

Modeling and control of manufacturing systems

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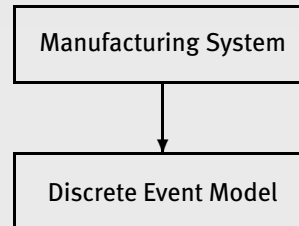
Outline

- Control framework
- Modeling

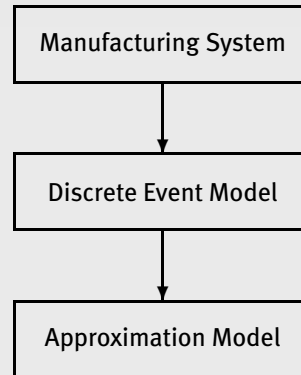
Control Framework

Manufacturing System

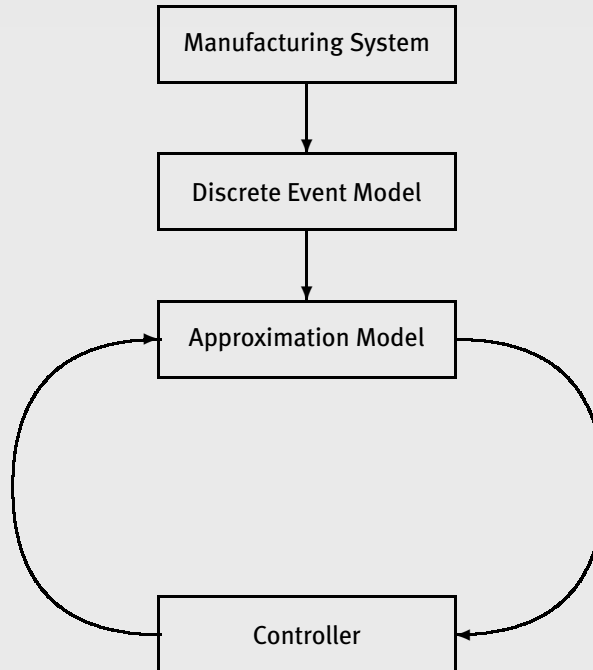
Control Framework



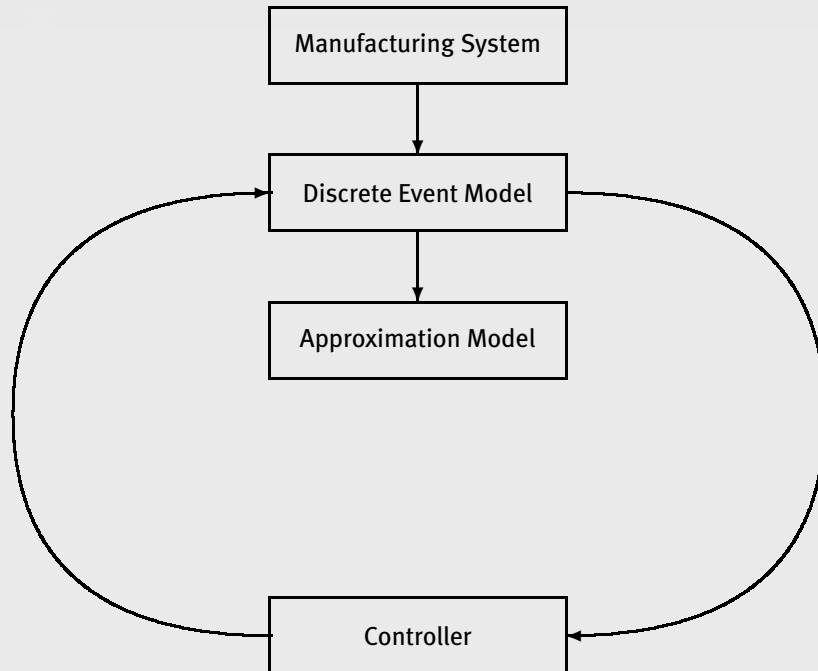
Control Framework



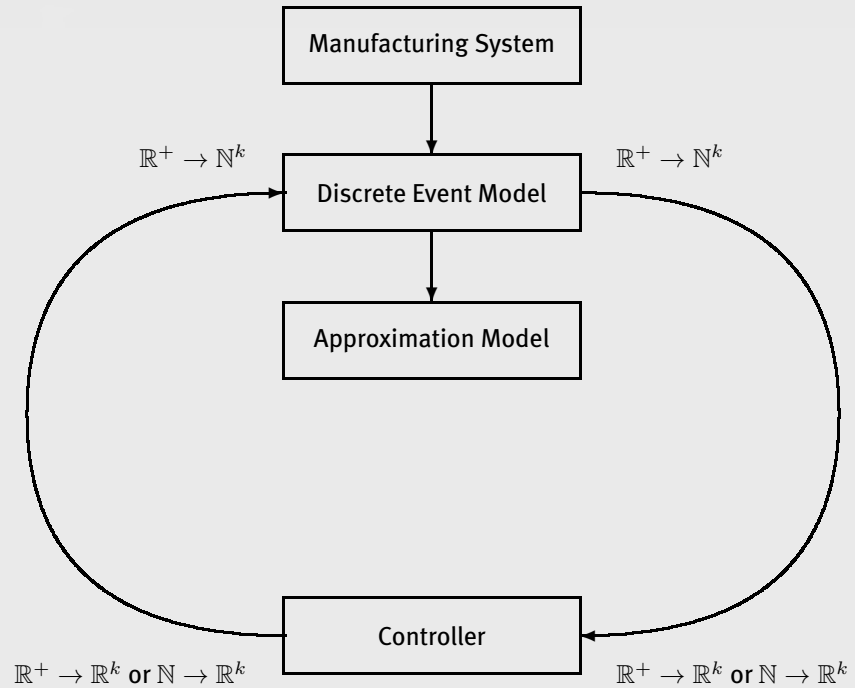
Control Framework



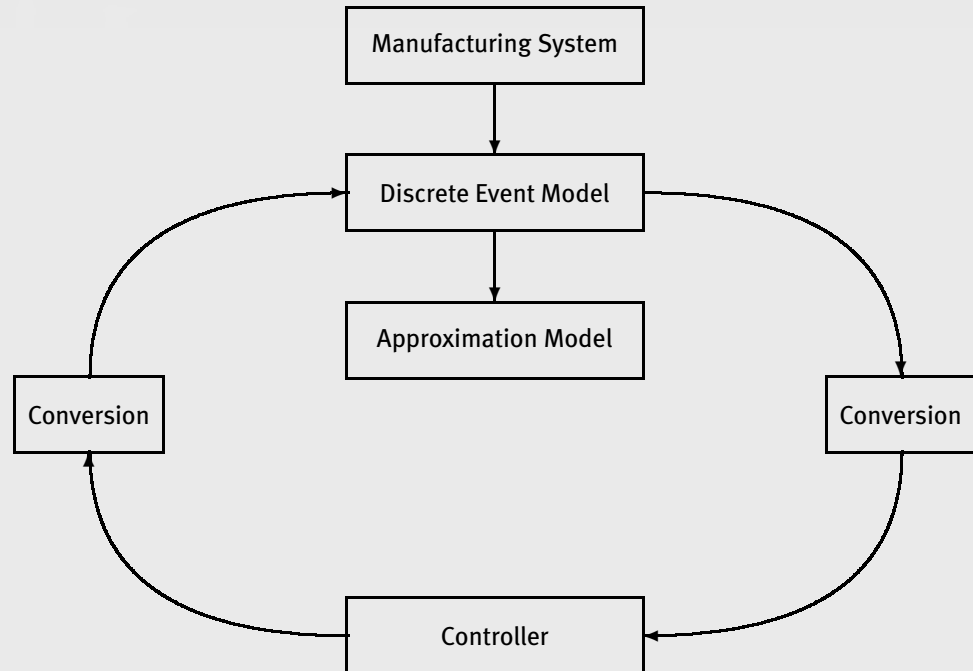
Control Framework



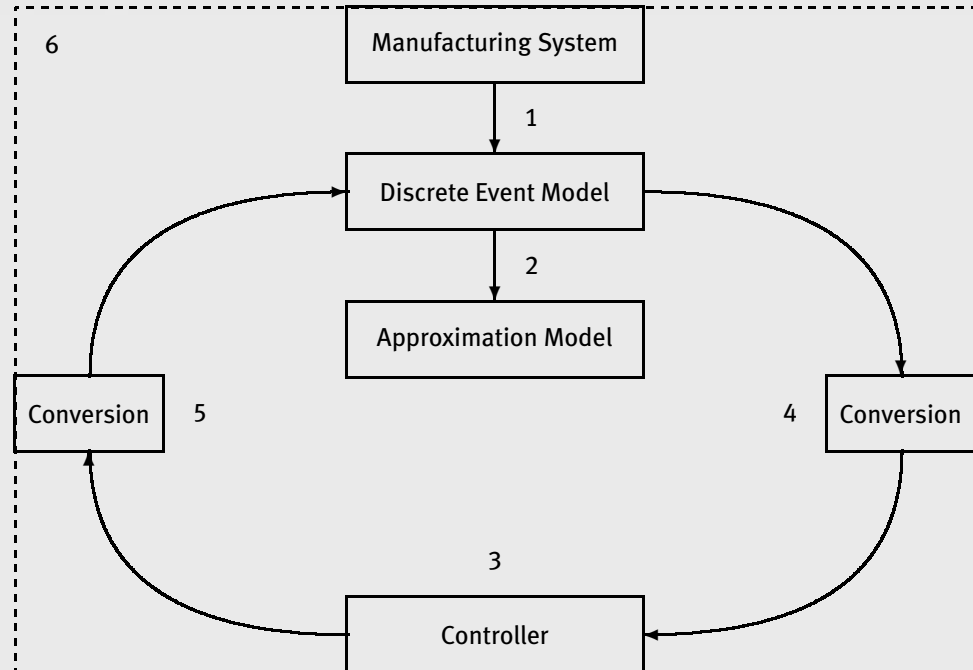
Control Framework



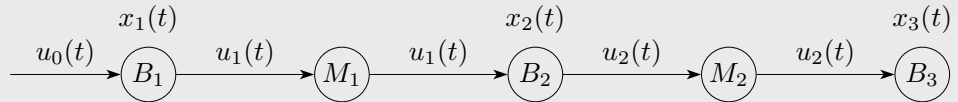
Control Framework



Control Framework (final)



Fluid models



$$\dot{x}_1 = u_0 - u_1$$

$$\dot{x}_2 = u_1 - u_2$$

$$\dot{x}_3 = u_2$$

or

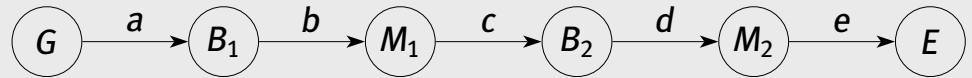
$$x_1(k+1) = x_1(k) + u_0(k) - u_1(k)$$

