On the adaptive tracking control of nonholonomic systems

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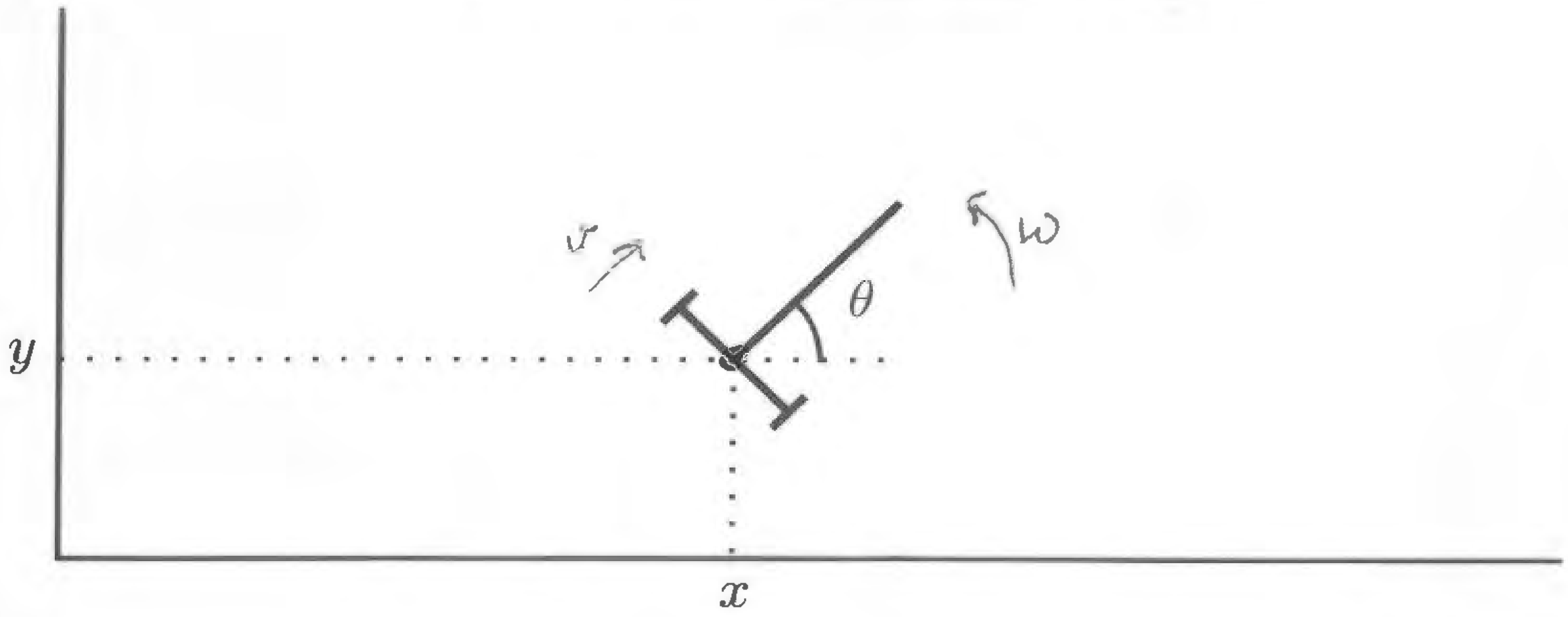
Erjen Lefeber (supervisor: Henk Nijmeijer)
Department of Systems Signals and Control, Faculty of
Mathematical Sciences, University of Twente, Enschede
{A.A.J.Lefeber,H.Nijmeijer}@math.utwente.nl

Outline

- A tracking control problem
- How to formulate the *adaptive* tracking control problem?
- A first attempt
- Evaluation
- Proposed formulation of adaptive tracking control problem
- Further research

A simple kinematic model



$$
\begin{aligned}
\dot{x} &= v\cos\theta \\
\dot{y} &= v\sin\theta \\
\dot{\theta} &= \omega
\end{aligned}
$$

Formulating the tracking problem

Reference robot:

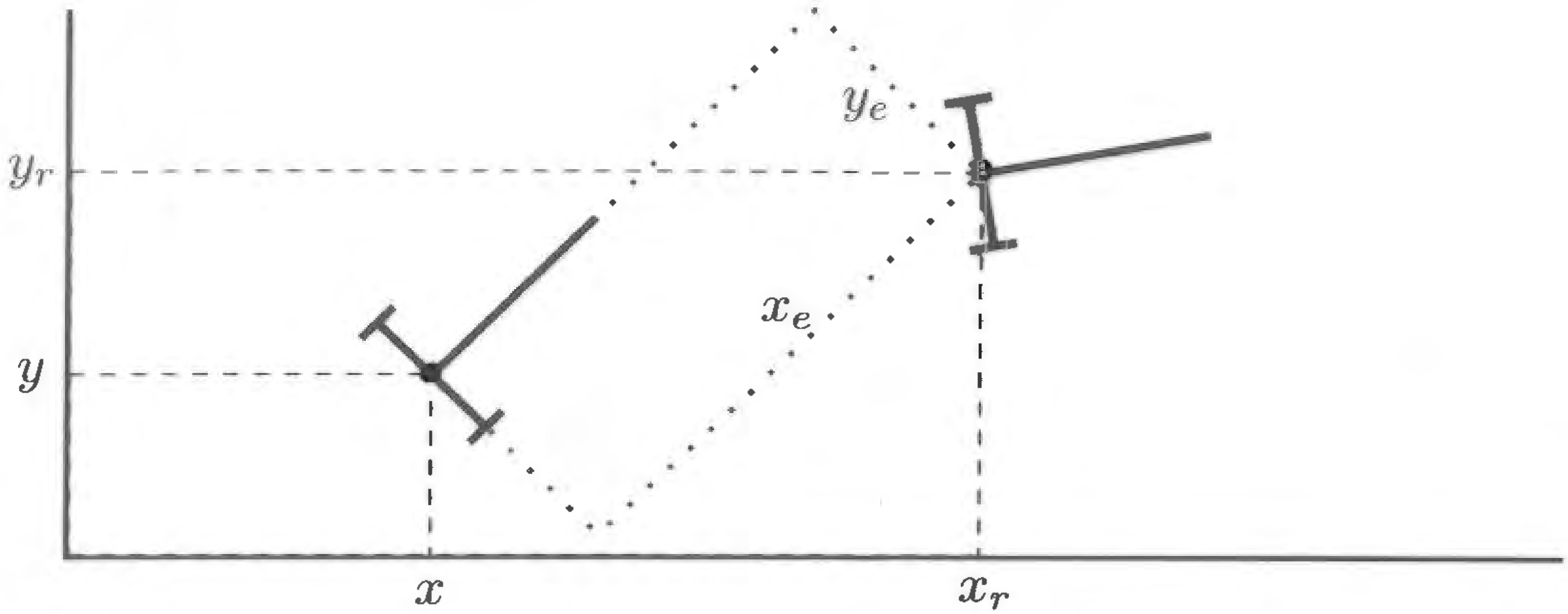
$$
\begin{aligned}
\dot{x}_r &= v_r \cos\theta_r \\
\dot{y}_r &= v_r \sin\theta_r \\
\dot{\theta}_r &= \omega_r
\end{aligned}
$$

Find control laws

$$
\begin{aligned}
v &\equiv v(\underbrace{x, y, \theta}, \underbrace{x_r, y_r, \theta_r, v_r, \omega_r}) \quad \dot{x}_r, \dot{y}_r, \ldots \\
\omega &\equiv \omega(\underbrace{x, y, \theta}, \underbrace{x_r, y_r, \theta_r, v_r, \omega_r})
\end{aligned}
$$

that yield

$$
\lim_{t\to\infty} |x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| = 0
$$



Define new coordinates (Kanayama, Kimura, Miyazaki, Noguchi (1990))

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}$$

Several solutions have been found, e.g Jiang, Nijmeijer :

