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Trajectory Tracking of Under-Actuated Marine Vehicles

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***Abstract*— A control strategy for trajectory tracking of straight line trajectories for autonomous surface vehicles (ASV) is presented in this paper. Our control strategy is based on input-output feedback linearization with the so called *hand position* point as output. This is motivated by a method previously used for ground autonomous vehicles, without external disturbances. The proposed control strategy may be used also for path following. The control approach proposed in this paper is furthermore able to deal with external disturbances, e.g. unknown irrotational ocean currents, and gives an estimate of the disturbance. Using Lyapunov analysis, *almost*-global asymptotic stability (*almost*-GAS) of the closed-loop system is proven. Simulation results are included to validate the theoretical result.**

I. Introduction

There is currently a large interest in developing autonomous vehicles for the execution of tasks which are dull, hard or impossible to execute for humans. For this reason autonomous vehicles are extensively studied and developed for use in different fields. We find unmanned vehicles for ground applications (UGV), for aerial applications (UAV), and marine applications, i.e., autonomous surface vehicles (ASV) and autonomous underwater vehicles (AUV). In their respective environments they represent a valid resource for the execution of several tasks, including mapping [1] or exploration of unknown environments [2]. Clearly, their use is even more relevant in hazardous environments, and environments which are impossible to reach for humans, e.g. space exploration [3] and seabed exploration in the Arctic [4]. In order to achieve reliable execution of tasks, control strategies which aim to improve the autonomy of unmanned vehicles, are needed.

An important control problem for ASVs and AUVs, which has received a lot of attention in the last years, is the trajectory tracking problem, i.e., reaching and following a given trajectory with a time constraint on the along trajectory position of the vehicle. This problem is particularly challenging for ASVs and AUVs since they are generally under-actuated [5], i.e., they are second order non-holonomic vehicles. The trajectory tracking problem for marine vehicles has been dealt with in several works and with different approaches during the last years [6]–[9].

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In [6], the trajectory tracking problem is approached by designing a feedback-linearizing controller, and $\mathscr{K}-$exponential stability for the closed-loop system is proven using cascaded system theory. The designed controller requires the well-known condition of persistence of excitation (PE), i.e., the angular velocity of the vehicle is required to be constantly excited. The PE condition is also considered in [7], where two different controllers for trajectory tracking are developed using Lyapunov's direct method. Both [6], [7] considered that the reference state is generated by a virtual ship, of which the parameters should be the same as for the real one. In practice this is a difficult condition to satisfy. This assumption is removed in [9], where the authors use cascaded system theory and backstepping methods in order to solve the trajectory tracking problem for under-actuated ships subjected to input saturation. The proposed controller is able to drive the position tracking error to a ball centered at the origin and whose radius may be made arbitrarily small. Also in [9] the PE condition is required.

In [8], the backstepping method is used in order to design a controller which makes the position and orientation errors globally asymptotically and locally exponentially converge to balls with radii depending on the desired states. The effect of the ocean current is also considered.

The main contribution of this paper is a feedback linearizing controller for under-actuated ASVs and AUVs moving in a horizontal plane, utilizing the concept of *hand position* which is discussed further below. The choice of the *hand position* as output provides an external dynamics which behaves as a double integrator, and a tracking dynamics which is shown to be *almost*-GAS. The fact that the external dynamics behaves as a double integrator simplifies the definition of a trajectory tracking controller. In fact, in this case it is not necessary to define geometric laws in order to point and converge to a path, e.g. integral line-of-sight (ILOS) [10], or geometric considerations [11]. Moreover, since control strategies for systems with a double integrator dynamics have been thoroughly studied in the past, this result makes it possible to extend well-known control strategies for double integrators to ASVs and AUVs, e.g. for multi-agent systems, for which few results exist for under-actuated systems, and this is the topic of future work.

Generally, in previous works on marine vessels, whether disturbance rejection is taken into account or not, the trajectory tracking problem is tackled by designing a controller in order to make either the center of mass or the pivot point converge to the desired trajectory [6]–[9], [12]–[14]. In this paper we use a different approach. In particular, we adapt the definition of hand position from works about ground vehicles [15]. Using the hand position point, the ground vehicles are seen to have holonomic kinematics properties [15]. Motivated by this, we extend the definition of hand position to marine vehicles. Defining the hand position as the output of our system, we use the input-output feedback-linearization method [16] to design our controller. Since the hand position point can be chosen with some freedom along the center line of the vehicle, choosing it as output of the system has other important advantages. For instance, assume that we want a camera to follow a specific trajectory in the space. If the camera can be placed only on the bow of the vehicle we can choose the bow as the hand position. Then with our approach it is easy to have the camera to follow the assigned trajectory. The task would require more complex path planning phase if the camera is on the bow and we control the position of the center of mass.

Based on the discussion above, we address the particular problem of trajectory tracking. We consider a complete model with non-zero off-diagonal terms in the damping and mass matrices. Furthermore, we take into account the effect of unknown constant and irrotational ocean currents. Finally, the control system is able to give an estimate of the disturbance (both the magnitude and the direction of the ocean current) when the system reaches the steady state. We present a proof which shows *almost*-global asymptotic stability (*almost*-GAS) for the closed-loop system. Furthermore, we present also a controller which solves the path following control problem. In this paper we restrict our analysis to straight line trajectories and constant desired forward velocity for the vehicle. However, this is not a significant disadvantage since it is common for marine operations to define a desired trajectory as a piecewise linear trajectory, i.e., desired trajectories are generally defined by several way-points and straight lines connecting these [5].

The paper is organized as follows: in Section II the model of the class of vehicles which are considered is introduced; our approach is analyzed in Section III; the control objectives are given in IV; Section V deals with the control design for the external dynamics; in Section VI the main results are introduced; Section VII a case study is presented in order to validate the theoretical results; finally in Section VIII the conclusions are given.

II. Vehicle Model

In this section a 3 degree of freedom (DOF) maneuvering model that describes the motion of an ASV or an AUV moving in the horizontal plane, is briefly described [5]. First, the assumptions on which the model is based are presented.

A. Assumptions

Assumption 1: The motion of the vehicle is described in 3 DOF, e.g. surge, sway, yaw.

Assumption 2: The vehicle is port-starboard symmetric.

Assumption 3: The hydrodynamic damping is linear.

Remark 1: Nonlinear damping is not considered to not increase the complexity of the controller. However, due to the passive nature of the damping forces, the stability of the vehicle should still be enforced in case of nonlinear damping.

