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A Spatial Approach to Control of Platooning Vehicles: Separating Path-Following from Tracking

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Abstract: We introduce a new way to look at the combined lateral and longitudinal control problem for platooning vehicles by studying these problems separately. The lateral control problem is approached as a path following problem in the spatial domain: based on the path of the preceding vehicle we determine a path for the following vehicle which converges to the given path of its predecessor. In particular if the following vehicle happens to be on the path of its predecessor, the generated path of the follower equals the path of its predecessor. This approach not only overcomes the problem of corner cutting, but also achieves appropriate following behavior in case of large initial errors. As a by-product of solving the lateral control problem, we obtain a mapping from the path of the follower to the path of its predecessor. Using this mapping we can consider the longitudinal control problem as controlling two points on the same path towards a required inter-vehicle distance, which is comparable to CACC, i.e., the problem of controlling two points on a straight line towards a required inter-vehicle distance. We illustrate our approach by means of simulation.

Keywords: Trajectory tracking and path following, Cooperative navigation, Multi-vehicle systems, Intelligent cruise control, Autonomous vehicles, Nonlinear control.

1. INTRODUCTION

The increasing need for transportation leads to traffic congestion on highways. Platooning for communicating road vehicles can safely increase highway capacity by reducing inter vehicle distances as concluded in Vahidi and Eskandarian (2003). This so-called Cooperative Adaptive Cruise Control (CACC), which is an extension of traditional ACC, relies on vehicle-to-vehicle (V2V) communication which provides a following vehicle with information about its preceding vehicle. The additional V2V communication not only allows for safely reducing inter vehicle distances, but also attenuates disturbances in upstream direction (string stability), see Naus et al. (2010a); Ploeg et al. (2011). The reduced inter vehicle distance also results in a reduction of aerodynamic drag force, in particular for heavy duty vehicles, resulting in a reduction of emissions and fuel consumption, see Shladover (2006); Alam et al. (2010).

Several strategies for fully automated platooning have been presented in literature. As in this paper, a separation of longitudinal and lateral control of automated vehicles has been introduced in Rajamani et al. (2000). On the one hand, the longitudinal control has been developed based on CACC, whereas the lateral control has been developed based on a lane-keeping method. A disadvantage of this approach is the need to embed each lane with magnets, rendering its usage impractical. Tunçer et al. (2010) propose a vision-based lane-keeping system for the lateral control which is commonly adopted. A problem with that approach in case of platooning is that the preceding vehicle blocks the lane markings, making it impossible for the vision system to track them, cf. Solyom et al. (2013).

Using the position and orientation of the preceding vehicle as a reference, direct vehicle following controllers for the lateral control problem have, amongst others, been introduced by Lu and Tomizuka (2003); Solyom et al. (2013). Since the following vehicle is controlled towards the preceding vehicle, in case of following a curved path, corner cutting appears, as observed by Gehrig and Stein (1998); Goi et al. (2010). This corner cutting depends on the curvature of the path and the inter-vehicle distance. Though Lu and Tomizuka (2003) claim that corner cutting is limited for typical highway driving (an inter-vehicle distance of $10[m]$ and curvature of $0.00125[m^{-1}]$ leads to a deviation with the path of the predecessor of only $5cm$), this does become a serious problem in case of larger platoon lengths, causing vehicles to depart from their lane. To avoid corner cutting, instead of controlling the following vehicle towards its predecessor, it has been controlled towards the path of its predecessor. To that end, Gehrig and Stein (1998) keep a buffer of predecessor data such as position, velocity, and yaw rate until the following vehicle reaches a certain point, after which that data can be removed from the history buffer since it is no more of interest. Keeping a similar buffer of predecessor data is also required for the method we propose in this paper. Several path following papers introduce a look-ahead point, where the vehicle is going to be based on the vehicles current heading, cf. Hsu and Tomizuka (1998); Hingwe and Tomizuka (1998); Rossetter (2003); Abdullah et al. (2006); Yazbeck et al. (2011). This look-ahead point is controlled towards the path of the predecessor and needs to be large enough, in particular for high velocities, otherwise the system might become unstable. If the look-ahead point equals the desired inter-vehicle distance, this approach is very similar to direct vehicle following, and therefore corner cutting also results for these methods. In Taylor et al. (1999); Tseng et al. (2005); Kritayakirana (2012) a combination of lateral offset and heading angle error has been used, but also that approach results in corner cutting, steering either too early or too late when switching from a straight line to a circular path. Using an extended look-ahead point, Bayuwindra et al. (2016) were able to significantly reduce corner cutting by introducing a virtual preceding vehicle as the new tracking objective.

