stackrel{\Delta}{=} \left[\int_0^{x_1} f_1(\zeta_1)d\zeta_1, \ldots, \int_0^{x_n} f_n(\zeta_n)d\zeta_n\right]^T$
- $f'(x) \stackrel{\Delta}{=} \mathrm{diag}(f_1'(x_1), \ldots, f_n'(x_n))$
- $f''(x) \stackrel{\Delta}{=} [f_1''(x_1), \ldots, f_n''(x_n)]$

The results of this note are established in [5], where an essential ingredient in the analysis is the so-called Matrosov theorem [2]. For completeness we give this important result as it will be used in the proofs given in this note.

**Theorem 1.1 (Matrosov)** *Consider the system $\dot{x} = f(t,x)$ with $f(t,0) = 0$ for all $t \ge 0$. Let $\Omega \in I\!R^n$ be an open connected region in $I\!R^n$ containing the origin. If there exist two $\mathcal{C}^1$ functions $V : [0,\infty) \times \Omega \to I\!R$, $W : [0,\infty) \times \Omega \to I\!R$; a $\mathcal{C}^0$ function $V^* : \Omega \to I\!R$; three class $\mathcal{K}$ functions $a, b, c$ such that, for every $(x,t) \in \Omega \times [0,\infty)$*

*M1.* $a(||x||) \le V(t,x) \le b(||x||)$

*M2.* $\dot{V}(t,x) \le V^*(x) \le 0$

*M3.* $|W(t,x)|$ *is bounded*

*M4.* $\max(d(x,E), |\dot{W}(t,x)|) \ge c(||x||)$

*M5.* $||f(t,x)||$ *is bounded*

*where $E \equiv \{x \in \Omega | V^*(x) = 0\}$; choosing $\alpha > 0$ such that $\bar{B}_\alpha \stackrel{\Delta}{=} \{x \in I\!R^n | \, ||x|| \le \alpha\} \subset \Omega$, define for all $t \in [0,\infty)$*
$$V_{t,\alpha}^{-1} = \{x \in \Omega : V(t,x) \le a(\alpha)\},$$
*then*

1. *For all $x_0 \in V_{t,\alpha}^{-1}$, $x(t)$ tends to zero uniformly in $0, x_0$ as $t$ tends to infinity.*
2. *The origin is uniformly asymptotically stable.*

**Remark 1.2** Since M2 is a weaker condition than known in standard Lyapunov theory, we have to make sure that will not get stuck in $E$ somewhere else but in the origin. Roughly speaking, the other conditions tell that near $E$ and away from zero, the rate of change of a second auxiliary function $W$ is of constant sign, becuase of M4. However, $W$ is bounded and therefore the system can only converge to $E$ if it converges to the origin.

**Lemma 1.3 ([8])** *Condition M4 of Theorem 1.1 is satisfied if the following conditions are satisfied:*

*M4'a.* *$\dot{W}(x,t)$ is continuous in both arguments and depends on time in the following way: $\dot{W}(x,t) = g(x,\beta(t))$ where $g$ is continuous in both of its arguments, $\beta$ is also continuous and its image lies in a bounded set $K_1$.*

*M4'b.* *There exists a class $\mathcal{K}$ function, $k$, such that $|\dot{W}(x,t)| \ge k(||x||)$ for all $x \in E$ and $t \ge 0$.*

**Remark 1.4** Following [8] we will say that $\dot{W}(x,t)$ depends on time continuously through a bounded function.

## 2 Composite controllers

Suppose one can find a controller that steers the system towards the origin, provided one starts within some (possibly small) region of attraction. In case we want to extend this controller to a global one, there are two ways to establish this. The first way is trying to modify the controller in such a way that global asymptotic stability of the error-dynamics is achieved. A second way consists of trying to find a (global) controller that steers the system into the region of attraction of the (locally) stabilizing controller. As soon as we are in its region of attraction, we can switch controllers and the composite controller then is a globally stabilizing controller. This second approach results in an easier problem, since we only have to find a controller that steers our system into some prescribed region, so we seek an ultimately uniformly bounded or practically stable controller, whereas in the first approach we have to find an asymptotically stable controller.

Therefore, in case we want to find a globally stabilizing controller, we can seperate this problem into two problems which are easier to solve. In case we are able to find both a locally stabilizing controller with some region of attraction and a controller that globally steers our system into that region of attraction, the composite controller will be a globally stabilizing one. Here the composition of both controllers consists of using the global controller until the system is inside the region of attraction of the stabilizing controller and then switch to this stabilizing controller.

