Modeling and Control of Manufacturing Systems

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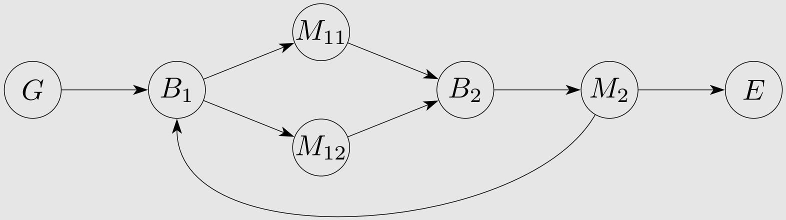
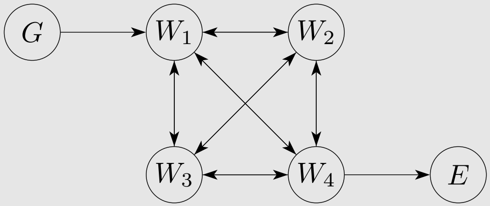
Outline

- Discrete Event Modeling
- Control Framework
- ODE Models
- Control (MPC)
- Modeling
  - PDE Models
  - Validation
- Control
  - Lyapunov based
  - Nonlinear MPC
- Conclusions

Manufacturing system





Manufacturing system: Issues

- setup
- finite buffers
- machine failure
- machine maintenance (software upgrade)
- operators (talking, breaks)

- WIP
- throughput
- flow time (cycle time, throughput time)

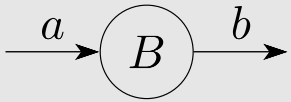
$$c_e = \frac{\sigma}{\mu}$$

Machine

$$\xrightarrow{a} M \xrightarrow{b}$$

$$
\begin{array}{l}
\mathsf{proc}\ M(a : ?\mathsf{lot}, b : !\mathsf{lot}, t_e, c_e^2 : \mathsf{real}) = \\
[\![\, u : \rightarrow \mathsf{real}, x : \mathsf{lot} \\
\mid u := \Gamma(t_e, c_e^2) \\
; *[\, \mathsf{true} \longrightarrow a?x; \Delta\sigma u; b!x \,] \\
]\!]
\end{array}
$$

Buffer



$$
\begin{array}{l}
\textsf{proc}\ B(a : ?\textsf{lot}, b : !\textsf{lot}) = \\
[\![\ x : \textsf{lot}, xs : \textsf{lot}^* \\
\mid xs := [\,] \\
;\ast[\ \textsf{true}; \qquad\qquad a?x \qquad \longrightarrow xs := xs +\!\!+ [x] \\
\quad [\!]\ \mathrm{len}(xs) > 0; b!\,\mathrm{hd}(xs) \longrightarrow xs := \mathrm{tl}(xs) \\
\quad] \\
]\!]
\end{array}
$$

Generator and Exit

type lot $=$ real

proc $G(a : !\text{lot}, t_a : \text{real}) = [\![*[\,\text{true} \longrightarrow a!\tau; \Delta t_a]\,]\!]$

proc $E(a : ?\text{lot}) =$
$[\![\, x : \text{lot}$
$|\ *[\,\text{true} \longrightarrow a?x$
$\quad ; !\text{“Flow time: ”}, x - \tau, \text{“\\ n”}$
$\quad]$
$]\!]$

Overall model

$$G \xrightarrow{a} B_1 \xrightarrow{b} M_1 \xrightarrow{c} B_2 \xrightarrow{d} M_2 \xrightarrow{e} E$$

$$
\begin{array}{l}
\textsf{clus}\ F() = \\
\ [\![\, a, b, c, d, e : -\textsf{lot} \\
\ \mid G(a, 3.0) \\
\parallel B(a, b) \parallel M(b, c, 1.0, 1.0) \\
\parallel B(c, d) \parallel M(d, e, 2.0, 1.0) \\
\parallel E(e) \\
\ ]\!]
\end{array}
$$

$$\textsf{xper} = [\![F()]\!]$$

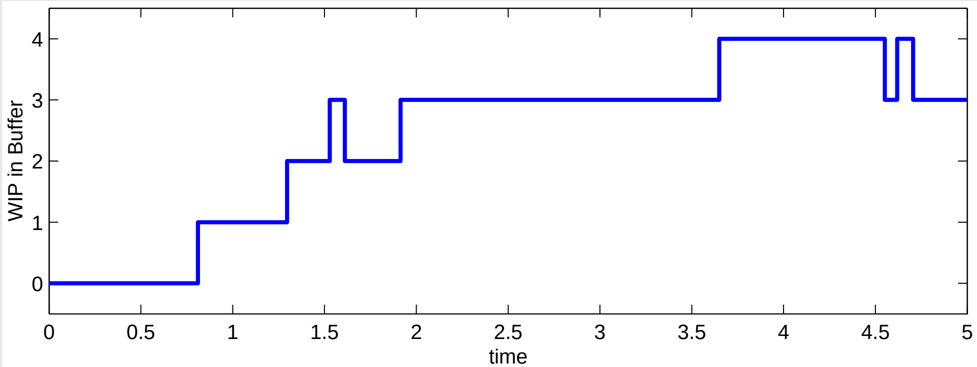
Modeling issues

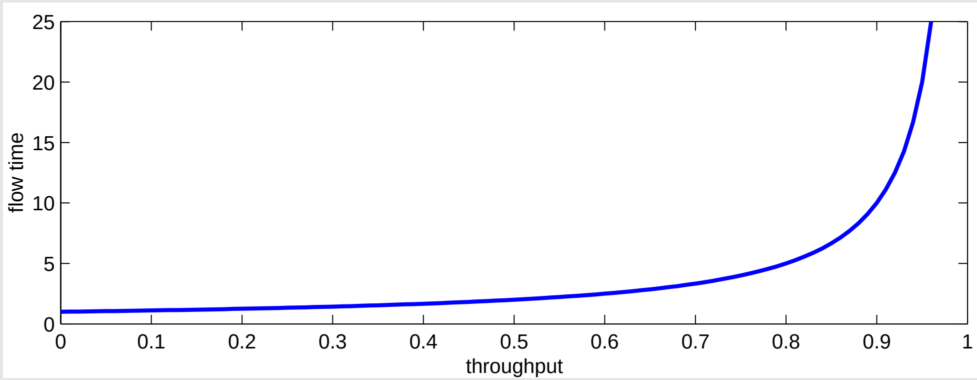
- setup
- finite buffers
- machine failure
- machine maintenance
- operators

