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Control of supply networks by robust optimal control and using observers

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Abstract: The control strategy to organize the flow of goods is extremely important in managing supply networks. Handling of uncertainty, e.g. customer demand, is often a considerable challenge. We introduce methods of constrained robust optimal control, a technique from control theory, to compute the explicit control strategy for small-sized demand-driven discrete-time controlled dynamical systems with uncertainties. This allows us to avoid any assumptions about the form of the policy a priori. We aim to show the applicability of the methods, instead of finding innovative policies as the resulting policies are well known. Another control problem arises when control actions depend on the information of multiple states and these states are not (completely) known, e.g. due to communication issues. We use observers, another technique from the control theory, to derive the required state information based on the in- and output. By means of an example, it is shown that by using these observers, the control policy can be perfectly utilized and no longer depends on communication.

Keywords: supply network, constrained robust optimal control, observer.

1. INTRODUCTION

A supply network is a generalization of the original concept of a supply chain where mainly linear or tree-structured facilities and material flows are considered. In today's network economy more complex structures exist where, in principle, any of the involved components can supply each other, and these are captured in the term 'supply network'.

Managers face increasing pressure to control inventories and costs along the network while maximizing customer service performance. Their most important task is organizing the flow of products and information within the supply network. This includes both the design of the network as well as the control of the material flow. Due to evolving networks, it remains a major challenge to achieve optimal performance. In particular, managing uncertainties, such as the buyers demand, material availability or variabilities in transport and service times, is of great importance. This problem is classified as a Supply Chain Operations Planning problem, with the objective to coordinate the release of items in the supply network at minimal cost, cf. de Kok and Fransoo (2003). They also derived optimization models considering either the network structure or control policies. Typically, a control strategy is given and influence of the parameters on the dynamics are studied, see for instance Daganzo (2003).

For the problem of finding an optimal control strategy without any strategy assumptions a priori we believe techniques from control theory are helpful. By way of illustration we consider a supply network where a retailer can order products at multiple sources with different lead times and costs, and faces an unknown customer demand. We introduce a well-known method from control theory to find optimal control strategies for flows in demand-driven supply networks, independent of structural assumptions about the network and therefore without any prior assumptions of a certain family of strategies. It is not our aim to derive new policies, as the resulting policies are well known, but our aim is to show that ideas from control theory can be applicable for supply networks and that these therefore can be a stepping stone for future results. Note that introducing control theory applications to a production-inventory problem is not a new phenomenon, cf. Ortega (2004), Laumanns and Lefeber (2006) and Ivanov et al. (2012) and references therein.

An additional complexity in the management of material flows through a supply chain is introduced by the organizational structure and barriers within the network. Nowadays multiple sites worldwide are working together to deliver a product, while reporting to different organizational units within the corporation. Each of these sites has its unique culture, constraints, and objectives. Therefore, for some networks complete centralized control of material flows would be optimal but may not be feasible. For these cases, global information is required throughout the network with proper communication between warehouses.

However, proper communication is not always possible, as for instance indicated in Cutting-Decelle et al. (2006) and Cutting-Decelle et al. (2007). One could think of complicated communication due to the infrastructure or competition. Therefore, the combination of a network with control laws concerning multiple states of the system and

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poor communication could yield non-optimal results. In control theory similar problems exist where the complete state of the system is not directly measurable. A solution to this problem is to derive *observers* which estimate the state based on measurements, see for instance O'Reilly (1983); Rugh (1996). We show that this method is also applicable in a supply network setting. Measuring inputs and outputs of the system for a while enables the observer to reconstruct unknown states. In other words, by estimating the unknown states in the supply network the control laws can be perfectly utilized. The estimation is performed using a state estimator which employs only the available directly measurable input and output signals. Luenberger (1964) introduced this method for linear systems. Hence, the problem of designing controllers for systems with incomplete state measurements is equivalent to constructing observer-based controllers.