In this paper, we introduce a new way to look at the combined lateral and longitudinal problem. We use different approaches for solving the lateral and the longitudinal control problem. The lateral control problem is approached as a path following problem. That is, based on the path of the preceding vehicle we determine a path for the following vehicle which converges to the given path of its predecessor. In particular if the following vehicle happens to be on the path of its predecessor, the generated path of the follower is equal to the path of its predecessor. This overcomes the problem of corner cutting. As a by-product of solving the lateral control problem, we obtain a mapping from points on the path of the follower to points on the path of its predecessor. Using this mapping we can consider the longitudinal control problem as controlling two points on the same path towards a required inter-vehicle distance, which is comparable to CACC, i.e., the problem of controlling two points on a straight line towards a required inter-vehicle distance as studied in Ploeg et al. (2011, 2014).

The remainder of this paper is organised as follows. Section 2 contains some preliminaries. In Section 3 we formulate the lateral and longitudinal control problems, for which we present solutions in sections 4 and 5 respectively. Section 6 contains results from a simulation study, and Section 7 concludes the paper.

2. PRELIMINARIES

Lemma 1. (Barbălat's Lemma, see Barbălat (1959)). Let $\phi : \mathbb{R}_+ \to \mathbb{R}$ be a uniformly continuous function. Suppose that $\lim_{t\to\infty} \int_0^t \phi(\tau)\mathrm{d}\tau$ exists and is finite. Then

$$\lim_{t\to\infty} \phi(t) = 0.$$

Lemma 2. ((Micaelli and Samson, 1993, Lemma 1)). Let $f : \mathbb{R}_+ \to \mathbb{R}$ be any differentiable function. If $f(t)$ converges to zero as $t \to \infty$ and its derivative satisfies

$$\dot{f}(t) = f_0(t) + \eta(t) \qquad t \geq 0,$$

where f_0 is a uniformly continuous function and $\eta(t)$ tends to zero as $t \to \infty$, then $\dot{f}(t)$ and $f_0(t)$ tend to zero as $t \to \infty$.

In the remainder, let $\sigma(\tau)$ denote a continuous monotone function, differentiable at $\tau = 0$, satisfying $\sigma(\tau)\tau > 0$ for $\tau \neq 0$, $|\sigma(\tau)| \leq 1$, and $\sigma'(0) > 0$.

3. PROBLEM FORMULATION

Consider a simple kinematic model of a mobile car with length $L > 0$, rear wheel driving and front wheel steering:

$$\begin{aligned}\dot{x}(t) &= v(t)\cos\theta(t),\\ \dot{y}(t) &= v(t)\sin\theta(t),\\ \dot{\theta}(t) &= \frac{v(t)}{L}\tan\phi(t).\end{aligned} \tag{1}$$

The forward velocity of the rear wheel v and the angle of the front wheel ϕ are considered as inputs, (x, y) is the center of the rear axle of the vehicle, and θ is the orientation of the body of the car.

We want to recast the lateral control problem into a path following problem. We therefore let $s(t)$ denote the travelled distance along the path. Since

$$v(t) = \frac{\mathrm{d}s(t)}{\mathrm{d}t},$$

we can characterise our path by

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}s}x(s(t)) &= \cos\theta(s(t)),\\ \frac{\mathrm{d}}{\mathrm{d}s}y(s(t)) &= \sin\theta(s(t)),\\ \frac{\mathrm{d}}{\mathrm{d}s}\theta(s(t)) &= \frac{1}{L}\tan\phi(s(t)) = \kappa(s(t)),\end{aligned} \tag{2}$$

where $\kappa(s(t))$ denotes the curvature at $s(t)$. Notice that the curve with a constant curvature κ is a circle with radius $1/\kappa$, and $\kappa = 0$ corresponds with a straight line. In the remainder we drop dependency of s on t for ease of exposition.

