Robust Adaptive Resource Allocation in Container Terminals

Marco Laumanns
IBM Research - Zurich, 8803 Rueschlikon, Switzerland, mlm@zurich.ibm.com

Rico Zenklusen, Kaspar Schüpbach
Institute for Operations Research, ETH Zurich, Switzerland,
{zenklusen,schuepbach}@ifor.math.ethz.ch

Maarten Hendriks, Erjen Lefeber
Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands,
{M.P.M.Hendriks,A.A.J.Lefeber}@tue.nl

Jan Tijmen Udding
PSA HNN, Antwerp, Belgium, JanTijmen.Udding@psahnn.be

ABSTRACT. This paper considers adaptability for robust periodic capacity allocation problems, using the example of crane capacity allocation in container terminals. A new model is presented that allows to compute an adaptive plan via formulation of a linear program over a scenario graph. Computational results are given that quantify to the value of adaptability for reducing the price of robustness, i.e, the total capacity that is necessary to guarantee all service times for all considered realizations of the disturbances.

KEYWORDS. Robust optimization; Adaptability; Periodic capacity allocation

1. INTRODUCTION

As transportation systems are operated closer to their resource margins, uncertainties play an increasing role. If not handled adequately, small disturbances or disruptions might propagate and become amplified, with severe consequences for the service level provided to the customers.

To cope with disturbances and disruptions, several related research directions have emerged in operations research and decision science, such as robust optimization, disruption management, or robust control. Their intention is to provide advanced planning methods that support planners in better managing systems whose operation is subject to uncertainty or disturbances.

While first works about incorporating robustness into planning problems appeared already in the 1970s, a particular research area on robust optimization emerged only during the last decade with the works of Ben-Tal and Nemirovski [2], El Ghaoui et al. [8] and Bertsimas and Sim [4, 5]. The main focus of these works is on identifying tractable robust counterparts, including efficient characterizations of uncertainty sets, of linear and convex optimization problems as well as characterizing the "price" of robustness in terms of its higher cost when compared to the solution of the nominal model.

In the transportation field, like in railway or airline operations planning, approaches to incorporation robustness are mainly concerned with allocating buffers times or additional resources, which can be used to absorb disturbances during operation [11, 6]. Yet, the two most prominent approaches to incorporate adaptivity into robust optimization, the affine adaptability approach of [1] and the finite adaptability approach of [3], have not yet been used in robust transportation planning models.

While affine adaptability reacts by definition to infinitesimal changes in the (uncertain) input parameters and in therefore only suited to continuous recourse variables, the finite adaptability approach works with piecewise constant reaction functions and would thus be suited to adapt, for instance, discrete structures. On the other end of the spectrum of available techniques is robust control, in particular discrete-time constrained robust control. For certain classes of linear systems with additive or polytopic disturbances, the optimal state-feedback control is known to be a piecewise affine function of the state, to which both the affine adaptability as well as the finite adaptability with piecewise constant functions can be seen as approximations. Of course, the state space of complex transportation systems is typically of such high dimension that robust control techniques are only in principle applicable but normally intractable for all practical purposes. Nevertheless, the open question remains how adaptability, or online reactivity, can be integrated into offline, pro-active planning models.

In this paper, we develop a new way to compute adaptive capacity allocation plan in a periodic, infinite-horizon setting. Using the example of quay crane allocation in container terminals, we first introduce the problem via the nominal (deterministic) problem and then present the static robust problem, which is, as discussed above, concerned with allocating extra capacity (buffers) in order to absorb a certain range of disturbances. In the robust problem, the central trade-off will be between the range of the uncertainties versus the necessary amount of allocated resources. We address this problem by minimizing the necessary resources (crane capacity) for a given fixed uncertainty set. Then an adaptive robust planning model is developed, based on the notion of a scenario graph. The scenario graph models the underlying stochastic process of the uncertain vessel arrivals and hence represents the information sets that the decisions can be adapted to. Computational results are given to quantify the value of adaptability, i.e., the reduction in resource usage that the adaptive robust model achieves over the static robust model for guaranteeing feasiblity for the same uncertainty sets.