The remainder of this paper is organized as follows. The supply network dynamics and constraints are presented in Section 2. Section 3 introduces constrained robust optimal control. An illustrative example of a supply network is presented in Section 4, used to determine the optimal control. State dependent control based on observers is presented with an example in Section 5. Conclusions are provided in Section 6.

2. SUPPLY NETWORK

The supply network consists of *production units*, introduced by Bertrand et al. (1990), separated by stock points. A production unit may be a single machine, a production line or a production facility. A *supply network* is a set of production units, separated by controlled stock points, to which production orders are released. We consider the supply network as a discrete time controlled dynamical system with uncertainty. Two representations of a network consisting of two stock points x_1 and x_2 with a production unit PU in between is presented in Figure 1. The upper representation in OR-framework is well known. The lower figure represents the network as a discrete time controlled dynamical system. An item in stock x_2 has *lead time* L, i.e., the time between the release of an order and availability of the goods in the next stock point x_1. This lead time is assumed to be deterministic and integer, and is divided over $L-1$ intermediate stock points $x_{1,L-1}, ..., x_{1,1}$, cf. Laumanns and Lefeber (2006).

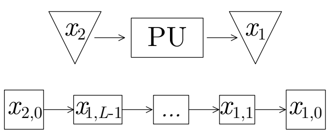

Fig. 1. Representation of two stock points x_1 and x_2 and production unit PU with lead time L in OR-framework (top) and as a discrete time dynamical system (bottom).

The dynamics of a supply network with uncertain demand can be written as

$$x(k+1) = Ax(k) + Bu(k) + Ed(k), \tag{1}$$

where $x(k)$ denotes the state vector. For a supply network one can think of states describing the physical inventory of an item at a warehouse, backlog of an item or the number of ordered items waiting to be received. The vector $u(k)$ denotes the input, which consists of order quantities of items at warehouses. This vector belongs to the set $\mathcal{U}(k) \subseteq \mathbb{R}^{n_u}$. The exogenous disturbance $d(k) \in \mathcal{D}(k) \subseteq \mathbb{R}^{n_d}$ describes the customer demand and is assumed to be restricted to $[0, d^{max}]$. Linear combinations between these vectors are implied by system matrix $A \in \mathbb{R}^{n_x \times n_x}$, input matrix $B \in \mathbb{R}^{n_x \times n_u}$ and disturbance matrix $E \in \mathbb{R}^{n_x \times n_d}$.

This system is constrained by

$$Fx(k) + Gu(k) \leq g,$$

with $F \in \mathbb{R}^{n_g \times n_x}$, $G \in \mathbb{R}^{n_g \times n_u}$, $g \in \mathbb{R}^{n_g}$ and $n_g \in \mathbb{N}_0$. These constraints are used to model lower bounds, e.g. nonnegativity constraints of the physical inventories or order quantities, or upper bounds, e.g limited storage capacity or maximum order quantity. By setting nonnegative inventories the allowed order quantities become state-dependent, e.g. a warehouse can not send more items than available.
We consider the problem of finding optimal inputs $u(k)$ to the system with respect to the constraints and costs on state and control variables.
The costs $C(k)$ are described by

$$C(k) = Qx(k) + Ru(k),$$

where one can denote costs on inventory, backlog etc. in state cost matrix $Q \in \mathbb{R}^{1 \times n_x}$ and cost depending on the input, e.g. order costs, in input cost matrix $R \in \mathbb{R}^{1 \times n_u}$.