Assumption 4: The ocean current in the inertial frame $\mathbf{V} = [V_x, V_y]^T$ is constant, irrotational and bounded, i.e., $\exists V_{\max} \geq 0$ such that $\sqrt{V_x^2 + V_y^2} \leq V_{\max}$.

B. The Vessel Model

In the following we use the *North-East-Down* frame (NED) [5] convention for the inertial frame i. The pose of the vehicle, i.e., the position and the orientation of the vehicle, in the NED frame is given by the vector $\eta = [x, y, \psi]^T$. The vector $\nu = [u, v, r]^T$ gives the the surge velocity, sway velocity and angular rate in the body frame. The rotation matrix $\mathbf{R} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]^T$, with $\mathbf{e}_1 = [\cos(\psi), \sin(\psi), 0]^T, \mathbf{e}_2 = [-\sin(\psi), \cos(\psi), 0]^T, \mathbf{e}_3 = [0, 0, 1]^T$, gives the rotation from the body frame to the inertial frame. The ocean current affecting the system in the NED frame is given by $\mathbf{V} = [V_x, V_y, 0]^T$, while $\mathbf{v}_c = \mathbf{R}^T [V_x, V_y, 0]$ is the vector of the current in the body frame. The motion of an ASV or an AUV moving in a horizontal plane, is given by the following 3 DOF maneuvering model given in [5]:

$$\dot{\eta} = \mathbf{R}\nu_r + \mathbf{V} \tag{1a}$$

$$\mathbf{M}\dot{\nu}_r + \mathbf{C}(\nu_r)\nu_r + \mathbf{D}(\nu_r)\nu_r = \mathbf{Bf}, \tag{1b}$$

where $\nu_r = [u_r, v_r, r]^T = \nu - \mathbf{v}_c$ is the vector of the relative velocities in the body frame. The vector $\mathbf{f} = [T_u, T_r]^T$ gives the control inputs T_u which is the thruster force and T_r which is the rudder angle. The vector $\mathbf{f} \in \mathbb{R}^2$ and therefore the vehicle is under-actuated in its configuration space $\mathbb{R}^3$. The structure of the matrices $\mathbf{M}, \mathbf{D}, \mathbf{B}$ is derived according to Assumptions 1, 2, 3, and is given by

$$\mathbf{M} \triangleq \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix}; \mathbf{D} \triangleq \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix}; \mathbf{B} \triangleq \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \\ 0 & b_{32} \end{bmatrix}. \tag{2}$$

The mass matrix $\mathbf{M} = \mathbf{M}^T > 0$ includes the hydrodynamic added mass. The matrix $\mathbf{D}$ gives the linear damping terms, and $\mathbf{B} \in \mathbb{R}^{3\times 2}$ is the actuator configuration matrix. The Coriolis matrix $\mathbf{C}$, which includes the Coriolis and centripetal effects, can be derived from $\mathbf{M}$ as shown in [5]. We consider the next assumption to hold:

Assumption 5: The body-fixed coordinated frame b (body frame) is located at a point $(x_P^*, 0)$, at a distance x_P^* from the vehicle's center of gravity (CG) along the center-line of the ship. The pivot point $(x_P^*, 0)$ is chosen such that $\mathbf{M}^{-1}\mathbf{B}f = [\tau_u, 0, \tau_r]^T$ when the model (1) is written with respect to this point.

Remark 2: The pivot point $(x_P^*, 0)$ satisfying Assumption 5 always exists for ships and AUVs with the center of mass located on the centerline of the vehicle [5]. This is implied by Assumption 2. Furthermore, the body-fixed frame can always be translated to a desired location x_P^* [5].

In the following we consider (1) in component form

$$\dot{x} = u_r\cos(\psi) - v_r\sin(\psi) + V_x \tag{3a}$$
$$\dot{y} = u_r\sin(\psi) + v_r\cos(\psi) + V_y \tag{3b}$$
$$\dot{\psi} = r \tag{3c}$$
$$\dot{u}_r = F_{u_r}(v_r) + \tau_u \tag{3d}$$
$$\dot{v}_r = X(u_r)r + Y(u_r)v_r \tag{3e}$$
$$\dot{r} = F_r(u_r, v_r, r) + \tau_r. \tag{3f}$$

The expressions for $F_{u_r}(u_r), F_r(u_r,v_r,r)$ are given in Appendix I. Furthermore, $X(u_r) = -X_1u_r + X_2, Y(u_r) = -Y_1u_r - Y_2$ and X_1, X_2, Y_1, Y_2 are reported in Appendix I. We consider the following assumption to hold:

Assumption 6: The following bounds hold on Y_1, Y_2

$$Y_1 > 0, \qquad Y_2 > 0. \tag{4}$$

Remark 3: The conditions $Y_1, Y_2 < 0$ imply $Y(u_r) < 0$, which is a natural assumption. In fact, $Y(u_r) \geq 0$ would result in an unstable sway dynamics, which is unfeasible for commercial marine vehicles by design. This is a common assumption for marine systems control design, e.g. [10].

III. Hand Position and Feedback Linearization

Before defining the output tracking problem, we analyze in this section the inherent dynamics of the system (3) in order to make a qualified choice of the system output. In previous works on output trajectory tracking of ASVs and AUVs the motion of the center of mass or of the pivot point $P = [x,y]^T$, which was the origin of the body-frame (cf. Figure 1), was chosen as output. Motivated by the ideas developed for ground vehicles in [15], we take a different approach and we choose instead the motion of a certain "hand position" point as the output of the system.

In [15], the hand position is defined as $\mathbf{h}_{gv} = [x_{gv}, y_{gv}]^T = [x_c + l\cos(\psi), y_c + l\sin(\psi)]^T$, where $[x_c, y_c]^T$ is the center of the wheel's axis, $l > 0$ is a constant and ψ is the yaw angle. The point $[x_c, y_c]^T$ has the same kinematic model as given in (3a-3c) but with $v_r = 0$, i.e., it has unicycle kinematics. The point $[x_c, y_c]^T$ has actuation in yaw and along the x-axis of the body-fixed frame. The point $\mathbf{h}_{gv} = [x_{gv}, y_{gv}]^T$ can be considered as a point indirectly actuated through $[x_c, y_c]^T$ along the x and y axis of the body-fixed frame as illustrated in Figure 1. In fact, from Figure 1, it is easy to see that the application of a positive surge velocity at the point P corresponds to a positive surge velocity at the point $\mathbf{h}$, while a positive (negative) yaw rate at the point P results in a positive (negative) side velocity rl at the point $\mathbf{h}$. Since the kinematic model of the ship is similar to the one of a ground vehicle, we aim to do the same with the choice of the hand position as $\mathbf{h} = [x_1, y_1]^T = [x + l\cos(\psi), y + l\sin(\psi)]^T$, where $l > 0$ is constant, $[x,y]^T$ is the pivot point and ψ the yaw angle (cf. Figure 1). Therefore, the point $\mathbf{h}$ is indirectly controlled through the actuation of the pivot point $[x,y]^T$. The main advantage of choosing the motion of the point $\mathbf{h}$ as output of our systems is that applying the output feedback linearization method [16], we see that its dynamic behavior is described