2. NOMINAL CRANE CAPACITY ALLOCATION MODEL

Consider a container terminal operator who serves a number of shipping lines by discharging and loading their periodically arriving container vessels according to an agreed timetable. Deviations of vessels' travel times lead to stochastic arrivals in the port around the scheduled arrival time. To manage these disturbances, the operator and each shipping line have contractual service agreements that we model as follows. If a vessel arrives within its window, the terminal operator guarantees to process this vessel within an agreed service time. If a vessel arrives out of its window, the operator is not bound to any guaranteed process time. Nevertheless, the terminal operator still tries to handle the vessel as soon as possible in a best effort, without jeopardizing his ability to meet the service times for other vessels that are on time.

Besides the planned arrival times (and corresponding arrival windows), the berth plan includes a resource allocation for each vessel to be served. Here, we consider a tactical berth plan that allocates a certain crane capacity for each vessel over time. The assignment of actual berth positions and cranes as well as constraints resulting from the operations in the yard (e.g. container stacking) are neglected here to focus on the techniques to incorporate robustness and keep the problem tractable also for the adaptive robust formulation.

In the nominal version of the resource allocation problem, we consider V vessels $v \in \mathcal{V} = \{1, 2, \dots, V\}$ with periodic[1] arrival times $a_v \in \mathcal{T} = \{1, \dots, T\}$, their contractual service times σ_v and load λ_v. The task is to allocate crane capacity $x_v(t)$ to each vessel v for all time steps $t \in \{a_v, \dots, a_v + \sigma_v\}$ such that the total installed crane capacity is minimized. This can be expressed as the linear program

[1] Whenever a time index is used in this paper, it is to be interpreted modulo the period length T. We generally refrain from stating the modulo operation to improve readability of the expressions.

$$
\begin{aligned}
\text{Minimize} \quad & c && && (1)\\
\text{subject to} \quad & \sum_{v \in \mathcal{V}} x_v(t) \leq c && \forall t \in \mathcal{T} && (2)\\
& \sum_{t=a_v}^{a_v+\sigma_v+1} x_v(t) \geq \lambda_v && \forall v \in \mathcal{V} && (3)\\
& x_v(t) \leq \rho_v && \forall v \in \mathcal{V}, \forall t \in \mathcal{T} && (4)
\end{aligned}
$$

The first set of inequalities ensures that the allocated capacity doesn't exceed the available capacity. The second set ensures that the cumulative capacity allocated to each vessel is sufficient for the total load of that vessel. Finally, the upper bounds ρ_v limit the maximal capacity that can be allocated to a vessel, depending on the vessel's size and superstructure. This formulation is a simplified version of the model proposed in [9].

3. THE STATIC ROBUST MODEL

The main goal we consider for a robust formulation of the berth scheduling problem is to be robust against a bounded set of disturbances of the arrival times of the vessels. These are in fact the major uncertainties faced in practise due to all kinds of events during travel (e.g., tailwind, adverse weather conditions, technical problems). Thus, vessels might arrive respectively earlier or later than their scheduled arrival time. To cope with these disturbances, the terminal operator and each of the vessel lines agree on an *arrival window*, which is placed around the scheduled arrival time.

In the following, we assume that each vessel has an agreed uncertainty range W_v, which is the maximum number of time steps a vessel is allowed to be delayed with respect to its earliest considered arrival time a_v. If a vessel arrives even earlier than a_v, it simply has to wait until it can for sure berth at time a_v. In practise, the terminal operator might allow a vessel to start berthing outside of its window it the situation allows and the other service agreements are not endangered. This, however, is an operational decision that is not modeled in the tactical planning models considered here.