In case we want to find *bounded* globally stabilizing controllers, the concept of composite controllers may become more important. Not only can we separate our problem into the two easier problems of finding both a locally stabilizing controller with some region of attraction and a bounded controller that globally steers our system into that region of attraction, but also the stabilizing controller does not neccessarily have to be a priori bounded. From the stability analysis of the stabilizing controller we usually know that all signals will remain bounded. The only problem is that those bounds depend on the initial conditions. Since we only switch to our stabilizing controller in case we are in a prescribed region, we can determine in advance from this stability analysis an upperbound on the control input of our stabilizing controller. Therefore only the controller that steers our system into that region has to be a priori bounded.

## 3 Using full state measurements

To solve the problem of tracking a desired trajectory $q_d \in \mathcal{C}^2$ we propose the control law:

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + G(q) - f_1(\dot{e}) - f_2(e) \qquad (3)$$

where $e \equiv q - q_d$ denotes the tracking error and $f_1, f_2 \in \mathcal{F}$. This control law results in the closed-loop system

$$M(q)\ddot{e} + C(q,\dot{q})\dot{e} + f_1(\dot{e}) + f_2(e) = 0 \qquad (4)$$

**Proposition 3.1** *Consider the system (1) together with the control law (3). If $f_2 \in \mathcal{C}^2$ then the resulting closed-loop system (4) is globally asymptotically stable.*

**Proof** This proof is a straightforward extension of the proof of Paden and Panja [8], where this proposition is proved in case $f_1(\dot{e}) = K_d\dot{e}$ and $f_2(e) = K_p e$. To prove this result we will use Matrosov's theorem (Theorem 1.1).

Consider the function

$$V(t,e,\dot{e}) = \frac{1}{2}\dot{e}^T M(q)\dot{e} + \sqrt{F_1(e)}^T\sqrt{F_1(e)}$$

which satisfies condition M1 of Theorem 1.1. Calculating its time-derivative along solutions of (4) results in

$$\dot{V}(t,e,\dot{e}) = -\dot{e}^T f_2(\dot{e})$$

Therefore, with

$$V^*(e,\dot{e}) = -\dot{e}^T f_2(\dot{e}) \qquad (5)$$

condition M2 has also been satisfied.

Since $V(t,e,\dot{e})$ is a decreasing function of time, we can conclude that $e$ and $\dot{e}$ and from (2) also $q$ and $\dot{q}$ are bounded. Since

$$\ddot{e} = -M(q)^{-1}[C(q,\dot{q})\dot{e} + f_1(\dot{e}) + f_2(e)] \qquad (6)$$

which is a continuous function of $e$, $\dot{e}$, $q$ and $\dot{q}$, we know that also $\ddot{e}$ and $\ddot{q}$ are bounded.

In anology with [8] we define $W(t,e,\dot{e}) = \ddot{V}(t,e,\dot{e})$:

$$W(t,e,\dot{e}) = -\ddot{e}^T f_2(\dot{e}) - \dot{e}^T f_2'(\dot{e})\ddot{e} \qquad (7)$$

where $\ddot{e}$ is given by (6) and obviously condition M3 is satisfied.

To verify condition M4 we use Lemma 1.3. Differentiating (7) with respect to time results in

$$\begin{aligned}\dot{W}(t,e,\dot{e}) \quad &= \quad -(\frac{d}{dt}\ddot{e})^T f_2(\dot{e}) - 2\ddot{e}^T f_2'(\dot{e})\ddot{e} \\ &\quad -\dot{e}^T f_2''(\dot{e})\ddot{e}^T\ddot{e} - \dot{e}^T f_2'(\dot{e})(\frac{d}{dt}\ddot{e})\end{aligned}$$

Since $f_2 \in \mathcal{C}^2$ we know that all arguments, except $\frac{d}{dt}\ddot{e}$, are continuous in the tracking error and depend continuously on time through a bounded function. That $\frac{d}{dt}\ddot{e}$ is continuous with respect to the tracking error and continuous with respect to time through a bounded function follows from differentiating (6) with respect to time and noticing that both $\frac{d}{dt}M(q)$ and $\frac{d}{dt}C(q,\dot{q})$ are continuous with respect to the tracking error and continuous with respect to time through a bounded function.

Furthermore for $(e,\dot{e}) \in \{(e,\dot{e})|V^*(e,\dot{e}) = 0\} = \{(e,\dot{e})|\dot{e} = 0\}$ it follows that

$$\dot{W}(t,e,\dot{e}) = -2\ddot{e}^T f_2'(0)\ddot{e}$$

Thus from Lemma 1.3 it follows that also the fourth condition has been fulfilled.