3. CONSTRAINED ROBUST OPTIMAL CONTROL

Any policy (sequence of control input decisions) produces a sequence of costs. The goal in constrained robust optimal control is to find inputs that guarantee satisfaction of the constraints for all possible combinations of disturbances and that are favorable with respect to the resulting cost distribution due to the disturbances. In many practical situations, a stochastic description of the uncertainty may not be available, and one may have information with less detailed structure, such as bounds on the magnitude of the uncertain quantities. Under these circumstances one may use a *min-max* approach, whereby the worst possible values of the uncertain disturbance within the given set are assumed to occur, see for instance Bertsekas (2001). For this system, the approach is to minimize the worst case cost via a minimum over u and maximum over d. Given that system (1) is in state $x(t_0)$ at time $t_0 = 0$ and given a horizon $K \in \mathbb{N}$, we are looking for an optimal control input sequence $(u^*(k))_{k=0}^{K-1}$, i.e., sequence of all $u^*(k)$ from $k = 0$ until $k = K-1$, such that the cost-to-go is defined by

$$J^*[k](x(k)) = \min_{u(k)} J[k](x(k), u(k)) \tag{2}$$

subject to

$$
\begin{aligned}
&Fx(k) + Gu(k) \le g,\\
&Ax(k) + Bu(k) + Ed(k) \in \mathcal{X}^{(k+1)},\\
&\forall d(k) \in [0, d^{max}].\\
J[k](x(k), u(k)) =&Qx(k) + Ru(k) + \max_{d(k)\in[0,d^{max}]} \ldots\\
&\ldots J^*[k+1](Ax(k) + Bu(k) + Ed(k))
\end{aligned}
$$

for $k = K-1, ..., 0$. Here $x(k)$ is the state vector at time $t_0 + k$ given the system started in $x_0 = x(t_0)$ and was exposed to input sequence $(u(j))_{j=0}^{k}$ and disturbance sequence $(d(j))_{j=0}^{k}$ and where the system remained in the set of *feasible* states $\mathcal{X}^{(k)}$. This set is described by

$$
\begin{aligned}
\mathcal{X}^{(k)} =&\{x \in \mathbb{R}^{n_x} | \forall d \in \mathcal{D} \exists u \in \mathbb{R}^{n_u} \text{ with}\\
&Fx^{(k)} + Gu^{(k)} \le g \wedge Ax^{(k)} + Bu^{(k)} + Ed^{(k)} \in \mathcal{X}^{(k+1)}\}
\end{aligned}
$$

meaning that for all possible disturbances there exists an input which respects the constraints and makes sure that the state at the next step is within the feasible set. As boundary condition we assume zero costs

$$J^*[K](x(K)) = 0$$

and

$$\mathcal{X}^{(k)} = \{x \in \mathbb{R}^{n_x} | Fx \le g\}.$$

Then, the optimal control law u^* can be computed by dynamic programming (DP) over k

$$
\begin{aligned}
J[K](x(K)) =&0,\\
J[k](x(k)) =& \min_{u(k)\in\mathcal{U}} Qx(k) + Ru(k) + \max_{d(k)\in[0,d^{max}]} \ldots\\
&\ldots J[k+1](Ax(k) + Bu(k) + Ed(k)),\\
&k = 0, 1, ..., K-1. \qquad (3)
\end{aligned}
$$

Applying this algorithm results in the optimal cost-to-go $J[0](x(0))$. Also, if $u^*(k)$ minimizes the right hand side of (3) for both $x(k)$ and k, the policy $(u^*(k))_{k=0}^{K-1}$ is optimal. In each iteration the DP algorithm gives the optimal cost-to-go for every possible state, denoted by $J^*[k]$.

Using this method, optimal control of a supply network is derived by multiparametric linear programming (mpLP). In the next section this method is presented by using an illustrative example of a supply network.