The decision variables in the static robust model are the same as in the nominal model, but it will become necessary to allocate more crane capacity overall in order to have sufficient capacity to meet the service time constraint for every vessel and every possible arrival time realization from its arrival window. The corresponding robust counterpart of the nominal problem 1 can then be stated as follows.

$$
\begin{aligned}
\text{Minimize} \quad & c && && (5)\\
\text{subject to} \quad & \sum_{v \in \mathcal{V}} x_v(t) \leq c && \forall t \in \mathcal{T} && (6)\\
& \sum_{t=\tau}^{\tau+\sigma_v+1} x_v(t) \geq \lambda_v && \forall v \in \mathcal{V}, \forall \tau \in \{a_v, \ldots, a_v + W_V\} && (7)\\
& x_v(t) \leq \rho_v && \forall v \in \mathcal{V}, \forall t \in \mathcal{T} && (8)
\end{aligned}
$$

The uncertainty in the arrival time enters the second set of constraints, where the cumulative allocated capacity must be sufficient for any realization of the arrival time from the arrival window $\{a_v, \ldots, a_v + W_V\}$. Thus, while the number of variables remains the same as in the nominal model, the number of constraints is increased by replacing one single constraint of the nominal problem with $W_v + 1$ constraints in the robust model. The robust model is called *static* as the capacity allocations are independently of the realization of the uncertain arrival time. Nevertheless, the constraints will hold for every possible arrival time realization and enforce that sufficient capacity is allocated to meet the service time bound.

4. AN ADAPTIVE ROBUST MODEL

The main property of the static robust model was to reserve a certain capacity of the available resources independently for each vessel. This capacity was exclusively dedicated for each vessel and never shared. It might happen, though, that there is no arrival scenario that uses the total available capacity c completely. In other words, there might always be some positive slack left in each possible situation, as the capacity allocation for one vessel does not take into account the realized arrival time of other vessels. To exploit this remaining slack is the motivation to devise a more flexible, *adaptive* capacity allocation plan.

We aim to integrate adaptivity by making the capacity allocation $x_v(t)$ dependent on the state of the stochastic arrival process at any given moment in time. To this end, finding a suitable notion of what constitutes an arrival state (which is the information set that a decision should be adapted to) is essential. Of course, the total number of arrival combinations realized over one period is exponential in the number of vessels, and it seems to be prohibitive to ask for a potentially different solution for each of these situations. But the potential benefit to base the capacity allocation on current information (whether other vessels have already arrived or not) is enticing. In fact, this is rather natural and therefore current practise in operation [10]. Moreover, the realized arrival times of the vessels it the crucial factor that determines at which time each vessel has to be processed to fulfill its service time agreements. A further difficulty is the periodic nature of the problem, which does neither allow for a unique starting state, nor for a termination of the system after a finite number of time steps. Hence, a standard multi-stage recourse formulation seems not possible.

To overcome these difficulties, we present a new periodic infinite-horizon recourse formulation using *scenario graphs*. The scenario graph describes the joint stochastic arrival process of all vessels with a finite number of vertices that represent the different states of the process at the different time steps to which the decisions can be adapted. Hence, the scenario graph is a compact description of all possible scenarios than can occur. The directed edges represent transitions between states at successive time steps that can occur with positive probability. Concrete probability values for the transition are not needed since the capacity allocation plan must be feasible for any possible sample path.

The scenario graph is constructed as follows. Let $\mathcal{T}_v = \{a_v, \ldots, a_v + W_V + \sigma_v\} \subset \mathcal{T}$ be the set of time steps where capacity can be allocated to vessel v and $\tilde{a}_v$ be the realization of the arrival time in the current period. We define the arrival state of vessel v at time t as a random variable $s_v(t) \in \mathcal{S}_v(t) = \{a_v, \ldots, a_v + W_v\} \cup \{0, -1, -2\}$ as

$$s_v(t) = \begin{cases} 0 & \text{if } t \notin \mathcal{T}_v \\ -1 & \text{if } t \in \mathcal{T}_v \text{ and } t < \tilde{a_v} \\ \tilde{a}_v & \text{if } t \in \mathcal{T}_v \text{ and } \tilde{a_v} \leq t \leq \tilde{a}_v + \sigma_v \\ -2 & \text{if } t \in \mathcal{T}_v \text{ and } t. > \tilde{a_v} \end{cases}$$