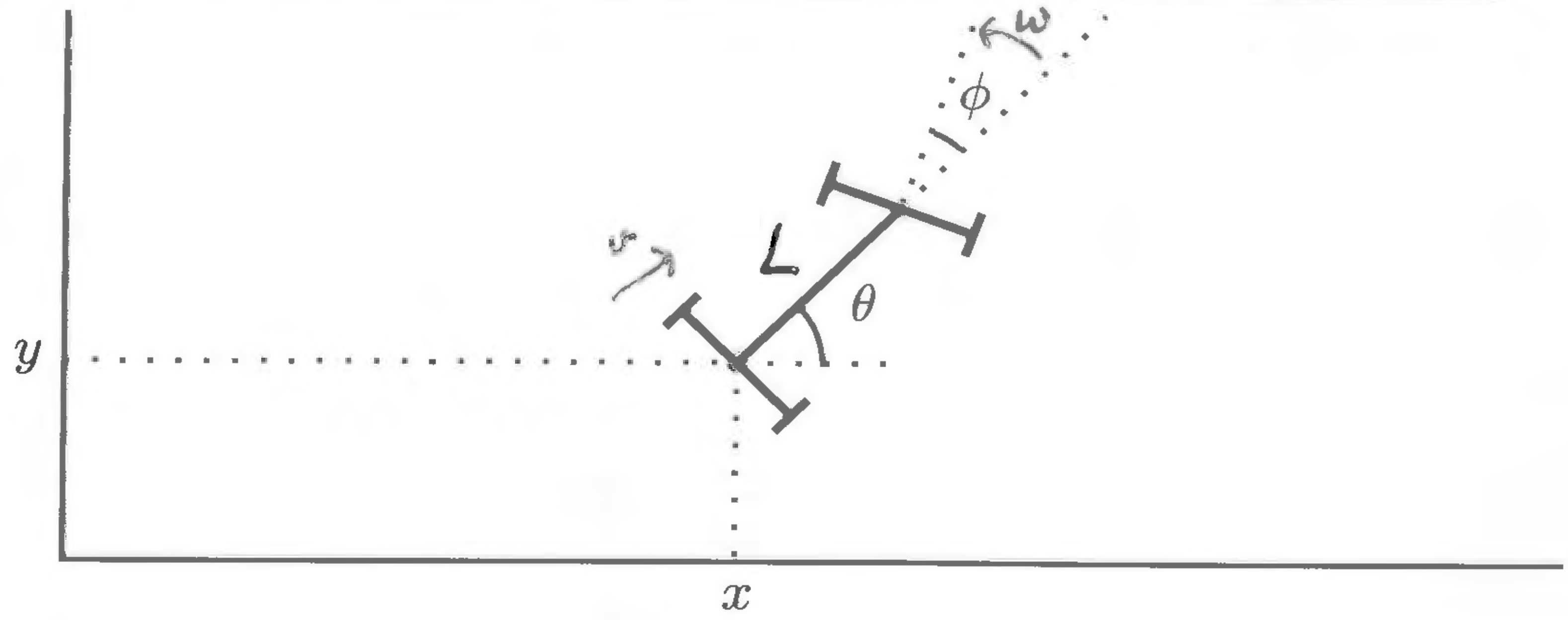
$$\begin{aligned} v &= \overset{v_r\cos\theta_e}{\cancel{v_r}} + c_x x_e + \gamma y_e v_r \int_0^1 \cos(s\theta_e)ds \qquad && \gamma, c_x > 0 \\ \omega &= \omega_r + c_\theta \theta_e && c_\theta > 0 \end{aligned}$$

yields

$$\lim_{t\to\infty} |x_e| + |y_e| + |\theta_e| = 0$$

provided either $v_r \not\to 0$ or $\omega_r \not\to 0$.

A simple kinematic model containing a parameter



$$\begin{aligned}\dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \frac{v}{L}\tan\phi \\ \dot{\phi} &= \omega\end{aligned}$$

Formulating the tracking problem

Reference robot:

$$
\begin{aligned}
\dot{x}_r &= v_r \cos\theta_r \\
\dot{y}_r &= v_r \sin\theta_r \\
\dot{\theta}_r &= \frac{v_r}{L}\tan\phi_r \\
\dot{\phi}_r &= \omega_r
\end{aligned}
$$

Find control laws

$$
\begin{aligned}
v &\equiv v(\underbrace{x, y, \theta, \phi}, \underbrace{x_r, y_r, \theta_r, \phi_r, v_r, \omega_r}) \\
\omega &\equiv \omega(\underbrace{x, y, \theta, \phi}, \underbrace{x_r, y_r, \theta_r, \phi_r, v_r, \omega_r})
\end{aligned}
$$

$\dot{x}_r, \dot{y}_r, \cdots$

that yield

$$
\lim_{t\to\infty} |x^{(k)} - x_r^{(k)}| + |y^{(k)} - y_r^{(k)}| + |\theta^{(k)} - \theta_r^{(k)}| + |\phi^{(k)} - \phi_r^{(k)}| = 0
$$

How to formulate the *adaptive* state tracking problem?

