

Way-point tracking control of ships

K. Y. Pettersen* and E. Lefeber**

* Department of Engineering Cybernetics
 Norwegian University of Science and Technology
 N-7491 Trondheim, Norway

E-mail: Kristin.Y.Pettersen@itk.ntnu.no

** Department of Mechanical Engineering
 Eindhoven University of Technology
 P.O. Box 513, NL-5600 MB Eindhoven
 The Netherlands

E-mail: A.A.J.Lefeber@tue.nl

Abstract *The paper considers way-point tracking control of ships using yaw torque control. A full state feedback control law is developed using a cascaded approach, and proved to globally asymptotically stabilize the heading and the cross-track error of the ship. Simulation results are presented.*

I. INTRODUCTION

The paper considers way-point tracking control of ships. We consider a ship that is moving at a constant surge velocity U , independently controlled by the main thruster control system, and the control aim is to make the ship follow a straight line between two way-points using yaw torque control. The applied torque is delivered by the differential action of two stern thrusters. In literature, several methods have been proposed, see for instance the survey paper [1].

One approach is to use a conventional *autopilot*, controlling the heading ψ using yaw torque control, and combine this with a Line of sight algorithm. In many cases, however, it is important to minimize the cross-track error, i.e. the shortest distance between the ship and the straight line. Choosing the Earth-fixed coordinate system such that its origin is at the previous way-point, and such that the x -axis points towards the next way-point, the cross-track error equals the sway position y of the ship. *Cross-track or path control* thus implies controlling the sway position y to zero. The ship dynamics can be modelled as [1] [2]

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \begin{bmatrix} 0 \\ \tau_r \end{bmatrix} \quad (1)$$

where $\nu = [v, r]^T$ denote the sway and yaw velocities, the matrices M , C , and D are the inertia, Coriolis/centripetal and damping matrices, respectively. The yaw control torque is denoted by τ_r . The ship kinematics are given by

$$\dot{y} = \sin(\psi)U + \cos(\psi)v \quad (2)$$

$$\dot{\psi} = r \quad (3)$$

where U is the constant forward speed of the ship. The ship model is thus a nonlinear system with four states (y, ψ, v, r) and one input τ_r .

When developing cross-track controllers, it is often assumed that ψ is small, such that the following assumption

can be made

$$\dot{y} \approx \psi U + v \quad (4)$$

$$\dot{\psi} = r \quad (5)$$

and thus linear kinematic equations are achieved. Also, linear dynamic models have often been used for tracking controller design. However, in order to achieve accurate control which is important for instance in restricted waters, controllers taking into account the nonlinear effects should be developed. Nonlinear models that have been used for control design have for instance been developed using the simplifying assumption that the sway velocity can be neglected, i.e. $v = 0$, or that $v = \alpha r$ where $\alpha = \text{constant}$ [3]. The sway dynamics are then neglected, and the dynamic model can be written

$$m_{33}\dot{r} + d(r)r = \tau_r \quad (6)$$

In this paper we will use the ship dynamics model (1) and kinematics (2-3) assuming that the inertia and damping matrices are diagonal. The model can then be written

$$\dot{y} = \sin(\psi)U + \cos(\psi)v \quad (7)$$

$$\dot{\psi} = r \quad (8)$$

$$\dot{v} = -\frac{m_{11}U}{m_{22}}r - \frac{d_{22}}{m_{22}}v \quad (9)$$

$$\dot{r} = \frac{(m_{11} - m_{22})U}{m_{33}}v - \frac{d_{33}}{m_{33}}r + \frac{1}{m_{33}}\tau_r \quad (10)$$

In this paper we seek to control both the heading ψ and the sway position y using the yaw torque control. Full state tracking control of ships have been considered in [4]–[6] where controllers are derived that stabilize both the position and heading of the ship to the desired trajectory. However, the trajectories considered in these works are trajectories having a non-zero curvature, and thus do not apply to the tracking of straight lines. In this paper we will derive a feedback control law that controls the cross-track error, i.e. the sway position y , to zero and the heading ψ to zero, such that the ship heading is tangential to the straight line trajectory. For this particular control problem, under the assumption of the ship moving at a constant forward velocity U , the ship model linearized about the origin is

controllable. A linear control law may therefore asymptotically stabilize the straight-line trajectory represented by $(y, \psi, v, r) = (0, 0, 0, 0)$, but the model and thus the linear analysis will of course only be locally valid.