Because all signals remain bounded and the closed-loop system (4) is a continuous function of those signals, the fifth and last condition of Theorem 1.1 has also been satisfied.

Since for arbitrary initial conditions we can determine $\alpha$ and $\Omega$ such that $x_0 \in V_{0,\alpha}^{-1}$ we can conclude from Matrosov's theorem that the origin is globally asymptotically stable. ∎

When we use

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + G(q) - f_1(\dot{e}) - f_2(e) \quad (8)$$

where $f_1, f_2 \in \mathcal{B}$ we almost have a bounded control law. Every term of (8) is bounded, except for $C(q,\dot{q})\dot{q}_d$, since $\dot{q}$ is not a priori bounded. However, from the proof of Proposition 3.1 we know that

$$||\dot{e}(t)||^2 \leq \frac{2}{M_m} V(0, e(0), \dot{e}(0))$$

which gives us a bound on (8), provided we know the initial conditions. However, the control effort increases as $||\dot{q}(0)||$ increases, and therefore (8), although it is a globally asymptotically stable controller, is not a globally bounded tracking controller.

How to obtain a globally bounded tracking controller? Following section 2 we now proceed to construct a bounded composite controller. That is, we have to seek for a globally bounded controller that steers our system into a region in which $e$ and $\dot{e}$ are within prescribed known bounds. As soon as we are in that region, we can switch to (8). Since we switch at a time $t_s$ at which $e(t_s)$ and $\dot{e}(t_s)$ are within bounds that we know in advance, we also have a bound on $V(t_s, e(t_s), \dot{e}(t_s))$ in advance, from which an in advance known bound on (8) follows. By using a globally bounded controller to steer our system into the region in which $e$ and $\dot{e}$ are within prescribed known bounds, the resulting composite controller is a globally bounded controller.

Fortunately, the problem of finding a globally bounded controller that steers our system into a region in which $e$ and $\dot{e}$ are within prescribed known bounds is not difficult to solve. In case we want to control our system towards the origin, e.g. $q_d(t) \equiv 0$, we know that (8), i.e.

$$\tau = G(q) - f_1(\dot{e}) - f_2(e)$$

where $f_1, f_2 \in \mathcal{B}$, will do the job.

Therefore for all $\epsilon > 0$ there exists a time $t_s \geq 0$ such that $||q(t)||, ||\dot{q}(t)|| \leq \epsilon$ for any $t \geq t_s$, and then also $||e(t)|| \leq B_0 + \epsilon$ and $||\dot{e}(t)|| \leq B_1 + \epsilon$ for any $t \geq t_s$.

**Proposition 3.2** *Consider the system (1). Then there exists a switching time $t_s \geq 0$ such that given any $\tilde{t}_s \geq t_s$ the composite controller*

$$\tau = \begin{cases} G(q) - f_1(\dot{q}) - f_2(q) & t < \tilde{t}_s \\ M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + G(q) - f_3(\dot{e}) - f_4(e) & t \geq \tilde{t}_s \end{cases} \quad (9)$$

*results in a globally asymptotically stable closed-loop system. Furthermore, if $f_1, f_2 \in \mathcal{B}$ we can determine a $\tau_{max}$ such that the controller (9) satisfies*

$$||\tau(t)|| \leq \tau_{max} \quad \forall t \geq 0. \quad (10)$$

**Proof** From Proposition 3.1 we know that for all $\epsilon > 0$ there exists a time $t_s \geq 0$ such that $||e(\tilde{t}_s)|| \leq B_0 + \epsilon$ and $||\dot{e}(\tilde{t}_s)|| \leq B_1 + \epsilon$. Furthermore we know that the first phase controller is global and if $f_1, f_2 \in \mathcal{B}$ we can also determine in advance a bound $\tau_{max,1}$ within which this first phase controller will remain.

For our second phase, we know from Proposition 3.1 that our resulting closed-loop system is asymptotically stable. Since $||e(\tilde{t}_s)|| \leq B_0 + \epsilon$ and $||\dot{e}(\tilde{t}_s)|| \leq B_1 + \epsilon$, we can determine a bound for $V(\tilde{t}_s, e(\tilde{t}_s), \dot{e}(\tilde{t}_s))$. Since $V(t, e, \dot{e})$ is a decreasing function of time, we can determine bounds on $e(t)$, $\dot{e}(t)$ for all $t \geq \tilde{t}_s$, from which also bounds on $q(t)$ and $\dot{q}(t)$ follow. Since $f_3$ and $f_4$ are continuous, we can determine a bound $\tau_{max,2}$ on $\tau$ for our second phase.