Remarks

- Language χ (deterministic) is *formal language*
- Possible to proof properties

Typical signal + Nonlinear relation





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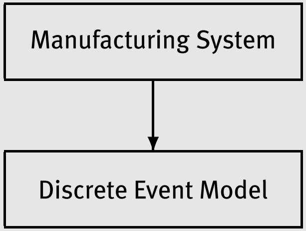
Control Framework

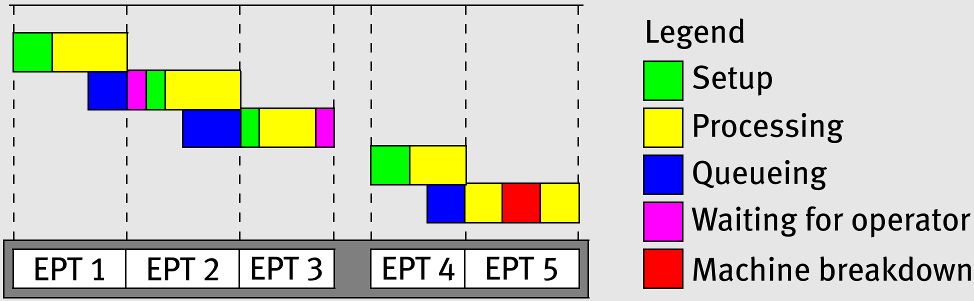
Manufacturing System

Control Framework

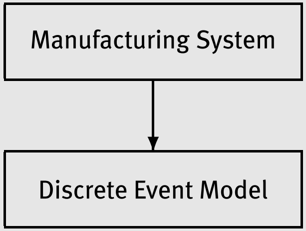


Effective Processing Time

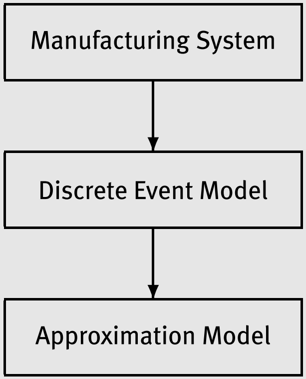
Time a lot experiences (from a logistic point of view)



