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Almost global decentralised formation tracking for multiple distinct UAVs

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Abstract: We consider formation control of multiple UAVs, not necessarily identical, by means of a decentralized state feedback controller. The attitude dynamics is formulated on SO(3) to avoid singularities of Euler angles and ambiguity of quaternions, thus allowing for large angular maneuvers, e.g., loopings). Since we explicitly take into account the constraint of non-zero total thrust in our controller design, the controller achieves uniform almost global asymptotic stability instead of only a local result. The trade off between individual trajectory tracking and formation forming is illustrated by means of simulations.

Keywords: Nonlinear control systems, Lyapunov methods, Distributed control

1. INTRODUCTION

The formation control of multiple unmanned aerial vehicles (UAVs) has received considerable interest in the nonlinear control community over the last decade. In this paper we do not consider a kinematic model of an UAV, but a dynamic model, using propeller thrusts as input. Furthermore, we deal with underactuated dynamics by considering four inputs: total thrust, and three angular momenta. Most of the currently existing controllers are framed in Euler angles, which exhibit singularities when the pitch angle becomes 90 degrees. Therefore, these models can not be used when large angular movements need to be achieved like loopings. As an alternative, attitude control using a quaternion representation has been considered in e.g., Abdessameud and Tayebi (2009). However, two quaternions can be used for describing the same attitude. This leads to ambiguities in the control action (the same control action should be required for both representations) and the resulting controller may exhibit unwinding behavior, see Bhat and Bernstein (2000).

In order to represent the attitude both globally and uniquely, formation control of UAVs should be considered directly on the special orthogonal group SO(3), as studied in e.g., Turpin et al. (2012); Miao et al. (2017); Dong et al. (2018). A prerequisite for these controllers is that the total thrust is non-zero/strictly positive. In the above mentioned papers, the authors show asymptotic stability of their system under the assumption that the thrust is non-zero. However, since the required thrust is specified by the controller, one can not assume a priori that this thrust is non-zero: this should be guaranteed by the controller. For the controllers in the above mentioned papers, a non-zero thrust can only be guaranteed when starting in a small neighborhood around the equilibrium point, reducing these results to local results.

The main contribution of this paper is that we present a decentralized controller which achieves uniform *almost global* asymptotic stability of the formation tracking error dynamics for multiple distinct UAVs while considering attitude dynamics on SO(3). That is, uniform global asymptotic stability except for initial conditions in a set of measure zero. Note that a global result can not be achieved on SO(3), cf. Bhat and Bernstein (2000). For our result we only need to assume that the total thrust for the *reference formation* stays away from zero. Our controller design then guarantees that the total thrust for the UAVs themselves stays away from zero.

The outline of this paper is as follows. In Section 2 we introduce definitions and theorems used in the remainder of the paper. In Section 3 we introduce the UAV dynamics, reference formation, formation tracking errors and the problem formulation. In Section 4 we derive a distributed position tracking controller under the assumption that we can use linear acceleration errors as (virtual) input. In Section 5 we aim to realize this virtual input by controlling the rotor thrusts. In Section 6 we analyze the stability of the cascaded system that we obtained. Section 7 contains simulation results with our proposed controllers and Section 8 concludes the paper.

2. PRELIMINARIES

In this section we introduce notation, definitions and theorems used in the remainder of this paper.

Let e_i for $i \in \{1, 2, 3\}$ denote the standard unit vector.

For definitions of uniform global (or local) asymptotic (or exponential) stability (UGAS, UGES, ULES), see Khalil (2002).

Definition 1. The origin of (1) is *uniformly almost globally asymptotically stable (UaGAS)* if it is UGAS, except for initial conditions in a set of measure zero.

In the remainder, let $\sigma : \mathbb{R}^n \to \mathbb{R}^n$ (possibly with subscripts) denote a vector-function $\sigma(e) = s(e^T e)e$, where $s : \mathbb{R}^+ \to \mathbb{R}^+$ is a twice continuously differentiable function

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satisfying $s(0) > 0$ and for which $\int s(x)\mathrm{d}x$ is radially unbounded. Note that $\sigma(-e) = -\sigma(e)$. Furthermore, let $V_\sigma(e) = \frac{1}{2}\int_0^{e^Te} s(x)\mathrm{d}x$, which is positive definite and radially unbounded. Possible candidates are $\sigma(e) = k_0 e$ and $\sigma(e) = \frac{k_0 k_\infty e}{\sqrt{k_\infty^2 + k_0^2 e^T e}}$, with $k_0 > 0$ and $k_\infty > 0$, where the latter function is bounded.

Definition 2. A function σ as considered above for which $|\sigma(e)| \le M$ for all $e \in \mathbb{R}^n$ is called a *saturation function*.