Now it is obvious that

$$\tau_{max} = \max\{\tau_{max,1}, \tau_{max,2}\}.$$

∎

**Remark 3.3** Notice that not necessarily $f_3, f_4 \in \mathcal{B}$.

# 4 Using only position measurements

In [6] Loria and Nijmeijer presented the controller

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q}_d)\dot{q}_d + G(q) - K_p f(e) - K_d f(z) \quad (11)$$

where $K_p$ and $K_d$ are $n \times n$ diagonal positive definite matrices, $z$ is generated from the observer

$$\begin{aligned} z &= w + Be \\ \dot{w} &= -Af(w + Be) \end{aligned} \quad (12)$$

and $f(\cdot) = \tanh(\cdot)$. The following was shown in [6]:

**Proposition 4.1** *Consider the system (1) together with the control law (11) and observer (12). Then the resulting closed-loop system is semi-globally stable, i.e. the resulting closed-loop system is locally asymptotically stable but its region of attraction can be arbitrarily enlarged by suitably selecting the observer gains $A$ and $B$.*

**Remark 4.2** The only properties of the function $f(x) = \tanh(x)$ being used in the proof given in [6] are that $\forall x \in \mathbb{R}$:

- $f \in \mathcal{F}$
- $|f(x)| \leq 1$.

- $\frac{F(x)}{f^2(x)} \geq \Gamma > 0$.
- $0 < f'(x) \leq \Delta$.

where $F(x) = \int_0^x f(\zeta)d\zeta$ and $\Gamma$, $\Delta$ are constants. Therefore other choices for $f$ are also possible.

Proposition 4.1 is a semi-global result. In case we want to extend this to a global result, we only need to find a globally bounded controller that steers the system into a specific region. Analogously with section 3 we can try to find a globally bounded controller that steers our system towards the origin, or more general, to a fixed point $(q_d, 0)$ (position control):

**Proposition 4.3** *Consider the system (1) together with the control law*

$$\tau = G(q) - f_1(\dot{\hat{e}}) - f_2(e) \quad (13)$$

*where $f_1, f_2 \in \mathcal{F}$, $e = q - q_d$ and $\dot{\hat{e}}$ is generated from the observer*

$$\dot{\hat{e}} = L_{p,1}e - L_{p,2}\hat{e} \quad (14)$$

*where $L_{p,1}$ and $L_{p,2}$ are $n \times n$ symmetric positive definite matrices. Then the resulting closed-loop system is globally asymptotically stable.*

**Proof** see [5]. ■

When we choose $f_1, f_2 \in \mathcal{B}$ we have that (13) is a globally bounded control law that steers our system towards the origin. Therefore:

**Proposition 4.4** *Consider the system (1). Then there exists a switching time $t_s \geq 0$ such that given any $\tilde{t}_s \geq t_s$ the composite control law*

$$\tau = \begin{cases} G(q) - f_1(z_1) - f_2(q) & t < \tilde{t}_s \\ \\ M(q)\ddot{q}_d + C(q,\dot{q}_d)\dot{q}_d + \\ +G(q) - K_d f(z_2) - K_p f(e) & t \geq \tilde{t}_s \end{cases} \quad (15)$$

*where $z_1$ and $z_2$ are given by*

$$\begin{array}{rcl} z_1 & = & L_{p,1}q - L_{p,2}\hat{q} \\ z_2 & = & w + Be \end{array} \quad (16)$$

*and $\hat{q}$ and $w$ are generated from the reduced observers*

$$\begin{array}{rcll} \dot{\hat{q}} & = & L_{p,1}q - L_{p,2}\hat{q} & t < \tilde{t}_s \\ \dot{w} & = & -Af(w + Be) & t \geq \tilde{t}_s \end{array} \quad (17)$$

*results in a globally asymptotically stable closed-loop system. Furthermore, if $f_1, f_2 \in \mathcal{B}$ we can determine a $\tau_{max}$ such that the controller (15) together with the reduced observer (16, 17) satisfies:*

$$||\tau(t)|| \leq \tau_{max} \quad \forall t \geq 0$$

**Proof** Analogously with Proposition 3.2. ■

## 5 Adaptive Controller

In case some of the parameters are unknown, we may use an adaptive controller. A useful property of the system (1) is the following:

- There exists a reparametrization of all unknown parameters into a parameter vector $\theta \in I\!R^p$ that enters linearly in the system dynamics (1). Therefore, the following holds:

$$\begin{array}{c} M(q,\theta)\ddot{q} + C(q,\dot{q},\theta)\dot{q} + G(q,\theta) = \\ M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q) + Y(q,\dot{q},\dot{q},\ddot{q})\theta \end{array}$$