Recently, several authors have considered the straight-line tracking control problem, with the aim of controlling both the course angle and the cross-track error, and to achieve global results. In [7] a control scheme is derived based on input-output linearization and sliding mode control. In order to use input-output linearization on this control problem an output redefinition is necessary, and it is shown that choosing the system output variable $z = y + k\psi$, the control scheme provides asymptotic stability of the straight-line trajectory. In [8] the 4. order ship model is developed in the Serret-Frenet frame, and the tracking of straight lines and circumferences in the presence of ocean currents are addressed. A control law is developed using feedback linearization and backstepping techniques, providing asymptotic convergence to the desired trajectory. Moreover, the effect of unknown ocean currents is taken into account, and a current estimator is included in the control scheme. In [9] a globally asymptotically stabilizing controller is developed for straight-line tracking, by defining a yaw velocity that steers the ship parallel to a given velocity vector field, designed such that an ideal point moving at this velocity will converge to the desired straight line. A yaw torque control law is then designed by integrator backstepping of the yaw velocity, and it is shown how sliding mode techniques can be used to deal with modeling parameter uncertainties.

In this paper we suggest to use an intuitive comprehension of the ship behavior and the action of a helmsman in order to track a straight line. In this way we define the desired ship course angle as a function of the cross-track error, and show using the cascaded system theory of [10] that any control law exponentially stabilizing the course angle to its desired value, will indeed globally asymptotically stabilize the overall system, i.e. stabilize both the course angle and the cross-track error such that the ship tracks the straight line. This approach provides a quite simple control law that is closely related to Line of sight methods [2], which are much used in ship control practice, but which are ad hoc methods for which, to the authors' best knowledge, stability and convergence of the cross-track error have not been proven.

The paper is organized as follows. In Section II a full state feedback control law is developed and proven to globally asymptotically stabilize the straight line trajectory. Simulation results illustrating the convergence of the ship to the desired trajectory are presented in Section III and some conclusions are given in Section IV.

II. THE WAY-POINT TRACKING CONTROL LAW

In this section we derive a nonlinear feedback control law based on the 4. order ship model

$$\dot{y} = \sin(\psi)U + \cos(\psi)v \quad (11)$$

$$\dot{\psi} = r \quad (12)$$

$$\dot{v} = -\frac{m_{11}U}{m_{22}}r - \frac{d_{22}}{m_{22}}v \quad (13)$$

$$\dot{r} = \frac{(m_{11} - m_{22})U}{m_{33}}v - \frac{d_{33}}{m_{33}}r + \frac{1}{m_{33}}\tau_r \quad (14)$$

in order to make the ship follow the straight line $y = 0$ and also making the ship heading parallel to the straight line tra-

jectory, i.e. $\psi = 0$. The variable y denotes the cross-track error, ψ the ship's course angle with respect to the earth-fixed x -axis, v the sway velocity (decomposed in the body-fixed coordinate system, having its origin at the Center of buoyancy, CB), and r the yaw rate. The forward velocity of the ship is denoted by U , and is assumed to be controlled to a constant positive value by the main thruster control system. The constant parameters $m_{ii} > 0$, $i = 1..3$, denote the ship inertia, including added mass. The constant parameters $d_{ii} > 0$ represent the hydrodynamic damping parameters.

Before designing the control law, we have to consider the control objectives. Obviously, we want y and ψ to converge to zero in order to track the straight-line between the way-points. However, in order to have good ship performance it is not indifferent *how* the ship converges, or how the transient behavior of the ship is. The idea used in this paper is that in order to steer the ship towards the straight line $y = 0$, a good helmsman will use the ship course angle ψ , (given that the ship has a positive forward velocity U) rather than making the ship glide sideways, i.e. rather than use the ship sway velocity v (cf. (11)). Moreover, the helmsman will choose the magnitude of ψ dependent on the distance from the straight-line, i.e. dependent on y . We therefore want the ship angle to be a function of y , for instance we could choose $\psi_d = -ky$, where k is a constant parameter. However, we do not want the ship to whirl round if the deviation y is large. We want the ship course to stay within certain bounds, typically $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and we therefore choose

$$\psi_d = -\arctan(y/\Delta) \quad (15)$$

The control parameter Δ can be interpreted as the distance ahead of the ship along the x -axis, i.e. the straight-line trajectory, that the ship should aim at, see Figure 1. For instance Δ could be chosen equal to two boat's length, corresponding to an usual choice in Line of sight algorithms. We could however choose ψ_d equal to any function $-\sigma_y(y)$ satisfying

$$y\sigma_y(y) > 0 \quad (16)$$

$$\sigma_y(0) = 0 \quad (17)$$

$$\sigma_y : y \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}] \quad (18)$$

Kinematically, i.e. neglecting the sway velocity assuming $v = 0$, we see from (11) that this choice of ψ_d will control y to zero, since $U > 0$ and $\sin(-\sigma_y(y))y < 0 \forall y \neq 0$. Due to the ship dynamics however, v will in general be non-zero. We will in the following show that designing a control law for τ_r based on the idea of making ψ converge to $\psi_d = -\arctan(y/\Delta)$, will indeed make the ship track the desired trajectory, i.e. both y , ψ , v and r will converge to zero.

