technische universiteit eindhoven

Validation of PDE Models for Supply Chain Modeling and Control

Erjen Lefeber, Roel van den Berg, Koos Rooda

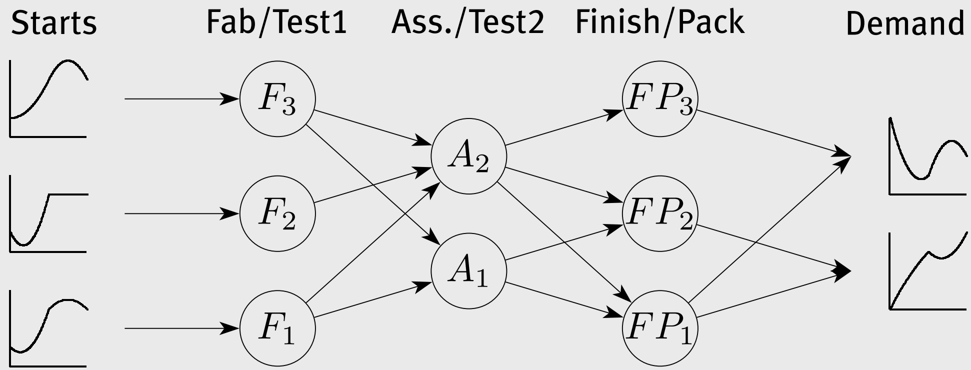
20 October 2003

/department of mechanical engineering

Outline

- Modeling problem
  - Shortcomings of available models
  - PDE models developed so far
- Validation studies
  - case description, setup
  - results, observations
- Issues in development of other models
- Conclusions

Modeling problem



Modeling for control (supply chain/mass production).

- Like to understand dynamics of factories
- Throughput, cycle time, variance of cycle time
- Answer questions like: How to perform ramp up?

Available models (I)

Discrete Event

- Advantages
  - Include dynamics
  - Throughput and cycle time related
- Disadvantage
  - Clearly infeasible for entire supply chain

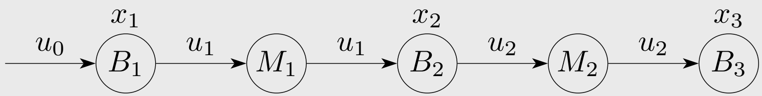
Available models (II)

Queueing Theory

- Advantages
  - Throughput and cycle time related
  - Computationally feasible (approximations)
- Disadvantage
  - Only steady state, no dynamics

Available models (III)

Fluid models



$$\begin{aligned} \dot{x}_1 &= u_0 - u_1 \\ \dot{x}_2 &= u_1 - u_2 \\ \dot{x}_3 &= u_2 \end{aligned} \qquad \text{or} \qquad \begin{aligned} x_1(k+1) &= u_0(k) - u_1(k) \\ x_2(k+1) &= u_1(k) - u_2(k) \\ x_3(k+1) &= u_2(k) \end{aligned}$$

- Advantages
  - Dynamical model
  - Computationally feasible
- Disadvantage
  - Only throughput incorporated, no cycle time

Available models (conclusion)

- Discrete Event: Not computationally feasible
  Queueing Theory: No dynamics
  Fluid models: No cycle time
- Need something else!
- Discrete event models (and queueing theory) have proved themselves. Can be used for verification!

Traffic flow: LWR model

Lighthill, Whitham ('55), and Richards ('56)

Traffic behavior on one-way road:

- density $\rho(x,t)$,
- speed $v(x,t)$,
- flow $u(x,t) = \rho(x,t)v(x,t)$.

Conservation of mass:

$$\frac{\partial \rho}{\partial t}(x,t) + \frac{\partial u}{\partial x}(x,t) = 0.$$

Static relation between flow and density:

$$u(x,t) = S(\rho(x,t)).$$

Modeling manufacturing flow (I)

- density $\rho(x,t)$,
- speed $v(x,t)$,
- flow $u(x,t) = \rho(x,t)v(x,t)$,
- Conservation of mass: $\frac{\partial \rho}{\partial t}(x,t) + \frac{\partial \rho v}{\partial x}(x,t) = 0.$
- Boundary condition: $u(0,t) = \lambda(t)$

Modeling manufacturing flow (II)

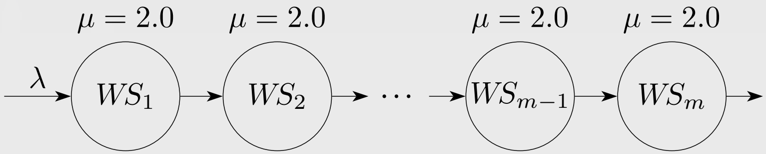
Armbruster, Marthaler, Ringhofer (2002):

- Single queue: $\frac{1}{v(x,t)} = \frac{1}{\mu}(1 + \int_0^1 \rho(s,t)\,\mathrm{d}s)$
- Single queue: $\frac{\partial \rho v}{\partial t}(x,t) + \frac{\partial \rho v^2}{\partial x}(x,t) = 0$
  $\rho v^2(0,t) = \frac{\mu \cdot \rho v(0,t)}{1+\int_0^1 \rho(s,t)\,\mathrm{d}s}$
- Re-entrant: $v(x,t) = v_0 \left(1 - \frac{\int_0^1 \rho(s,t)\,\mathrm{d}s}{W_{\mathsf{max}}}\right)$

Lefeber (2003):