Lemma 3. If $a_{ij} = a_{ji}$ and $\sigma_{ij}(x) = \sigma_{ji}(x)$, then

$$\sum_{i=1}^n \sum_{j=1}^n a_{ij}\sigma_{ij}(x_i - x_j) = 0$$

Proof. Since $\sigma(-e) = -\sigma(e)$ we have

$$\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n a_{ij}\sigma_{ij}(x_i - x_j) + \frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n a_{ij}\sigma_{ij}(x_i - x_j)$$
$$= \frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n a_{ij}\sigma_{ij}(x_i - x_j) - \frac{1}{2}\sum_{j=1}^n\sum_{i=1}^n a_{ji}\sigma_{ji}(x_j - x_i) = 0$$

Lemma 4. (cf. (Ren, 2008, Lemma 3.1)). If $a_{ij} = a_{ji}$ and $\sigma_{ij}(x) = \sigma_{ji}(x)$, then

$$\sum_{i=1}^n\sum_{j=1}^n a_{ij}(x_i - x_j)^T\sigma_{ij}(y_i - y_j) = 2\sum_{i=1}^n\sum_{j=1}^n a_{ij}x_i^T\sigma_{ij}(y_i - y_j)$$

Theorem 5. (Corollary of (Loría et al., 2005, Theorem 1)). Consider the dynamical system

$$\dot{x} = f(t,x) \qquad x(t_0) = x_0 \qquad f(t,0) = 0, \tag{1}$$

with $f : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ locally bounded, continuous and locally uniformly continuous in t.

If there exist j differentiable functions $V_i : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$, bounded in t, and continuous functions $Y_i : \mathbb{R}^n \to \mathbb{R}$ for $i \in \{1, 2, \ldots j\}$ such that

- V_1 is positive definite and radially unbounded,
- $\dot{V}_i(t,x) \le Y_i(x)$, for all $i \in \{1, 2, \ldots, j\}$,
- $Y_i(x) = 0$ for $i \in \{1, 2, \ldots, k-1\}$ implies $Y_k(x) \le 0$, for all $k \in \{1, 2, \ldots, j\}$,
- $Y_i(x) = 0$ for all $i \in \{1, 2, \ldots, j\}$ implies $x = 0$,

then the origin $x = 0$ of (1) is uniformly globally asymptotically stable (UGAS).

Theorem 6. (cf. Sanyal and Chaturvedi (2008)). Consider the dynamics

$$\dot{\tilde{R}} = \tilde{R}S(\tilde{\omega}) \qquad J\dot{\tilde{\omega}} = -K_\omega\tilde{\omega} + \sum_{i=1}^3 k_i(e_i \times \tilde{R}^T e_i),$$

where $R \in \mathrm{SO}(3) = \{R \in \mathbb{R}^{3\times3} \mid R^T R = I, \det R = 1\}$, $\omega \in \mathbb{R}^3$, $J = J^T > 0$ and

$$S(a) = -S(a)^T = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \tag{2}$$

If $K_\omega = K_\omega^T > 0$, and $k_i > 0$ are distinct ($k_1 \ne k_2 \ne k_3 \ne k_1$), then the resulting equilibrium point $(I, 0)$ is ULES and UaGAS, i.e., let $E_c = \{I, \mathrm{diag}(1,-1,-1), \mathrm{diag}(-1,1,-1), \mathrm{diag}(-1,-1,1)\}$. Then $\tilde{R}$ converges to E_c and $\tilde{\omega}$ converges to zero. The equilibria $(R, 0)$ where $R \in E_c \setminus \{I\}$ are unstable and the set of all initial conditions converging to the equilibrium $(R, 0)$, where $R \in E_c \setminus \{I\}$ form a lower dimensional manifold.

Theorem 7. (cf. Panteley and Loría (1998)). Let the system (1) be written as

$$\dot{x}_1 = f_1(t, x_1) + g(t, x_1, x_2)x_2 \tag{3a}$$
$$\dot{x}_2 = f_2(t, x_2), \tag{3b}$$

where $x_1 \in \mathbb{R}^n$, $x_2 \in \mathbb{R}^m$, $f_1(t, x_1)$ is continuously differentiable in (t, x_1) and $f_2(t, x_2)$, $g(t, x_1, x_2)$ are continuous in their arguments, and locally Lipschitz in x_2 and (x_1, x_2) respectively. This system is a cascade of the systems

$$\dot{x}_1 = f_1(t, x_1) \tag{4}$$

and (3b). If the origins of the systems (4) and (3b) are UGAS and solutions of (3) remain bounded, then the origin of the system (3) is UGAS. In addition, if the systems (4) and (3b) are ULES, then the system (3) is ULES.