Reference robot:

$$
\begin{aligned}
\dot{x}_r &= v_r \cos\theta_r \\
\dot{y}_r &= v_r \sin\theta_r \\
\dot{\theta}_r &= \frac{v_r}{L}\tan\phi_r \\
\dot{\phi}_r &= \omega_r
\end{aligned}
$$

Find control laws for v and ω that yield

$$
\lim_{t\to\infty} |x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| + |\phi(t) - \phi_r(t)| = 0
$$

where L is an unknown parameter.

Problem

How to specify the dynamics of the reference robot:

$$\begin{aligned}
\dot{x}_r &= v_r \cos\theta_r && (1)\\
\dot{y}_r &= v_r \sin\theta_r && (2)\\
\dot{\theta}_r &= \frac{v_r}{L}\tan\phi_r && (3)\\
\dot{\phi}_r &= \omega_r && (4)
\end{aligned}$$

Not by specifying $x_r(t)$, $y_r(t)$, $\theta_r(t)$, $\phi_r(t)$! Since:

$$\begin{aligned}
v_r &= \dot{x}_r \cos\theta_r + \dot{y}_r \sin\theta_r\\
L &= \frac{v_r}{\dot{\theta}_r}\tan\phi_r
\end{aligned}$$

A first attempt

We know that $[x_r(t), y_r(t)]$ is flat output, i.e.

$$[x_r, y_r, \theta_r, \phi_r, v_r, \omega_r] = f(x_r, \dot{x}_r, \ddot{x}_r, x_r^{(3)}, y_r, \dot{y}_r, \ddot{y}_r, y_r^{(3)})$$

This can be seen as follows:

$$\begin{aligned} \dot{x}_r &= v_r \cos\theta_r \\ \dot{y}_r &= v_r \sin\theta_r \\ \dot{\theta}_r &= \frac{v_r}{L}\tan\phi_r \\ \dot{\phi}_r &= \omega_r \end{aligned} \qquad \begin{aligned} \theta_r &= \arctan(\dot{y}_r/\dot{x}_r) \\ v_r &= \sqrt{\dot{x}_r^2 + \dot{y}_r^2} \\ \phi_r &= \arctan(L\dot{\theta}_r/v_r) \\ \omega_r &= \dot{\phi}_r \end{aligned}$$

So by specifying $[x_r(t), y_r(t)]$ we can recover the entire state.

What signals can we use in control law?

If we specify $x_r(t), y_r(t)$ we obtain

$$\begin{aligned}
\theta_r &= \arctan(\dot{y}_r/\dot{x}_r) \\
v_r &= \sqrt{\dot{x}_r^2 + \dot{y}_r^2} \\
\phi_r &= \arctan(L\dot{\theta}_r/v_r) \\
\omega_r &= \dot{\phi}_r
\end{aligned}$$

What signals can we 'use' if L is unknown?

- $x_r(t)$, $y_r(t)$, $\theta_r(t)$, $v_r(t)$ are independent of L
- $\phi_r(t)$, $\omega_r(t)$ dependent of L.

A way to formulate the adaptive tracking problem

By specifying $x_r(t), y_r(t)$ we specify the entire reference dynamics:

- $x_r(t)$, $y_r(t)$, $\theta_r(t)$, $v_r(t)$ independent of L
- $\phi_r(t)$, $\omega_r(t)$ dependent of L.

Find control laws

$$
\begin{aligned}
v &\equiv v(x, y, \theta, \phi, x_r, y_r, \theta_r, v_r) \\
\omega &\equiv \omega(x, y, \theta, \phi, x_r, y_r, \theta_r, v_r)
\end{aligned}
$$

$\dot{x}_r, \dot{y}_r, \dots$

NOT $\phi_r, \omega_r, \dot{\phi}_r, \dots$!

that yield

$$\lim_{t\to\infty} |x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| + |\phi(t) - \phi_r(t)| = 0$$

↳ $\phi \to \phi_r$ even though we don't know ϕ_r (since it "depends" on the unknown L)

Do we need flatness?

Two properties of a flat output are:

- Dimension of flat output = Number of (independent) inputs,
- We can reconstruct the state and inputs without integrating.

Example 1

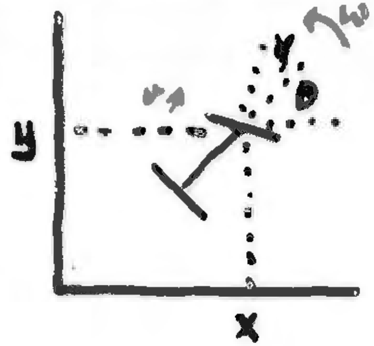
Consider the system

$$
\begin{aligned}
\dot{x} &= v\cos\theta \\
\dot{y} &= v\sin\theta \\
\dot{\theta} &= \frac{v}{L}\tan\phi \\
\dot{\phi} &= \omega
\end{aligned}
$$

where v, L are parameters, ω (only) input.

