Exponential tracking control of a mobile car using a cascaded approach

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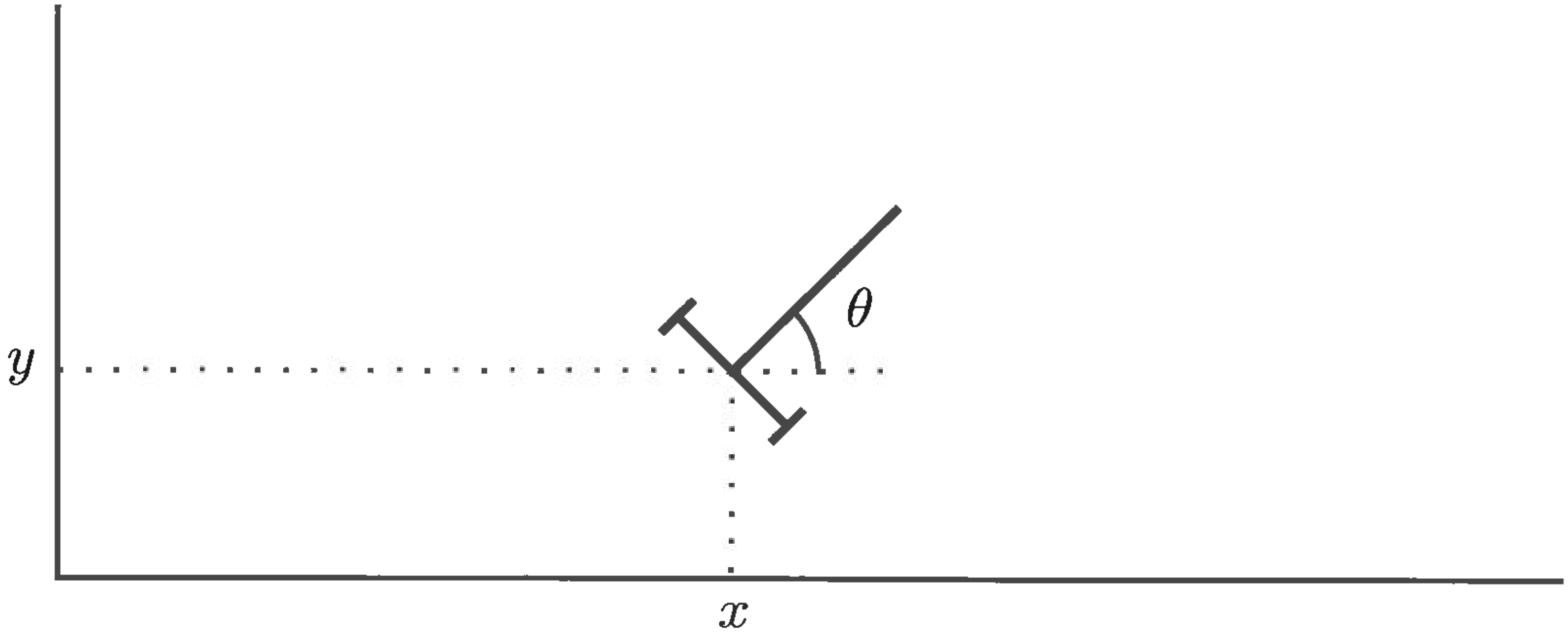
Outline

- Model and Problem formulation
- Cascaded systems
- Derivation of linear tracking controller
- Simulations
- (Dynamic extension)
- Conclusions

A simple kinematic model



$$
\begin{aligned}
\dot{x} &= v\cos\theta \\
\dot{y} &= v\sin\theta \\
\dot{\theta} &= \omega
\end{aligned}
$$

Problem formulation (I)

Reference robot:

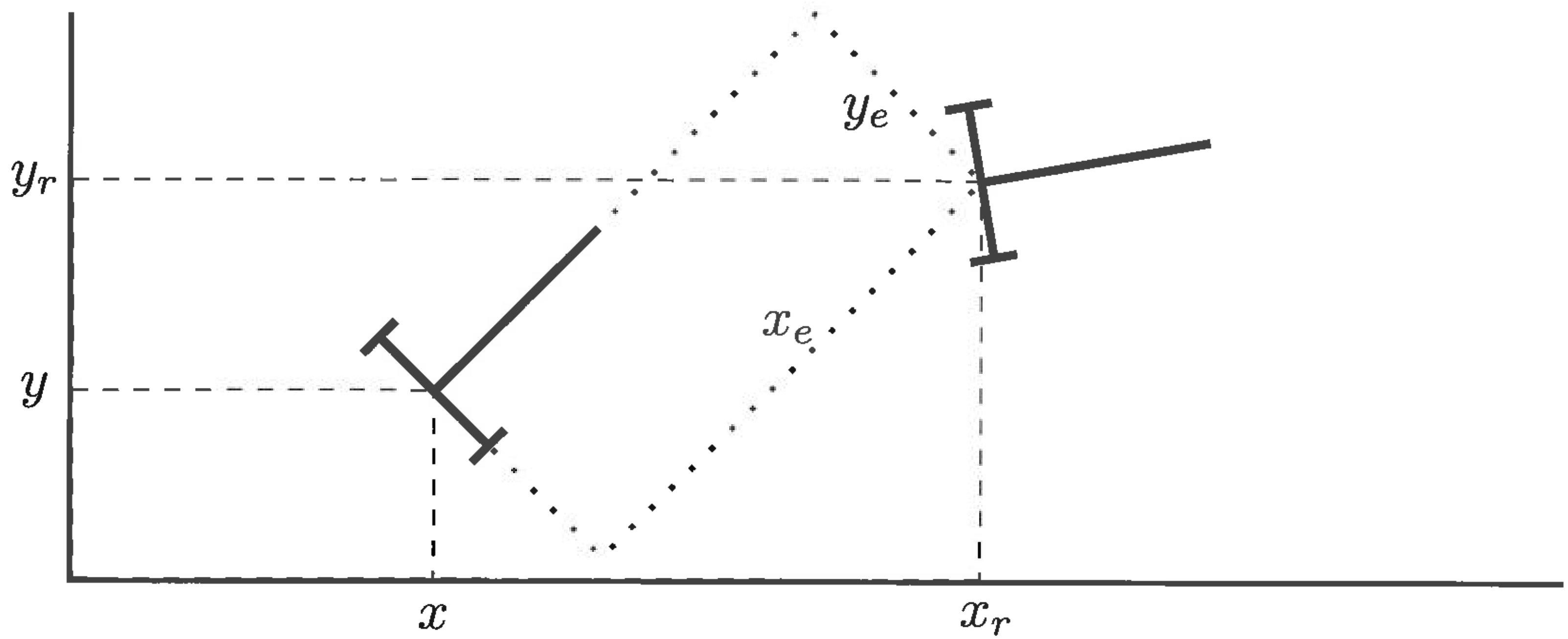
$$\begin{aligned}
\dot{x}_r &= v_r \cos\theta_r \\
\dot{y}_r &= v_r \sin\theta_r \\
\dot{\theta}_r &= \omega_r
\end{aligned}$$

Find control laws

$$\begin{aligned}
v &\equiv v(x, y, \theta, x_r, y_r, \theta_r, v_r, \omega_r) \\
\omega &\equiv \omega(x, y, \theta, x_r, y_r, \theta_r, v_r, \omega_r)
\end{aligned}$$

that yield

$$\lim_{t\longrightarrow\infty} |x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| = 0$$



Define new coordinates (cf. Kanayama et al. (1990))

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}$$

Problem formulation (II)

Resulting error dynamics

$$\begin{aligned}\dot{x}_e &= \omega y_e - v + v_r \cos\theta_e \\ \dot{y}_e &= -\omega x_e + v_r \sin\theta_e \\ \dot{\theta}_e &= \omega_r - \omega\end{aligned}$$