- Line of many identical queues: $v(x,t) = \frac{\mu}{1+\rho(x,t)}$

Validation studies



- Identical workstations, infinite buffers (FIFO)
- Number of workstations: $m = 10$, $m = 50$
- Processing times: exponential (mean 0.5)
- Inter arrival times: exponential (mean $1/\lambda$)
- From one steady state to the other
  - ramp up: from initially empty to 25%, 50%, 75%, 95% utilization
  - ramp down: from 50%, 75%, 90%, 95% utilization to 25% utilization

Performance measures

- mean WIP (in steady state): w_{ss}
- mean throughput (in steady state): δ_{ss}
- mean cycle time (in steady state): φ_{ss}
- time for reaching 99% of steady state WIP
- time for reaching 99% of steady state throughput
- time for reaching 99% of steady state cycle time
- cycle time for first lot inserted at $t = 0$

- Batches of 100 experiments
- Repeat until in each buffer 95% two sided confidence interval smaller than 2% of mean

Results (ramp up)

	m=10	m=50	m=10	m=50	m=10	m=50
φ_{ss}	$++$	$++$	$++$	$++$	$++$	$++$
time to φ_{ss}	0	0	$+$	$--$	$-$	0
φ 1st lot	$-$	0	$+$	0	$-$	0
δ_{ss}	$++$	$++$	$++$	$++$	$++$	$++$
time to δ_{ss}	0	0	0	$-$	0	0
w_{ss}	$++$	$++$	$++$	$++$	$++$	$++$
time to w_{ss}	0	0	0	$--$	$-$	0

$++$	$<$5%
$+$	5% – 10%
0	10% – 50%
$-$	50% – 100%
$--$	$>$100%

Results (ramp down)

	m=10	m=50	m=10	m=50	m=10	m=50
φ_{ss}	$+$	$+$	$+$	$+$	$+$	$+$
time to φ_{ss}	0	0	0	$-$	0	0
φ 1st lot	0	0	$+$	$++$	$+$	$++$
δ_{ss}	$++$	$++$	$++$	$++$	$++$	$++$
time to δ_{ss}	0	0	$+$	$-$	0	0
w_{ss}	$++$	$++$	$++$	$++$	$++$	$++$
time to w_{ss}	0	0	0	0	0	0

$++$	$<$5%
$+$	5% – 10%
0	10% – 50%
$-$	50% – 100%
$--$	$>$100%

General observations

- Steady state performance well described
- Time to reach steady state ill described
- Amount of lots produced before reaching steady state (most cases) relatively small
- Homogeneous velocity results in ill described behavior of throughput
- Simulation run Discrete Event: 4 minutes
  Batch run Discrete Event: 7 hours
  Simulation run PDE: 1 minute

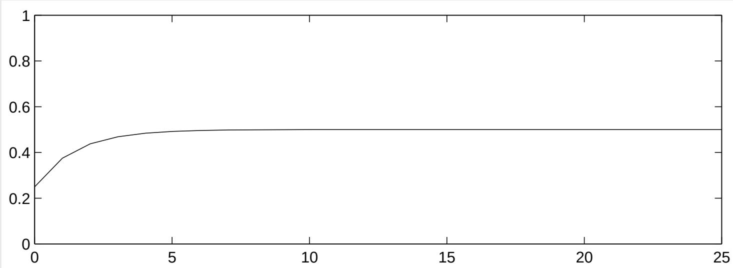
Extensions: Properties needed

- No backward-flow allowed (cf. Daganzo '95)
- No negative density
- Stable steady states
  - constant feed rate $\longrightarrow$ equilibrium
  - equilibrium meets relations queueing theory

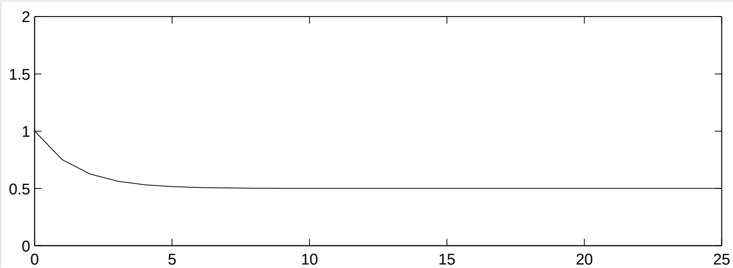
Extensions: considerations (I)

100 machines, $\mu = 1$, exponential. Utilization: 50%.

- Regular arrivals: $c_a^2 = 0$



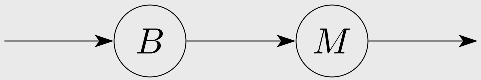
- Irregular arrivals: $c_a^2 = 3$



Extensions: considerations (II)

Variability needs to be included. However, . . .

1 machine, $\mu = 1$, exponential



- Push control: exponential arrivals. Utilization 50%
  - Throughput: 0.5 lots per unit time
  - Cycle time: 2 hours
  - Mean WIP: 1 lot
- CONWIP control: WIP=1
  - Throughput: 1 lots per unit time
  - Cycle time: 1 hours
  - Mean WIP: 1 lot

Conclusions

Need for computationally feasible dynamical models incorporating both throughput and cycle time.

- NOT: Discrete event, Queueing theory, Fluid models
- Possible: PDE-models
  - Correct steady state behavior
  - Better description transient needed
  - Second moment and correlation needs to be included
  - Queueing theory, discrete event models can be used for validation of PDE models
- Next step: PDE-based controller design