In other words, we denote the situation that a vessel has not yet arrived with the value -1 and the situation that a vessel has already left with -2, while outside the interval $\mathcal{T}_v$ the vessel cannot be there in any case, in which case the vessel is in state 0. The joint arrival state of all vessels at a given time t

is then described by the vector $s(t) = (s_1(t), s_2(t), \ldots, s_V(t)) \in \mathcal{S}(t) := \bigcup_{v \in \mathcal{V}} \mathcal{S}_v(t)$. Transitions are defined straightforward between the states of successive time steps the combinations following events for all vessels (assuming that $t \in \mathcal{T}_v$):

- if $s_v(t-1) \in \{0, -1\}$ and $t < a_v + W_v$ then $s_v(t) \in \{-1, t\}$ (a vessel might arrive at time t)
- if $s_v(t-1) \in \{0, -1\}$ and $t = a_v + W_v$ then $s_v(t) = t$ (a vessel must arrive at time t)
- if $s_v(t-1) > 0$ and $s_v(t-1) + \sigma_v < t$ then $s_v(t) = s_v(t-1)$ (a vessel stays)
- if $s_v(t-1) > 0$ and $s_v(t-1) + \sigma_v \geq t$ then $s_v(t) = -2$ (a vessel must depart if the service time has elapsed).

We now give a linear programming formulation for the adaptive robust model in which the capacity allocation decisions are adapted to the stochastic arrival process that is described by the above scenario graph. Hence, a decision variable $x_v(s)$ is associated with each vertex $s \in \mathcal{S} := \bigcup_{t \in \mathcal{T}} \mathcal{S}(t)$.

To formulate the service time constraints, we have to enforce, for each vessel v, that the cumulative allocated capacity is sufficient over all possible chains of length σ_v in the scenario graph that originate in arrival vertices of v. Let $\mathcal{A}_V \subset \mathcal{S}$ the set of such vertices where $s_v(t) > 0$ and in all predecessor vertices $s(t-1) \leq 0$ holds. Furthermore, let $\mathcal{P}_v$ be the set of paths of length σ_v that originate in vertices of $\mathcal{A}_v$. The adaptive robust LP can now be stated as:

$$\text{Minimize} \quad c \tag{9}$$

$$\text{subject to} \quad \sum_{v \in \mathcal{V}} x_v(s) \leq c \qquad \forall s \in \mathcal{S} \tag{10}$$

$$\sum_{s \in \Pi} x_v(s) \geq \lambda_v \qquad \forall v \in \mathcal{V}, \forall \Pi \in \mathcal{P}_v \tag{11}$$

$$x_v(s) \leq \rho_v \qquad \forall v \in \mathcal{V}, \forall s \in \mathcal{S} \tag{12}$$

To illustrate the effect and the benefit of an adaptive plan, Figure 1 gives the results for a typical berth schedule of $V = 15$ vessels for increasing arrival uncertainty W_v, which was chosen identical for all vessels. It can be noticed that the adaptive solution can significantly reduce the price of robustness, i.e., the necessary total capacity to fulfill all contractual service times for different uncertainty ranges.

The size of the linear program for the adaptive model is determined by the size of the scenario graph, which grows exponentially in the maximum number of overlapping berthing time windows. For the test instances considered here, the number of variables (respectively constraints) grow from 607 (548) for the instance with $W_v = 1$ to around 470'000 (1.1 million) for $W_v = 12$. The computation time for the latter was around 1 hour using IBM ILOG CPLEX 12.1 as the LP solver. We used the program to generate the scenario graph developed in [7].

CONCLUSIONS

We have proposed the concept of adaptive robustness as a potential route to integrate reactive actions into an tactical robust planning problem and thereby to reduce the conservatism of the strict static robust optimization approach. A linear programming formulation over a scenario graph was presented to compute adaptive plans, which can be seen as infinite-stage stochastic programming recourse model or, from a different viewpoint, as a simultaneous formulation of a stochastic control problem. The computational results show that the adaptive plan is able to reduce the price of robustness significantly, compared to the static robust solution.