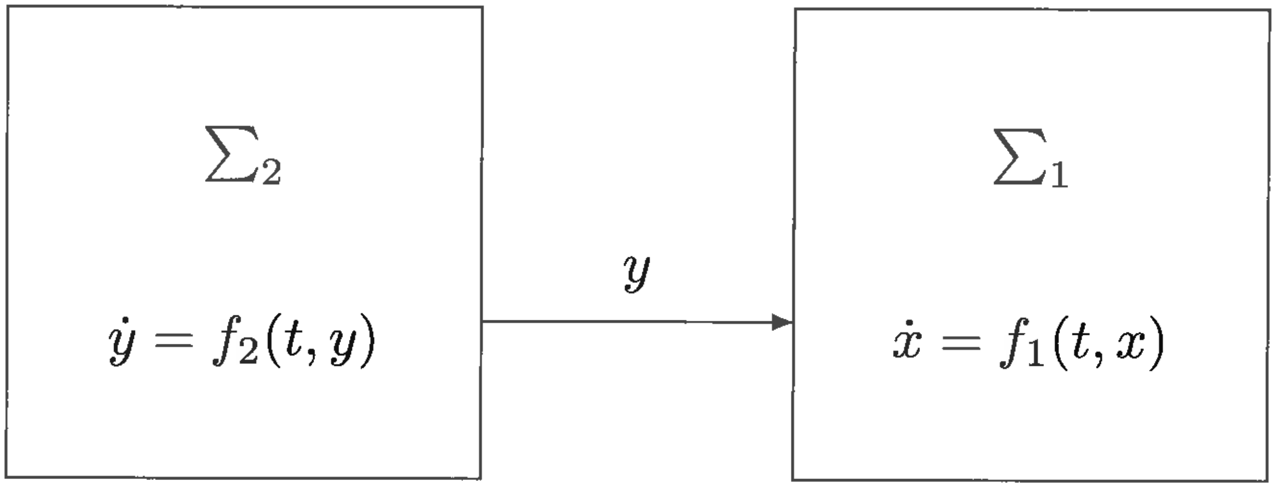
Find control laws

$$\begin{aligned}v &\equiv v(x_e, y_e, \theta_e, x_r, y_r, \theta_r, v_r, \omega_r) \\ \omega &\equiv \omega(x_e, y_e, \theta_e, x_r, y_r, \theta_r, v_r, \omega_r)\end{aligned}$$

that yield

$$\lim_{t\to\infty} |x_e(t)| + |y_e(t)| + |\theta_e(t)| = 0$$

Cascaded systems



$$
\begin{aligned}
\dot{x} &= f_1(t, x) + g(t, x, y)y \\
\dot{y} &= f_2(t, y)
\end{aligned}
$$

Conditions

E. Panteley en A. Loría (S&CL 33(2), 1998):

Cascade Globally Uniformly Asymptotically Stable (GUAS) when

- Σ_1 GUAS, polynomial Lyapunov function
- $g(t, x, y)$ at most linear in x
- Σ_2 GUAS, $y(t)$ integrable

Derivation of controller

Error dynamics

$$\begin{aligned}
\dot{x}_e &= \omega y_e - v + v_r \cos\theta_e \\
\dot{y}_e &= -\omega x_e + v_r \sin\theta_e \\
\dot{\theta}_e &= \omega_r - \omega
\end{aligned}$$

Now use

$$\omega = \omega_r + c_1\theta_e \qquad\qquad c_1 > 0$$

Substituting $\theta_e \equiv 0$ yields ($\omega = \omega_r$):

$$\begin{aligned}
\dot{x}_e &= \omega_r y_e - v + v_r \\
\dot{y}_e &= -\omega_r x_e
\end{aligned}$$

Which can be rewritten as

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} 0 & \omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} (v - v_r)$$

When we use $v = v_r + c_2 x_e$ with $c_2 > 0$ we get

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} -c_2 & \omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \end{bmatrix}$$