Let the path of a preceding vehicle be given by

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}s_l}x_l(s_l) &= \cos\theta_l(s_l),\\ \frac{\mathrm{d}}{\mathrm{d}s_l}y_l(s_l) &= \sin\theta_l(s_l),\\ \frac{\mathrm{d}}{\mathrm{d}s_l}\theta_l(s_l) &= \kappa_l(s_l),\end{aligned} \tag{3}$$

with bounded $\kappa_l(s_l)$. Then we formulate the lateral control problem as follows.

Problem 3. (Lateral control). *Given a feasible path of a preceding vehicle* $[x_l(s_l), y_l(s_l), \theta_l(s_l), \kappa_l(s_l)]$, *i.e., a path satisfying* (3) *with bounded* $\kappa_l(s_l)$, *determine a diffeomorphism* $\alpha : \mathbb{R}^+ \to \mathbb{R}^+$, $s_l = \alpha(s)$, *and an appropriate control law* $\kappa(s)$ *such that for the resulting closed-loop system* (2, 3)*:*

$$\begin{aligned}\lim_{s_l\to\infty} x_l(s_l) - x(\alpha^{-1}(s_l)) &= 0,\\ \lim_{s_l\to\infty} y_l(s_l) - y(\alpha^{-1}(s_l)) &= 0,\\ \lim_{s_l\to\infty} \theta_l(s_l) - \theta(\alpha^{-1}(s_l)) &= 0,\end{aligned}$$

or equivalently:

$$\begin{aligned}\lim_{s\to\infty} x_l(\alpha(s)) - x(s) &= 0,\\ \lim_{s\to\infty} y_l(\alpha(s)) - y(s) &= 0,\\ \lim_{s\to\infty} \theta_l(\alpha(s)) - \theta(s) &= 0.\end{aligned}$$

Remark 4. Observe that in case of an initial error, the path of the following vehicle is different from the path of the preceding vehicle. In particular the travelled distance to a certain point is different. That is why we require a diffeomorphism from s to s_l to formulate asymptotic convergence of one path to the other.

In platooning, the objective is to follow the preceding vehicle at a desired distance d, which is known as the spacing policy. The constant time gap spacing policy, cf. Ploeg et al. (2011, 2014), can be formulated as

$$d = r + hv, \tag{4}$$

where d is the desired distance between the preceding vehicle and its follower, r is the standstill distance and v is the velocity of the following vehicle. The constant time gap spacing policy improves string stability compared with the constant spacing policy (i.e., (4) with $h = 0$), see e.g., Naus et al. (2010b,a), and is the reason why we use it in this paper.

If the preceding vehicle and following vehicle are on exactly the same path, the distance d is given by $s_l - s$. However, since in general the paths are different, we have to measure distance differently. From solving the longitudinal control problem we have a mapping $s_l = \alpha(s)$ which maps a driven distance s on the path of the follower to an associated driven distance s_l along the path of the predecessor. Using this mapping we can transform the location of the following vehicle to the location of a virtual following vehicle on the path of the preceding vehicle and vice versa. We can therefore consider the distance between the preceding vehicle and the virtual follower on the path of the preceding vehicle: $s_l-\alpha(s)$, or the distance between the virtual preceding vehicle and the follower on the (planned) path of the follower: $\alpha^{-1}(s_l) - s$. From the point of view of the follower, the latter seems the most natural choice. Now we can define the longitudinal control problem as