We view the ship model (11–14) as consisting of two subsystems: The system Σ_1 consists of (11) and (13) and the system Σ_2 consists of (12) and (14). Furthermore, we view ψ as an input to the Σ_1 system. Motivated by the above discussion, we choose the coordinate transformation

$$z = \psi - \psi_d = \psi + \arctan(y/\Delta) \quad (19)$$

$$\dot{z} = \dot{\psi} - \dot{\psi}_d = r + \frac{\Delta\dot{y}}{\Delta^2 + y^2} \quad (20)$$

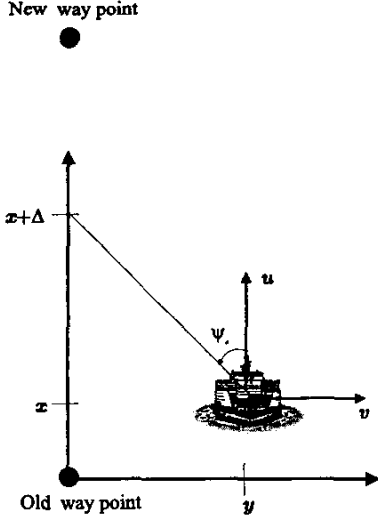


Fig. 1. An interpretation of ψ_d and Δ .

The system equations can then be written

$$\dot{y} = \frac{\sin(z)(\Delta U + yv)}{\sqrt{y^2 + \Delta^2}} + \frac{\cos(z)(\Delta v - Uy)}{\sqrt{y^2 + \Delta^2}} \quad (21)$$

$$\dot{v} = -\frac{d_{22}}{m_{22}}v - \frac{m_{11}U}{m_{22}}\left(\dot{z} - \frac{\Delta\dot{y}}{\Delta^2 + y^2}\right) \quad (22)$$

$$\dot{z} = \dot{z} \quad (23)$$

$$\ddot{z} = \frac{(m_{11} - m_{22})U}{m_{33}}v - \frac{d_{33}}{m_{33}}\left(\dot{z} - \frac{\Delta\dot{y}}{\Delta^2 + y^2}\right) + \frac{1}{m_{33}}\tau_r - \left(\frac{2\Delta y(\dot{y})^2}{(\Delta^2 + y^2)^2} - \frac{\Delta\ddot{y}}{\Delta^2 + y^2}\right) \quad (24)$$

As we will show later, when substituting $z = 0$ and $\dot{z} = 0$ in the Equations (21–22), the resulting (y, v) -system is globally asymptotically stable. Therefore, we would like to design τ_r such that it makes both z and \dot{z} converge to zero, and conjecture that we then can use cascade system theory to show that the overall system then will be globally asymptotically stable. From Equation (24) we see that the control law

$$\begin{aligned} \tau_r = & -(m_{11} - m_{22})Uv + d_{33}\left(\dot{z} - \frac{\Delta\dot{y}}{\Delta^2 + y^2}\right) \\ & + m_{33}\left(\frac{2\Delta y(\dot{y})^2}{(\Delta^2 + y^2)^2} - \frac{\Delta\ddot{y}}{\Delta^2 + y^2}\right) \\ & - m_{33}k_1\dot{z} - m_{33}k_0z \end{aligned} \quad (25)$$

or in the original coordinates

$$\begin{aligned} \tau_r = & -(m_{11} - m_{22})Uv - (k_1m_{33} - d_{33})r - k_0m_{33}\psi \\ & + m_{33}\left(\frac{2\Delta y(\sin(\psi)U + \cos(\psi)v)^2}{(\Delta^2 + y^2)^2}\right. \\ & - \frac{\Delta(Ur \cos(\psi) - vr \sin(\psi) + \cos(\psi)(-\frac{m_{11}U}{m_{22}}r - \frac{d_{22}}{m_{22}}v))}{\Delta^2 + y^2} \\ & \left. - k_1\frac{\Delta(\sin(\psi)U + \cos(\psi)v)}{\Delta^2 + y^2} - k_0 \arctan\left(\frac{y}{\Delta}\right)\right) \end{aligned} \quad (26)$$

where the control parameters $k_0 > 0$ and $k_1 > 0$, gives the

closed-loop system equations

$$\dot{y} = \frac{\sin(z)(\Delta U + yv)}{\sqrt{y^2 + \Delta^2}} + \frac{\cos(z)(\Delta v - Uy)}{\sqrt{y^2 + \Delta^2}} \quad (27)$$

