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Feedback control for a multi-product flow line

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Abstract

The objective of this report is to develop a method to control a manufacturing system using feedback control. The manufacturing system of interest is a three machine multi-product flow line with stochastic process times, producing eight different product types in large quantities. The performance of the feedback controlled flow line is compared to the performance of the flow line controlled by several common control methods: push, pull, conwip, and POLCA control. The main performance criteria for the flow line are assumed to be the throughput of the system and the inventory level.

1 Introduction

The performance of a manufacturing system strongly depends on the methods used to control the flow of goods and information in the manufacturing system. In the second half of the twentieth century, various new concepts for controlling the flow of inventory have been developed. Examples of these concepts are Material Requirements Planning (MRP), Just In Time (JIT) and Zero Inventories. The use of the concept MRP explosively grew in the 1970's ([1]). In the 1980's, methods such as Just In Time and Zero Inventories, originating from Japan, achieved successes ([5]). Currently, numerous variants of control methods have been developed, trying to combine the best aspects of both MRP and JIT.

Various tools may be used for the design and analysis of control methods for a manufacturing system. The science of queueing theory provides methods to perform computations on the queueing behavior of relatively simple systems. The rise of computer technology enabled the use of simulation experiments to test more complicated designs of control methods and manufacturing systems. For example, discrete event models may be used to predict the performance a manufacturing system and control method. Nevertheless, designing a control method mainly remains a case of experience, rules of thumb and a process of trial-and-error.

An alternative method to design a control method for a manufacturing system may be the use of control theory and feedback control. The objective of control theory is to cause a system's variable to follow a reference signal. Of a feedback controlled system, the output variables are measured, and the knowledge of the output variables is used to determine an appropriate input signal for the system. Control theory is a well-developed field of science, intensively studied since World War II. It is widely used for controlling various types of systems, such as mechanical systems or chemical process systems. Less research has been done in the use of feedback control for controlling manufacturing systems.

The objective of this report is to develop a method for controlling the flow of goods in a manufacturing system, using feedback control. The performance of the feedback controlled manufacturing system should be compared to the performance of common control methods. As a test case to measure the performance of the control methods, an imaginary three machine multi-product flow line is defined. For this report, the main performance criterion for a controlled manufacturing system is the relation between inventory level and throughput. A given throughput should be reached with a minimum of inventory, or, a maximum throughput should be achieved for a given amount of inventory. The high throughput level and low inventory level may be used as a reference signal for a feedback controller. Objective of the feedback controller is to make the manufacturing system follow this reference trajectory.

The method to design a feedback controller for a manufacturing system may be divided into the following aspects:

- The feedback controller: The type of feedback controller chosen to control the manufacturing system is a Model Predictive Control (MPC) controller. As for most types of feedback controllers, designing an MPC controller requires a continuous dynamic model of the system that is to be controlled.
- The continuous dynamic model: A continuous dynamic model of a system is a set of differential (or difference) equations describing the response of the system out puts to the system inputs. In this report, the method of System Identification is used to construct a dynamic model of the manufacturing system.
- The conversion of signals: The input and output signals of a feedback controller are generally continuous variables. A discrete event model of a manufacturing system de-

scribes the behavior of the system using discrete events. For this report, a feedback controller is to be connected to a discrete event model. The incompatibility of the continuous and discrete signals requires a conversion of the continuous controller output signal into discrete events.

Simulation experiments with discrete event models are used to measure the performance of both the feedback controlled manufacturing system and the same manufacturing system controlled by common control methods. The results of the simulation experiments are compared.

Structure of the report

In Section 2, some preliminary knowledge is presented. Section 3 describes the concept of the framework discussed in this report. In Section 4, a test case is introduced. The concept of Section 3 is illustated by applying it to this test case. Simple suggestions for the implementation of the components of the framework are given in Section 5. The results of simulation experiments with this implementation are discussed in Section 6, and conclusions are presented in Section 7.

2 Preliminaries

This report presents a framework for designing a feedback controller for a discrete event model of a manufacturing system, using an approximation model. Signal conversions enable the application of the controller to the model. For those unfamiliar with MPC, this method from control theory is briefly summarized. After that, some basic manufacturing systems terminology is discussed.

The MPC Controller

In Section 5, the method of Model Predictive Control (MPC) is used for designing a feedback controller. As many control theory methods, MPC attempts to make the output variables of a system follow a reference trajectory. For that purpose, the output is measured, and this information is used to determine an appropriate input signal to be applied to the system.

MPC is a discrete time method, using an internal model of the system to predict future system responses. A receding horizon is used: at each sample, an optimal input trajectory is determined for a number of future steps, but only the first step is implemented. The optimal system input trajectory is the trajectory minimizing $V(k)$ in (1):

$$V(k) = \sum_{i=H_w}^{H_p} ||\hat{z}(k+i|k) - r(k+i)||^2_{Q(i)} + \sum_{i=0}^{H_u-1} ||\Delta\hat{u}(k+1|k)||^2_{R(i)}, \tag{1}$$

with

$$\Delta\hat{u}(k) = \hat{u}(k) - \hat{u}(k-1). \tag{2}$$

H_u is the control horizon, H_p the prediction horizon, and output deviations are penalized only from step H_w. Variable $\hat{z}$ represents the prediction of the system output and r the system output reference, both over the prediction horizon. The assumed system input trajectory is denoted as $\hat{u}$. Q and R are weighing factors, defining the ratio between the penalties for system output errors (i.e. the first term of (1)) and steps in system input (i.e. the second term of (1)).