Problem 5. (Longitudinal control). *Given the trajectory of a preceding vehicle, i.e., both a velocity profile $v_l(t)$ and a path $[x_l(s_l), y_l(s_l), \theta_l(s_l), \kappa_l(s_l)]$, as well as a solution to the lateral control problem, Problem 3, determine a velocity profile $v(t)$ such that*

$$\lim_{t\to\infty} \alpha^{-1}(s_l(t)) - s(t) - hv(t) - r = 0. \tag{5}$$

4. LATERAL CONTROLLER DESIGN

For solving the lateral control problem, we consider a virtual vehicle driving along the trajectory generated by the predecessor, which has travelled a distance $s_l = \alpha(s)$ along the path of the preceding vehicle when the following vehicle has travelled a distance s along its path. Let $\bar{v}(s) = \frac{\mathrm{d}\alpha(s)}{\mathrm{d}s}$ and define $\bar{x}_l(s) = x_l(\alpha(s))$, $\bar{y}_l(s) = y_l(\alpha(s))$, $\bar{\theta}_l(s) = \theta_l(\alpha(s))$, and $\bar{\kappa}_l(s) = \kappa_l(\alpha(s))$. Then we have for the virtual vehicle:

$$\begin{aligned}\frac{\mathrm{d}\bar{x}_l(s)}{\mathrm{d}s} &= \bar{v}(s)\cos\bar{\theta}_l(s),\\ \frac{\mathrm{d}\bar{y}_l(s)}{\mathrm{d}s} &= \bar{v}(s)\sin\bar{\theta}_l(s),\\ \frac{\mathrm{d}\bar{\theta}_l(s)}{\mathrm{d}s} &= \bar{v}(s)\bar{\kappa}_l(s).\end{aligned}$$

Inspired by Kanayama et al. (1990), we express the error coordinates in the frame of the following vehicle, i.e

$$\begin{bmatrix} x_e(s)\\ y_e(s)\\ \theta_e(s)\end{bmatrix} = \begin{bmatrix} \cos\theta(s) & \sin\theta(s) & 0\\ -\sin\theta(s) & \cos\theta(s) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix} \bar{x}_l(s) - x(s)\\ \bar{y}_l(s) - y(s)\\ \bar{\theta}_l(s) - \theta(s)\end{bmatrix}. \tag{6}$$

Dropping the dependency of all signals on s and using $'$ for $\frac{\mathrm{d}}{\mathrm{d}s}$ we obtain for the error dynamics:

$$\begin{aligned} x_e' &= \kappa y_e + \bar{v}\cos\theta_e - 1,\\ y_e' &= -\kappa x_e + \bar{v}\sin\theta_e,\\ \theta_e' &= -\kappa + \bar{v}\bar{\kappa}_l.\end{aligned} \tag{7}$$

Consider initial conditions satisfying $|\theta_e(0)| < \pi/2$. Differentiating the function $V = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 - \frac{1}{c_3}\log(\cos\theta_e)$, which is positive definite for $c_3 > 0$, along solutions of (7) results in

$$V' = x_e(\bar{v}\cos\theta_e - 1) + y_e(\bar{v}\sin\theta_e) + \frac{1}{c_3}\tan\theta_e(-\kappa + \bar{v}\bar{\kappa}_l).$$

Using the controller

$$\bar{v} = \frac{1 - c_1\sigma_1(x_e)}{\cos\theta_e} \tag{8a}$$

$$\kappa = c_3 y_e(1 - c_1\sigma_1(x_e)) + \bar{v}\bar{\kappa}_l + c_2\sigma_2(\theta_e) \tag{8b}$$

with $0 < c_1 < 1$ and $0 < c_2$ results in

$$V' = -c_1 x_e\sigma_1(x_e) - \frac{c_2}{c_3}\sigma_2(\theta_e)\tan\theta_e \le 0. \tag{9}$$

This implies that x_e, y_e, and $-\log(\cos\theta_e)$ are bounded, which implies that $|\theta_e| \le M < \pi/2$ and $\bar{v}$ is bounded. Since κ_l is bounded by assumption, we have that $\bar{\kappa}_l$ is bounded, and also κ is bounded (from (8)) and as a result x_e', y_e', and θ_e' are bounded, implying that x_e, y_e, and θ_e are uniformly continuous functions of s. From Lemma 1 applied to (9), we then have that $\lim_{s\to\infty} x_e(s) = \lim_{s\to\infty}\theta_e(s) = 0$, and from Lemma 2 applied to θ_e in (7, 8): $\lim_{s\to\infty} y_e(s) = 0$.