Globally Exponentially Stable, provided $\omega_r(t)$ persistently exciting, i.e. there exist $\delta, k > 0$ such that

$$\int_t^{t+\delta} \omega_r(t)^2 d\tau \geq k \quad \forall t \geq t_0$$

To summarize

Consider the error dynamics

$$
\begin{aligned}
\dot{x}_e &= \omega y_e - v + v_r \cos\theta_e \\
\dot{y}_e &= -\omega x_e + v_r \sin\theta_e \\
\dot{\theta}_e &= \omega_r - \omega
\end{aligned}
$$

in closed loop with the controller

$$
\begin{aligned}
\omega &= \omega_r + c_1\theta_e \qquad && c_1 > 0 \\
v &= v_r + c_2 x_e \qquad && c_2 > 0
\end{aligned}
$$

Then the closed loop is globally exponentially stable provided $\omega_r(t)$ is persistently exciting.

Simulations

The system

$$\begin{aligned} \dot{x}_e &= \omega y_e - v + v_r \cos\theta_e \\ \dot{y}_e &= -\omega x_e + v_r \sin\theta_e \\ \dot{\theta}_e &= \omega_r - \omega \end{aligned}$$

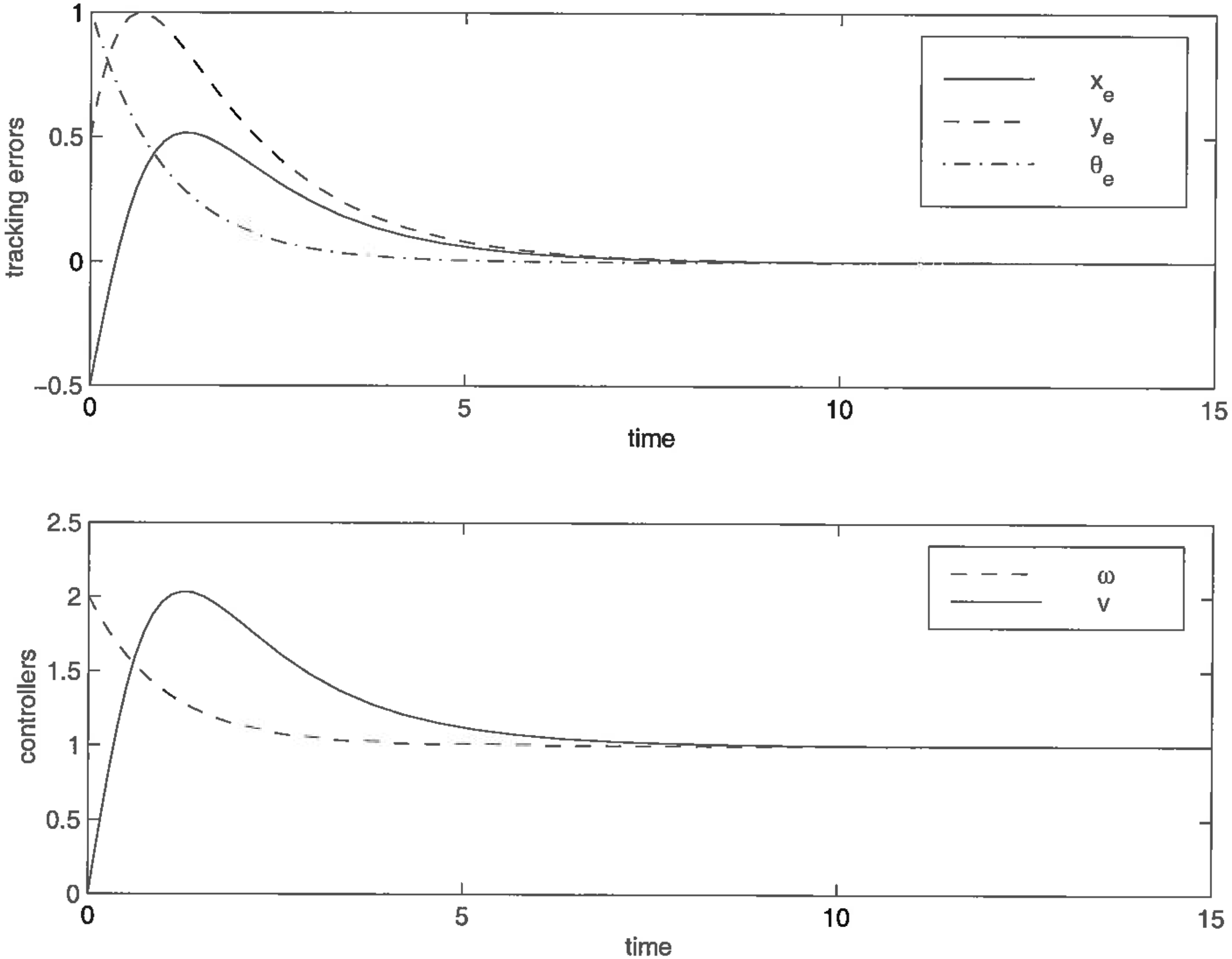
in closed loop with controller

$$\begin{aligned} v &= v_r + 2x_e \\ \omega &= \omega_r + \theta_e \end{aligned}$$

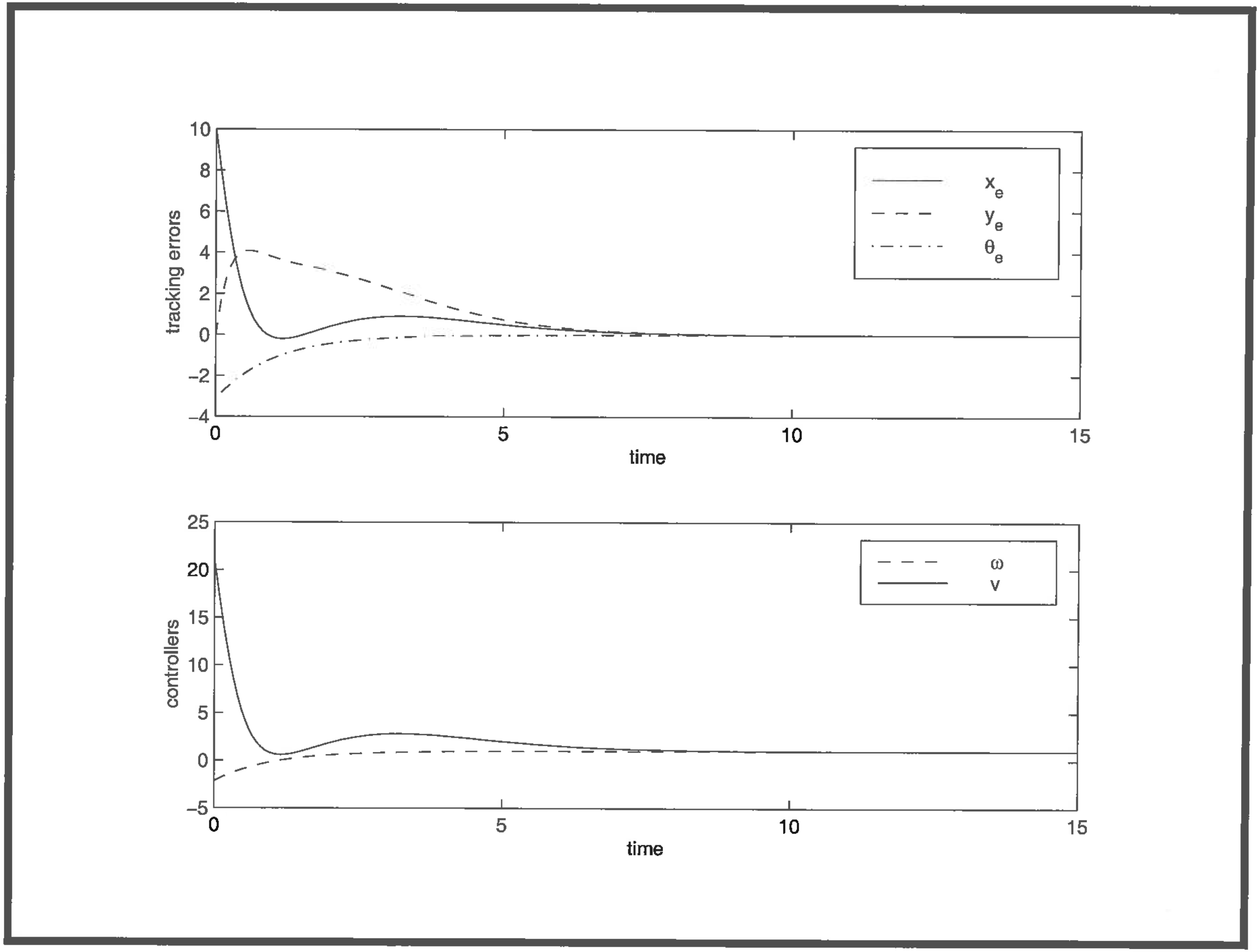
Reference trajectory (circle):