3. DYNAMICS AND PROBLEM FORMULATION

3.1 UAV dynamics

Consider a group of n not necessarily identical UAVs. For $i \in \{1, 2, \ldots, n\}$, let $\rho_i \in \mathbb{R}^3$ denote the position of the centre of mass of UAV i relative to a North-East-Down (NED) inertial frame. Let $R_i \in \mathrm{SO}(3)$ denote the rotation matrix from the body-fixed frame of UAV i to the inertial frame. Furthermore, let $\nu_i \in \mathbb{R}^3$ and $\omega_i \in \mathbb{R}^3$ denote the body-fixed linear and angular velocities of UAV i. Then the dynamics of UAV i can be described as:

$$\dot{\rho}_i = R_i\nu_i \tag{5a}$$
$$\dot{\nu}_i = -S(\omega_i)\nu_i + gR_i^T e_3 - \frac{f_i}{m_i}e_3 \tag{5b}$$
$$\dot{R}_i = R_i S(\omega_i) \tag{5c}$$
$$J_i\dot{\omega}_i = S(J_i\omega_i)\omega_i + \tau_i, \tag{5d}$$

where m_i denotes the total mass of UAV i, $J_i = J_i^T > 0$ the inertia matrix of UAV i with respect to its body-fixed frame (note that m_i and J_i can be different for each UAV), the matrix S is given by (2), and $f_i \in \mathbb{R}$ and $\tau_i \in \mathbb{R}^3$ denote respectively the total thrust magnitude and the total moment vector in the body-fixed frame, which are assumed to be the inputs.

3.2 Reference formation and formation tracking errors

Consider a feasible reference for these n UAVs, i.e., for each UAV of the formation a feasible reference trajectory is given satisfying

$$\dot{\rho}_{r,i} = R_{r,i}\nu_{r,i} \tag{6a}$$
$$\dot{\nu}_{r,i} = -S(\omega_{r,i})\nu_{r,i} + gR_{r,i}^T e_3 - \frac{f_{r,i}}{m_i}e_3 \tag{6b}$$
$$\dot{R}_{r,i} = R_{r,i}S(\omega_{r,i}) \tag{6c}$$
$$J_i\dot{\omega}_{r,i} = S(J_i\omega_{r,i})\omega_{r,i} + \tau_{r,i}, \tag{6d}$$

where $0 < f_{r,i}^{\min} \le f_{r,i}(t)$, for $i \in N = \{1, 2, \ldots, n\}$. Since we want to consider a formation, we usually do not specify reference trajectories for each UAV individually in an inertial frame, but with respect to a formation frame, see e.g. Gutiérrez et al. (2017); Sadowska et al. (2011).

Let a formation centered frame $\mathcal{F}$ be located at a fixed but free to choose virtual center of the formation. Let $\rho_f \in \mathbb{R}^3$ denote the position of this frame relative to a

NED inertial frame, and $R_f \in \mathrm{SO}(3)$ the rotation matrix from the frame $\mathcal{F}$ to the inertial frame. Then we can also express the reference position for each of the UAVs in the formation frame $\mathcal{F}$:

$$p_{r,i} = R_f^T(\rho_{r,i} - \rho_f).$$

Typically, for a formation the vectors $p_{r,i}$ are constant, or given as a function of time multiplying a constant vector, though we allow for arbitrary functions of time. The movement of the formation then is described by ρ_f and R_f, which we both assume to be twice continuously differentiable.

Similarly, we can also express the position p_i of UAV i in the formation frame $\mathcal{F}$: $p_i = R_f^T(\rho_i - \rho_f)$, as well as the (individual) position and velocity tracking errors expressed in the formation frame $\mathcal{F}$:

$$p_{e,i} = p_{r,i} - p_i = R_f^T(\rho_{r,i} - \rho_i) \tag{7a}$$

$$v_{e,i} = R_f^T(R_{r,i}\nu_{r,i} - R_i\nu_i). \tag{7b}$$

3.3 Problem formulation

We are interested in what is called mutual (internal) synchronisation in Nijmeijer and Rodriguez-Angeles (2003), where synchronous behavior is obtained as a result of interactions between the UAVs in the system on equal terms (cooperative systems), instead of e.g. master-slave following.

Therefore, we have to define a communication topology specifying these interconnections. Let $a_{ij} = a_{ji} = 1$ when UAV i and j exchange position information, and 0 otherwise. Similarly, let $b_{ij} = b_{ji} = 1$ when UAV i and j exchange velocity information, and 0 otherwise. Usually we have $a_{ij} = b_{ij}$, but this is not required.

Assumption 8. The (undirected) graphs with incidence matrices $A = [a_{ij}]$, respectively $B = [b_{ij}]$ are connected.

Assumption 9. Each UAV knows the position ρ_f and the orientation R_f of the formation frame $\mathcal{F}$, as well as its reference state and input satisfying (6). In particular this means that each UAV also knows its own $p_{e,i}$, $v_{e,i}$, $R_{r,i}$, $\omega_{r,i}$, $\dot{\omega}_{r,i}$, and $f_{r,i}$.