The method for computing the optimal input trajectory is different for constrained and unconstrained MPC. For constrained MPC, each control action requires solving a quadratic programming problem. The optimal solution for unconstrained MPC follows from a matrix multiplication, using controller matrix K_{MPC} which can be determined once, off-line. For further information about MPC and the used notation is refered to [4].

Properties of a flow line

Important properties for describing and controlling a manufacturing system, and especially a mass production system, are throughput δ, flow time φ, and inventory or work-in-process (WIP) level w. This subsection briefly describes these properties.

The throughput δ describes the speed of products passing a certain section of the manufacturing system. In this report, the smallest unit for products is a lot. Because the throughput is a derivative or a rate, it is measured by taking an average over a certain period of time.

The flow time φ of a lot is the period of time it takes a lot to flow through (a part of) the manufacturing system. Therefore, φ not only consists of process time, but also e.g. queueing time. Flow time can be measured for individual lots.

The WIP level w is the amount of work-in-process in the manufacturing system or subsystem of the manufacturing system at a certain moment. It consists of all lots present in that part of the system, so both of the lots being currently processed and of the queued lots. The WIP level is measured at a certain moment for a certain (sub)system.

For a manufacturing system in a steady state situation, the relation between these three properties is given by (3), Little's Law:

$$\bar{w} = \bar{\varphi} \cdot \delta, \tag{3}$$

with $\bar{w}$ and $\bar{\varphi}$ denoting the mean w and φ, respectively.

3 Concept

Production processes are becoming increasingly complex. Therefore, computer models are crucial for the design and optimization of advanced manufacturing systems. For example, a discrete event model (DEM) may be used to describe the complex behavior of a manufacturing system. A disadvantage of discrete event models is their computational expense. Furthermore, for realistically sized problems, discrete event models are mainly suitable for experimentally determining the system's performance rather than for analytically determining an optimal method to control the system.

The objective of this report is to present a framework to enable the use of standard methods from control theory to complex simulation models such as discrete event models. The framework consists of the following components:

The approximation model: Of the complex discrete event model, an approximation model is created. The type of approximation model depends on which aspects of the complex simulation model should be included, on the demands of the method from control

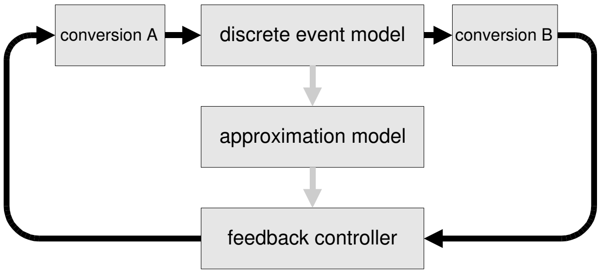

Figure 1: The framework

theory and on the control goal.

The feedback controller: Control theory provides methods to derive a suitable feedback controller, based on the approximation model. The feedback controller may be tuned by closed-loop analysis of the controller and the approximation model.

Interconnection of the controller and the DEM: The output of the DEM can be connected to the input of the controller, and the output of the controller to the input of the DEM. Depending on the type of feedback controller and the properties of the discrete event model, signal conversions are required.

The discrete event model, the approximation model, the controller, and the flow of signals are presented in Figure 1. The two signal conversions are named conversion A and conversion B.

4 The three machine flow line

To illustate the concept described in Section 3, a test case is introduced. This section describes the manufacturing system to be used as a test case. In Section 5, the concept is applied to the case.

The manufacturing system serving as case is a flow line, similar to the flow line used in [3]: it consists of three workstations in series, each consisting of a buffer and a single machine. The machines are single lot machines, i.e. machines sequentially processing one lot at a time. Eight different product types are produced in large quantities, with the demand equally divided over the product types. The machines are assumed not to have setup times and not to break down. Processing of lots never fails and lots do not re-enter the system. Processing times are assumed to be exponentially distributed, with a mean of 1 [units time] for each machine and product type. Raw material supply, buffer capacities and customer demand are all assumed to be infinite.

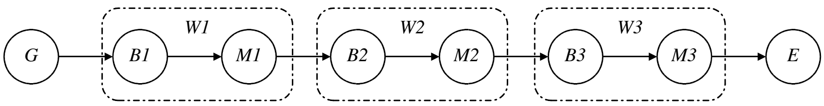

Figure 2: The three machine multi-product flow line

The manufacturing system is described by a discrete event model (DEM). A graphical representation of the system is given in Figure 2. Buffers, machines and workstations are denoted by B, M and W respectively. A generator process G is responsible for releasing new lots into the line. Finished lots are removed from the system by exit process E.