4. ILLUSTRATIVE EXAMPLE

A supply network is considered to illustrate the method of finding optimal inputs with robust optimal control as described in the previous section. A graphical representation of this network is given in Figure 2, where nodes are stages in the network, solid arcs denote that an upstream stage supplies a downstream stage and intersected arcs denote the upstream orders. The network consists of a retailer R, the manufacturer M as first source and the subcontractor S as the second source. Note that this is not a convergent system, i.e., there is no assembly of items. For each step k, the retailer faces an uncertain demand $d(k)$ by its customers and can choose to order $u_1(k)$ number of items with lead time 1 from the manufacturer and $u_2(k)$ number of items with lead time 2 from the subcontractor. The inventory level of the retailer is denoted by x_R. The intermediate states x_{R1} and x_{R2} denote the number of intermediate items, i.e., $x_{R2}(k)$ are the number of items ordered at M at time $k-1$ and $x_{R1}(k)$ are the number of items ordered at M at time $k-2$ plus the number of items ordered at S at time $k-1$. The number of items supplied to the customer is denoted by $s(k)$. Furthermore, the retailer supplies items to the customer with a lead time of 1, and $x_d(k)$ denotes the number of ordered items at time $k-1$.

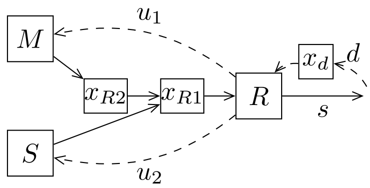

Fig. 2. Supply network consisting of a retailer (R) and two suppliers (M and S). The retailer faces demand d, orders u_1 and u_2, and sells s items.

The dynamics of the system are described by

$$x(k+1) = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} d(k),$$

with $x = [x_R\ x_{R1}\ x_{R2}\ x_d]^\top \in \mathbb{R}_+^4$ with $\mathbb{R}_+ = [0, \infty)$ and $u = [u_1\ u_2]^\top \in \mathbb{R}_+^2$. The orders $u_j(k)$ (j=1,2) are bounded by $[0, u_j^{max}]$, representing for instance a maximal transportation capacity. Demand $d(k)$ is bounded by $[0, d^{max}]$. We assume that the retailer always meets the customers demand, i.e., $s = x_d$. Therefore, all buffer levels are nonnegative, i.e., no backlog. This introduces an extra constraint on the system, $d^{max} \le u_1^{max} + u_2^{max}$ since the demand can always be fulfilled by both suppliers. Furthermore, we assume the manufacturer and subcontractor to have an infinite stock, i.e., retailer orders can always be supplied. The retailer faces c^+ inventory costs per item on inventory in stock (x_R) and order costs c_{u_1} and c_{u_2} per ordered item for ordering at the manufacturer and the subcontractor, respectively. Total costs are expressed by

$$C(k) = c^+ x_R(k) + c_{u_1} u_1(k) + c_{u_2} u_2(k). \qquad (4)$$

Throughout the paper we assume the following parameters: $d^{max} = 8$, $u_1^{max} = 9$, $u_2^{max} = 5$, $c^+ = 1$, $c_{u_1} = 1$ and $c_{u_2} = 4$.

By solving optimization problem (2), it follows that the feasible region, i.e., set of feasible states $\mathcal{X}^\infty$, is given by

$$x_R + x_{R1} - x_d \ge 0, \qquad (5)$$
$$x_R + x_{R1} + x_{R2} - x_d \ge 3, \qquad (6)$$

where (5) makes sure that the physical inventory of x_R is nonnegative at a single order with maximal demand and (6) makes sure that the physical inventory of x_R

remains nonnegative at a sequence of orders with maximal demand. The resulting robust optimal control is given by

$$u_1^*(k) = \min[\max(14 - z(k), 0), 6], \quad (7)$$
$$u_2^*(k) = \max(8 - z(k), 0), \quad (8)$$

with *echelon inventory position* $z(k) = x_R(k) + x_{R1}(k) + x_{R2}(k) - x_d(k)$. In this policy, items are ordered from the manufacturer M with a maximal amount of 6 items when the inventory falls below threshold $14 - z(k)$, see (7). Items are ordered from the subcontractor S when the inventory falls below $8 - z(k)$, see (8). In the next subsection these order-up-to levels are determined analytically. Note that this is not a new policy, as Tan and Gershwin (2004) proved that this dual-basestock policy is optimal for this system. However, the robust optimal control approach leads to the optimal ordering policy without any prior assumption about the structure of the policy. This indicates that the control theory approach is promising.