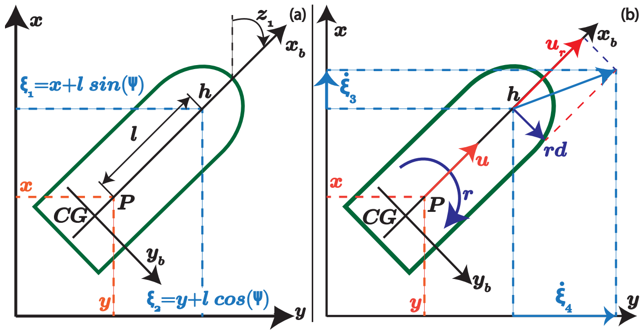

Fig. 1: a) The center of gravity (CG), the pivot point (P) and the hand position $\mathbf{h}$. b) Relative velocities in the NED frame.

by a double integrator together with a tracking dynamics which is *almost*-GAS.

We now apply the output feedback linearization method [16], choosing the hand position point $\mathbf{h} \in \mathbb{R}^2$ as output. First, we need to check if (3) is input-output feedback linearizable with output $\mathbf{h}$, i.e, we need to check if the vector relative degree $\rho = [\rho_{x_1} \rho_{y_1}]^T$ is well defined. In order to check this condition we derive twice the expression of $\mathbf{h} = [x_1 = x + l\cos(\psi),\ y_1 = y + l\sin(\psi)]^T$ and we get

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} F_u(v,r) - vr - lr^2 \\ ur + X(u)r + Y(u)v + F_r(u,v,r)l \end{bmatrix} + \underbrace{\begin{bmatrix} \cos(\psi) & -l\sin(\psi) \\ \sin(\psi) & l\cos(\psi) \end{bmatrix}}_{\mathbf{B}(\psi)} \begin{bmatrix} \tau_u \\ \tau_r \end{bmatrix}. \tag{5}$$

From (5), we see that the system has a well-defined vector relative degree since $\rho_{x_1} = \rho_{y_1} = 2$ for $l \neq 0$ since $\mathbf{B}(\psi)$ is non-singular for $l \neq 0$. Note that $l = 0$ makes $\mathbf{B}(\psi)$ singular and therefore the pivot point cannot be chosen as output.

In order to perform an input-output feedback linearization we define the change of coordinates

$$z_1 = \psi \tag{6a}$$
$$z_2 = r \tag{6b}$$
$$\xi_1 = x_1 \tag{6c}$$
$$\xi_2 = y_1 \tag{6d}$$
$$\xi_3 = u_r\cos(\psi) - v_r\sin(\psi) - rl\sin(\psi) \tag{6e}$$
$$\xi_4 = u_r\sin(\psi) + v_r\cos(\psi) + rl\cos(\psi). \tag{6f}$$

Note that we cannot take $\xi_3 = \dot{\xi}_1, \xi_4 = \dot{\xi}_2$ since this would imply that our change of coordinates incorporates the knowledge of the ocean current. Since we assume that we do not know the ocean current, our change of coordinates results in $\xi_3 = \dot{\xi}_1 - V_x, \xi_4 = \dot{\xi}_2 - V_y$. Therefore, ξ_3, ξ_4 are the relative velocities of the vehicle in the global frame.

Using the new coordinates, (3) can be rewritten as

$$\dot{z}_1 = z_2 \tag{7a}$$
$$\dot{z}_2 = F_{z_2}(z_1, \xi_3, \xi_4) + \tau_r \tag{7b}$$
$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} \xi_3 \\ \xi_4 \end{bmatrix} + \begin{bmatrix} V_x \\ V_y \end{bmatrix} \tag{7c}$$
$$\begin{bmatrix} \dot{\xi}_3 \\ \dot{\xi}_4 \end{bmatrix} = \begin{bmatrix} F_{\xi_3}(z_1, \xi_3, \xi_4, V_x, V_y) \\ F_{\xi_4}(z_1, \xi_3, \xi_4, V_x, V_y) \end{bmatrix} + \begin{bmatrix} \cos(z_1) & -l\sin(z_1) \\ \sin(z_1) & l\cos(z_1) \end{bmatrix} \begin{bmatrix} \tau_u \\ \tau_r \end{bmatrix} \tag{7d}$$

where

$$\begin{bmatrix} F_{\xi_3}(z_1,\xi_3,\xi_4,V_x,V_y) \\ F_{\xi_4}(z_1,\xi_3,\xi_4) \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} F_{u_r}(v_r,r)-v_r r-dr^2 \\ u_r r+X(u_r)r+Y(u_r)v_r+F_r(u_r,v_r,r)l \end{bmatrix} \quad (8)$$

and $F_{z_2}(z_1,\xi_3,\xi_4)$ is obtained from $F_r(u_r,v_r,r)$ substituting $u_r = \xi_3\cos(z_1)+\xi_4\sin(z_1)$, $v_r = -\xi_3\sin(z_1)+\xi_4\cos(z_1)-z_2 l$, and $r = z_2$. Now we apply the following changed input in order to linearize the external dynamics

$$\begin{bmatrix} \tau_u \\ \tau_r \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -l\sin(\psi) \\ \sin(\psi) & l\cos(\psi) \end{bmatrix}^{-1} \begin{bmatrix} -F_{\xi_3}(z_1,\xi_3,\xi_4)+\mu_1 \\ -F_{\xi_4}(z_1,\xi_3,\xi_4)+\mu_2 \end{bmatrix}. \quad (9)$$