From (6) we then also obtain

$$\begin{aligned}\lim_{s\to\infty} x_l(\alpha(s)) - x(s) &= 0,\\ \lim_{s\to\infty} y_l(\alpha(s)) - y(s) &= 0,\\ \lim_{s\to\infty} \theta_l(\alpha(s)) - \theta(s) &= 0.\end{aligned}$$

Furthermore, since $1 - c_1\sigma(x_e) \ge 1 - c_1 > 0$ and $|\theta_e| \le M < \pi/2$ we have that $\bar{v}(s) \ge 1 - c_1 > 0$, and therefore $s_l = \alpha(s)$ is a diffeomorphism, where $\alpha(s)$ is obtained from

$$\frac{\mathrm{d}\alpha(s)}{\mathrm{d}s} = \bar{v}(s), \qquad \alpha(0) = 0. \tag{10}$$

We can summarise the above in the following:

Proposition 6. Let $c_3 > 0$, then the controller (8) solves the lateral control problem, Problem 3, for all initial states satisfying $|\theta_e(0)| < \pi/2$, where the function α is given in (10). Furthermore, the resulting closed-loop system is locally exponentially stable (in the distance driven).

Proof. That the proposed controller solves the lateral control problem has been shown above. Local exponential stability in the driven distance follows from asymptotic stability of the linearisation of the dynamics (7, 8) around $x_e = y_e = \theta_e = 0$. $\square$

Remark 7. Using the Lyapunov function candidate

$$V = \frac{1}{2}\log(1 + x_e^2 + y_e^2) - \frac{1}{c_3}\log(\cos\theta_e), \tag{11}$$

which is positive definite for $c_3 > 0$, one can show along the same lines that replacing (8b) by

$$\kappa = c_3 \frac{(1 - c_1\sigma_1(x_e))y_e}{1 + x_e^2 + y_e^2} + \bar{v}\bar{\kappa}_l + c_2\sigma_2(\theta_e) \tag{12}$$

with $0 < c_2$, solves the lateral control problem for $|\theta_e(0)| < \pi/2$. The controller (12) can be useful for forcing the required curvature κ to be a priori bounded. This assures safe and comfortable lateral accelerations.

Corollary 8. Let $\max_s |\kappa_l(s)| = \kappa_l^{\max}$, and assume the additional requirement $|\kappa(s)| \leq \kappa^{\max}$ where $\kappa_l^{\max} < \kappa^{\max}$. Assume an initial condition for which $|\theta_e(0)| < \arccos(\kappa_l^{\max}/\kappa^{\max})$. Then gains $c_2 > 0$, $c_3 > 0$ exist, such that the controller (12) not only solves the lateral control problem, but also guarantees that $|\kappa(s)| \leq \kappa^{\max}$.

Proof. Since the function V given by (11) is non-increasing, we have

$$\bar{v}(s) \leq \frac{1}{\cos(\theta_e(s))} \leq e^{\frac{c_3}{2}\log(1+x_e(0)^2+y_e(0)^2)} \frac{1}{\cos(\theta_e(0))}$$
$$< e^{\frac{c_3}{2}\log(1+x_e(0)^2+y_e(0)^2)} \kappa^{\max}/\kappa_l^{\max}.$$

Note that for $c_3 = 0$ this gives us $\bar{v}\kappa_l < \kappa^{\max}$. Therefore, by choosing $c_3 > 0$ sufficiently small, we can still guarantee that $\bar{v}\kappa_l < \kappa^{\max}$. Since $|\sigma_2(\theta_e)| \leq 1$ and $\frac{|y_e|}{1+x_e^2+y_e^2} \leq \frac{1}{2}$, we can pick $c_2 > 0$ and $c_3 > 0$ in (12) sufficiently small to guarantee $|\kappa| \leq \kappa^{\max}$. $\square$

5. LONGITUDINAL CONTROLLER DESIGN

Since the constant time gap spacing policy (4) contains the forward velocity v, we extend the model (1) by adding the equation

$$\dot{v}(t) = a(t), \tag{13}$$

which boils down to taking the acceleration $a(t)$ as an input instead of the velocity $v(t)$. Extending the dynamics as in Ploeg et al. (2011) can straightforwardly be done.