4.1 Order-up-to level

In order to process all orders without backlog we have $u_1^{max} + u_2^{max} \geq d^{max}$ and $u_1 + u_2 \leq d^{max}$. Let us denote the desired order quantity at manufacturer M by $\bar{u}_1$ and the maximal remaining order quantity at subcontractor S by $\bar{u}_2 = d^{max} - \bar{u}_1$. Given that backlog is not allowed, the lower bound on the order-up-to level of the retailer is given by

$$L = 2d^{max} - \bar{u}_2 = d^{max} + \bar{u}_1. \quad (9)$$

Note that this lower bound is also the desired level since a higher order-up-to level increases the inventory costs. The order policy is given by

$$u_1(k) = \min[\max(L - z(k), 0), \bar{u}_1)], \quad (10a)$$
$$u_2(k) = \min[\max(L - \bar{u}_1 - z(k), 0), \bar{u_2}], \quad (10b)$$

where the retailer orders between zero and $\bar{u}_1$ products at M and the remainder, if necessary, at S. If $c_{u_1} < c_{u_2}$, one is inclined to order the products at the cheaper manufacturer M. However, due to the larger lead time, ordering more products at M results in a larger order-up-to level, see (9), and complementary higher inventory costs. Therefore, a trade-off exists between order costs and inventory costs, which are related to $\bar{u}_1$. The desired order quantities $\bar{u}_1$ can be derived by regarding the extreme cases: maximal order costs C_O and maximal inventory costs C_I. With robust control the maximal costs $(\max(C_O, C_I))$ are minimized. The maximal cost of ordering is given by

$$C_O = (c_{u_1} - c_{u_2})\bar{u}_1 + c_{u_2} d^{max}, \quad (11)$$

which can be derived by regarding ordering at maximal demand. Note that in this case, the inventory costs are zero. The maximal costs spent on inventory, denoted by C_I, depend on the maximal inventory level. The inventory level is maximal if $d = 0$ for at least two time-units and is equal to the order-up-to level L. Therefore, maximal costs spent on inventory are:

$$C_I = c^+ L. \quad (12)$$

It can be seen that C_O is decreasing and C_I is increasing for increasing $\bar{u}_1$. Therefore, maximal costs are minimal when $C_O = C_I$, this results in

$$\bar{u}_1 = \frac{(c_{u_2} - c^+) d^{max}}{c_{u_2} + c^+ - c_{u_1}}. \quad (13)$$

If the retailer is able to order the maximal order quantity from the manufacturer $\bar{u}_1 \leq u_1^{max}$ and the maximal remaining quantity from the subcontractor $\bar{u}_2 \leq u_2^{max}$, the basestock policy is given by (10). This policy was also used in the numerical example.
Otherwise, if the retailer is not able to order the maximum order quantity from the manufacturer the basestock level follows from $\bar{u}_1 = u_1^{max}$ and solving $C_O = C_I$:

$$L = \frac{(c_{u_1} - c_{u_2}) u_1^{max} + c_{u_2} d^{max}}{c^+} - z, \quad \text{if } \bar{u}_1 > u_1^{max}.$$

Furthermore, if the retailer is not able to order $\bar{u}_2$ from the subcontractor the basestock level grows such that backlog is prevented. In this case $\bar{u}_2 = u_2^{max}$, therefore $\bar{u}_1 = d^{max} - u_2^{max}$ and from (9) it follows that

$$L = 2d^{max} - u_2^{max}, \quad \text{if } \bar{u}_2 < d^{max} - \bar{u}_1. \quad (14)$$

The feasible region of this network is given by

$$x_R + x_{R2} - x_d \geq 0, \quad (15)$$
$$z \geq d^{max} - u_2^{max}, \quad (16)$$

For completeness, optimal control for $c_{u_1} \geq c_{u_2}$ is presented here. This might occur due to special offers from the subcontractor. Logically, the maximal amount of products are ordered at the cheaper and faster subcontractor. Therefore, $\bar{u}_2 = \min(d^{max}, u_2^{max})$ and $\bar{u}_1 = \max(d^{max} - \bar{u}_2, 0)$.