$$\begin{aligned} \dot{v} = & -\frac{d_{22}}{m_{22}}v - \frac{m_{11}U}{m_{22}}\left(\dot{z} \right. \\ & \left. - \frac{\Delta}{\Delta^2 + y^2}(\sin(z)\frac{U\Delta + yv}{\sqrt{y^2 + \Delta^2}} + \cos(z)\frac{\Delta v - yU}{\sqrt{y^2 + \Delta^2}})\right) \end{aligned} \quad (28)$$

$$\dot{z} = \dot{z} \quad (29)$$

$$\ddot{z} = -k_1\dot{z} - k_0z \quad (30)$$

The objective of the coordinate transformation (19–20) and of the choice of control law (25) was to obtain system equations in cascaded form and to steer ψ to ψ_d exponentially. The cascaded structure is easily seen when rewriting Equations (27–30) as follows (Note that $\frac{\sin(z)}{z}$ and $\frac{1-\cos(z)}{z}$ are well defined, continuous functions with $\lim_{z \rightarrow 0} \frac{\sin(z)}{z} = 1$ and $\lim_{z \rightarrow 0} \frac{1-\cos(z)}{z} = 0$)

$$\dot{y} = -\frac{U}{\sqrt{y^2 + \Delta^2}}y + \frac{\Delta}{\sqrt{y^2 + \Delta^2}}v + h_{11}z + h_{12}\dot{z} \quad (31)$$

$$\begin{aligned} \dot{v} = & -\frac{m_{11}U^2\Delta}{m_{22}(\Delta^2 + y^2)^{\frac{3}{2}}}y - \left(\frac{d_{22}}{m_{22}} - \frac{m_{11}U\Delta^2}{m_{22}(y^2 + \Delta^2)^{\frac{3}{2}}}\right)v \\ & + h_{21}z + h_{22}\dot{z} \end{aligned} \quad (32)$$

$$\dot{z} = \dot{z} \quad (33)$$

$$\ddot{z} = -k_1\dot{z} - k_0z \quad (34)$$

where

$$h_{11}(y, v, z, \dot{z}) = \frac{\sin(z)}{z} \frac{\Delta U + yv}{\sqrt{y^2 + \Delta^2}} + \frac{1 - \cos z}{z} \frac{Uy - \Delta v}{\sqrt{y^2 + \Delta^2}} \quad (35)$$

$$h_{12}(y, v, z, \dot{z}) = 0 \quad (36)$$

$$\begin{aligned} h_{21}(y, v, z, \dot{z}) = & \frac{m_{11}U}{m_{22}} \frac{\Delta}{\Delta^2 + y^2} \left(\frac{\sin(z)}{z} \frac{U\Delta + yv}{\sqrt{y^2 + \Delta^2}} \right. \\ & \left. + \frac{(1 - \cos(z))}{z} \frac{Uy - \Delta v}{\sqrt{y^2 + \Delta^2}} \right) \end{aligned} \quad (37)$$

$$h_{22}(y, v, z, \dot{z}) = -\frac{m_{11}U}{m_{22}} \quad (38)$$

i.e.

$$\begin{aligned} \begin{bmatrix} \dot{y} \\ \dot{v} \end{bmatrix} = & \begin{bmatrix} -\frac{U}{\sqrt{y^2 + \Delta^2}} & \frac{\Delta}{\sqrt{y^2 + \Delta^2}} \\ -\frac{m_{11}U^2\Delta}{m_{22}(\Delta^2 + y^2)^{\frac{3}{2}}} & -\left(\frac{d_{22}}{m_{22}} - \frac{m_{11}U\Delta^2}{m_{22}(y^2 + \Delta^2)^{\frac{3}{2}}}\right) \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix} \\ & + \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} \end{aligned} \quad (39)$$

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_0 & -k_1 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} \quad (40)$$

and from (40) it is seen that $z = \psi - \psi_d$ converges exponentially to zero.

Proposition 1: The origin $(y, \psi, v, r) = (0, 0, 0, 0)$ of the closed loop system (11–14) with controller (25) and control parameters $k_0, k_1 > 0$ and

$$\Delta > \frac{3m_{11}U}{2d_{22}} \quad (41)$$

is globally asymptotically stable.

