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Adaptive tracking control of nonholonomic systems: an example

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Abstract

We study an example of an adaptive (state) tracking control problem for a four-wheel mobile robot, as it is an illustrative example of the general adaptive state-feedback tracking control problem. It turns out that formulating the adaptive state-feedback tracking control problem is not straightforward, since specifying the reference state-trajectory can be in conflict with not knowing certain parameters. Our example illustrates this difficulty and we propose a problem formulation for the adaptive state-feedback tracking problem that meets the natural prerequisite that it reduces to the state-feedback tracking problem if the parameters are known. A general methodology for solving the problem is derived.

1 Introduction

In recent years a lot of interest has been devoted to (mainly) stabilization and tracking of nonholonomic dynamic systems, see e.g. [1, 6, 11, 14] and references therein. One of the reasons for the attention is the lack of a continuous static state feedback control since Brockett's necessary condition for smooth stabilization is not met, see [3]. The proposed solutions to this problem follow mainly two routes, namely discontinuous and/or time-varying control. For a good overview, see the survey paper [10].

Less studied is the adaptive control of nonholonomic systems. Results on adaptive stabilization can be found in [2, 7]. In [4, 5, 12, 15] the adaptive tracking problem is studied, but all papers are either concerned with adaptive *output* tracking, or the state trajectory to be tracked is feasable for any possible parameter. However, it is possible that specifying a reference-state trajectory and not knowing certain parameters are in conflict with each other. The question then arises how to formulate the adaptive tracking problem in such a way that it reduces to the state feedback tracking problem in case the parameters are known.

In this paper we consider a simple academic example that clearly illustrates the above mentioned conflict. We propose a formulation for the adaptive (state) tracking control problem and derive a general methodology for solving this problem.

The example we study is the kinematic model of a mobile car with rear wheel driving and front wheel steering:

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \frac{v}{L}\tan\phi \\ \dot{\phi} &= \omega \end{aligned} \tag{1}$$

The forward velocity of the rear wheel v and the angular velocity of the front wheel ω are considered as inputs, (x, y) is the center of the rear axis of the vehicle, θ is the orientation of the body of the car, ϕ is the angle between front wheel and car and $L > 0$ is a constant that denotes the length of the car (see also Figure 1), and is assumed to be unknown.

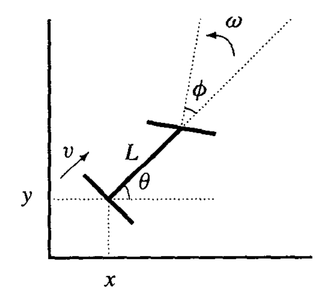

Figure 1: The mobile car

The organization of the paper is as follows. Section 2 contains the problem formulation of the tracking problem and illustrates the difficulties in arriving at the problem formulation for the adaptive tracking problem. Section 3 contains some definitions and preliminary results. Section 4 addresses the tracking problem and prepares for Section 5 in which the adaptive tracking problem is considered. Finally, Section 6 concludes the paper.

2 Problem formulation

2.1 Tracking control problem

Since we want the adaptive tracking control problem to reduce to the tracking problem for known L, we first have to formulate the tracking problem for the case L is known.

Consider the problem of tracking a feasible reference trajectory, i.e. a trajectory $([x_r, y_r, \theta_r, \phi_r]^T, [v_r, \omega_r]^T)$ satisfying

$$\begin{aligned} \dot{x}_r &= v_r \cos\theta_r \\ \dot{y}_r &= v_r \sin\theta_r \\ \dot{\theta}_r &= \frac{v_r}{L} \tan\phi_r \\ \dot{\phi}_r &= \omega_r \end{aligned} \tag{2}$$

This reference trajectory can be generated by any of the motion planning techniques available from literature. The tracking control problem then can be formulated as