In this section we have shown by example that using the method of constrained robust optimal control for a given supply network results in an optimal control policy. This policy was derived without any knowledge of policies a priori. However, a drawback of the method is the limitation to consider small networks. Due to DP, it involves an enormous amount of storage to even record the solution to a moderate complicated problem. For larger networks, the optimal solution can not be solved any more with DP, and a possible solution could be to use an approach of the cost-to-go function, e.g. quadratic function instead of piecewise linear function, where the solution remains close to the optimal solution. Another option is to consider MPC to derive controllers, see for instance Garcia et al. (1989).

5. OBSERVERS: STATE DEPENDENT CONTROL

Often control depends on multiple states. To achieve optimal performance, communication within the network is very important. Ideally, the retailer and suppliers should treat each other as strategic partners in the supply chain with flawless communication. However, as Cutting-Decelle et al. (2006) and Cutting-Decelle et al. (2007) also indicate, in many cases communication is a problem. For instance, the retailer can get inaccurate or no information about the supplier stock levels due to competition, organization or infrastructure. Therefore, we introduce the use of *observers* in the supply chain network to derive the control based on local information. An observer predicts unobservable or unmeasurable states based on observable/measurable parameters. For instance, in an airplane it could be difficult to measure the velocity of the plane directly while measuring time and position are easy. By measuring the time and position of the plane for a while, one can observe the velocity. Analogous for the supply network, by measuring the order and supply quantity long enough, the stock levels can be observed. We illustrate this approach by expanding the supply network from the previous example.

5.1 Example

In order to create a lack of state information due to communication issues we extend the example from Section 4 with warehouses that supply the manufacturer M and subcontractor S, as illustrated in Figure 3. Inventory levels of the retailer, manufacturer and subcontractor are denoted by x_R, x_M and x_S, respectively. Products ordered at the warehouses have a lead time of 2. Ordered products that have not been received by M or S are stored in intermediate states x_{M1} and x_{S1}, respectively. Resulting from the robust optimal control (2)-(3), both the manufacturer M and subcontractor S order their items with a basestock policy at a warehouse with infinite stock (∞) via channels u_M and u_S, respectively.

$$\begin{aligned} u_M(k) &= L_M - x_M(k) - x_{M1}(k), \\ u_S(k) &= L_S - x_S(k) - x_{S1}(k), \end{aligned}$$

where L_i are the basestock levels for x_i, $i \in \{M, S\}$.

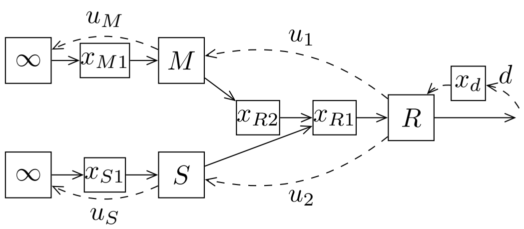

Fig. 3. Supply network consisting of a retailer (R) and two suppliers (M and S). The retailer faces demand d, orders u_1 and u_2. The suppliers order via u_M and u_S from a warehouse with infinite stock (∞).

The retailer uses the policy

$$u_1(k) = \min[\max(L_R - z(k), 0), x_M(k)], \tag{17a}$$
$$u_2(k) = \min[\max(d^{max} - z(k), 0), x_S(k)], \tag{17b}$$

with basestock level L_R. The resulting controller for the retailer clearly depends on the stock levels of both suppliers. Therefore, proper communication between retailer and both suppliers is necessary to fulfill the customers demand.