In (9), the terms μ_1, μ_2 are new inputs which are defined in Section V in order to solve the trajectory tracking problem. After substituting (9) in (7) we obtain

$$\dot{z}_1 = z_2 \quad (10a)$$

$$\begin{aligned}\dot{z}_2 =& -\left(\left(Y_1 - \tfrac{X_1-1}{l}\right)(\xi_3\cos(z_1)+\xi_4\sin(z_1)) + Y_2 + \tfrac{X_2}{l}\right) z_2 - \\ &-\left(\tfrac{Y_1}{l}(\xi_3\cos(z_1)+\xi_4\sin(z_1)) + \tfrac{Y_2}{l}\right)(\xi_3\sin(z_1)-\xi_4\cos(z_1)) + \\ &-\tfrac{\mu_1\sin(z_1)}{l} + \tfrac{\mu_2\cos(z_1)}{l}\end{aligned} \quad (10b)$$

$$\dot{\xi}_1 = \xi_3 + V_x \quad (10c)$$

$$\dot{\xi}_2 = \xi_4 + V_y \quad (10d)$$

$$\dot{\xi}_3 = \mu_1 \quad (10e)$$

$$\dot{\xi}_4 = \mu_2 \quad (10f)$$

Since the state z_1 in (10b) appears only as argument of trigonometric functions of period 2π, we can consider (10a) to (10b) to take values on the manifold $\mathbb{M} = \mathbb{S} \times \mathbb{R}$ where $\mathbb{S}$ is the one dimensional sphere.

IV. Control Objectives

In this section we formalize the straight line trajectory tracking problem for ASVs and AUVs in the presence of an ocean current of unknown magnitude and direction. Our control objective is to make the point **h** follow a desired trajectory. As already mentioned above, we focus on straight line trajectories and constant forward velocity. Without loss of generality we choose the trajectory aligned along the global x axis resulting in $\xi_{1_d} = u_d t, \xi_{2_d} = 0$, where u_d is a chosen positive constant and t is the time. From (10), we need $\xi_3 \to u_d - V_x, \xi_4 \to -V_y$. The control objectives are thus

$$\lim_{t\to\infty}(\xi_1 - u_d t) = 0 \quad (11a)$$

$$\lim_{t\to\infty}\xi_2 = 0 \quad (11b)$$

$$\lim_{t\to\infty}(\xi_3 - (u_d - V_x)) = 0 \quad (11c)$$

$$\lim_{t\to\infty}(\xi_4 - (-V_y)) = 0. \quad (11d)$$

Notice that the control objectives are based on the knowledge of V_x, V_y. However, we have already assumed that the vehicle does not know **V**. For this reason, we include an integral action in the controller in order to compensate for and estimate the ocean current. We consider the following assumption to hold

Assumption 7: The linear velocity is such that $u_d > V_x$. Furthermore, the vehicle's thrusters provide enough power in order to overcome the ocean current disturbance.

Remark 4: This is a necessary assumption in order to have forward motion of the vehicle.

Remark 5: Note that controlling the position of **h**, the pivot point (x,y) may be as far as l from the trajectory. However, this is not a problem, since, if we choose **h** in correspondence of a sensor (e.g., a camera) which has to follow a given trajectory, our goal is to have **h** to track the desired trajectory.

V. The Controller

In this section we design the control law $\mu = [\mu_1, \mu_2]^T$ in (10) in order to fulfill the control objectives (11).

In order to make the output track the reference trajectory while compensating for the unknown ocean current disturbance, we choose the following new inputs

$$\mu_1 = -k_{v_x}(\xi_3 - \xi_{3_d}) - k_{p_x}(\xi_1 - \xi_{1_d}) - k_{I_x}(\xi_{1_I} - \xi_{1_{d_I}}) \quad (12a)$$

$$\mu_2 = -k_{v_y}(\xi_4 - \xi_{4_d}) - k_{p_y}(\xi_2 - \xi_{2_d}) - k_{I_y}(\xi_{2_I} - \xi_{2_{d_I}}) \quad (12b)$$

where $k_{p_x}, k_{p_y}, k_{v_x}, k_{v_y}, k_{I_x}, k_{I_y}$ are positive real gains, $\xi_{i_I} = \int \xi_i$ where $i \in \{1, 2, 1_d, 2_d\}$, and $\xi_{2_d} = \xi_{4_d} = 0$. Based on disturbance rejection theory for linear systems [17], the integral states ξ_{1_I}, ξ_{2_I} are used in order to compensate for the ocean current disturbance and obtain an estimate of **V**.

VI. Main Result

In this section we present the main result. In the following theorem the conditions under which the controller (9) makes the system achieve the control objectives (11) are given.

Theorem 1: Consider an under-actuated marine vehicle described by the model (3). Consider the hand position point $\mathbf{h} = [x_1, y_1]^T = [x + l\cos(\psi), y + l\sin(\psi)]^T$, where $[x,y]^T$ is the position of the pivot point of the ship, l is a positive constant and ψ is the yaw angle of the vehicle. Then define $U_d = \sqrt{(u_d - V_x)^2 + V_y^2} > 0$ as the desired relative velocity magnitude and $\phi = \arctan\left(\frac{-V_y}{u_d - V_x}\right)$ as the crab angle. If Assumptions 1-7 are satisfied and if

$$0 < U_d < \frac{Y_2}{Y_1} \quad (13)$$

$$k_{v_i} > 0,\ k_{p_i} > 0,\ k_{I_i} > 0,\ i \in \{x,y\} \quad (14)$$

$$k_{v_i} k_{p_i} > k_{I_i} \quad i \in \{x,y\} \quad (15)$$

$$l > \max\left\{\frac{m_{22}}{m_{23}}, -\frac{X_2}{Y_2}\right\} \quad (16)$$

then the controller (9), where the new inputs μ_1, μ_2 are given by (12), guarantees the achievement of the control objectives (11). In particular, $(z_1, z_2, \xi_1, \xi_2, \xi_3, \xi_4) \to (\phi, 0, u_d t, 0, u_d - V_x, -V_y)$ *almost*-globally asymptotically. Furthermore, the steady state values of the integral variables give an estimate of the ocean current:

$$V_x = \frac{k_{I_x}(\xi_{1_I} - \xi_{1_{I_d}})}{k_{v_x}} + u_d, \quad V_y = \frac{k_{I_y}(\xi_{2_I} - \xi_{2_{I_d}})}{k_{v_y}}. \quad (17)$$

Remark 6: From Figure 2 it is clear that the crab angle ϕ is the yaw angle the ship has to move with in order

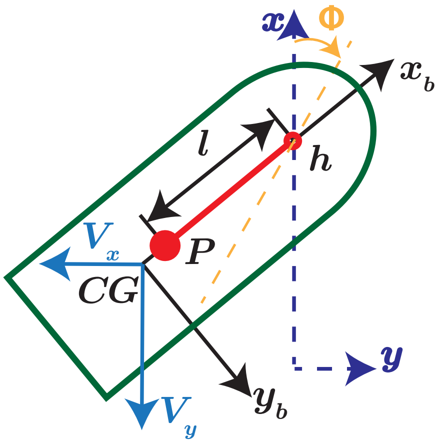

Fig. 2: Internal dynamics and pendulum dynamics similarities. When $\mathbf{h}$ is on the path the ocean current makes P swing around $\mathbf{h}$.

to counteract the ocean current effect. This is how an experienced helmsman acts to compensate for ocean currents. Typically this has been set as a constant crab angle, and if chosen too small the ASV/AUV drifts, and chosen too large, the ASV/AUV uses more energy than necessary since it is hydrodynamically less energy efficient for a ship/torpedo shaped form to travel with a heading transverse to the path. Notice that we assume an unknown ocean current and therefore also ϕ is in general unknown. However, the integral action in (9) takes care of compensating for the unknown value of the constant disturbance.