Problem 2.1 (Tracking control problem) *Given a feasible reference trajectory* $([x_r, y_r, \theta_r, \phi_r]^T, [v_r, \omega_r]^T)$, *find appropriate control laws* v *and* ω *of the form*

$$v = v(t, x, y, \theta, \phi), \quad \omega = \omega(t, x, y, \theta, \phi) \tag{3}$$

such that for the resulting closed-loop system (1,3)

$$\lim_{t\to\infty} \big(|x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| + |\phi(t) - \phi_r(t)|\big) = 0$$

Remark 2.2 *Notice that in general, the control laws (3) are not only a function of* x, y, θ, *and* ϕ, *but also of* $v_r(t)$, $\omega_r(t)$, $x_r(t)$, $y_r(t)$, $\theta_r(t)$, $\phi_r(t)$, *and possibly their derivatives with respect to time. This explains the time-dependency in (3).*

Remark 2.3 *Notice that the tracking control problem we study here is not the same as an output tracking problem of the flat output* $[x_r(t), y_r(t)]^T$. *First of all, by specifying* $x_r(t)$ *and* $y_r(t)$ *the reference trajectory can not be uniquely specified (e.g.* $v_r(t)$ *can be either positive or negative). But more important is the fact that tracking of* $x_r(t)$ *and* $y_r(t)$ *does not guarantee tracking of the corresponding* $\theta_r(t)$ *and* $\phi_r(t)$.

2.2 Adaptive tracking control problem

In case the parameter L is unknown, however, we can not formulate the adaptive tracking problem in the same way. This is due to the fact that for unknown L we can not specify a feasible reference trajectory $([x_r, y_r, \theta_r, \phi_r]^T, [v_r, \omega_r]^T)$, satisfying (2). In specifying $v_r(t)$, $\phi_r(t)$ and $\theta_r(t)$ we have to make sure that

$$\dot{\theta}_r = \frac{v_r}{L} \tan\phi_r \tag{4}$$

in order to obtain a feasible reference trajectory. This is in conflict with the assumption that we do not know L, since once $v_r(t)$, $\phi_r(t)$ and $\theta_r(t)$ are specified it is possible to determine L from (4).

So the question is how to formulate the adaptive tracking problem for the nonholonomic system (1) in such a way that it reduces to the state-feedback tracking control problem for the case L is known? Appearently we can not both specify v_r, θ_r and ϕ_r as functions of time, and assume that L is unknown.

When generating a feasible reference trajectory satisfying (2), one usually generates some sufficiently smooth reference signals, e.g. $x_r(t)$ and $y_r(t)$, and then all other signals are derived from the equations (2). Notice that it is possible to specify $v_r(t)$, $x_r(t)$, $y_r(t)$, and $\theta_r(t)$ without assuming anything on L. These signals mainly cover the behaviour of the mobile car. However, as mentioned in Remark 2.3, tracking of the output x_r, y_r, θ_r is not what we are interested in, since it is possible to have $x(t) - x_r(t)$, $y(t) - y_r(t)$, and $\theta(t) - \theta_r(t)$ converge to zero as t goes to infinity, but $\phi(t)$ not converge to $\phi_r(t)$. Actually, $\phi(t)$ can even grow unbounded. That is why we insist on looking at the *state* tracking problem.

In case we know L it is possible, once $v_r(t)$, $x_r(t)$, $y_r(t)$ and $\theta_r(t)$ are given, to determine $\phi_r(t)$ uniquely. Notice that we can determine $\tan\phi_r(t)$, from which $\phi_r(t)$ is uniquely determined (since $\dot{\theta}_r$ has to exist and therefore $\phi_r \in]-\frac{\pi}{2}, \frac{\pi}{2}[$). Once $\phi_r(t)$ is known, also $\omega_r(t)$ can be uniquely determined using (2).

When L is unknown we still know that once $v_r(t)$, $x_r(t)$, $y_r(t)$ and $\theta_r(t)$ are given, $\phi_r(t)$ and $\omega_r(t)$ are uniquely determined. The only problem is that these signals are unknown, due to the fact that L is unknown. This is something we illustrate throughout by writing $\phi_r^L(t)$ and $\omega_r^L(t)$. Therefore, we can assume that a feasible reference trajectory $([x_r, y_r, \theta_r, \phi_r^L]^T, [v_r, \omega_r^L]^T)$, satisfying (2) is given and study the problem of finding a state-feedback law that assures tracking of this reference state.