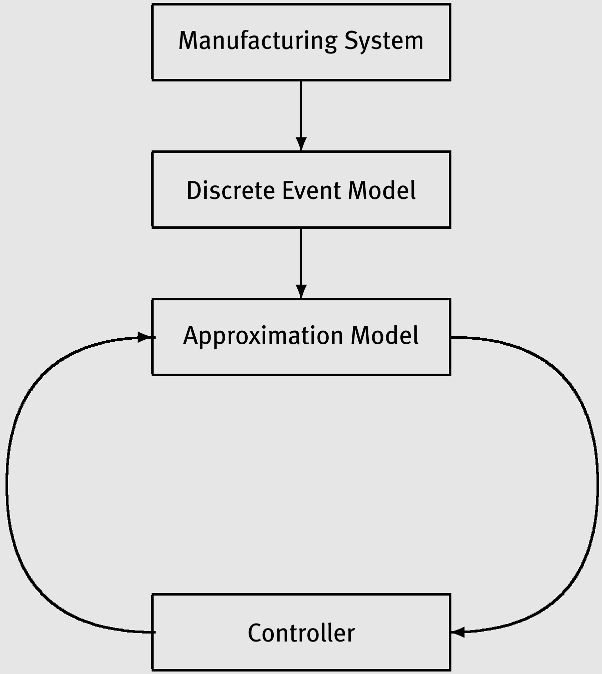
Control Framework



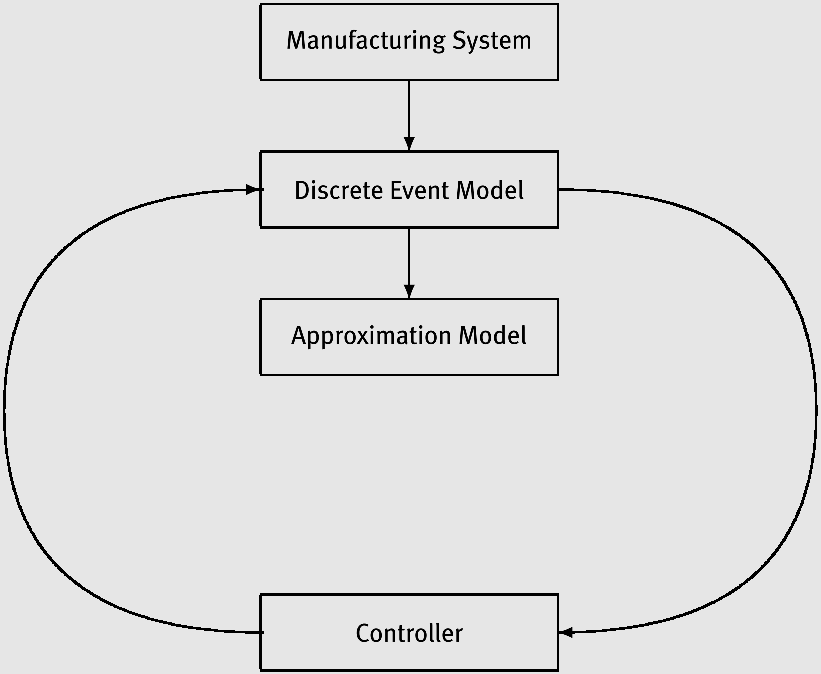
Control Framework



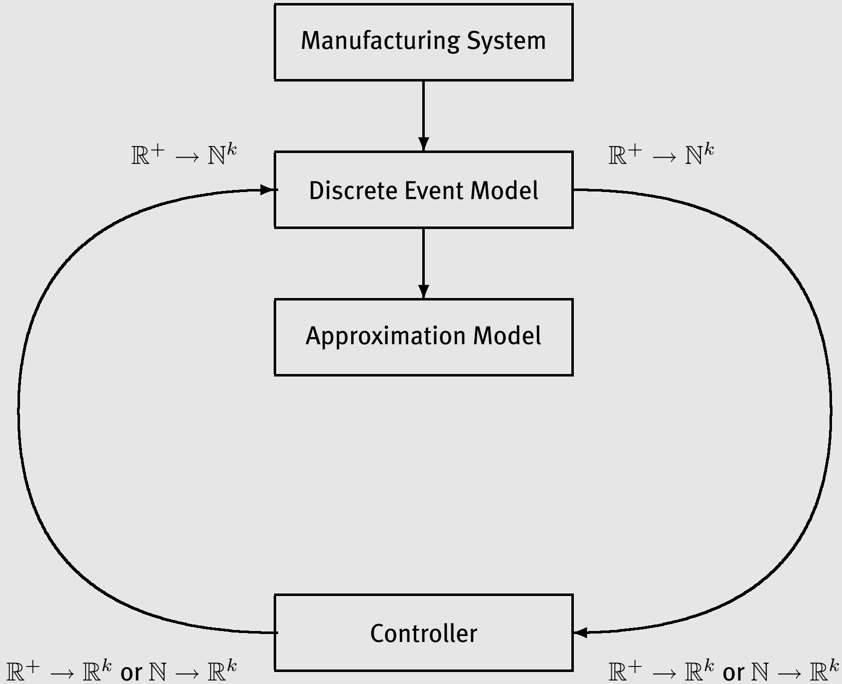
Control Framework



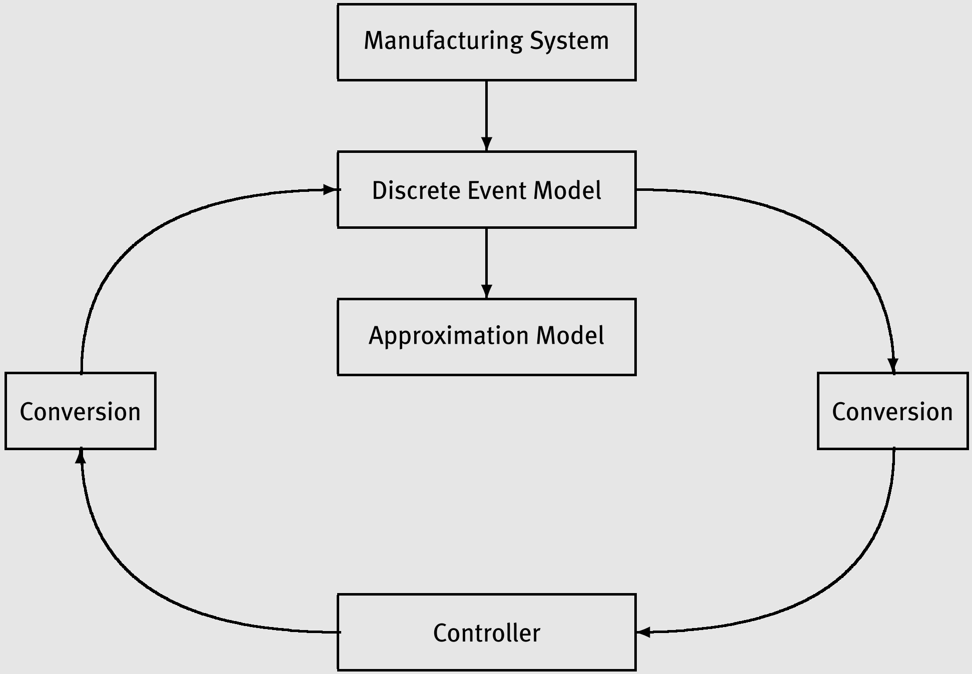
Control Framework



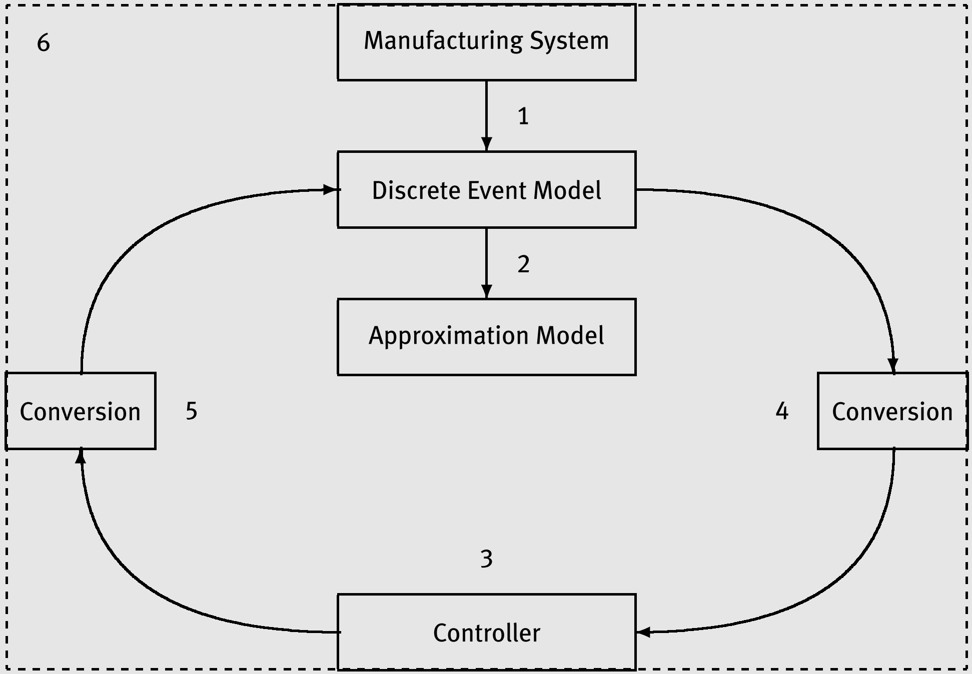
Control Framework



Control Framework



Control Framework



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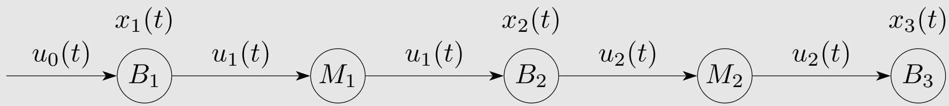
ODE Models (Fluid models)

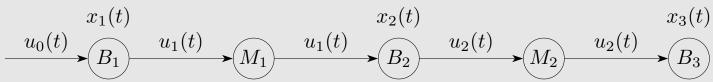
- Kimemia and Gershwin: Flow model
- Queueing theorists: Fluid models/Fluid queues



$$
\begin{aligned}
\dot{x}_1(t) &= u_0(t) - u_1(t) \\
\dot{x}_2(t) &= u_1(t) - u_2(t) \\
\dot{x}_3(t) &= u_2(t)
\end{aligned}
$$

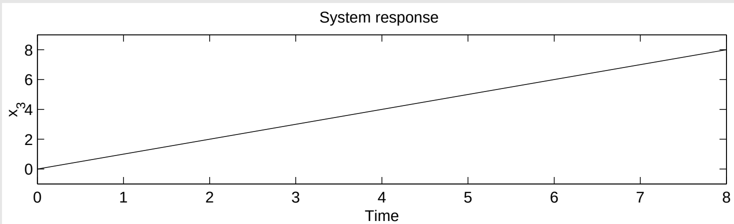
- Cassandras: Stochastic Fluid Model

Ramp up of fluid model



- Initially empty fab, $u_0 = 1$, $\mu_1 = \mu_2 = 1$.
- Machine produces whenever possible:

$$u_i = \begin{cases} 1 & \text{if } y_i > 0 \\ 0 & \text{otherwise} \end{cases} \qquad i \in \{1, 2\}.$$



Example: MPC

Model-based Predictive Control

- Discrete time model

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= h(x(k), u(k)) \end{aligned}$$

- Costs $\min_{u(\cdot)} J(y(k), u(k), k)$
- Prediction horizon (p)
- Control horizon (c, $c \geq p$)
- Yields $u(k), u(k+1), \ldots, u(k+p-1)$. Apply $u(k)$.
- At $k+1$: redo

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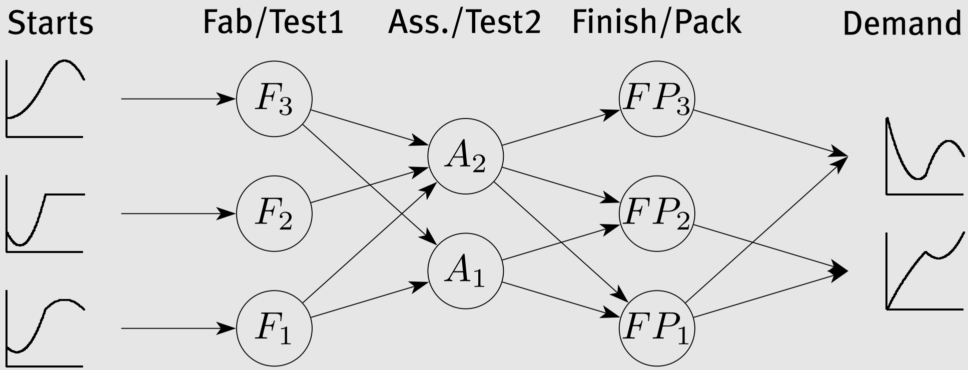
Modeling problem



Modeling for control (supply chain/mass production).

- Like to understand dynamics of factories
- Throughput, flow time, variance of flow time
- Answer questions like: How to perform ramp up?

Modeling problem

Some observations from practice:

- Quick answers (“What if . . . ”).
- A factory is (almost) never in steady state
- Throughput and flow time are related

We look for an approximation model that

- is computationally feasible,
- describes dynamics, and
- incorporates both throughput and flow time

Available models

Discrete Event

- Advantages
  - Include dynamics
  - Throughput and flow time related
- Disadvantage
  - Clearly infeasible for entire supply chain

Available models

Queueing Theory

- Advantages
  - Throughput and flow time related
  - Computationally feasible (approximations)
- Disadvantage
  - Only steady state, no dynamics

Available models

Fluid models (ODE)

- Advantages
  - Dynamical model
  - Computationally feasible
- Disadvantage
  - Only throughput incorporated in model, no flow time
    - No processing delay
    - Any throughput possible with zero inventory

Available models (conclusion)

- Discrete Event: Not computationally feasible
  Queueing Theory: No dynamics
  Fluid models: No flow time
- Need something else!
- Discrete event models (and queueing theory) have proved themselves. Can be used for verification!