Remark 7: Note that ϕ is constant, and this implies $\dot{\phi} = 0$.

Proof: Applying the change of variables $\tilde{z}_1 = z_1 - \phi, \tilde{z}_2 = z_2 - \dot{\phi} = z_2, \tilde{\xi}_{1_I} = \xi_{1_I} - \xi_{1_{I_d}}, \tilde{\xi}_{2_I} = \xi_{2_I} - \xi_{2_{I_I}}, \tilde{\xi}_1 = \xi_1 - u_d t, \tilde{\xi}_2 = \xi_2, \tilde{\xi}_3 = \xi_3 - (u_d - V_x), \tilde{\xi}_4 = \xi_4 + V_y$ and defining the vectors $\tilde{z} = [\tilde{z}_1, \tilde{z}_2]^T, \tilde{\xi} = [\tilde{\xi}_{1_I}, \tilde{\xi}_{2_I}, \tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3, \tilde{\xi}_4]^T$, the closed-loop system can be written as

$$\dot{\tilde{z}} = H_{\tilde{z}}(\tilde{z}_1)\tilde{z} + G(\tilde{z}, \tilde{\xi}_3, \tilde{\xi}_4)\tilde{\xi} \tag{18a}$$

$$\dot{\tilde{\xi}} = H_{\tilde{\xi}}\tilde{\xi} \tag{18b}$$

where $G(\cdot)$ is reported in Appendix I and

$$H_{\tilde{z}}(\tilde{z}) = \begin{bmatrix} 0 & 1 \\ -(c\cos(z_1)+d) & -(a\cos(z_1)+b) \end{bmatrix} \tag{19}$$

$$a = \left(Y_1 - \frac{X_1 - 1}{l}\right) U_d \qquad b = Y_2 + \frac{X_2}{l} \tag{20}$$

$$c = \frac{Y_1 U_d^2}{l} \qquad d = \frac{Y_2 U_d}{l} \tag{21}$$

$$H_{\tilde{\xi}} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -k_{V_x} & 0 & -k_{p_x} & 0 & -k_{v_x} & 0 \\ 0 & -k_{V_y} & 0 & -k_{p_y} & 0 & -k_{v_y} \end{bmatrix}. \tag{22}$$

where, according to Assumption 6, we have $c, d > 0$ and (13) implies $d > c$. Furthermore, the condition (16) implies also $a, b > 0$. We now study the stability properties of the external dynamics (18b) and the tracking dynamics (Equation (18a) with $G(\tilde{z}, \tilde{\xi}_3, \tilde{\xi}_4)\tilde{\xi} = 0$) and then the stability properties of the overall system (18).

A. The external dynamics

The equilibrium point of (18b) is $(0,0,0,0,0,0)$. The matrix $H_{\tilde{\xi}}$ is Hurwitz for $k_{v_i}, k_{p_i}, k_{I_i}$ respecting (14-15). Thus, we have that the origin is globally exponentially stable, and thus that $[\xi_{1_I}, \xi_{2_I}, \xi_1, \xi_2, \xi_3, \xi_4]^T \to [\xi_{1_{I_d}} + k_{v_x}V_x/k_{I_x}, \xi_{24_{I_d}} + k_{v_y}V_y/k_{I_y}, u_d t, 0, u_d - Vy, -V_y]^T$, globally exponentially.

Remark 8: Notice that from the integral states we obtain an estimate of the unknown ocean current when the steady state condition is reached. In particular, we have

$$V_x = \frac{k_{I_x}(\xi_{1_I} - \xi_{1_{I_d}})}{k_{v_x}} + u_d, \quad V_y = \frac{k_{I_y}(\xi_{2_I} - \xi_{2_{I_d}})}{k_{v_y}}. \tag{23}$$

B. The tracking dynamics

Let us now focus on the tracking dynamics:

$$\dot{z}_1 = z_2 \tag{24a}$$

$$\dot{z}_2 = -(a\cos(z_1) + b)z_2 - (c\cos(z_1) + d)\sin(z_1). \tag{24b}$$

As said in Section III, the system (24) can be studied on the manifold $\mathbb{M} = \mathbb{S} \times \mathbb{R} = \{(\cos(\theta), \sin(\theta), r) \mid \theta \in \mathbb{R}, r \in \mathbb{R}\}$. The system (24) has two equilibria , and they are

$$E_s = (1,0,0) \in \mathbb{M} \quad E_u = (-1,0,0) \in \mathbb{M} \tag{25}$$

where E_s is a stable node, while, due to the assumption $d > c$, E_u is a saddle point, which implies that it is an hyperbolic equilibrium. Let us now define $\tilde{z}_s = [\sin(\tilde{z}_1), \tilde{z}_2]^T$ and the following Lyapunov function candidate (LFC)

$$W = \frac{1}{2}\tilde{z}_s^T \underbrace{\begin{bmatrix} a^2+c & a \\ a & 1 \end{bmatrix}}_{P_{z_s}} \tilde{z}_s + (ab + d)(1 - \cos(\tilde{z}_1)). \tag{26}$$

We have that $W > 0 \; \forall (\cos(\tilde{z}_1), \sin(\tilde{z}_1), \tilde{z}_2) \in \mathbb{M} - \{[1,0,0]\}$ and $W = 0$ only for $(\cos(\tilde{z}_1), \sin(\tilde{z}_1), \tilde{z}_2) = (1,0,0)$. The derivative is

$$\dot{W} = -\tilde{z}_s^T \underbrace{\begin{bmatrix} b & 0 \\ 0 & a(d + c\cos(\tilde{z}_1)) \end{bmatrix}}_{Q_{\tilde{z}}} \tilde{z}_s \tag{27}$$

where

$$\dot{W} \leq 0 \; \forall (\sin(\tilde{z}_1), \tilde{z}_2) \neq (0,0). \tag{28}$$