Dependent on how much emphasis each UAV puts to tracking its reference trajectory, the minimum we want to achieve is that the formation is formed, so that all UAVs have the same formation tracking error, i.e., for all $i, j \in \{1, 2, \ldots, n\}$:

$$\lim_{t\to\infty} \|p_{e,i}(t) - p_{e,j}(t)\| = 0 \quad \lim_{t\to\infty} \|v_{e,i}(t) - v_{e,j}(t)\| = 0. \tag{8}$$

Furthermore, in case we do obtain tracking of the reference formation, i.e.

$$\lim_{t\to\infty} \|p_{e,i}(t)\| = \lim_{t\to\infty} \|v_{e,i}(t)\| = 0, \tag{9}$$

we also want tracking of the reference attitude, i.e., we additionally require that the tracking errors on SO(3):

$$R_{e,i} = R_i^T R_{r,i} \qquad \omega_{e,i} = \omega_{r,i} - R_{e,i}^T\omega_i \tag{10}$$

satisfy

$$\lim_{t\to\infty} R_{e,i} = I_3 \qquad \lim_{t\to\infty} \omega_{e,i} = 0. \tag{11}$$

Problem 10. Consider the dynamics (5), reference (6) and formation tracking errors (7). Design control laws

$$f_i = f_i^0(p_{e,i},v_{e,i},t) + \sum_{j=1}^{n} a_{ij} f_{ij}^a(p_{e,i},p_{e,j},t) + b_{ij} f_{ij}^b(v_{e,i},v_{e,j},t)$$

$$\tau_i = \tau_i^0(p_{e,i},v_{e,i},t) + \sum_{j=1}^{n} a_{ij} \tau_{ij}^a(p_{e,i},p_{e,j},t) + b_{ij} \tau_{ij}^b(v_{e,i},v_{e,j},t)$$

which guarantee (8), and in case of (9) also (11).

Remark 11. Notice that since each UAV knows the reference trajectory it should track, one could design for each UAV a separate tracking controller, cf. Lefeber et al. (2017) and references therein, which achieve flying in formation even without mutual communication, i.e., for $a_{ij} = b_{ij} = 0$. This results in a non-social behavior, where each UAV only cares about its reference trajectory and completely ignores the desired formation. We are therefore particularly interested in solutions resulting in social behavior, i.e., in which UAVs prefer flying in formation over flying the reference trajectories.

4. POSITION TRACKING CONTROL

Along the lines of Lefeber et al. (2017) we first consider the position tracking control. For each of the UAVs we have

$$\dot{p}_{e,i} = -S(\omega_f)p_{e,i} + v_{e,i} \tag{12a}$$

$$\dot{v}_{e,i} = -S(\omega_f)v_{e,i} + u_{e,i}, \tag{12b}$$

where ω_f denotes the angular velocity of the formation frame $\mathcal{F}$, and $u_{e,i}$ is a virtual input satisfying

$$u_{e,i} = -R_f^T\left(\frac{f_{r,i}}{m_i}R_{r,i} - \frac{f_i}{m_i}R_i\right)e_3. \tag{13}$$

Inspired by Ren (2008), consider the following inputs

$$u_{e,i} = -\sum_{j=1}^{n}\left[a_{ij}\sigma_{p_{ij}}(p_{e,i} - p_{e,j}) + b_{ij}\sigma_{v_{ij}}(v_{e,i} - v_{e,j})\right] - \alpha_i\sigma_{p_i}(p_{e,i}) - \beta_i\sigma_{v_i}(v_{e,i}). \tag{14}$$

Then we can prove the following two propositions:

Proposition 12. Consider the dynamics (12) in closed-loop with the (virtual) input (14) where $\sigma_{p_{ij}}(x) = \sigma_{p_{ji}}(x)$, $\alpha_i = 0$, and $\beta_i = 0$. If Assumption 8 is satisfied, and ω_f and $\dot{\omega}_f$ are bounded, then the resulting closed-loop system with states $p_{e,i} - p_{e,j}$ and $v_{e,i} - v_{e,j}$ is UGAS, i.e., we obtain (8).

Proposition 13. Consider the dynamics (12) in closed-loop with the (virtual) input (14) where $\sigma_{p_{ij}}(x) = \sigma_{p_{ji}}(x)$, $\alpha_i \geq 0$, $\beta_i \geq 0$, $\sum_{i=1}^{n}\alpha_i > 0$, and $\sum_{i=1}^{n}\beta_i > 0$. If Assumption 8 is satisfied, and ω_f and $\dot{\omega}_f$ are bounded, then the resulting closed-loop system with states $p_{e,i}$ and $v_{e,i}$ is UGAS and the UAVs track the formation, i.e., we obtain (9).

Remark 14. Note that for $\omega_f(t) = 0$ we have three separate integrators in parallel and the above results reduce to those in Ren (2008).