$$\begin{aligned} v_r &= 1 \\ \omega_r &= 1 \end{aligned}$$

Initial condition $[x_e(0), y_e(0), \theta_e(0)]^T = [-0.5, 0.5, 1]^T$.
Initial condition $[x_e(0), y_e(0), \theta_e(0)]^T = [-10, 0, -\pi]^T$.



tracking errors
time
x_e
y_e
θ_e
controllers
time
ω
v



10
8
6
4
2
0
−2
−4
tracking errors
0
5
10
15
time
x_e
y_e
θ_e
25
20
15
10
5
0
−5
controllers
0
5
10
15
time
ω
v

Simulations (II)

Brockett (1983): No continuous state feedback for stabilization.

Therefore:

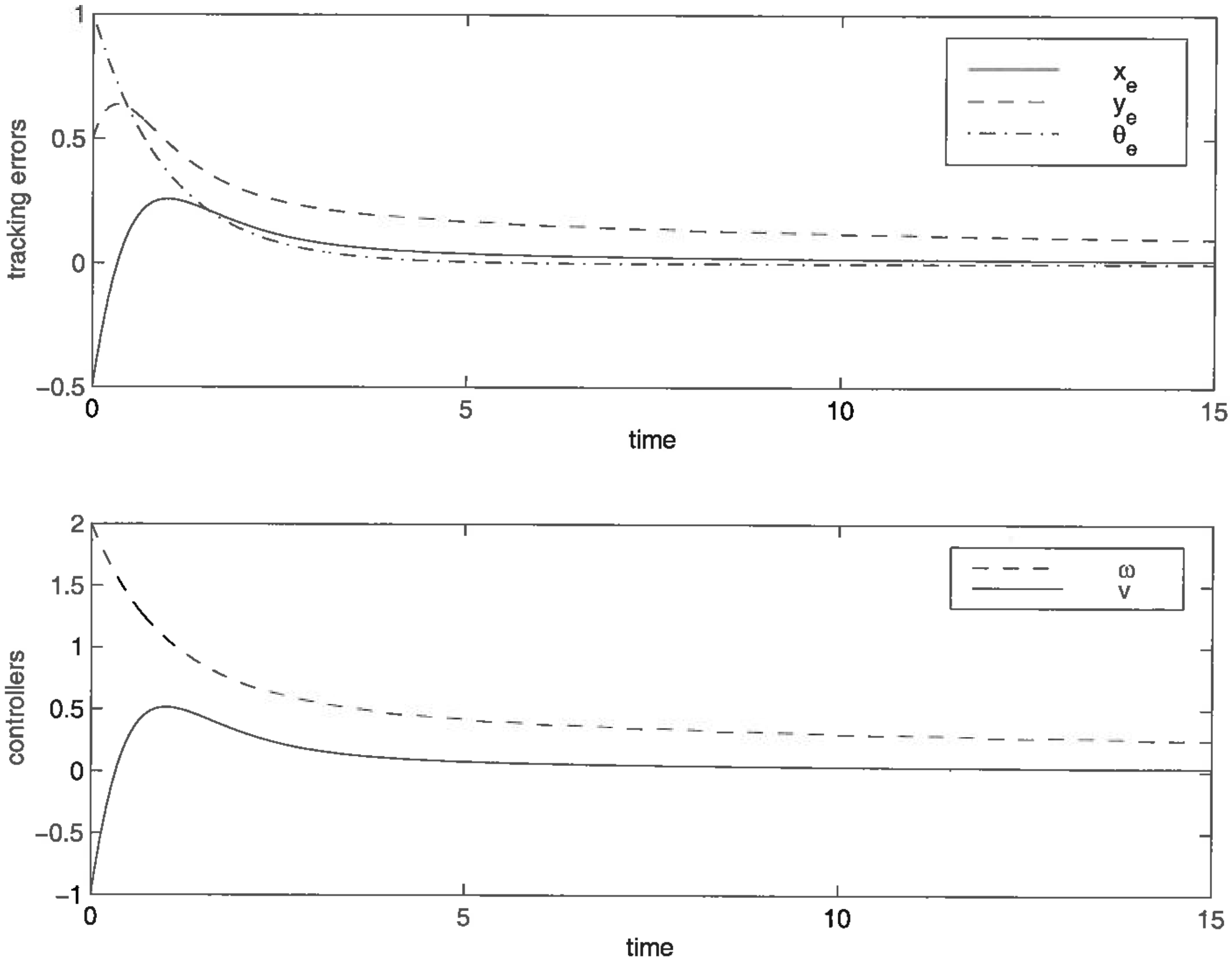
- $v_r(t) \not\to 0$ or $\omega_r(t) \not\to 0$ (cf. Jiang and Nijmeijer)
- ω_r persistently exciting.