Proof. In order to show that the origin of the closed-loop system is globally asymptotically stable, we will apply [10, Lemma 1]. Assumption A1 of [10, Lemma 1] is a linearity assumption on the interconnection term

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} \quad (42)$$

Noting that $\frac{|y|}{\sqrt{\Delta^2 + y^2}} < 1$ and $\frac{\Delta}{\sqrt{\Delta^2 + y^2}} < 1$ it is easily seen that

$$\|h_{11}z + h_{12}\dot{z}\| \leq 2U\|\xi\| + 2\|\xi\|\|x\| \quad (43)$$

$$\|h_{21}z + h_{22}\dot{z}\| \leq \frac{m_{11}}{m_{22}}U(2U + 1)\|\xi\| + 2\frac{m_{11}}{m_{22}}U\|\xi\|\|x\| \quad (44)$$

where $\xi = [z, \dot{z}]^T$ and $x = [y, v]^T$. The linearity assumption A1 is thus satisfied.

Also, the exponential stability of the system (40) is guaranteed. We need to verify that there exists a Lyapunov function $W(y, v)$ which establishes global asymptotic stability of the system (39) when $z = 0$ and $\dot{z} = 0$, i.e. of the system

$$\begin{bmatrix} \dot{y} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -\frac{U}{\sqrt{y^2 + \Delta^2}} & \frac{\Delta}{\sqrt{y^2 + \Delta^2}} \\ -\frac{m_{11}U\Delta}{m_{22}(\Delta^2 + y^2)^{\frac{3}{2}}} & -\left(\frac{d_{22}}{m_{22}} - \frac{m_{11}U\Delta^2}{m_{22}(y^2 + \Delta^2)^{\frac{3}{2}}}\right) \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix} \quad (45)$$

Furthermore, if this Lyapunov function W is positive definite, radially unbounded and a polynomial function of y and v , then Assumption A2 of [10, Lemma 1] is satisfied.

Consider the Lyapunov function candidate

$$W = \frac{1}{2}y^2 + \frac{\gamma}{2}v^2 \quad (46)$$

where the constant γ is

$$\gamma = \frac{2m_{11}m_{22}(1 - \alpha)}{\alpha^3 d_{22}^2} > 0 \quad (47)$$

and

$$\alpha = \frac{m_{11}U}{d_{22}\Delta} < 1 \quad (48)$$

The derivative of W along the system trajectories of the system (39) when $z = 0$ and $\dot{z} = 0$, i.e. of the system (45), is

$$\begin{aligned} \dot{W} &= -\frac{\alpha d_{22}\Delta}{m_{11}\sqrt{y^2 + \Delta^2}}y^2 + \frac{\Delta}{\sqrt{y^2 + \Delta^2}}yv \\ &\quad - \frac{\gamma\alpha^2 d_{22}^2 \Delta^3}{m_{11}m_{22}(\Delta^2 + y^2)^{\frac{3}{2}}}yv - \left(\frac{d_{22}\gamma}{m_{22}} - \frac{\gamma\alpha d_{22}\Delta^3}{m_{22}(y^2 + \Delta^2)^{\frac{3}{2}}}\right)v^2 \end{aligned} \quad (49)$$

$$\begin{aligned} &= -\frac{\alpha d_{22}\Delta}{m_{11}\sqrt{y^2 + \Delta^2}}\left(y^2 - \left(\frac{m_{11}}{\alpha d_{22}} - \frac{\gamma\alpha d_{22}\Delta^2}{m_{22}(\Delta^2 + y^2)}\right)yv\right) \\ &\quad + \frac{1}{4}\left(\frac{m_{11}}{\alpha d_{22}} - \frac{\gamma\alpha d_{22}\Delta^2}{m_{22}(\Delta^2 + y^2)}\right)^2 v^2 \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{4}\frac{\alpha d_{22}\Delta}{m_{11}\sqrt{y^2 + \Delta^2}}\left(\frac{m_{11}}{\alpha d_{22}} - \frac{\gamma\alpha d_{22}\Delta^2}{m_{22}(\Delta^2 + y^2)}\right)^2 v^2 \\ &- \left(\frac{d_{22}\gamma}{m_{22}} - \frac{\gamma\alpha d_{22}\Delta^3}{m_{22}(y^2 + \Delta^2)^{\frac{3}{2}}}\right)v^2 \end{aligned} \quad (50)$$

$$\begin{aligned} &= -\frac{\alpha d_{22}\Delta}{m_{11}\sqrt{y^2 + \Delta^2}}\left(y - \frac{1}{2}\left(\frac{m_{11}}{\alpha d_{22}} - \frac{\gamma\alpha d_{22}\Delta^2}{m_{22}(\Delta^2 + y^2)}\right)v\right)^2 \\ &- \left(\frac{d_{22}\gamma}{m_{22}} + \frac{\gamma\alpha d_{22}\Delta^3}{2m_{22}(\Delta^2 + y^2)^{\frac{3}{2}}} - \frac{\Delta m_{11}}{4\alpha d_{22}\sqrt{y^2 + \Delta^2}}\right. \\ &\quad \left. - \frac{\gamma^2\alpha^3 d_{22}^3 \Delta^5}{4m_{11}m_{22}^2(\Delta^2 + y^2)^{\frac{3}{2}}} - \frac{\gamma\alpha d_{22}\Delta^3}{m_{22}(y^2 + \Delta^2)^{\frac{3}{2}}}\right)v^2 \end{aligned} \quad (51)$$