Proof of Proposition 12. Differentiating

$$V_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}V_{\sigma_{p_{ij}}}(p_{e,i} - p_{e,j}) + \frac{1}{2n}(v_{e,i} - v_{e,j})^T(v_{e,i} - v_{e,j})$$

along the closed-loop dynamics (12), (14) results in

$$\dot{V}_1 = -\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(v_{e,i} - v_{e,j})^T\sigma_{v_{ij}}(v_{e,i} - v_{e,j}) = Y_1(v_{e,i} - v_{e,j})$$

Differentiating the function

$$V_2 = -\sum_{i=1}^{n}\sum_{j=1}^{n}(v_{e,i}-v_{e,j})^T(\dot{v}_{e,i}-\dot{v}_{e,j})$$

along the closed-loop dynamics (12), (14) results in

$$\dot{V}_2 \leq -\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\sum_{k=1}^{n}a_{ik}\sigma_{p_{ik}}(p_{e,i}-p_{e,k})-\sum_{k=1}^{n}a_{jk}\sigma_{p_{jk}}(p_{e,j}-p_{e,k})\right]^2 + M\sum_{i=1}^{n}\sum_{j=1}^{n}\|v_{e,i}-v_{e,j}\| = Y_2(p_{e,i}-p_{e,j}, v_{e,i}-v_{e,j}).$$

Note that $Y_1 = 0$ implies $v_{e,i} = v_{e,j}$ for all $i,j \in \{1,2,\dots,n\}$ since the graph with incidence matrix $B = [b_{ij}]$ is connected. If both $Y_1 = 0$ and $Y_2 = 0$, we have for all $i,j \in \{1,2,\dots,n\}$:

$$\sum_{k=1}^{n}a_{ik}\sigma_{p_{ik}}(p_{e,i}-p_{e,k}) = \sum_{k=1}^{n}a_{jk}\sigma_{p_{jk}}(p_{e,j}-p_{e,k})$$

and therefore

$$n\sum_{k=1}^{n}a_{ik}\sigma_{p_{ik}}(p_{e,i}-p_{e,k}) = \sum_{j=1}^{n}\sum_{k=1}^{n}a_{jk}\sigma_{p_{jk}}(p_{e,j}-p_{e,k}) = 0$$

which implies

$$0 = 2\sum_{i=1}^{n}p_{e,i}^T\sum_{j=1}^{n}a_{ij}\sigma_{p_{ij}}(p_{e,i}-p_{e,j}) = = \sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(p_{e,i}-p_{e,j})^T\sigma_{p_{ij}}(p_{e,i}-p_{e,j})$$

resulting in $p_{e,i} = p_{e,j}$ for all $i,j \in \{1,2,\dots,n\}$ since the graph with incidence matrix $A = [a_{ij}]$ is connected. Applying Theorem 5 completes the proof. □

Proof of Proposition 13. Differentiating

$$V_1 = \sum_{i=1}^{n}\alpha_i V_{\sigma_{p_i}}(p_{e,i}) + \sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}V_{\sigma_{p_{ij}}}(p_{e,i}-p_{e,j}) + \frac{1}{2}\sum_{i=1}^{n}v_{e,i}^Tv_{e,i} \tag{15}$$

along the closed-loop dynamics (12), (14) results in

$$\dot{V}_1 = -\sum_{i=1}^{n}\beta_i v_{e,i}^T\sigma_{v_i}(v_{e,i}) - \sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}(v_{e,i}-v_{e,j})^T\sigma_{v_{ij}}(v_{e,i}-v_{e,j}).$$

Differentiating the function

$$V_2 = -\sum_{i=1}^{n}v_{e,i}^T\dot{v}_{e,i}$$

along the closed-loop dynamics (12), (14) results in

$$\dot{V}_2 \leq -\sum_{i=1}^{n}\left[\alpha_i\sigma_{p_i}(p_{e,i}) + \sum_{j=1}^{n}a_{ij}\sigma_{p_{ij}}(p_{e,i}-p_{e,j})\right]^2 + \sum_{i=1}^{n}M\|v_{e,i}\| = Y_2(p_{e,i}, v_{e,i}).$$

Let $Y_1 = \dot{V}$. Then $Y_1 = 0$ implies $v_{e,i} = 0$ for all $i \in \{1,2,\dots,n\}$ since the graph with incidence matrix $B = [b_{ij}]$ is connected. If both $\dot{V} = Y_1 = 0$ and $Y_2 = 0$, we have for all $i \in \{1,2,\dots,n\}$:

$$\alpha_i\sigma_{p_i}(p_{e,i}) + \sum_{j=1}^{n}a_{ij}\sigma_{p_{ij}}(p_{e,i}-p_{e,j}) = 0$$

and therefore

$$\begin{aligned}0 &= \sum_{i=1}^{n}\alpha_i p_{e,i}^T\sigma_{p_i}(p_{e,i}) + \sum_{j=1}^{n}a_{ij}p_{e,i}^T\sigma_{p_{ij}}(p_{e,i}-p_{e,j})\\ &= \sum_{i=1}^{n}\alpha_i p_{e,i}^T\sigma_{p_i}(p_{e,i}) + \sum_{j=1}^{n}a_{ij}(p_{e,i}-p_{e,j})^T\sigma_{p_{ij}}(p_{e,i}-p_{e,j}),\end{aligned}$$

which implies $p_{e,i} = 0$ for all $i \in \{1,2,\dots,n\}$ since the graph with incidence matrix $A = [a_{ij}]$ is connected. Applying Theorem 5 completes the proof. □