As mentioned in Section 3, we can use the diffeomorphism α obtained from solving the lateral control problem to transform the position of the preceding vehicle to a position on the (planned) path of the follower. From (5) it seems straightforward to define a longitudinal tracking error as

$$e(t) = \alpha^{-1}(s_l(t)) - s(t) - hv(t) - r$$

and try to control this error towards zero. However, at $t = 0$ we have both $s_l(0) = s(0) = \alpha(0) = 0$, implying $e(0) = -hv(0) - r$. So even when the follower is behind the preceding vehicle, this error definition implies a negative initial error which would cause the follower to decelerate even when it is far behind its predecessor. This is undesirable, so to overcome this problem, we assume that not only the follower has planned a future path, but also the preceding vehicle has planned a future path, for a distance of Δ into the future, and communicates its planned path to the following vehicle using V2V communication.

The planned trajectory of the follower is converging to the trajectory of the predecessor, since it is a solution to the lateral control problem. So if we move a distance Δ along both the trajectory of the predecessor and the follower, those two points are closer to each other for larger Δ. We can use this insight to redefine the error. We move a distance Δ along the future path of the predecessor, i.e. to $s_l + \Delta$. The associated point on the path of the follower is given by $\alpha^{-1}(s_l(t) + \Delta)$. From that point we move Δ back along the path of the follower, and compare that position to the position of the follower, which leads to the following error definition:

$$e(t) = \alpha^{-1}(s_l(t) + \Delta) - (s(t) + \Delta) - hv(t) - r.$$

Using (10) and (13) we obtain

$$\dot{e}(t) = \frac{v_l(t)}{\bar{v}(\alpha^{-1}(s_l(t) + \Delta))} - v(t) - ha(t).$$

Taking the controller

$$a(t) = \frac{1}{h}\left[\frac{v_l(t)}{\bar{v}(\alpha^{-1}(s_l(t) + \Delta))} - v(t) + k\sigma(e(t))\right], \tag{14}$$

with $k > 0$, results in

$$\dot{e}(t) = -k\sigma(e(t)) \qquad k > 0$$

which clearly is a globally asymptotically stable and locally exponentially stable system. We therefore obtain the following:

Proposition 9. Consider the system (1, 13), a solution to the lateral control problem, as well as the path of a preceding vehicle planned for a distance Δ into the future. Then the controller (14) achieves

$$\lim_{t\to\infty} \alpha^{-1}(s_l(t) + \Delta) - (s(t) + \Delta) - hv(t) - r =$$
$$\lim_{t\to\infty} \alpha^{-1}(s_l(t)) - s(t) - hv(t) - r = 0.$$

Furthermore, the resulting closed-loop system is locally exponentially stable.

Remark 10. Implementing the controller (14) in real time is rather straightforward. Assume that the preceding vehicle communicates at time t: $x_l(s_l(t) + \Delta)$, $y_l(s_l(t) + \Delta)$, $\theta_l(s_l(t) + \Delta)$, and $\kappa_l(s_l(t) + \Delta)$. Furthermore, let $\tilde{x}(t) = x(\alpha^{-1}(s_l(t) + \Delta))$, $\tilde{y}(t) = y(\alpha^{-1}(s_l(t) + \Delta))$, and $\tilde{\theta}(t) = \theta(\alpha^{-1}(s_l(t) + \Delta))$. Then we have

$$\dot{\tilde{x}}(t) = \frac{v_l(t)\cos\tilde{\theta}(t)}{\tilde{v}(t)} \quad \dot{\tilde{y}}(t) = \frac{v_l(t)\sin\tilde{\theta}(t)}{\tilde{v}(t)} \quad \dot{\tilde{\theta}}(t) = \frac{v_l(t)\tilde{\kappa}(t)}{\tilde{v}(t)}$$

where, using (8),

$$\tilde{v}(t) = \frac{1 - c_1\sigma_1(\tilde{x}_e(t))}{\cos\tilde{\theta}_e(t)}$$
$$\tilde{\kappa}(t) = c_3\tilde{y}_e(t)\big(1 - c_1\sigma_1(\tilde{x}_e(t))\big) +$$
$$+ \tilde{v}(t)\kappa_l(s_l(t) + \Delta) + c_2\sigma_2(\tilde{\theta}_e(t))$$