Problem 2.4 (Adaptive tracking control problem) *Let a feasible reference trajectory* $([x_r, y_r, \theta_r, \phi_r^L]^T, [v_r, \omega_r^L]^T)$ *be given (i.e.* $x_r(t)$, $y_r(t)$, $\theta_r(t)$ *and* $v_r(t)$ *are known time-functions, but* $\phi_r^L(t)$ *and* $\omega_r^L(t)$ *are unknown, due to the fact that* L *is unknown). Find appropriate control laws* v *and* ω *of the form*

$$v = v(t, x, y, \theta, \phi), \quad \omega = \omega(t, x, y, \theta, \phi) \tag{5}$$

such that for the resulting closed-loop system

$$\lim_{t\to\infty} \big(|x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| + |\phi(t) - \phi_r^L(t)|\big) = 0$$

Remark 2.5 *Notice that the time-dependency in (5) allows for using* $v_r(t)$, $x_r(t)$, $y_r(t)$, $\theta_r(t)$ *in the control laws (as well as their derivatives with respect to time), but in this case we can **not** use* $\omega_r^L(t)$ *or* $\phi_r^L(t)$.

Remark 2.6 *It is clear that once* L *is known this problem formulation reduces to that of the tracking problem for known* L. *Then also* $\phi_r(t)$ *and* $\omega_r(t)$ *can be used in the control laws again, since these signals are just functions (depending on* L*) of* $v_r(t)$, $\theta_r(t)$ *and their derivatives with respect to time.*

In order to be able to solve the (adaptive) tracking control problem, we need to make the following assumptions on the reference trajectory

Assumption 2.7 *First of all, the reference dynamics need to have a unique solution, which is why we need $\phi_r(t) \in]-M, M[$ with $M < \frac{\pi}{2}$. This is equivalent to assuming that $\frac{\dot{\theta}_r}{v_r}$ is bounded.*

Second, we assume that the reference is always moving in a forward direction with a bounded velocity, i.e. there exist constants v_r^{min} and v_r^{max} such that

$$0 < v_r^{min} \leq v_r(t) \leq v_r^{max}$$

Furthermore, we assume that the forward and angular acceleration, i.e. $\dot{v}_r$ and $\ddot{\theta}_r$, are bounded.

3 Preliminaries

In this section we introduce the definitions and theorems used in the remainder of this paper.

Definition 3.1 *We call $w(t) = [w_1(t), \ldots, w_n(t)]^T$* persistently exciting *if there exist constants $\delta, \epsilon_1, \epsilon_2 > 0$ such that for all $t > 0$:*

$$\epsilon_1 I \leq \int_t^{t+\delta} w(\tau)w(\tau)^T d\tau \leq \epsilon_2 I$$

Lemma 3.2 (cf. e.g. [9, 16]) *Consider the system*

$$\begin{bmatrix} \dot{e} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} A_m & b_m w^T(t) \\ -\gamma w(t) c_m^T & 0 \end{bmatrix} \begin{bmatrix} e \\ \phi \end{bmatrix} \qquad (6)$$

where $e \in \mathbb{R}^n$, $\phi \in \mathbb{R}^m$, $w \in \mathbb{R}^k$, $\gamma > 0$. Assume that $M(s) = c_m^T(sI - A_m)^{-1}b_m$ is a strictly positive real transfer function, then $\phi(t)$ is bounded and

$$\lim_{t\to\infty} e(t) = 0.$$

If in addition $w(t)$ and $\dot{w}(t)$ are bounded for all $t \geq t_0$, and $w(t)$ is persistently exciting then the system (6) is globally exponentially stable.

Lemma 3.3 ([13]) *Let $f : \mathbb{R}_+ \to \mathbb{R}$ be any differentiable function. If $f(t)$ converges to zero as $t \to \infty$ and its derivative satisfies*

$$\dot{f}(t) = f_0(t) + \eta(t) \qquad t \geq 0$$

where f_0 is a uniformly continuous function and $\eta(t)$ tends to zero as $t \to \infty$, then $\dot{f}(t)$ and $f_0(t)$ tend to zero as $t \to \infty$.