Note that

$$0 \leq \frac{\Delta}{\sqrt{\Delta^2 + y^2}} \leq 1 \quad \forall y \quad (52)$$

The derivative of W thus satisfies

$$\begin{aligned} \dot{W} &\leq -\frac{\alpha d_{22}\Delta}{m_{11}\sqrt{y^2 + \Delta^2}}\left(y - \frac{1}{2}\left(\frac{m_{11}}{\alpha d_{22}} - \frac{\gamma\alpha d_{22}\Delta^2}{m_{22}(\Delta^2 + y^2)}\right)v\right)^2 \\ &- \left(\frac{d_{22}\gamma}{m_{22}} - \frac{m_{11}}{4\alpha d_{22}} - \frac{\gamma^2\alpha^3 d_{22}^3}{4m_{11}m_{22}^2} - \frac{\gamma\alpha d_{22}}{m_{22}}\right)v^2 \end{aligned} \quad (53)$$

We can conclude that \dot{W} is negative definite if

$$\begin{aligned} \frac{d_{22}\gamma}{m_{22}} - \frac{m_{11}}{4\alpha d_{22}} - \frac{\gamma^2\alpha^3 d_{22}^3}{4m_{11}m_{22}^2} - \frac{\gamma\alpha d_{22}}{m_{22}} &> 0 \quad (54) \\ \Downarrow \gamma = \frac{2m_{11}m_{22}(1 - \alpha)}{\alpha^3 d_{22}^2} & \\ \alpha < 1 & \end{aligned}$$

$$\begin{aligned} \frac{2m_{11}(1 - \alpha)}{d_{22}\alpha^3} - \frac{m_{11}}{4\alpha d_{22}} & \\ - \frac{m_{11}(1 - \alpha)^2}{\alpha^3 d_{22}} - \frac{2m_{11}(1 - \alpha)}{\alpha^2 d_{22}} &> 0 \quad (55) \end{aligned}$$

$$2(1 - \alpha) - \frac{1}{4}\alpha^2 - (1 - \alpha)^2 - 2\alpha(1 - \alpha) > 0 \quad (56)$$

$$\frac{3}{4}(\alpha - 2)\left(\alpha - \frac{2}{3}\right) > 0 \quad (57)$$

$$0 < \alpha < \frac{2}{3} \quad (58)$$

$$\Delta > \frac{3m_{11}U}{2d_{22}} \quad (59)$$

If the inequality (59) is satisfied, the system thus satisfies the assumptions of [10, Lemma 1], and we can then conclude that the origin of the closed-loop system is globally asymptotically stable.

Remark 1. The cascaded approach used here show that any control law τ_r that makes ψ converge to ψ_d exponentially, i.e. any controller that makes z and \dot{z} converge globally asymptotically and locally exponentially to zero, provides global asymptotic stability of the overall system. One

special case could be where this can be achieved by a controller/observer combination which uses filtered signals.

Remark 2. Note that a PD-controller for a Line of sight algorithm based on (15) would include the last two terms of (25). Equation (25) thus shows which (relatively simple) terms that can be added in order to guarantee convergence of both the cross track error and the yaw angle to zero, and which give global convergence.

III. SIMULATIONS

The ship model used in the simulations describes Cyber-ship I which is a model of an offshore supply vessel scale 1:70, having a mass of 17.6 kg and a length of 1.2 m.

To control the surge velocity dynamics

$$m_{11}\dot{u} = m_{22}vr - d_{11}u + \tau_u \quad (60)$$

we chose the control

$$\tau_u = -m_{22}vr + d_{11}u - m_{11}k_u(u - U) \quad (61)$$

where U is the desired forward speed. The control parameter $k_u = 10$ and the desired forward speed $U = 10$ cm/s.