and

$$\begin{bmatrix} \tilde{x}_e(t) \\ \tilde{y}_e(t) \\ \tilde{\theta}_e(t) \end{bmatrix} = \begin{bmatrix} \cos\tilde{\theta}(t) & \sin\tilde{\theta}(t) & 0 \\ -\sin\tilde{\theta}(t) & \cos\tilde{\theta}(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_l(s_l(t) + \Delta) - \tilde{x}(t) \\ y_l(s_l(t) + \Delta) - \tilde{y}(t) \\ \theta_l(s_l(t) + \Delta) - \tilde{\theta}(t) \end{bmatrix}.$$

6. SIMULATION RESULTS

To illustrate our results we considered MATLAB® simulations for a platoon of four vehicles with $L = 3[m]$. The first vehicle starts at $x = 0[m]$, $y = 0[m]$, $\theta = 0[rad]$ with a velocity $v = 0[m/s]$. It accelerates with $a = 2[m/s^2]$ until it reaches a velocity of $33.3[m/s]$, after which it drives with constant velocity. At $t = 20[s]$ it starts driving a half circle with a radius of $800[m]$, after which it continues straight. From $t = 100[s]$ to $t = 105[s]$ a deceleration of $a = -2$ is used, and from $t = 105[s]$ to $t = 110[s]$ an acceleration of $a = 2$ is used, after which the first vehicle drives at constant velocity. The second, third and fourth vehicle are all initiated with $v = 0[m/s]$ at respectively $(x, y, \theta) = (-10, 0, 0)$, $(-10, -10, 1.5)$, and $(-20, -10, 0)$.

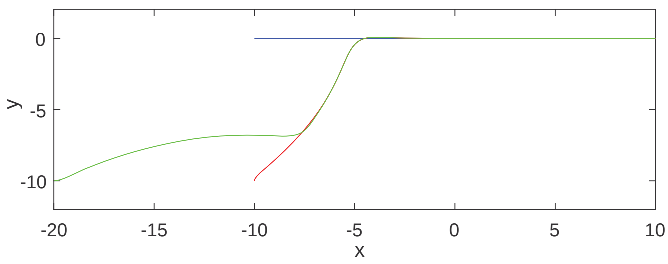

Fig. 1. Resulting paths for platoon of four vehicles.

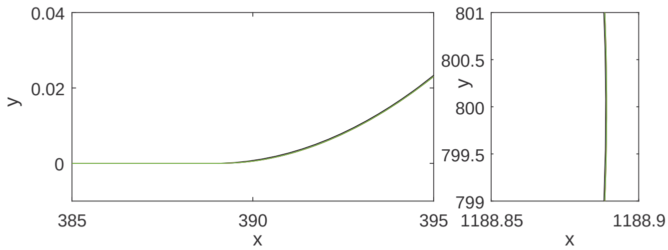

Fig. 2. Zooming in to the transition point from straight line to circle clearly shows that paths remain indistinguishable: corners are not cut. The same holds halfway the circle.

The following vehicles all use the lateral controller (8) with

$$c_1\sigma(x_e) = 0.99\,\mathrm{sat}\left[\frac{2}{0.99}x_e\right] \quad c_2\sigma_2(\theta_e) = 4\,\mathrm{sat}(\theta_e) \quad c_3 = 4,$$

where

$$\mathrm{sat}(x) = \begin{cases} -1 & \text{for } x \leq -1, \\ x & \text{for } -1 \leq x \leq 1, \\ 1 & \text{for } 1 \leq x, \end{cases}$$

placing the poles of the linearisation of the error dynamics for a straight line at -2. Note that each vehicle uses only information of its predecessor, resulting in a distributed controller for the platoon.

The resulting paths are depicted in Fig. 1, where we use black for the first vehicle, blue for the second, red for the third, and green for the fourth vehicle.

As can be seen, the second vehicle perfectly follows the first vehicle. The path of the third vehicle nicely converges toward the path of the second vehicle, and the path of the fourth vehicle converges to that of the third. From the position $(x, y) = (-4, 0)$ the resulting paths are indistinguishable. Also when switching from a straight line to a circle the trajectories clearly show no corner cutting, cf. Fig. 2. The same holds when zooming in on halfway the circle.