Assuming that the retailer can get no information on the stock levels of both suppliers, the retailer faces a problem to ensure that the customers demand is met. We assume that the policies and base stock levels of the suppliers are known by the retailer. However, the actual stock levels are unknown which introduces a problem concerning the amount of products to order at the suppliers. To solve this problem we introduce observers, which are well-known in the control community, see for instance O'Reilly (1983); Rugh (1996). First, consider the affine dynamics of the manufacturer M:

$$\begin{bmatrix} x_M(k+1) \\ x_{M1}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_M(k) \\ x_{M1}(k) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} u_1(k) + \begin{bmatrix} 0 \\ L_M \end{bmatrix} \tag{18}$$
$$y(k) = [0 \ 0] \begin{bmatrix} x_M(k) \\ x_{M1}(k) \end{bmatrix} + [1]\, u_1(k), \tag{19}$$

where u_1 is the input from the retailer and y the output from the manufacturer.

A useful property to determine the states is *detectability*. A system is detectable when the states are asymptotically determined by the in- and outputs. Detectability of system (18)-(19) is addressed by considering the difference between the actual states and estimated states. This is denoted by the error dynamics

$$e(k) = |x(k) - \hat{x}(k)|,$$

where $\hat{x}$ indicates the estimated state. For system (18)-(19) the error dynamics are given by

$$e(k+1) = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} e(k). \tag{20}$$

It can be seen from the error dynamics for x_2 and x_{M1} in (20) that after two iterations the errors are zero. Therefore, system (18)-(19) is detectable. Furthermore, this indicates that the states x_M and x_{M1} can be exactly observed after two steps:

$$\begin{aligned} x_M(k+2) &= L_M - u_1(k+1) - u_1(k), \\ x_{M1}(k+2) &= u_1(k). \end{aligned}$$

Similar results are obtained for states x_S and x_{S1} of subcontractor S via input u_2:

$$\begin{aligned} x_S(k+2) &= L_S - u_2(k+1) - u_2(k), \\ x_{S1}(k+2) &= u_2(k). \end{aligned}$$

With this approach, the retailer can imply the control defined in (17) using the estimation of the states of both the manufacturer and subcontractor:

$$\begin{aligned} u_1(k) = \min[&\max(L_R - z(k), 0), ... \\ &L_M - u_1(k-1) - u_1(k-2)], \end{aligned} \tag{21a}$$
$$\begin{aligned} u_2(k) = \min[&\max(d^{max} - z(k), 0), ... \\ &L_S - u_2(k-1) - u_2(k-2)], \end{aligned} \tag{21b}$$

The resulting-observer based controller (21) does not depend on communication anymore. The observers estimate the unknown supplier stock levels in two steps, whereafter the controller operates as desired.

6. CONCLUSION

First, we presented the use of techniques from robust optimal control to derive optimal control policies for discrete-time controlled dynamical systems with uncertainty. For the considered example of a supply network this led to the traditional, optimal ordering policy. However, this approach differs from other approaches by not assuming a certain strategy, or family of strategies, a priori. Also, instead of considering a fixed parameter setting, it is possible to derive optimal control for a network with unknown parameters. Due to use of Dynamic Programming, it involves an enormous amount of storage to even record the solution to a moderate complicated problem. For larger networks, the optimal solution can not be solved any more with Dynamic Programming, and a possible solution could be to use an approach of the cost-to-go function, e.g. quadratic function instead of piecewise linear function, where the solution remains close to the optimal solution. Another promising option from control theory is to consider the multi-variable control algorithm Model Predictive Control to derive controllers.

Second, the use of observers in supply networks is presented to derive explicit state-feedback control. The performance in a supply network for which the control of each state depends on more than the state itself can be non-optimal, due to a lack of information or incomplete information about other states in the network. Using observers in a observable/detectable supply network reduces the control to an explicit state-feedback control, i.e., there is no need for communication. An example is presented to illustrate this approach.

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