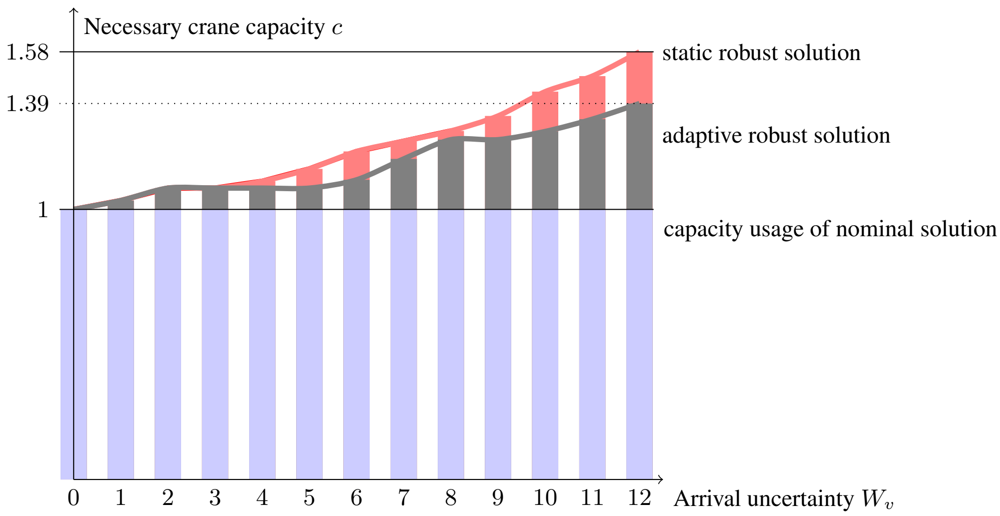

Figure 1: Optimal value of the adaptive robust model versus the static robust model for increasing arrival uncertainty W_v.

References

[1] A. Ben-Tal, A. Goryashko, E. Guslitzer, and A. Nemirovski. Adjustable robust solutions of uncertain linear programs. *Mathematical Programming*, 99:351–376, 2003.

[2] A. Ben-Tal and A. Nemirovski. Robust convex optimization. *Mathematics of Operations Research*, (23):769–805, 1998.

[3] D. Bertsimas and C. Caramanis. Finite adaptability in linear optimization. Technical report, University of Texas, 2007.

[4] D. Bertsimas and M. Sim. Robust discrete optimization and network flows. *Mathematical Programming B*, 98:48–71, 2003.

[5] D. Bertsimas and M. Sim. The price of robustness. *Operations Research*, 52:35–53, 2004.

[6] M. Fischetti and M. Monaci. Robust optimization through branch-and-price. In *AIRO Proceedings*, 2006.

[7] J. Jung. Robust resource allocation and scheduling in container terminals. Bachelor Thesis, Institute for Operations Research, ETH Zurich, 2009.

[8] L. El Ghaoui, F. Oustry, and H. Lebret. Robust solutions to uncertain semidefinite programs. *SIAM Journal on Optimization*, 9(1):33–52, 1998.

[9] M. Hendriks. Multi-step optimization of logistics networks: Strategic, tactical, and operational decisions. Dissertation, Eindhoven University of Technology, 2009.

[10] M. Hendriks, G. Karsemakers, E. Lefeber, and J. Udding. Joint vessel position, container stacking and quay crane allocation. Technical Report 2008-05, Systems Engineering Group, Deparment of Mechanical Engineering, Eindhoven Technical University, 2008.

[11] L. G. Kroon, R. Dekker, and M.J.C.M. Vromans. Cyclic railway timetabling: a stochastic optimization approach. In F. Geraets et al., editors, *Algorithmic Methods for Railway Optimization*, LNCS 4359, pages 41–66. Springer, 2007.