Not (dynamic) feedback linearizable $\Rightarrow$ no flat output.

However, we still have that by specifying $x_r(t), y_r(t)$ we specify the entire reference dynamics.



Example 2

Consider the system

$$\dot{x} = v\cos(\theta+\phi) \tag{5}$$

$$\dot{y} = v\sin(\theta+\phi) \tag{6}$$

$$\dot{\theta} = \frac{v}{L}\sin\phi \tag{7}$$

$$\dot{\phi} = \omega \tag{8}$$

with inputs v and ω.

From $x(t)$ and $y(t)$ we obtain $v(t)$ and $(\theta+\phi)(t)$. Then (7,8) yields:

$$\frac{d}{dt}(\theta+\phi)(t) = \frac{v(t)}{L}\sin\phi + \dot{\phi}$$

which (knowing $\phi(0)$) gives $\phi(t)$. That also gives $\theta(t)$ and $\omega(t)$.

By specifying $x_r(t), y_r(t)$ we specify the entire reference dynamics:

- $x_r(t)$, $y_r(t)$, $(\theta_r + \phi_r)(t)$, $v_r(t)$ independent of L
- $\phi_r(t)$, $\theta_r(t)$, $\omega_r(t)$ dependent of L.

Find control laws

$$\begin{aligned} v &\equiv v(x, y, \theta, \phi, x_r, y_r, \theta_r + \phi_r, v_r) \\ \omega &\equiv \omega(x, y, \theta, \phi, x_r, y_r, \theta_r + \phi_r, v_r) \end{aligned}$$

that yield

$$\lim_{t \to \infty} |x - x_r| + |y - y_r| + |\theta - \theta_r| + |\phi - \phi_r| = 0$$

Example 3

Consider again the system

$$\dot{x} = v\cos(\theta + \phi) \tag{9}$$

$$\dot{y} = v\sin(\theta + \phi) \tag{10}$$

$$\dot{\theta} = \frac{v}{L}\sin\phi \tag{11}$$

$$\dot{\phi} = \omega \tag{12}$$

with inputs v and ω.

We can also prescribe $\phi(t)$ and $v(t)$.

Then we also obtain $\omega(t)$, $\theta(t)$ (using $\theta(0)$), $x(t)$ and $y(t)$ (using $x(0)$ and $y(0)$.

By specifying $x_r(t), y_r(t)$ we specify the entire reference dynamics:

- $\phi_r(t)$, $v_r(t)$, $\omega_r(t)$ independent of L.
- $x_r(t)$, $y_r(t)$, $\theta_r(t)$ dependent of L

Find control laws

$$\begin{aligned} v &\equiv v(x, y, \theta, \phi, \phi_r, v_r, \omega_r) \\ \omega &\equiv \omega(x, y, \theta, \phi, \phi_r, v_r, \omega_r) \end{aligned}$$

that yield

$$\lim_{t \to \infty} |x - x_r| + |y - y_r| + |\theta - \theta_r| + |\phi - \phi_r| = 0$$

Proposed formulation of adaptive tracking problem

Consider a system $\dot{x} = f(x, u, \theta)$ with $\theta \in \Theta$ a vector of unknown constant parameters

- Specify $y_r(t)$ such that

$$y_r(t), x_r(0), u_r(0) \Rightarrow x_r^\theta(t), u_r^\theta(t)$$

- Define a class $\mathcal{Z}$ of signals determined by $y_r(t)$ and $x_r(0), u_r(0)$ that are θ-independent (in a sense $\frac{\partial z}{\partial \theta} = 0$).

Find a controller $u(t)$ depending on $x(t)$ and elements of $\mathcal{Z}$, i.e. $u \equiv u(x, z)$, such that

$$\lim_{t \to \infty} \|x(t) - x_r^\theta(t)\| = 0$$

Remaining questions for further research

- What to choose for y?
- How to define $\mathcal{Z}$?
- How to determine the largest $\mathcal{Z}$ for given y?
- choice of $y \Leftrightarrow$ richness of $\mathcal{Z}$
- **How to solve the adaptive tracking problem?**