Using standard techniques it is easy to show that

Lemma 3.4 *Assume that origin of the system*

$$\dot{x} = f(t,x) \qquad f(t,0) = 0 \quad \forall t$$

where $x \in \mathbb{R}^n$ is globally exponentially stable. Then the disturbed system

$$\dot{x} = f(t,x) + \Delta(t)$$

where $\Delta(t)$ is a bounded vanishing disturbance, i.e.

$$\sup_t \|\Delta(t)\| \leq M \quad \text{and} \quad \lim_{t\to\infty} \Delta(t) = 0$$

is globally asymptotically stable.

Remark 3.5 *Throughout this paper we use the expressions $\frac{x\cos x - \sin(x)}{x^2}$, $\frac{x-\sin(x)}{x^2}$, $\frac{\cos(x)-1}{x}$, $\frac{1-x\sin x-\cos(x)}{x^2}$, $\frac{\cos(x)-1}{x^2}$, and $\frac{\sin(x)}{x}$. These functions are discontinuous in $x = 0$, but if we define their values for $x = 0$ as respectively 0, 0, 0, $-\frac{1}{2}$, $-\frac{1}{2}$, and 1 it is easy to verify that all functions are continuous and bounded.*

4 A tracking controller

First we consider the tracking problem for the case L is known. To overcome the problem that the errors $x - x_r$ and $y - y_r$ depend on how we choose the inertial reference frame, we define errors in a body reference frame, i.e. in a coordinate-frame attached to the car (cf. [8]):

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \qquad (7)$$

In order to be able to control the orientation θ of our mobile car by means of the input ω, we prefer to have $v(t) \neq 0$ for all $t \geq 0$. Since $v_r(t) \geq v_r^{\min} > 0$ we know that if $\sigma(\cdot)$ is a function that fulfills

$$\sigma(x) > -v_r^{\min} \qquad \forall x \in \mathbb{R}$$

the control law

$$v = v_r + \sigma(x_e) \qquad (8)$$

automatically guarantees $v(t) > 0$ for all $t \geq 0$. Furthermore, we assume that $\sigma(x)$ is continuously differentiable and satisfies

$$x\sigma(x) > 0, \quad \forall x \neq 0$$

Examples of possible choices for $\sigma(x)$ are

$$\begin{aligned} \sigma(x) &= v_r^{\min} \cdot \tanh(x) \\ \sigma(x) &= v_r^{\min} \cdot \frac{x}{1+|x|} \end{aligned}$$

With the control law (8) the dynamics in the new coordinates (7) and ϕ become

$$\begin{aligned} \dot{x}_e &= y_e \tfrac{v_r+\sigma(x_e)}{L} \tan\phi + v_r(\cos\theta_e - 1) - \sigma(x_e) \\ \dot{y}_e &= -x_e \tfrac{v_r+\sigma(x_e)}{L} \tan\phi + v_r \sin\theta_e \\ \dot{\theta}_e &= \tfrac{v_r}{L} \tan\phi_r - \tfrac{v_r+\sigma(x_e)}{L} \tan\phi \\ \dot{\phi} &= \omega \end{aligned} \qquad (9)$$

Differentiating the function $V_1 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2$ along the solutions of (9) yields

$$\dot{V}_1 = -x_e\sigma(x_e) + v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right)\theta_e$$

When we consider ϕ as a *virtual control* we could design an intermediate control law for ϕ that achieves ($k_1, k_2 > 0$):

$$\dot{\theta}_e = -k_1\theta_e - k_2 v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right)$$

Using the Lyapunov function candidate $V_2 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{1}{2k_2}\theta_e^2$ and similar reasoning as in [6], we can then claim that x_e, y_e and θ_e converge to zero, provided that Assumption 2.7 is satisfied.