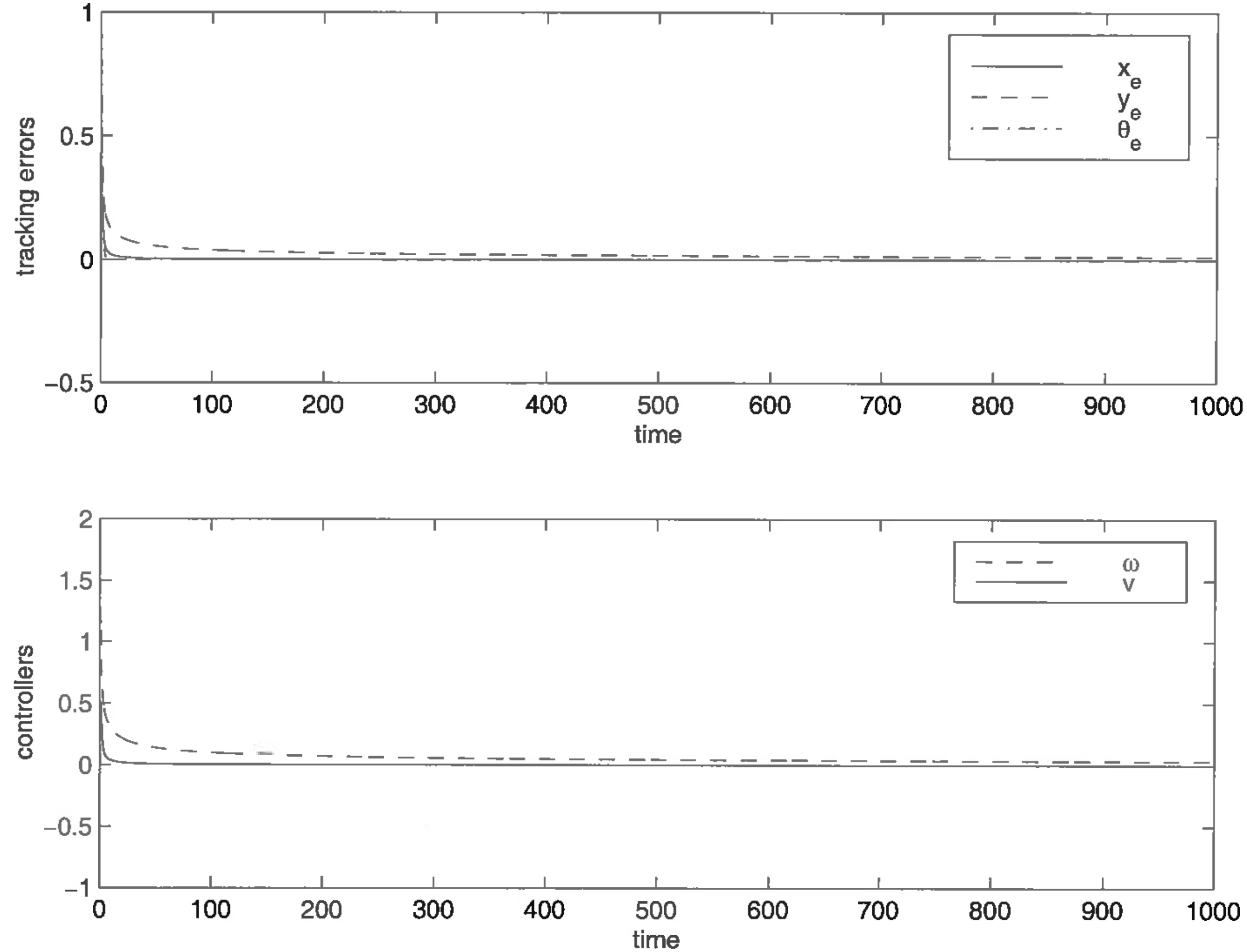
However $\displaystyle\int_{t_0}^{\infty} \omega_r(\tau)^2 d\tau = \infty$ suffices for asymptotic stability.