5. ATTITUDE CONTROL

Since $u_{e,i}$ is a virtual input, and not the actual input, we need to design the control laws for f_i and τ_i to let $f_iR_{r,i}^TR_ie_3$ converge to $f_{r,i}e_3 + m_iR_{r,i}^TR_fu_{e,i}$. Following Lefeber et al. (2017), we first observe that for a feasible reference trajectory $0 < f_{r,i}^{\min} \leq f_{r,i}(t)$ and therefore by properly selecting all functions σ_{p_i}, $\sigma_{p_{ij}}$, σ_{di}, and σ_{dij} we can not only tune our controller, but also guarantee that $\|u_{e,i}\| \leq \frac{f_{r,i}^{\min}-\epsilon_i}{m_i}$ for any $0 < \epsilon_i < f_{r,i}^{\min}$ by using saturation functions.

As a result, we have $0 < \epsilon_i \leq f_i(t)$ for

$$f_i = \|f_{r,i}e_3 + m_iR_{r,i}^TR_fu_{e,i}\|. \tag{16}$$

Since $0 < \epsilon_i \leq f_i$, we can define

$$f_{d,i} = \begin{bmatrix}f_{d,i_1}\\ f_{d,i_2}\\ f_{d,i_3}\end{bmatrix} = \frac{f_{r,i}e_3 + m_iR_{r,i}^TR_fu_{e,i}}{\|f_{r,i}e_3 + m_iR_{r,i}^TR_fu_{e,i}\|} \tag{17a}$$

as the desired thrust direction, satisfying $f_{d,i_3} > 0$. We let

$$R_{d,i} = \begin{bmatrix}1-\dfrac{f_{d,i_1}^2}{1+f_{d,i_3}} & -\dfrac{f_{d,i_1}f_{d,i_2}}{1+f_{d,i_3}} & f_{d,i_1}\\ -\dfrac{f_{d,i_1}f_{d,i_2}}{1+f_{d,i_3}} & 1-\dfrac{f_{d,i_2}^2}{1+f_{d,i_3}} & f_{d,i_2}\\ -f_{d,i_1} & -f_{d,i_2} & f_{d,i_3}\end{bmatrix} \in \mathrm{SO}(3) \tag{17b}$$

denote the rotation matrix which rotates the desired thrust vector to the thrust vector of the reference (i.e., e_3) in the plane containing both vectors. This also gives

$$\omega_{d,i} = \begin{bmatrix}-\dot{f}_{d,i_2} + \dfrac{f_{d,i_2}\dot{f}_{d,i_3}}{1+f_{d,i_3}}\\ \dot{f}_{d,i_1} - \dfrac{f_{d,i_1}\dot{f}_{d,i_3}}{1+f_{d,i_3}}\\ \dfrac{f_{d,i_2}\dot{f}_{d,i_1} - f_{d,i_1}\dot{f}_{d,i_2}}{1+f_{d,i_3}}\end{bmatrix}. \tag{18}$$

Define the following attitude error and angular velocity errors:

$$\tilde{R}_i = R_{d,i}^T(R_{r,i}^TR_i) \quad \tilde{\omega}_i = \omega_i - R_i^TR_{r,i}\omega_{r,i} - \tilde{R}_i^T\omega_{d,i}, \tag{19}$$

then we obtain

$$\begin{aligned}\dot{\tilde{R}}_i &= \tilde{R}_iS(\tilde{\omega}_i)\\ J_i\dot{\tilde{\omega}}_i &= S(J_i\omega_i)\omega_i + \tau_i - J_iR_i^TR_{r,i}\dot{\omega}_{r,i} + J_iS(\tilde{\omega}_i)[\omega_i - \tilde{\omega}_i]\\ &\quad + J_i\tilde{R}_i^T[S(\omega_{d,i})R_{d,i}^T\omega_{r,i} - \dot{\omega}_{d,i}]\end{aligned}$$

Using Theorem 6 it follows that the controller

$$\tau_i = -S(J_i\omega_i)\omega_i + J_i R_i^T R_{r,i}\dot{\omega}_{r,i} - J_i S(\tilde{\omega}_i)[\omega_i - \tilde{\omega}_i] - K_{\omega i}\tilde{\omega}_i - J_i\tilde{R}_i^T[S(\omega_{d,i})R_{d,i}^T\omega_{r,i} - \dot{\omega}_{d,i}] + \sum_{j=1}^{3} k_{ji}(e_j \times \tilde{R}_i^T e_j), \quad (20)$$

with $K_{\omega i} = K_{\omega i}^T > 0$, and $k_{ji} > 0$ distinct (i.e., $k_{1i} \neq k_{2i} \neq k_{3i} \neq k_{1i}$), renders the resulting equilibrium point $(\tilde{R}_i, \tilde{\omega}_i) = (I, 0)$ ULES and UaGAS.