Traffic flow: LWR model

Lighthill, Whitham ('55), and Richards ('56)

Traffic behavior on one-way road:

- density $\rho(x,t)$,
- speed $v(x,t)$,
- flow $u(x,t) = \rho(x,t)v(x,t)$.

Conservation of mass:

$$\frac{\partial \rho}{\partial t}(x,t) + \frac{\partial u}{\partial x}(x,t) = 0.$$

Static relation between flow and density:

$$u(x,t) = S(\rho(x,t)).$$

Modeling manufacturing flow

- density $\rho(x,t)$,
- speed $v(x,t)$,
- flow $u(x,t) = \rho(x,t)v(x,t)$,
- Conservation of mass: $\frac{\partial \rho}{\partial t}(x,t) + \frac{\partial \rho v}{\partial x}(x,t) = 0.$
- Boundary condition: $u(0,t) = \lambda(t)$

Modeling manufacturing flow

Armbruster, Marthaler, Ringhofer (2002):

- Single queue: $v(x,t) = \frac{\mu}{1+\int_0^1 \rho(s,t)\,\mathrm{d}s}$
- Single queue: $\frac{\partial \rho v}{\partial t}(x,t) + \frac{\partial \rho v^2}{\partial x}(x,t) = 0$
  $\rho v^2(0,t) = \frac{\mu \cdot \rho v(0,t)}{1+\int_0^1 \rho(s,t)\,\mathrm{d}s}$
- Re-entrant: $v(x,t) = v_0 \left(1 - \frac{\int_0^1 \rho(s,t)\,\mathrm{d}s}{W_{\text{max}}}\right)$
- Re-entrant: $\frac{\partial \rho v}{\partial t}(x,t) + \frac{\partial \rho v^2}{\partial x}(x,t) = 0$
  $\rho v^2(0,t) = v_0 \left(1 - \frac{\int_0^1 \rho(s,t)\,\mathrm{d}s}{W_{\text{max}}}\right) \cdot \rho v(0,t)$

Lefeber (2003):

- Line of m identical queues: $v(x,t) = \frac{\mu}{m+\rho(x,t)}$

Validation studies: Study I

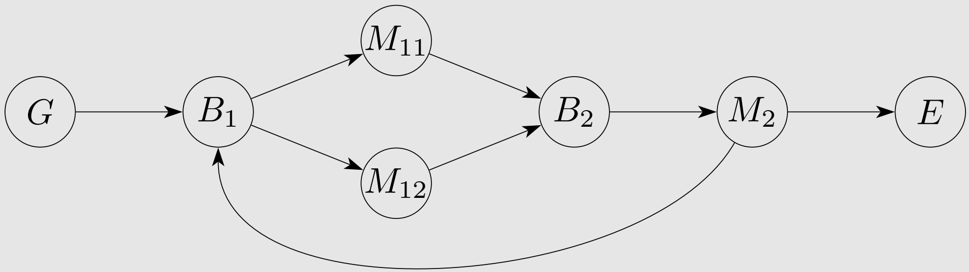
- Line of 15 identical workstations
- infinite buffers (FIFO)
- Processing times: exponential (mean 1.0)
- Inter arrival times: exponential (mean $1/\lambda$)
- From one steady state to the other
  - ramp up: from initially empty to 25%, 50%, 75%, 90%, 95% utilization
- Batches of 1000 experiments
- 1000 batches (99% confidence interval: relative width less than 0.01 for utilization of 95%)
- MOVIES

General observations

- Steady state performance well described
- Time to reach steady state ill described
- Amount of lots produced before reaching steady state (most cases) relatively small
- Homogeneous velocity (model 1) results in strange behavior of throughput (for manufacturing line)
- Some typical numbers:

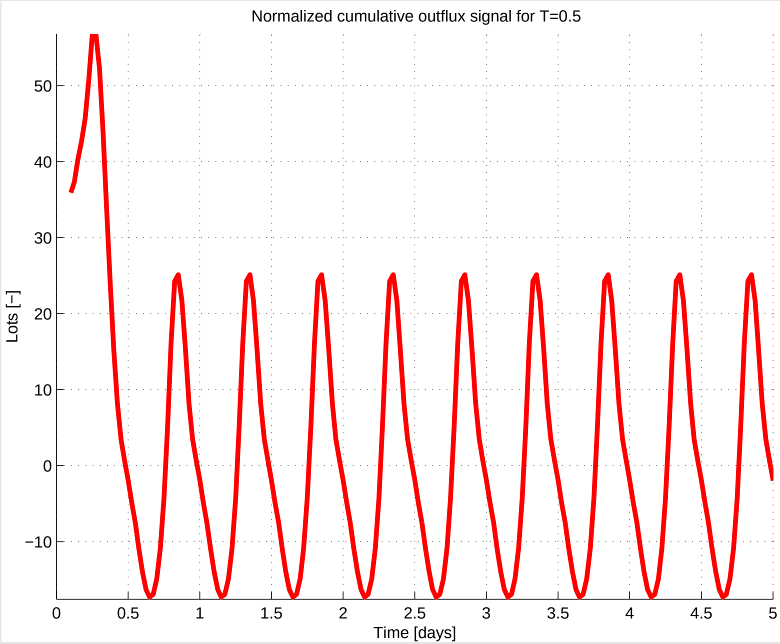
Batch run Discrete Event:	15 minutes
Simulation run Discrete Event:	250 hours
Simulation run PDE:	15 seconds