It would be the 'standard procedure' to define the *error variable*

$$\begin{aligned}\bar{z} &= \frac{v_r}{L}\tan\phi_r - \frac{v_r + \sigma(x_e)}{L}\tan\phi + k_1\theta_e + \\ &\quad + k_2 v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right)\end{aligned}$$

However, for simplicity of analysis we prefer to consider the error variable $z = L\bar{z}$, i.e. we define ($c_1, c_2 > 0$):

$$z = v_r\tan\phi_r - v\tan\phi + c_1\theta_e + c_2 v_r\left(\tfrac{\cos\theta_e - 1}{\theta_e}x_e + \tfrac{\sin\theta_e}{\theta_e}y_e\right) \qquad (10)$$

With this definition the error-dynamics (9) now become

$$\dot{x}_e = y_e\frac{v}{L}\tan\phi + v_r(\cos\theta_e - 1) - \sigma(x_e) \qquad (11a)$$

$$\dot{y}_e = -x_e\frac{v}{L}\tan\phi + v_r\sin\theta_e \qquad (11b)$$

$$\dot{\theta}_e = -\frac{c_1}{L}\theta_e - \frac{c_2}{L}v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right) + \frac{1}{L}z \qquad (11c)$$

$$\dot{z} = \frac{v}{\cos^2\phi}\left(\frac{1}{L}\alpha(t)\sin\phi\cos\phi - \omega\right) + \beta(t) \qquad (11d)$$

where

$$\begin{aligned}\alpha(t) &= y_e\tan\phi + c_1 - c_2 v_r\left(\tfrac{1-\cos\theta_e}{\theta_e^2}x_e + \tfrac{\theta_e - \sin\theta_e}{\theta_e^2}y_e\right) \\ \beta(t) &= \dot{v}_r\tan\phi_r + \frac{v_r\omega_r}{\cos^2\phi_r} + (v_r\cos\theta_e - v)\tan\phi + \\ &\quad + c_2(\dot{v}_r x_e - v v_r - v_r^2)\tfrac{\cos\theta_e - 1}{\theta_e} + c_2\dot{v}_r y_e\tfrac{\sin\theta_e}{\theta_e} + \\ &\quad + c_1\tfrac{v_r}{L}\tan\phi_r + c_2 v_r\left(\tfrac{1-\theta_e\sin\theta_e - \cos\theta_e}{\theta_e^2}x_e + \right. \\ &\quad \left. + \tfrac{\theta_e\cos\theta_e - \sin\theta_e}{\theta_e^2}y_e\right)\tfrac{v_r}{L}\tan\phi_r\end{aligned}$$

When we choose the input ω

$$\omega = \frac{1}{L}\alpha(t)\sin\phi\cos\phi + \frac{\cos^2\phi}{v}\left(\beta(t) + c_3 z\right) \qquad (12)$$

we obtain

$$\dot{z} = -c_3 z$$

Consider the Lyapunov function candidate

$$V_3 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{L}{2c_2}\theta_e^2 + \frac{1}{2c_1c_2c_3}\bar{z}^2 \qquad (13)$$

Differentiating (13) along solutions of (11,12) yields

$$\begin{aligned}\dot{V}_3 &= -x_e\sigma(x_e) - \tfrac{c_1}{c_2}\theta_e^2 + \tfrac{1}{c_2}\theta_e z - \tfrac{1}{c_1c_2}z^2 \\ &\leq -x_e\sigma(x_e) - \tfrac{c_1}{2c_2}\theta_e^2 - \tfrac{1}{2c_1c_2}z^2 \leq 0\end{aligned} \qquad (14)$$

We establish the following result

Proposition 4.1 *Assume that Assumption 2.7 is satisfied. Then all trajectories of (11,12) are globally uniformly bounded. Furthermore, all closed-loop solutions converge to zero, i.e.*

$$\lim_{t\to\infty}\left(|x_e(t)| + |y_e(t)| + |\theta_e(t)| + |z(t)|\right) = 0$$