For the longitudinal controller the following vehicles all four use (14) with $h = 0.3[s]$, $r = 4.5[m]$, $k\sigma(e) = \mathrm{sat}(e)$, and $\Delta = 10[m]$: the required inter vehicle distance for a velocity of $v = 33.3[m/s]$. In Fig. 3 we depict for each vehicle respectively the velocity $v(t)$ (top) and acceleration $a(t)$ (bottom). Again we use black for the first vehicle, blue for the second, red for the third, and green for the fourth vehicle. The left two figures depict the velocity and accelerations initially used, whereas the right two figures depict the velocity and accelerations during the deceleration and acceleration of the first vehicle. In particular we see that for following vehicles the velocity

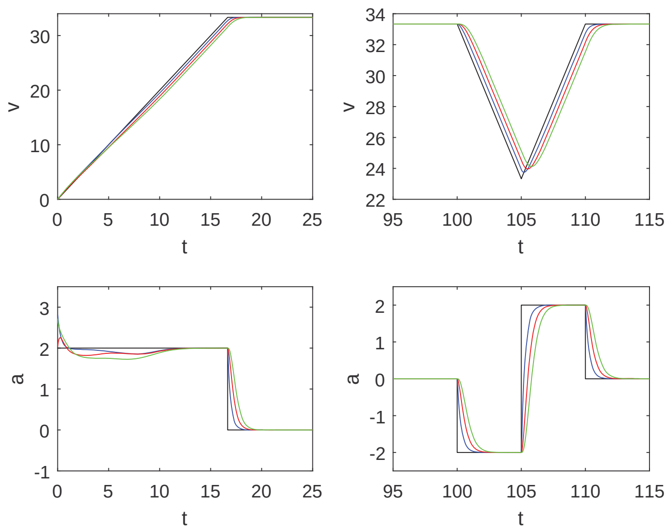

Fig. 3. The velocity v and acceleration a for each of the four vehicles in the platoon as a function of time, using respectively black, blue, red, and green for the consecutive vehicles.

reduction reduces downstream, implying string stability of the platoon.

7. CONCLUSION

In this paper we considered a new approach for solving the vehicle platooning problem incorporating both lateral and longitudinal control. To that end, we dealt with the lateral and longitudinal control problems separately. We considered the lateral control problem as a path following problem. That is, for the following vehicle we determined a path which converges to that of the preceding vehicle. We solved the lateral control problem by assuming that a virtual vehicle is driving along the trajectory of the preceding vehicle, adjusting its velocity for the follower to catch up with it. By associating the position of the following vehicle with the position of the virtual vehicle, we obtain a mapping from a point on the path of the following vehicle to a point on the path of the preceding vehicle and vice versa. Subsequently, this mapping is used when solving the longitudinal control problem. By means of this mapping we can translate the position of the preceding vehicle to a position on the curve of the follower. As a result, we can consider the longitudinal control problem along the planned path of the following vehicle, effectively reducing it to a standard CACC problem of controlling two points on a straight line towards a required inter vehicle distance. An advantage of our approach is that once the following vehicle is on the path of the preceding vehicle, it stays on that path. In particular this implies that for curved paths corners are not cut. We illustrated the benefits of our approach by means of simulation.

A disadvantage of our approach is that two separate differential equations need to be solved, one in the spatial domain and the other in the time domain. Therefore, our next step is to implement our controller in an experimental setup. A special point of interest will be the communication of planned trajectories to downstream vehicles. In our simulations we communicate all points generated by the

ODE solver and use splines for fitting the intermediate points required downstream. How to obtain satisfactory results while keeping the required communication to a minimum is in particular worth investigating. Another point of interest is to improve the lateral controller. Similar to tracking control of marine vessels, we could define a desired orientation of the following vehicle pointing towards the virtual vehicle. By controlling the orientation of the following vehicle towards this desired orientation we focus even more on first converging towards the desired path and only then on following the path. Next, we want to improve the longitudinal controller by incorporating constraints on velocity and acceleration. A final point of interest is in updating the mapping between the paths of preceding and following vehicle based on relative position measurements.

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