This proves that $(\sin(\tilde{z}_1), \tilde{z}_2) = (0,0)$ is GAS. However, $\sin(\tilde{z}_1) = 0$ corresponds either to $\cos(\tilde{z}_1) = 1$ or $\cos(\tilde{z}_1) = -1$ on the one-dimension unit sphere. But, as said above E_u is unstable and hyperbolic. Then, according to [18, Theorem 3.2.1] we have that E_u has a stable and an unstable manifold $\mathscr{W}_u^s, \mathscr{W}_u^u$, respectively. The unstable manifold $\mathscr{W}_u^u$ is tangent to the eigenspace spanned by the positive real part eigenvalue of the Jacobian matrix of the system (24) evaluated at E_u. This manifold is therefore one-dimensional and converges to the only other equilibrium point of the system, that is E_s. The stable manifold $\mathscr{W}_u^s$ is also one-dimensional since it is spanned by the negative real part eigenvalue of the Jacobian matrix of (24). Since the system (24) evolves on the manifold $\mathbb{M} = \mathbb{S} \times \mathbb{R}$, which is 2-dimensional (it is a "pipe-shaped" manifold, that is, it is a cylindrical surface in the space), we have that $\mathscr{W}_u^s$ has one dimension less than $\mathbb{M}$ and has therefore zero Lebesgue measure.

At this point we can conclude that all the trajectories which do not start on $\mathscr{W}_u^s$ converge to the point E_s. Furthermore, since $\mathscr{W}_u^s$ has zero Lebesgue measure, we can say that E_s is *almost*-GAS.

C. Stability of the complete system

Since (18b) is GES, we have two positive definite matrices P_ξ, Q_ξ such that they satisfy the Lyapunov equation $H_\xi^T P_\xi + P_\xi H_\xi = -Q_\xi$. Thus, we choose the following LFC

$$V = W + \kappa\, \tilde{\xi}^T P_\xi \tilde{\xi} \tag{29}$$

where W is the same as in (26), and $\kappa > 0$ still to be determined. Deriving (29) along the direction of (18) we obtain

$$\dot{V} \leq -\tilde{z}_s^T Q_{\tilde{z}} \tilde{z}_s - \kappa\, \tilde{\xi}^T Q_\xi \tilde{\xi} + \frac{\partial W}{\partial \tilde{z}} G(\cdot)\tilde{\xi}. \tag{30}$$

For $G(\cdot)$ and W it holds that

$$G(\tilde{z}, \tilde{\xi}_3, \tilde{\xi}_4) \leq G_1(\|\tilde{\xi}\|)\|\tilde{z}_s\| + G_2(\|\tilde{\xi}\|) \leq \bar{G}_1\|\tilde{z}_s\| + \bar{G}_2$$
$$\left\|\frac{\partial W}{\partial \tilde{z}}\right\| \leq \|\tilde{z}_s\| \left\| \begin{bmatrix} a^2 + c + \frac{ab+d}{2} & a \\ a & 1 \end{bmatrix} \right\| \leq \alpha_1 \|\tilde{z}_s\|,$$

where $\bar{G}_1 = G_1(\bar{\xi})$, $\bar{G}_2 = G_2(\bar{\xi})$, and $\bar{\xi}$ is the upperbound of $\|\xi\|$. Let $\lambda_{P_{z_s}}^{\min}$, $\lambda_{P_\xi}^{\min}$, $\lambda_{Q_{\tilde{z}}}^{\min}$, $\lambda_{Q_\xi}^{\min}$ denote the minimal eigenvalue of P_{z_s}, P_ξ, $Q_{\tilde{z}}$, Q_ξ respectively. Since (18b) is GES, there exists a time t^* such that for all $t \geq t^*$: $\|\tilde{\xi}(t)\| \leq \lambda_{Q_{\tilde{z}}}^{\min}/(2\alpha_1\bar{G}_1)$. For $t \leq t^*$ and

$$\kappa > \alpha_1^2 \bar{G}_2^2 \left(\lambda_{Q_{\tilde{z}}}^{\min} \lambda_{Q_\xi}^{\min} + \frac{2\lambda_{Q_{\tilde{z}}}^{\min} \alpha_1 \bar{G}_1 \bar{\xi} \lambda_{P_\xi}^{\min}}{\lambda_{P_{z_s}}^{\min}} \right)^{-1}$$

we have

$$\begin{aligned}
\dot{V} &\leq -\tilde{z}_s^T Q_{\tilde{z}} \tilde{z}_s - \kappa\, \tilde{\xi}^T Q_\xi \tilde{\xi} + \alpha_1 \|\tilde{z}_s\|(\bar{G}_1\|\tilde{z}_s\| + \bar{G}_2)\tilde{\xi} \\
&\leq \alpha_1 \bar{G}_1 \bar{\xi} \|\tilde{z}_s\|^2 - \lambda_{Q_{\tilde{z}}}^{\min} \|\tilde{z}_s\|^2 + \alpha_1 \bar{G}_2 \|\tilde{z}_s\| \|\tilde{\xi}\| - \kappa \lambda_{Q_\xi}^{\min} \|\tilde{\xi}\|^2 \\
&\leq \alpha_1 \bar{G}_1 \bar{\xi} \|\tilde{z}_s\|^2 + \frac{2\alpha_1 \bar{G}_1 \bar{\xi} \kappa \lambda_{P_\xi}^{\min}}{\lambda_{P_{z_s}}^{\min}} \|\tilde{\xi}\|^2 \leq \frac{2\alpha_1 \bar{G}_1 \bar{\xi}}{\lambda_{P_{z_s}}^{\min}} V,
\end{aligned}$$

so $V(t)$ remains bounded for $t \leq t^*$. For $t \geq t^*$ we have

$$\dot{V} \leq -\tilde{z}_s^T Q_{\tilde{z}} \tilde{z}_s - \kappa\, \tilde{\xi}^T Q_\xi \tilde{\xi} + \alpha_1 \|\tilde{z}_s\|(\bar{G}_1\|\tilde{z}_s\| + \bar{G}_2)\tilde{\xi} \tag{31}$$
$$\leq -\frac{1}{2}\lambda_{Q_{\tilde{z}}}^{\min} \|\tilde{z}_s\|^2 + \alpha_1 \bar{G}_2 \|\tilde{z}_s\| \|\tilde{\xi}\| - \kappa \lambda_{Q_\xi}^{\min} \|\tilde{\xi}\|^2 \tag{32}$$

which is negative definite for $\kappa > 2\alpha_1^2 \bar{G}_2^2/(\lambda_{Q_{\tilde{z}}}^{\min} \lambda_{Q_\xi}^{\min})$. Thus the system always converge to the equilibrium $\left(\sin(\tilde{z}_1), \tilde{z}_2, \tilde{\xi}\right) = (0, 0, \mathbf{0}_{1\times 6})$. The state $\tilde{z}_1$ converge either to $\tilde{z}_1 = 0$, of $\tilde{z}_1 = \pm\pi$, so we can conclude that $(\tilde{z}, \tilde{\xi}) = (\mathbf{0}_{1\times 2}, \mathbf{0}_{1\times 6})$ is *almost*-GAS. ■