Proof: Since V is positive-definite and radially unbounded, we conclude from (14) that x_e, y_e, θ_e and z are uniformly bounded. From (10) and Assumption 2.7 it follows that also v, $\tan\phi$ and as a result also ω and $\phi - \phi_r$ are uniformly bounded. Also the derivatives of all these signals are bounded. With Barbalat's Lemma it follows that x_e, θ_e and z converge to zero as t goes to infinity. Using Lemma 3.3 with $f = \theta_e$, $f_0 = -k_2 v_r y_e$ and $\eta = -k_1\theta_e - k_2 v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + (\frac{\sin\theta_e}{\theta_e} - 1)y_e\right) + z$ gives also that y_e tends to zero as t goes to infinity. ■

Corollary 4.2 *Consider the system (1) in closed loop with the control laws (8,12) where the reference trajectory satisfies (2) and Assumption 2.7. For the resulting closed-loop system we have*

$$\begin{aligned}\lim_{t\to\infty} \; &\left(|x(t) - x_r(t)| + |y(t) - y_r(t)| + \right. \\ &\left. + |\theta(t) - \theta_r(t)| + |\phi(t) - \phi_r(t)|\right) = 0\end{aligned}$$

Proof: Using (7) it follows from x_e and y_e tending to zero that also $x - x_r$ and $y - y_r$ converge to zero. It only remains to show that $\phi(t) - \phi_r(t)$ tends to zero as t tends to infinity. This comes down to showing that $\tan\phi(t) - \tan\phi_r(t)$ tends to zero as t tends to infinity, which is a direct result from the fact that z tends to zero (and x_e, y_e and θ_e). ■

5 An adaptive tracking controller

From now on we assume that the parameter L is unknown. As mentioned in section 2 we have the difficulty that not only L is unknown, but also the reference signals $\phi_r^L(t)$ and $\omega_r^L(t)$ (that appear in the expression $\beta(t)$) can not be used in the control law.

Fortunately, we are not only allowed to use x_e, y_e, θ_e and ϕ, but also $\dot{\theta}_r$ and $\ddot{\theta}_r$. Notice that in (10) we can replace the occurence of ϕ_r^L by means of the signal $\dot{\theta}_r$:

$$z = L\dot{\theta}_r - v\tan\phi + c_1\theta_e + c_2 v_r\left(\tfrac{\cos\theta_e - 1}{\theta_e}x_e + \tfrac{\sin\theta_e}{\theta_e}y_e\right) \quad (15)$$

However, using the variable z as defined in (10) or (15) makes it hard to design a controller using conventional adaptive techniques because z includes the unknown parameter L. Therefore, we define

$$\hat{z} = \hat{L}\dot{\theta}_r - v\tan\phi + c_1\theta_e + c_2 v_r\left(\tfrac{\cos\theta_e - 1}{\theta_e}x_e + \tfrac{\sin\theta_e}{\theta_e}y_e\right) \quad (16)$$

which can be seen as an estimate for z. Using $\hat{z}$ the tracking error dynamics (9) can be expressed as

$$\begin{aligned} \dot{x}_e &= y_e\frac{v}{L}\tan\phi + v_r(\cos\theta_e - 1) - \sigma(x_e) && (17a)\\ \dot{y}_e &= -x_e\frac{v}{L}\tan\phi + v_r\sin\theta_e && (17b)\\ \dot{\theta}_e &= -\frac{c_1}{L}\theta_e - \frac{c_2}{L}v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right) + \\ &\quad + \frac{1}{L}\hat{z} - \tilde{L}\frac{1}{L}\dot{\theta}_r && (17c)\\ \dot{\hat{z}} &= \frac{v}{\cos^2\phi}(\varrho\alpha(t)\sin\phi\cos\phi - \omega) + \hat{\beta}(t) && (17d) \end{aligned}$$

where we introduced the parameter $\varrho = \frac{1}{L}$. Furthermore, we defined $\tilde{L} = \hat{L} - L$ and