6. CASCADE ANALYSIS

In the previous two sections we determined respectively a desired control action for the position tracking error dynamics, and a controller for f_i and τ_i which asymptotically achieves this desired control action for the position tracking error dynamics. As a final step in our analysis we need to analyse stability of the cascaded system of attitude controller and desired position controller.

Consider the system dynamics (5) and the reference dynamics (6) in closed loop with the inputs (13), (16) and (20). Using (7) the resulting closed-loop system can be written as

$$\dot{p}_{e,i} = -S(\omega_f)p_{e,i} + v_{e,i} \quad (21a)$$

$$\dot{v}_{e,i} = -S(\omega_f)v_{e,i} - \alpha_i\sigma_{p_i}(p_{e,i}) - \beta_i\sigma_{v_i}(v_{e,i}) - \sum_{j=1}^{n}\left[a_{ij}\sigma_{p_{ij}}(p_{e,i} - p_{e,j}) + b_{ij}\sigma_{v_{ij}}(v_{e,i} - v_{e,j})\right] - \frac{f_i}{m_i}(I - \tilde{R}_i^T)e_3 \quad (21b)$$

$$\dot{\tilde{R}}_i = \tilde{R}_i S(\tilde{\omega}_i) \quad (21c)$$

$$J_i\dot{\tilde{\omega}}_i = -K_{\omega i}\tilde{\omega}_i + \sum_{j=1}^{3} k_{ji}(e_j \times \tilde{R}_i^T e_j). \quad (21d)$$

Differentiating (15) along solutions of (21) result in

$$\dot{V} \leq \sum_{i=1}^{n} v_{e,i}^T \frac{f_i}{m_i}(I - \tilde{R}_i)e_3 \leq M\sqrt{V}\sum_{i=1}^{n}\|I_3 - \tilde{R}_i\|.$$

Since ((21c),(21d)) is ULES we have

$$\sqrt{V(t)} - \sqrt{V(t_0)} \leq \bar{M}(\tilde{R}_i(t_0), \tilde{\omega}_i(t_0)).$$

So V is bounded, and therefore solutions of (21) are bounded. As a result, we obtain the following.

Proposition 15. Let $\sigma_{p_{ij}}(x) = \sigma_{p_{ji}}(x)$ be saturation functions guaranteeing that $\|u_{e,i}\| \leq \frac{f_{r,i}^{\min} - \epsilon_i}{m_i}$ for some $0 < \epsilon_i < f_{r,i}^{\min}$, $\alpha_i \geq 0$, $\beta_i \geq 0$, $\sum_{i=1}^{n}\alpha_i > 0$, and $\sum_{i=1}^{n}\beta_i > 0$. If Assumption 8 is satisfied, and ω_f and $\dot{\omega}_f$ are bounded, then the resulting closed-loop system (21) is UaGAS and the UAVs track the formation, i.e., we obtain (9). Furthermore, we obtain attitude tracking, i.e., (11).

Proof. Let G denote the almost global region of attraction of ((21c),(21d)). Restricting our stability analysis to $\mathbb{R}^{6n} \times G$ we can view (21) as a cascade and apply Theorem 7 to conclude UGAS on $\mathbb{R}^{6n} \times G$, and therefore UaGAS of (21) on $R^{6n} \times \mathrm{SO}(3)^{3n} \times R^{3n}$. Furthermore, from (14) we have $\tilde{u}_{e,i} \to 0$, which implies using (17) that $R_{d,i} \to I_3$ and therefore, using (19) and (10): $\tilde{R}_i \to I_3$. Similarly, since $\dot{\tilde{u}}_{e,i} \to 0$ also $\tilde{\omega}_i \to 0$.

Remark 16. Observe that for implementing the control action (20) we need, amongst others, $\dot{\omega}_{d,i}$ which, using (16) and (17), depends on $\ddot{u}_{e,i}$ for which we can find an analytic expression using (12) and (13). To that end we need to know both the signals $p_{e,j}$ and $v_{e,j}$, so implementing (20) can straightforwardly be done in case the graphs A and B are equal. In case the graphs A and B are not equal, we can replace the unavailable $v_{e,j}$ with estimates $\hat{v}_{e,j}$ coming from an observer.

The latter is also a solution in case only position estimates are communicated and no velocity information is shared among the UAVs. Then we take $B = A$ and use $\hat{v}_{e,j}$ instead of $v_{e,j}$.

7. SIMULATION RESULTS

In this section we demonstrate our theoretical results by means of a simple case study in which we illustrate a trade off between individual trajectory tracking and formation forming.