$$x_2(k+1) = x_2(k) + u_1(k) - u_2(k)$$

$$x_3(k+1) = x_3(k) + u_2(k)$$

Extensions

- Add a delay



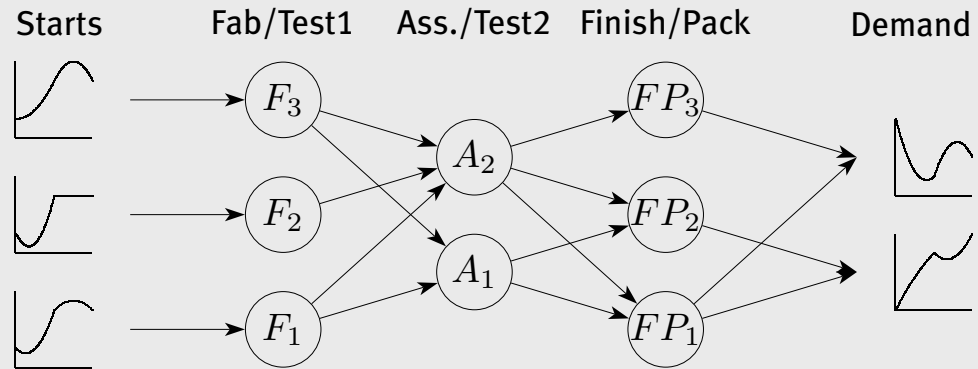
$$\dot{x}_1(t) = u_0(t) - u_1(t)$$

$$\dot{x}_2(t) = u_1\left(t - \frac{1}{\mu_1}\right) - u_2(t)$$

$$\dot{x}_3(t) = u_2\left(t - \frac{1}{\mu_2}\right)$$

- Padé approximation
- Hybrid model (switching logic)
- Stochastic differential equations (add noise)

Modeling problem



Modeling for control (supply chain/mass production).

- Like to understand dynamics of factories
- Throughput, cycle time, variance of cycle time
- Answer questions like: How to perform ramp up?

Available models

- Discrete Event: Not computationally feasible
- Queueing Theory: No dynamics
- Fluid models: No cycle time
- Need something else!
- Discrete event models (and queueing theory) have proved themselves. Can be used for verification!

PDE-model

- density $\rho(x, t)$,
- speed $v(x, t)$,
- flow $u(x, t) = \rho(x, t)v(x, t)$,
- Conservation of mass: $\frac{\partial \rho}{\partial t}(x, t) + \frac{\partial \rho v}{\partial x}(x, t) = 0$.
- Boundary condition: $u(0, t) = \lambda(t)$
- Static relation: $v(x, t) = \frac{\mu}{1 + \rho(x, t)}$

Validation studies showed: Much work need to be done!

Max-Plus algebra

Define

$$p \oplus q = \max(p, q)$$

$$p \otimes q = p + q$$

Notice that

$$p + \max(q, r) = \max(p + q, p + r)$$

$$p \otimes (q \oplus r) = (p \otimes q) \oplus (p \otimes r)$$

We can write

$$x(k+1) = A \otimes x(k) \oplus B \otimes u(k)$$

$$y(k) = C \otimes x(k) \oplus D \otimes u(k)$$

Future Research

- Derive ‘good’ models (throughput, (variance) cycle time)
- controller design
- observer design (E/A conversion)
- A/E conversion (scheduling)
- Trade-offs