Validation studies: Study II

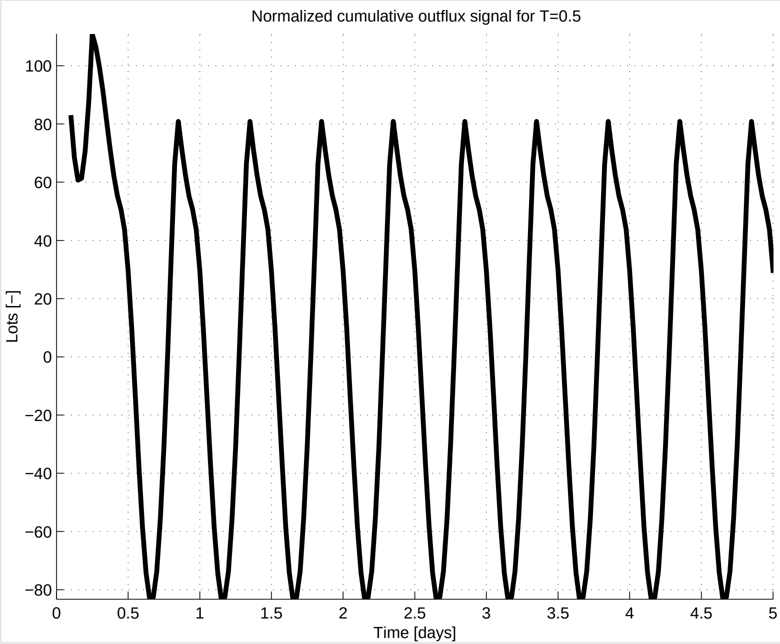


- Rentrant: 4 times
- 5 WS: 22 identical machines (WS 3: 21)
- Deterministic processing times
- Oscillating inflow, different frequencies
- Buffer policies: FIFO, push, pull

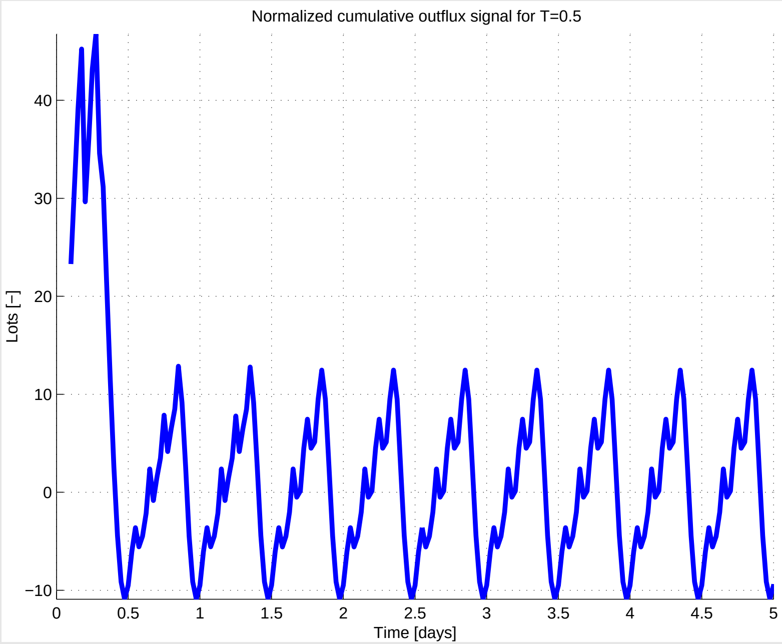
Outs: FIFO, period 0.5 day



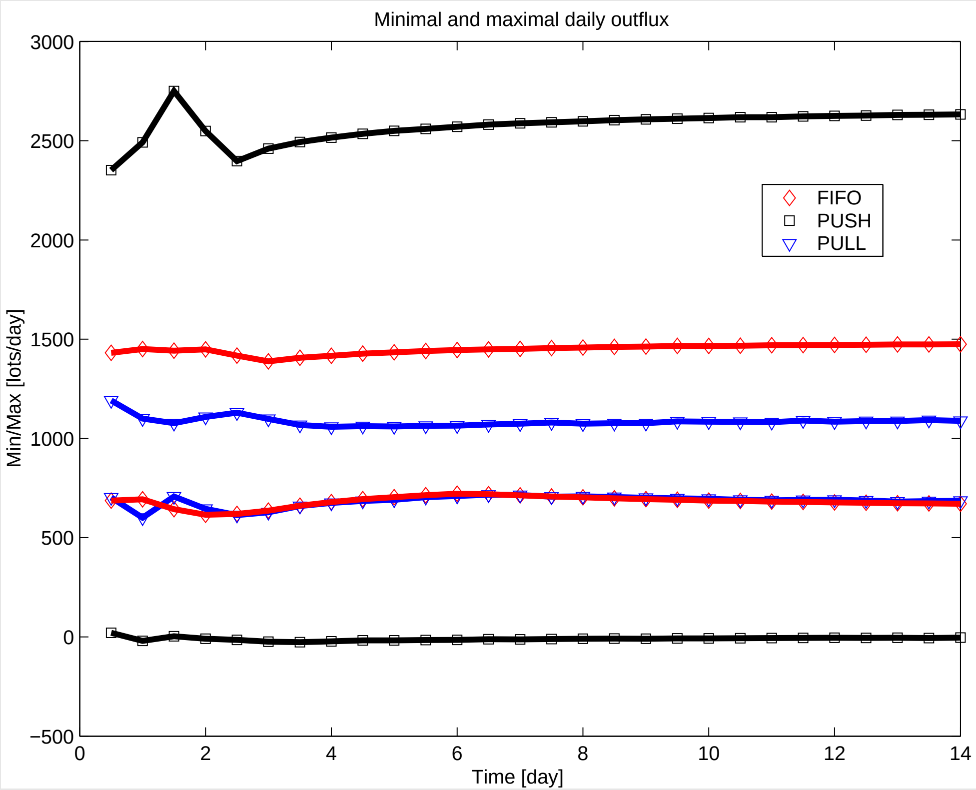
Outs: push, period 0.5 day



Outs: pull, period 0.5 day



Results (outflux)



Validation study II: conclusions

- Outflux is oscillating (with frequency of influx)
- Almost no resonance effects
- Buffer policy *does* matter

Conclusion of validation studies

Search for valid PDE models continues...