Let a formation frame $\mathcal{F}$ be given which moves with a velocity of 1 in the x-direction and is aligned with the inertial frame, i.e., $R_f = I_3$. Within this frame, let the desired formation shape be given by $p_{r,i}(t) = (3 - i)\,[1\ 1\ 0]^T$ for UAV $i \in \{1,2,3,4,5\}$. Assume identical UAVs with $m_i = 1$ and $J_i = 1$. This almost specifies the reference trajectories (6), except for the reference yaw rates of the UAVs. Since from the above we do have the final column of the rotation matrices $R_{d,i}$, we specify $R_{d,i}$ along the lines of (17b). For the communication topology, assume only information exchange between UAVs i and $i+1$, i.e., $a_{i,i+1} = b_{i,i+1} = 1$ and all other a_{ij} and b_{ij} are equal to 0. For the controller, assume all functions $\sigma.$ are given by $\sigma.(e) = e/\sqrt{1 + e^T e}$, and consider (attitude) controller gains $K_{\omega i} = I_3$, $k_{1i} = 0.9$, $k_{2i} = 1$, and $k_{3i} = 1.1$. Assume that all UAVs are initially at their reference trajectory, except for UAV 5 which initially is at $[-4\ -2\ 1]^T$.

We consider two scenarios. In the first scenario we take $\alpha_i = \beta_i = 3$, so that each UAV puts three times as much emphasis on following its own reference than on keeping the formation. In the second scenario we take $\alpha_i = \beta_i = 1/3$, so that each UAV puts three times as much emphasis on keeping the formation than on following its own reference. The resulting spatial trajectories are given in respectively Fig. 1 and Fig. 2. We clearly see in Fig. 1 that the emphasis of each UAV on its own trajectory results in non-social system behavior, whereas in Fig. 2 the UAVs that are already on their reference decide to leave their reference in order to form the desired formation first, after which they converge as a formation to the reference trajectory. Also observe that this holds for all UAVs, even though only UAV 4 receives the information of UAV 5 not being on its reference.

8. CONCLUSIONS

In this paper we presented a decentralized controller which achieves *uniform* almost global asymptotic stability of the formation tracking error dynamics for multiple UAVs while considering attitude dynamics on SO(3). By considering the attitude of a UAV on SO(3), we avoid singularities of Euler angles and ambiguity of quaternions, allowing for large angular maneuvers. Furthermore, by explicitly

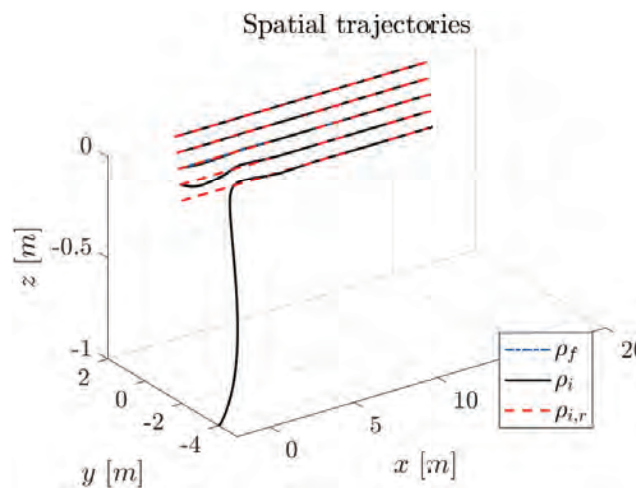

Fig. 1. Emphasis on own reference: no social behavior

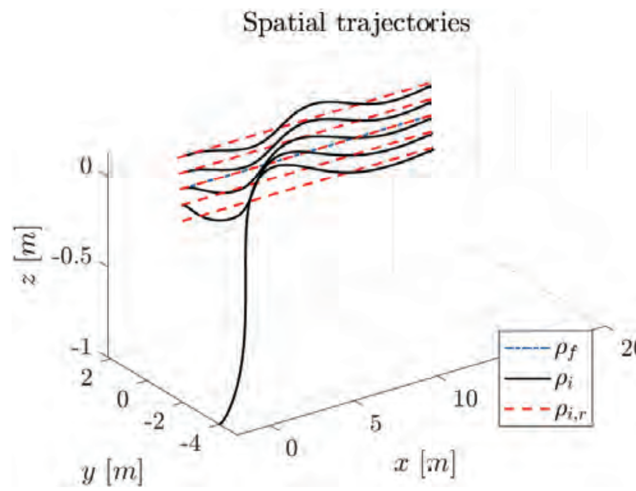

Fig. 2. Emphasis on formation: more social behavior

taking into account the constraint of non-zero total thrust in our controller design, our presented controller achieves an almost global result instead of only a local result.

We demonstrated our controller by means of simulations in which we illustrated a trade off between individual trajectory tracking and formation forming, i.e., between non-social and social behavior.

We currently have some students implementing the controller on an experimental setup. As mentioned in Remark 16 there is no need to exchange velocity information between UAVs since we can use an observer for reconstructing those. However, we also want to extend the state feedback controller to an output feedback controller, removing the need for measuring both translational and angular velocities.

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