$$\begin{aligned} \alpha(t) &= y_e\tan\phi + c_1 - c_2 v_r\left(\tfrac{1-\cos\theta_e}{\theta_e^2}x_e + \tfrac{\theta_e - \sin\theta_e}{\theta_e^2}y_e\right)\\ \hat{\beta}(t) &= \dot{\hat{L}}\dot{\theta}_r + \hat{L}\ddot{\theta}_r + (v_r\cos\theta_e - v)\tan\phi + \\ &\quad + c_2(\dot{v}_r x_e - v v_r - v_r^2)\tfrac{\cos\theta_e - 1}{\theta_e} + c_2\dot{v}_r y_e\tfrac{\sin\theta_e}{\theta_e} + \\ &\quad + c_1\dot{\theta}_r + c_2 v_r\left(\tfrac{1-\theta_e\sin\theta_e - \cos\theta_e}{\theta_e^2}x_e + \right.\\ &\quad \left. \tfrac{+\theta_e\cos\theta_e - \sin\theta_e}{\theta_e^2}y_e\right)\dot{\theta}_r \end{aligned}$$

When we choose the input

$$\omega = \hat{\varrho}\alpha(t)\sin\phi\cos\phi + \frac{\cos^2\phi}{v}\left(\hat{\beta}(t) + k_3\hat{z}\right) \quad (18)$$

we obtain ($\tilde{\varrho} = \hat{\varrho} - \varrho$):

$$\dot{\hat{z}} = -k_3\hat{z} - \tilde{\varrho}\alpha(t)v\tan\phi$$

Consider the Lyapunov function candidate ($\gamma_1, \gamma_2 > 0$)

$$V_4 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{L\theta_e^2}{2c_2} + \frac{\hat{z}^2}{2c_1c_2c_3} + \frac{\tilde{L}^2}{2c_2\gamma_1} + \frac{\tilde{\varrho}^2}{2c_1c_2c_3\gamma_2} \quad (19)$$

Differentiating (19) along solutions of (17,18) yields

$$\begin{aligned} \dot{V}_4 &\le -x_e\sigma(x_e) - \frac{c_1}{2c_2}\theta_e^2 - \frac{1}{2c_1c_2}\hat{z}^2 + \\ &\quad + \frac{1}{c_2\gamma_1}\left(\dot{\hat{L}} - \gamma_1\theta_e\dot{\theta}_r\right)\tilde{L} + \\ &\quad + \frac{1}{c_1c_2c_3\gamma_2}\left(\dot{\hat{\varrho}} - \gamma_2\hat{z}\alpha(t)v\tan\phi\right)\tilde{\varrho} \end{aligned}$$

So, if we define the parameter-update-laws

$$\begin{aligned} \dot{\hat{L}} &= \gamma_1\theta_e\dot{\theta}_r && (20a)\\ \dot{\hat{\varrho}} &= \gamma_2\hat{z}\alpha(t)v\tan\phi && (20b) \end{aligned}$$

we get

$$\dot{V}_4 \le -x_e\sigma(x_e) - \frac{c_1}{2c_2}\theta_e^2 - \frac{1}{2c_1c_2}\hat{z}^2 \le 0$$

and can establish the following result

Proposition 5.1 *Assume that Assumption 2.7 is satisfied. Then all trajectories of (17,18,20) are globally uniformly bounded. Furthermore,*

$$\lim_{t\to\infty}|x_e(t)| + |\theta_e(t)| + |\hat{z}(t)| + |\phi(t) - \phi_r(t)| = 0$$

If in addition $\dot{\theta}_r(t)$ is persistently exciting, we also have that

$$\lim_{t\to\infty}\left(|y_e(t)| + |\tilde{L}(t)| + |\tilde{\varrho}(t)|\right) = 0$$