$v_r(t) = 0$

$\omega_r(t) = \frac{1}{\sqrt{t+1}}$

with initial condition $[x_e(0), y_e(0), \theta_e(0)]^T = [-0.5, 0.5, 1]^T$


tracking errors
time
x_e
y_e
θ_e
controllers
time
ω
v
1
0.5
0
−0.5
0
5
10
15
2
1.5
1
0.5
0
−0.5
−1



tracking errors
time
x_e
y_e
θ_e
controllers
time
ω
v

Dynamic extension

Consider

$$\begin{aligned}\dot{x}_e &= \omega y_e - v + v_r \cos\theta_e \\ \dot{y}_e &= -\omega x_e + v_r \sin\theta_e \\ \dot{\theta}_e &= \omega_r - \omega \\ \dot{v} &= u_1 \\ \dot{\omega} &= u_2\end{aligned}$$

Define

$$\begin{aligned}v_e &= v - v_r \\ \omega_e &= \omega - \omega_r\end{aligned}$$

Then we obtain

$$\begin{bmatrix} \dot{x}_e \\ \dot{v}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} 0 & -1 & \omega_r(t) \\ 0 & 0 & 0 \\ -\omega_r(t) & 0 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ v_e \\ y_e \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} (u_1 - \dot{v}_r) +$$

$$+ \begin{bmatrix} v_r - v_r \cos\theta_e + y_e \omega_e \\ 0 \\ v_r \sin\theta_e - x_e \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_e \\ \dot{\omega}_e \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_e \\ \omega_e \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u_2 - \dot{\omega}_r)$$

The controller

$$\begin{aligned} u_1 &= \dot{v}_r + c_3 x_e - c_4 v_e \\ u_2 &= \dot{\omega}_r + c_5 \theta_e - c_6 \omega_e \end{aligned}$$

yields global asymptotic stability, provided $\omega_r(t)$ is persistently exciting.

Conclusions

- Simple (linear) controllers for (nonlinear) mobile robot. Both kinematic model and dynamic extension.
- Globally (not based on linearization), exponential.
- Similar in case of saturated control inputs.
- Similar result for general chained form systems.