We chose $\Delta = 2.4$ m which equals two ship lengths, which satisfy (41). The controller (25) was tuned using linear control techniques applied to (40). We chose the control parameters

$$k_1 = 0.1 \quad (62)$$

$$k_2 = 0.5 \quad (63)$$

In the simulations we assumed that the origin of the Earth-fixed coordinate system was at the current way-point, with the x -axis directed towards the next way-point, and that the ship was passing by the current way-point at a distance of 2 m ($x = 0$ m, $y = 2$ m) with forward speed $u = U = 10$ cm/s. In order to illustrate that the control system handles situation with large yaw angle values we chose $\psi = \frac{\pi}{2}$ as the initial course angle. This means that initially, the ship was moving away from the first way point, in a direction perpendicular to the desired straight line trajectory, at a speed of 10 cm/s. The initial states were accordingly chosen as

$$y(0) = 2 \text{ m} \quad (64)$$

$$\psi(0) = 90 \text{ deg} \quad (65)$$

$$v(0) = 0 \text{ m/s} \quad (66)$$

$$r(0) = 0 \text{ deg/s} \quad (67)$$

Figure 2 shows the trajectory of the ship in the xy -plane. The circles indicates the two way points. Figures 3–8 show the time evolution of the state variables (y, ψ, v, r) and the error variables (z, \dot{z}). The speed controller (61) kept the surge velocity u constant equal to U . Figure 9 shows the yaw control torque. The simulations illustrate how the ship converges to the desired straight-line trajectory between two way-points.

IV. CONCLUSIONS

In this paper we have developed a yaw torque control law for the ship way-point tracking control problem. The control law development was based on an intuitive comprehension of the ship behavior and the action of a helmsman in order to track a straight line between way-points. In this way

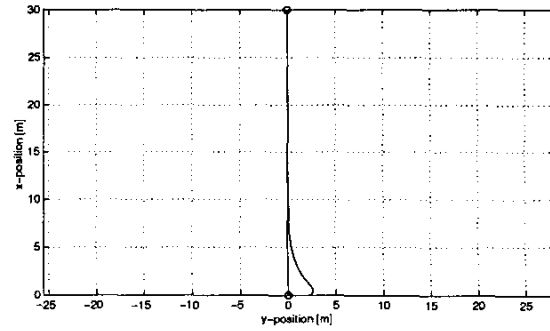


Fig. 2. The trajectory in the xy -plane [m].

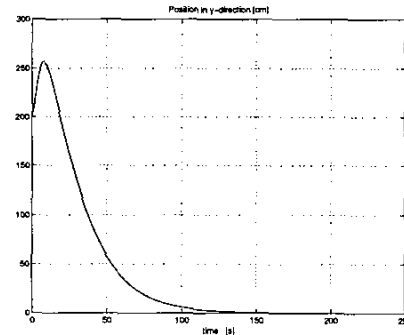


Fig. 3. The position variable y [cm].

we defined the desired ship course angle as a function of the cross-track error. A control law was designed to make the ship course angle converge exponentially to this value. By Lyapunov analysis it was then shown that this desired course angle function indeed stabilized a sub-system including the dynamics of the cross-track error and the sway velocity. Using cascaded control theory by [10] it was then shown that the overall system was globally asymptotically stable, and in fact that any control law exponentially stabilizing the course angle to this desired value would indeed globally asymptotically stabilize the overall system, i.e. stabilize both the course angle and the cross-track error such that the ship tracks the straight line. Simulation results were presented, illustrating how the ship converged to the desired

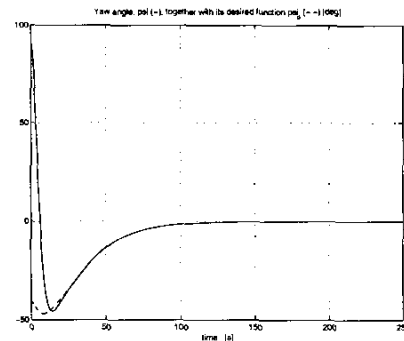


Fig. 4. The yaw angle ψ (-) together with its desired function ψ_d (- -) [deg].

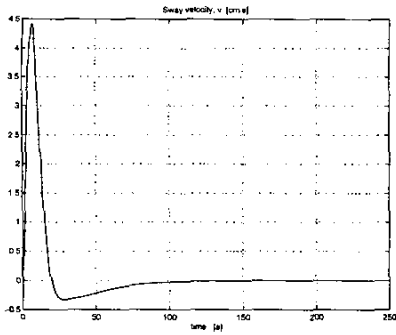


Fig. 5. The sway velocity v [cm/s].

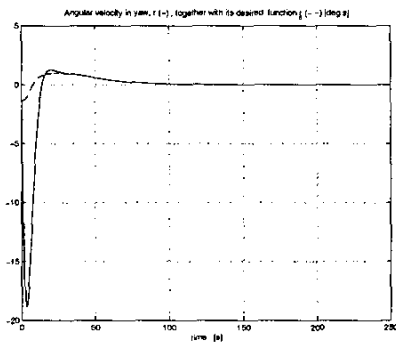


Fig. 6. The yaw velocity r (-) together with its desired function $\dot{\psi}_d$ (- -) [deg/s].

straight line.