D. Path following control

The path following problem consists of a geometric task which is fulfilled when the vehicle reaches a straight-line path. In addition, the ship should move along this path with a desired constant forward relative velocity. The main difference with the trajectory tracking stays in the fact that there is not a time constraint on the along-path position. In fact, the along path position ξ_1 is left uncontrolled [10]. With our approach we can fulfill also the path following task slightly adapting the auxiliary controller (12). We consider the NED frame such that its x-axis is aligned along the path. This choice does not cause any loss of generality. We describe a linear path as the set $\mathscr{P} = \{(\xi_1, \xi_2) \in \mathbb{R}^2 | \xi_2 = 0\}$. The path following problem is then described by the control objectives (11b), (11d) and

$$\lim_{t\to\infty} (\xi_3 - u_d) = 0 \tag{33}$$

and $u_d > V_x$. Note that since we do not have a time constraint on ξ_1 we do not consider it in our control objectives. In order to fulfill (11b), (11d) and (33), we choose

$$\mu_1 = -k_{v_x}(\xi_3 - \xi_{3_d}) \tag{34a}$$
$$\mu_2 = -k_{v_y}(\xi_4 - \xi_{4_d}) - k_{p_y}(\xi_2 - \xi_{2_d}) - k_{I_y}(\xi_{2_I} - \xi_{2_{d_I}}) \tag{34b}$$

where $\xi_{3_d} = u_d, \xi_{4_d} = 0, \xi_{2_{d_I}} = \int \xi_{2_d}$. For the path following task we have the following corollary of Theorem 1

Corollary 1: Consider an under-actuated marine vehicle described by the model (3b-3f). Consider the hand position point $\mathbf{h} = [x_1, y_1]^T = [x + l\cos(\psi), y + l\sin(\psi)]^T$, where $[x, y]^T$ is the position of the pivot point of the ship, l is a positive constant and ψ is the yaw angle of the vehicle. Then define $\bar{U}_d = \sqrt{u_d^2 + V_y^2}$ as the desired relative velocity and $\bar{\phi} = \arctan\left(\frac{-V_y}{u_d}\right)$ as the crab angle. If Assumptions 1-7 are satisfied and if

$$\bar{U}_d < \tfrac{Y_2}{Y_1} \tag{35}$$
$$k_{v_i} > 0,\ k_{p_i} > 0,\ k_{I_i} > 0,\ i \in \{x, y\} \tag{36}$$
$$k_{v_i} k_{p_i} > k_{I_i} \quad i \in \{x, y\} \tag{37}$$
$$l > \max\left\{\frac{m_{22}}{m_{23}}, -\frac{X_2}{Y_2}\right\} \tag{38}$$

then the controller (9), where the new inputs μ_1, μ_2 are given by (12), guarantees the achievement of the control objectives (11). In particular, $(z_1, z_2, \xi_2, \xi_3, \xi_4) \rightarrow (\bar{\phi}, 0, 0, u_d, -V_y)$ *almost*-globally asymptotically.

Remark 9: It is common for path following control laws (e.g. ILOS [10]) to control the body frame forward velocity, i.e., the surge velocity $u \to u_d$ instead of the along-path velocity, and leave the position along the x axis of the NED frame uncontrolled. In the presence of an unknown ocean current, the vehicle has to keep an unknown crab angle $\bar{\phi}$ in order to stay on the path due to the under-actuation. Therefore, the velocity u is misaligned with the path. This results in an absolute velocity in the NED frame $U_{ss} = u_d\cos(\bar{\phi})$ in the steady state, i.e., when the vehicle is on the path. With our approach, i.e., defining $\xi_3 = u_d$

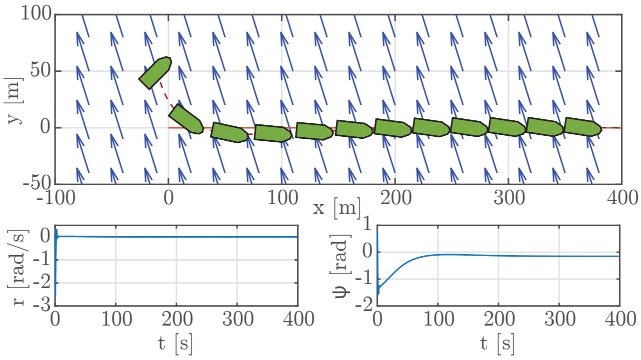

Fig. 3: Ship's motion and internal dynamics states.

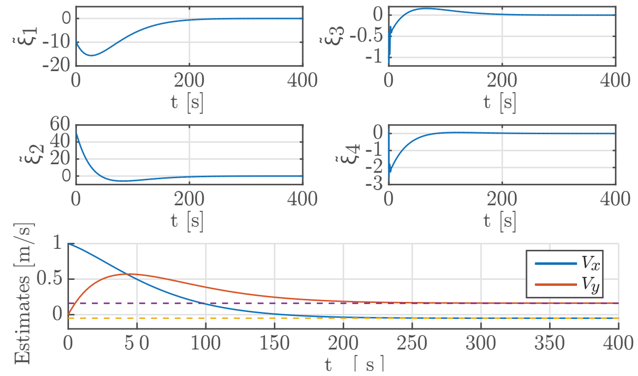

Fig. 4: Time evolution of the error states and ocean current estimates.

and leaving ξ_1 uncontrolled, we are assigning the desired along-path velocity. This implies that the body frame forward velocity $u_r = u_d \cos(\bar{\phi}) - V_y \sin(\bar{\phi})$.