Consider the system (1) together with the controller

$$\begin{array}{rcl} \tau & = & M(q,\hat{\theta})\ddot{q}_r + C(q,\dot{q},\hat{\theta})\dot{q}_r + \\ & & +G(q,\hat{\theta}) - f_1(e) - f_2(s_1) \\ & = & M_0(q)\ddot{q}_r + C_0(q,\dot{q})\dot{q}_r + G_0(q) + \\ & & Y(q,\dot{q},\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\theta} - f_1(e) - f_2(s_1) \end{array} \quad (18)$$

where $\dot{q}_r = \dot{q}_d - f_3(e)$, $e = q - q_d$, $s_1 = \dot{q} - \dot{q}_r = \dot{e} + f_3(e)$, $f_1, f_2, f_3 \in \mathcal{F}$ and $\hat{\theta}$ is an estimation for the vector of unknown (but constant) system parameters.

The controller (18) results in the closed-loop system

$$\begin{array}{r} M(q,\theta)\dot{s}_1 + C(q,\dot{q},\theta)s_1 + f_1(e) + f_2(s_1) = \\ = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\theta} \end{array} \quad (19)$$

where $\tilde{\theta} \equiv \hat{\theta} - \theta$. When we use the parameter update law

$$\dot{\hat{\theta}} = -\Gamma_0 Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)s_1 \quad (20)$$

where $\Gamma_0$ is a positive definite symmetric matric, we obtain

**Proposition 5.1** *Consider the system (1) together with the controller (18) and the parameter update law (20). Then the resulting closed-loop system (19) is globally asymptotically stable with respect to $e$ and $\dot{e}$.*

**Proof** See [5]. ■

Although the controller (18) is a globally asymptotically stable controller, it is not a globally bounded tracking controller. We can again contruct a globally bounded tracking controller along the lines of section 2. We have to contruct a globally bounded controller that steers the system (1) into a prescribed region. Therefore we consider a controller that steers our system towards a desired fixed position $(q_d, 0)$ (position control).

**Proposition 5.2** *Consider the system (1) together with the control law*

$$\tau = G(q_d) - f_1(\dot{e}) - \alpha f_2(e) \quad (21)$$

*where $e = q - q_d$ and $f_1, f_2 \in \mathcal{F}$. Then the resulting closed-loop system is globally asymptotically stable, provided $\alpha$ is choosen large enough.*

**Proof** See [5]. ■

**Remark 5.3** Analogously we can prove that the controller

$$\tau = G(q_d) - f_1(\dot{\hat{e}}) - \alpha f_2(e)$$

where $e = q - q_d$, $f_1, f_2 \in \mathcal{F}$ and $\dot{\hat{e}}$ is generated from the observer (14) results in a globally asymptotically stable closed-loop system, provided $\alpha$ is choosen large enough.

When we choose $f_1, f_2 \in \mathcal{B}$ and $q_d = 0$, we know that $G(q_d) = 0$, so that (21) is parameter-independent and globally bounded. Therefore:

**Proposition 5.4** *Consider the system (1). Then there exists a switching time $t_s \geq 0$ such that given any $\tilde{t}_s \geq t_s$ the composite control law*

$$\tau = \begin{cases} -f_1(\dot{e}) - \alpha f_2(e) & t < \tilde{t}_s \\ \\ M(q,\hat{\theta})\ddot{q}_r + C(q,\dot{q},\hat{\theta})\dot{q}_r + \\ +G(q,\hat{\theta}) - f_1(e) - f_2(s_1) & t \geq \tilde{t}_s \end{cases} \qquad (22)$$

*where $\hat{\theta}$ is updated by (20), results in a globally asymptotically stable closed-loop system, provided that $\alpha$ is choosen large enough. Furthermore, if $f_1, f_2 \in \mathcal{B}$ we can determine a $\tau_{max}$ such that the controller (22) together with the parameter-update-law (20) satisfies:*

$$||\tau(t)|| \leq \tau_{max} \quad \forall t \geq t_0$$

**Proof** Analogously with Proposition 3.2. ■

# 6 Conclusion

The result as presented in this note states that by using a composite controller tracking can be achieved by using bounded inputs. Although the result is seemingly of practical importance, a drawback still exists in that we use a switching controller. Specifically at a certain time instant we discontinuously change the control input which is certainly not feasible in practice. However, we will show in [5] that the here occuring discontinuous nature of the proposed controllers can be overcome by introducing a smooth time varying controller that is a weighted composition of the two controllers we use here. We will also give in [5] simulations based on this idea, thereby providing some possible choices for selection of functions $f_i$ in the composite controllers.

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