The choice of the desired course angle is closely related to Line of sight methods, which are much used in ship control practice, but which are ad hoc methods for which, to the authors' best knowledge, stability and convergence of the cross-track error have not been proven. In future work we would like to perform experiments to validate the result. Moreover, we would like to include environmental disturbances in the analysis, possibly including adaptation in the control scheme.

REFERENCES

- [1] T. I. Fossen, "A survey on nonlinear ship control: From theory to prac-

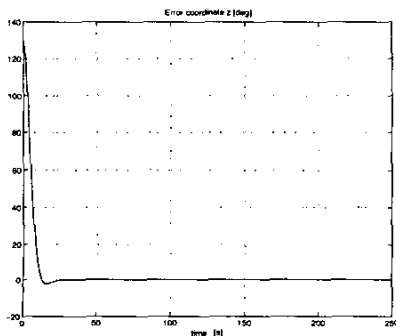


Fig. 7. The error coordinate $z = \psi - \psi_d$ [deg].

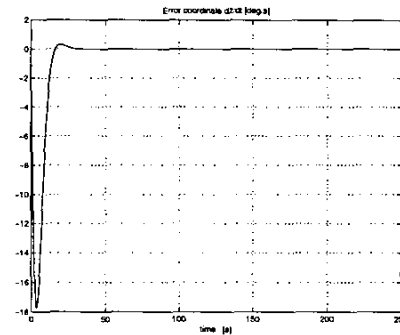


Fig. 8. The error coordinate $\dot{z} = \dot{r} - \dot{\psi}_d$ [deg/s].

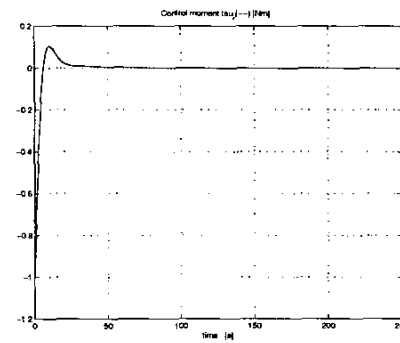


Fig. 9. The yaw control torque τ_r [Nm].

- rice," in *Proc. 5th IFAC Conf. Manoeuvring and Contr. Of Marine Crafts*, (Aalborg, Denmark), Aug. 2000.
- [2] T. I. Fossen, *Guidance and Control of Ocean Vehicles*. Chichester: John Wiley & Sons Ltd, 1994.
- [3] T. Holzhüter, "LQG approach for the high-precision track control of ships," *IEE Proceedings*, vol. 144, pp. 121-127, March 1997.
- [4] K. Y. Pettersen and H. Nijmeijer, "Global practical stabilization and tracking for an underactuated ship - a combined averaging and backstepping approach," *Modeling, Identification and Control*, vol. 20, no. 4, pp. 189-199, 1999.
- [5] K. Pettersen and H. Nijmeijer, "Underactuated ship tracking control: Theory and experiments," to appear in the *International Journal of Control*, 2001.
- [6] E. Lefeber, K. Y. Pettersen, and H. Nijmeijer, "Tracking control of an under-actuated ship," in preparation, 2001.
- [7] R. Zhang, Y. Chen, S. Zengqi, S. Fuchun, and X. Hanzhen, "Path control of a surface ship in restricted waters using sliding mode," in *Proc. 37th IEEE Conference on Decision and Control*, pp. 3195-3200, Dec. 1998.
- [8] P. Encarnação, A. Pascoal, and M. Arcaç, "Path following for autonomous marine craft," in *Proc. 5th IFAC Conf. On Manoeuvring and Contr. Of Marine Craft* (M. M. A. P. Eds. M. Blanke and Z. Z. Vukic, eds.), (Aalborg, Denmark), pp. 117-122, Aug. 2000.
- [9] G. Indiveri, M. Aicardi, and G. Casalino, "Robust global stabilization of an underactuated marine vehicle on a linear course by smooth time-invariant feedback," in *Proc. 39th IEEE Conference on Decision and Control*, (Sydney, Australia), Dec. 2000.
- [10] M. Jankovic, R. Sepulchre, and P. V. Kokotovic, "Constructive Lyapunov stabilization of nonlinear cascade systems," *IEEE Trans. on Aut. Contr.*, vol. 41, pp. 1723-1735, Dec. 1996.