VII. Simulation Results

In this section a simple case study is presented in order to validate the theoretical results given in Section VI. Using the model of the LAUV (Light Autonomous Underwater Vehicle) given in [19] we define a desired straight-line trajectory fixed along the global x-axis. The desired linear velocity in the inertial frame is $u_d = 1$[m/s] and $\xi_{1_d} = 1t$[m]. The initial orientation of the vehicle is $\psi_1|_{t=0} = z_1|_{t=0} = 45^\circ$ and the initial position is $[x_1|_{t=0}, y_1|_{t=0}]^T = [\xi_1|_{t=0}, \xi_2|_{t=0}]^T = [-10, 50]^T$[m]. The controller gains are chosen as $k_{v_x} = k_{v_y} = 10$, $k_{p_x} = k_{p_y} = .5$ and $k_{I_x} = k_{I_y} = 0.007$. We consider an ocean current (unknown to the control system) $\mathbf{V} = [V_x, V_y]^T = [-0.05, 0.16]^T$[m/s]. From Figure 3 we can see how the ship approaches the desired trajectory with a smooth motion, and how the ψ, r reach a steady state condition. Note that $\psi \to \phi \approx -8.66^\circ$. From Figure 4 we can see that all the control objectives (9) are achieved smoothly, and how at the steady state condition we can exploit the integral state in the controller (12) in order to obtain an estimate of the unknown ocean current. Finally from Figure 5

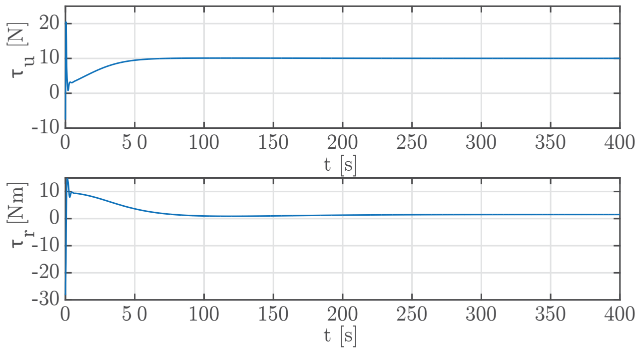

Fig. 5: Control inputs τ_u, τ_r.

we can see the time evolution of the control inputs τ_u, τ_r, they set to the steady state values $\tau_u = 10$[N] and $\tau_r = 1.5$[kN].

VIII. Conclusions and Future Work

In this paper we have presented a new approach for trajectory tracking of straight lines for ASVs and AUVs moving in a horizontal plane. We take into account the effect of unknown irrotational ocean currents. We adopt from recent results for ground vehicles the definition of hand position to marine vehicles, and then we use the feedback linearization method to reduce the problem of controlling a second order non-holonomic vehicle, i.e., the ASV or AUV, to the problem of controlling a simple double integrator. In particular, the resulting system has a double integrator as external dynamics, and we prove that the resulting internal dynamics is *almost*-GAS. The main advantage of our approach is given by the fact that we can choose the hand-position as a point located on the center line of the vehicle, maybe in correspondence of a certain sensor (e.g., a camera) and steer this point to a straight line trajectory using a simple PID controller. In future work we build on this result to extend well-known control strategies for double integrators for consensus based formation control. We furthermore propose a straight-line trajectory tracking controller and prove that the closed-loop system is *almost*-GAS. The controller is also shown to provide an estimate of the ocean current.

Appendix I
Equations

$$F_{u_r}(v_r, r) \triangleq \frac{1}{m_{11}}(m_{22}v_r + m_{23}r)r - \frac{d_{11}}{m_{11}}u_r, \tag{39}$$

$$X_1(\mathbf{M}) \triangleq \frac{m_{11}m_{33} - m_{23}^2}{m_{22}m_{33} - m_{23}^2} \quad X_2(\mathbf{M}, \mathbf{D}) \triangleq \frac{d_{33}m_{23} - d_{23}m_{33}}{m_{22}m_{33} - m_{23}^2} \tag{40}$$

$$Y_1(\mathbf{M}) \triangleq \frac{(m_{11} - m_{22})m_{23}}{m_{22}m_{33} - m_{23}^2} \quad Y_2(\mathbf{M}, \mathbf{D}) \triangleq \frac{d_{22}m_{33} - d_{32}m_{23}}{m_{22}m_{33} - m_{23}^2} \tag{41}$$

$$X(u_r) \triangleq -X_1 u_r + X_2 \quad Y(u_r) \triangleq -Y_1 u_r - Y_2, \tag{42}$$

$$\begin{aligned} F_r(u_r, v_r, r) \triangleq\ & \frac{m_{23}d_{22} - m_{22}(d_{32} + (m_{22} - m_{11})u_r)}{m_{22}m_{33} - m_{23}^2} v_r \\ & + \frac{m_{23}(d_{23} + m_{11}u_r) - m_{22}(d_{33} + m_{23}u_r)}{m_{22}m_{33} - m_{23}^2} r, \end{aligned} \tag{43}$$

$$G(\tilde{z}, \tilde{\xi}_3, \tilde{\xi}_4) \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sin(\tilde{z}_1)}{l} & \frac{\cos(\tilde{z}_1)}{l} & 0 & 0 & \alpha(\tilde{z}, \tilde{\xi}_3) & \beta(\tilde{z}, \tilde{\xi}_3, \tilde{\xi}_4) \end{bmatrix} \tag{44}$$

$$\alpha(\tilde{z}, \tilde{\xi}_3) \triangleq -\Big((Y_1 U_d \cos(\tilde{z}_1)^2 + Y_1 U_d \cos(\tilde{z}_1) \sin(\tilde{z}_1) + Y_2 \sin(\tilde{z}_1)$$

$$+Y_2\tilde{\xi}_3\sin(\tilde{z}_1)\cos(\tilde{z}_1)+\left(-Y_1+\tfrac{X_1-1}{d}\right)\tilde{z}_2\cos(\tilde{z}_1)\Big) \qquad (45)$$

$$\begin{aligned}\beta(\tilde{z},\tilde{\xi}_3,\tilde{\xi}_4) \triangleq\; & Y_1U_d\sin(\tilde{z}_1)\cos(\tilde{z}_1)+Y_1U_d\cos(\tilde{z}_1)^2+Y_2\cos(\tilde{z}_1)\\ & +Y_1\tilde{\xi}_3\sin(\tilde{z}_1)+Y_1\cos(\tilde{z}_1)^2\tilde{\xi}_3-Y_1\cos(\tilde{z}_1)\sin(\tilde{z}_1)\tilde{\xi}_4+\\ & +\left(-Y_1+\tfrac{X_1-1}{d}\right)\tilde{z}_2\sin(\tilde{z}_1)\end{aligned}$$

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