Proof: Similar to the proof of Proposition 4.1 we can show uniform boundedness of all signals and their derivatives with respect to time. From Barbalat's Lemma it follows that x_e, θ_e, and $\hat{z}$ converge to zero as t goes to infinity. From (16) we conclude that $\hat{L} - v\tan\phi + c_2 v_r y_e$ converges to zero too. Using Lemma 3.3 we can conclude that also $c_2 v_r y_e + \tilde{L}\dot{\theta}_r$ converges to zero. Combining these two results, we obtain that $L\dot{\theta}_r - v\tan\phi$ and therefore $v_r[\tan\phi_r^L - \tan\phi]$ converges to zero. As a result

$$\lim_{t\to\infty}\left|\phi(t) - \phi_r^L(t)\right| = 0$$

Assume that in addition $\dot{\theta}_r(t)$ is persistently exciting. Notice that the (x_e, y_e)-dynamics (17a,17b) can also be seen as a LTV subsystem with an additional disturbance that is bounded and goes to zero as t goes to infinity:

$$\begin{bmatrix}\dot{x}_e\\ \dot{y}_e\end{bmatrix} = \underbrace{\begin{bmatrix}-k & \dot{\theta}_r(t)\\ -\dot{\theta}_r(t) & 0\end{bmatrix}\begin{bmatrix}x_e\\ y_e\end{bmatrix}}_{\text{LTV subsystem}} + \underbrace{\begin{bmatrix}f_1(t)\\ f_2(t)\end{bmatrix}}_{\text{disturbance}} \quad (21)$$

From Lemma 3.2 we know that the LTV subsystem of (21) is globally exponentially stable and therefore, from Lemma 3.4 that also y_e tends to zero as t tends to infinity.

Also, the $(\theta_e, \tilde{L})$ dynamics can be seen as a cascade of a LTV subsystem with an additional disturbance that is bounded and goes to zero as t goes to infinity:

$$\begin{bmatrix}\dot{\theta}_e\\ \dot{\tilde{L}}\end{bmatrix} = \begin{bmatrix}-\frac{c_1}{L} & -\frac{1}{L}\dot{\theta}_r(t)\\ \gamma_1\dot{\theta}_r(t) & 0\end{bmatrix}\begin{bmatrix}\theta_e\\ \tilde{L}\end{bmatrix} + \begin{bmatrix}f_3(t)\\ f_4(t)\end{bmatrix}$$

In the same way we can conclude that also $\tilde{L}$ tends to zero as t tends to infinity.

Since we have shown that y_e tends to zero, also the $(\hat{z}, \tilde{\varrho})$ dynamics can be seen as a cascade of a LTV subsystem with an additional disturbance that is bounded and goes to zero as t goes to infinity:

$$\begin{bmatrix} \dot{\tilde{z}} \\ \dot{\tilde{\varrho}} \end{bmatrix} = \begin{bmatrix} -k_3 & -c_1 L\dot{\theta}_r(t) \\ \gamma_2 L\dot{\theta}_r(t) & 0 \end{bmatrix} \begin{bmatrix} \tilde{z} \\ \tilde{\varrho} \end{bmatrix} + \begin{bmatrix} f_5(t) \\ f_6(t) \end{bmatrix}$$

Therefore, also $\tilde{\varrho}$ tends to zero as t tends to infinity, which concludes the proof. ■

Corollary 5.2 *Consider the system (1) in closed loop with the control laws (8,18) where the parameter estimates $\hat{L}$ and $\hat{\varrho}$ are updated accoding to (20) and assume that the reference trajectory satisfies (2), Assumption 2.7, and that $\dot{\theta}_r$ is persistently exciting. For the resulting closed-loop system we have*

$$\lim_{t\to\infty} \Big(|x(t)-x_r(t)|+|y(t)-y_r(t)|+ \\ +|\theta(t)-\theta_r(t)|+\left|\phi(t)-\phi_r^L(t)\right|\Big) = 0$$

and convergence of the parameter-estimates to their true value, i.e.

$$\lim_{t\to\infty}\left(\left|\hat{L}(t)-L\right|+\left|\hat{\varrho}(t)-\frac{1}{L}\right|\right)=0$$

6 Concluding remarks

In this paper we addressed the problem of adaptive state tracking control for a four wheel mobile robot with unknown length. This simple example clearly illustrates that for the general state tracking problem specifying the state trajectory to be tracked and not knowing certain parameters can be in conflict with each other. We propose a formulation for the adaptive tracking problem that is such that it reduces to the tracking problem in case the parameters are known. Not only did we formulate the problem, also a solution was derived.

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