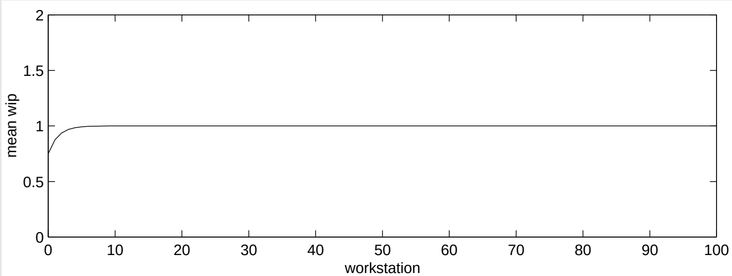
Desired Properties

- No backward-flow allowed
- No negative density
- Stable steady states
  - constant feed rate $\longrightarrow$ equilibrium
  - equilibrium meets relations queueing theory

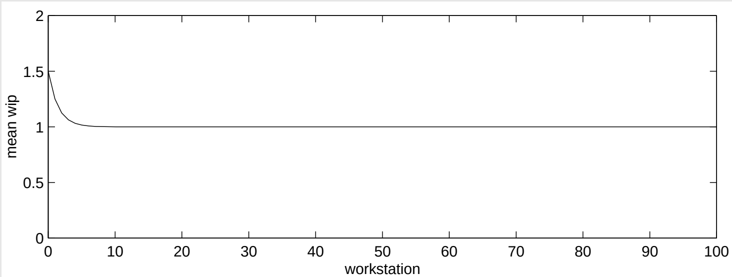
Desired Properties (II)

100 machines, $\mu = 1$, exponential. Utilization: 50%.

- Regular arrivals: $c_a^2 = 0$



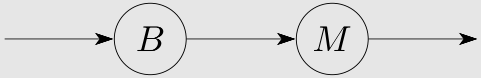
- Irregular arrivals: $c_a^2 = 3$



Desired Properties (III)

Variability needs to be included. However, ...

1 machine, $\mu = 1$, exponential



- Push control: exponential arrivals. Utilization 50%
  - Throughput: 0.5 lots per unit time
  - Cycle time: 2 hours
  - Mean WIP: 1 lot
- CONWIP control: WIP=1
  - Throughput: 1 lots per unit time
  - Cycle time: 1 hours
  - Mean WIP: 1 lot

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Control: example

- Conservation of mass: $\frac{\partial\rho}{\partial t}(x,t) + \frac{\partial\rho v}{\partial x}(x,t) = 0.$
- Line of m identical queues: $v(x,t) = \frac{\mu}{m+\rho(x,t)}$
- Initial condition: $\rho(x,0) = \rho_0(x)$

- Input: $u(0,t) = \lambda_{\text{in}}(t)$
- Outputs: $\lambda_{\text{out}}(t) = u(1,t)$, $w(t) = \int_0^1 \rho(x,t)\,\mathrm{d}t$

How to reach desired steady state?

Control: $\lambda_{\text{in}}(t) = f(\lambda_{\text{out}}(t), w(t))$

Lyapunov based controller design

Lyapunov function candidate

$$V = \frac{2}{3\mu m}\int_0^1 \left[(m+\rho(s,t))^3 - (m+\rho_{\mathsf{ss}})^3\right]^2 \mathrm{d}s,$$

Differentiating along dynamics

$$\dot{V} = [m+\rho_0(t)]^4 - [m+\rho_1(t)]^4 + 4[m+\rho_{\mathsf{ss}}]^3[\rho_1(t)-\rho_0(t)]$$

Minimizing $\dot{V}$ w.r.t. $\rho_0(t)$ yields

$$\rho_0(t) = \rho_{\mathsf{ss}}$$

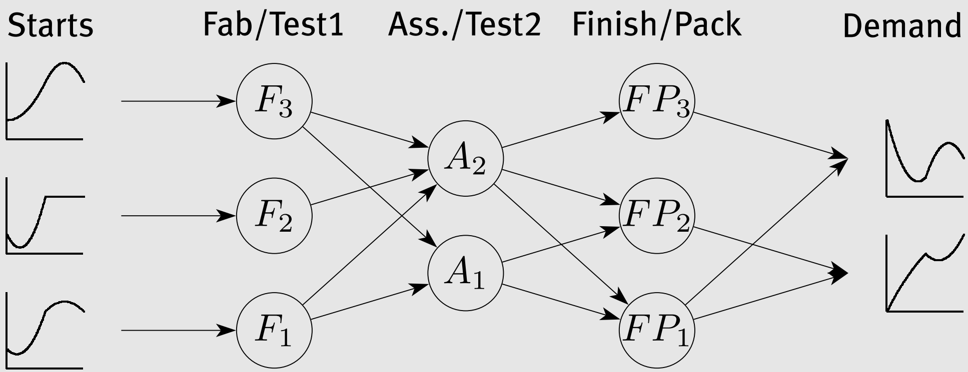
resulting in

$$\dot{V} = -\frac{1}{3}[\rho_1(t)-\rho_{\mathsf{ss}}]^4 - \frac{2}{3}[\rho_1(t)+2\rho_{\mathsf{ss}}+3m]^2[\rho_1(t)-\rho_{\mathsf{ss}}]^2 \leq 0$$

Promising developments

A.J. van der Schaft, B. Maschke (2003):

Hamiltonian framework (boundary control of PDEs)



MPC based controller design

Approximation model (nonlinear)

$$x_1(k+1) = x_1(k) - \frac{\mu x_1(k)}{m + x_1(k)} + \lambda_{\mathsf{in}}(k)$$

$$x_2(k+1) = x_2(k) - \frac{\mu x_2(k)}{m + x_2(k)} + \frac{\mu x_1(k)}{m + x_1(k)}$$

$$\vdots$$

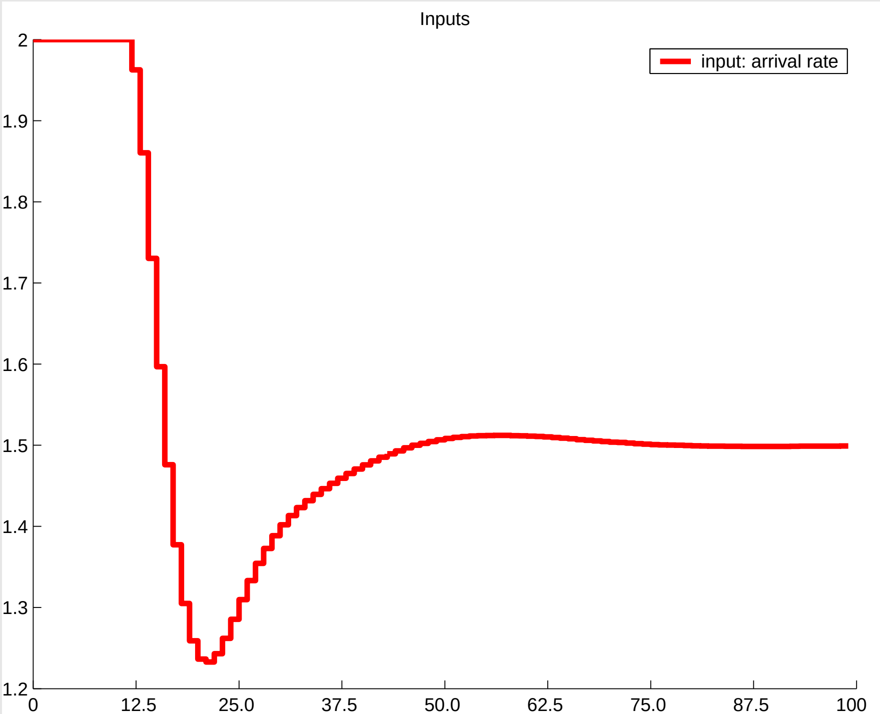
$$x_m(k+1) = x_m(k) - \frac{\mu x_m(k)}{m + x_m(k)} + \frac{\mu x_{m-1}(k)}{m + x_{m-1}(k)}$$

$$y(k) = \frac{\mu x_m(k)}{m + x_m(k)}$$

MPC based controller design

- Number of machines $m = 10$
- Mean processing time: 0.5h
- Desired $u = 0.75$ (1.5 lot per h)
- Initial WIP $x_i(0) = 0$
- Prediction horizon $p = 100$h
- Control horizon $p = 5$h
- Control constant over periods of 1h
- Time sampling: 40 steps per 1h

MPC based controller design



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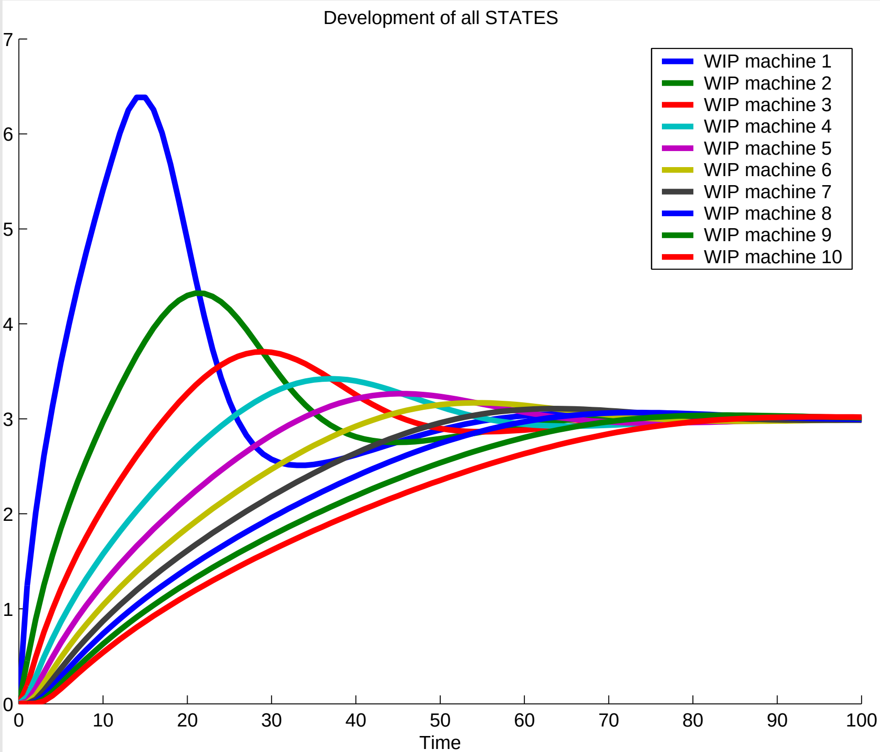
MPC based controller design



Conclusions

- Control framework (EPT)
- Modeling
  - NOT: Discrete event, Queueing theory, Fluid models
  - Possible: PDE-models
    - Correct steady state behavior
    - Better description transient needed
    - Resonance needs better study
    - Second moment and correlation needs to be included
    - Queueing theory, discrete event models can be used for validation of PDE models
- Next step: PDE-based controller design