The controllable events or free events are both the release of new lots into the line, and the authorization of machines to start processing a lot. The sequence of controllable events is considered to be the input signal of the DEM. The throughput (δ) and the inventory level (WIP, w) are important performance criterions for a flow line. Therefore, these variables may be considered as the outputs of the DEM. Controlling the DEM lets the system behave in a desired way, by offering an appropriate input signal to the system. An appropriate signal may for example be a signal yielding a high throughput with a low level of inventory.

5 The concept applied to the multi product flow line

This section provides simple suggestions for the implementation of all components of the framework in Figure 1. These suggestions illustrate the concept by means of the test case presented in Section 4. Clearly, other choices for the implementation can also be made. Model Predicted Control (MPC), a standard method from control theory, is selected for designing the controller. This section describes a simple approximation model suitable for MPC design, and a possible solution for the implementation of the required signal conversions. Experiments with the system consisting of the described implementations of the components are performed in the next section, Section 6.

The approximation model

A simple approximation model, describing the material flow, is the model proposed in [2]:

$$\dot{x} = u_{\text{in}}(t) - u_{\text{out}}(t), \tag{4}$$

with state variable x representing the buffer contents, and u_{in} and u_{out} the controlled material processing rate, of respectively the upstream and downstream resource. A similar model is used in this report as an example of an approximation model of the DEM. Since MPC requires a discrete time model, a sampled version of model (4) is used:

$$\begin{aligned} x_m(k+1) &= x_m(k) + Bu_m(k), \\ y_m(k) &= x_m(k). \end{aligned} \tag{5}$$

In (5), input vector u_m contains the material release rate, and the processing rate for all machines, per product type. Model output vector y_m contains the WIP levels of all buffers, and the cumulative realized production, per product type. The throughput of the line is the derivative of the cumulative production. Matrix B contains the products' routing, i.e. the information of which buffers and machines are connected. Note that the state variables x_m equal the output variables.

Model (5) is used in the MPC controller design. For a brief summary of MPC is referred to Section 2. The incompatibility of signal types disables direct interconnection of the designed MPC controller to the DEM. Therefore, the next subsection discusses the signal types, to determine which conversions are required.

Signal types

As mentioned in Section 3, conversions are required for interconnecting the controller and the DEM. Figure 3 shows the signal types. This section discusses the signals to show which conversions are required.

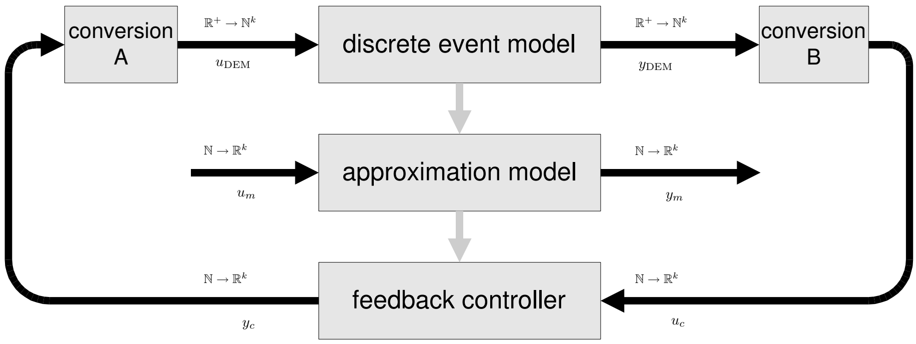

Figure 3: Signal types in the framework

The DEM input signal u_{DEM}: As mentioned in Section 4, the DEM input signal consists of the controllable events. This signal may be represented by a continuous time, integer valued signal ($\mathbb{R}^+ \to \mathbb{N}^k$).

The DEM output signal y_{DEM}: The inventory levels and cumulative production are selected as the DEM output variables: inventory is specified per buffer, and both inventory and cumulative production are specified per product type. This output signal is a continuous time, integer valued signal ($\mathbb{R}^+ \to \mathbb{N}^k$).

The MPC controller input signal u_c and output signal y_c: Since MPC is a discrete time control method, both its input and output signals are real valued signals in discrete time: $\mathbb{N} \to \mathbb{R}^k$.

As shown by Figure 3, both conversion A and conversion B are required. The next two subsections present a basic suggestion for both conversions.

Conversion A

Conversion A, shown in Figure 3, converts the controller output signal $y_c(k)$ into the DEM input signal u_{DEM}. Because $y_c(k)$ consists of target material release rates and processing rates, a target production can be computed. The target production during a sample interval equals sample interval t_s times $y_c(k)$. As long as the actual production has not reached the target production, and lots are available in the upstream buffer, lots are ordered to be processed (or released). As soon as the target production has been met, no more lots are processed or inserted, until the controller demands to do so. In case of different product types, the type with the largest shortage is processed first.

The algorithm is implemented as follows: target production $p^*(k)$ denotes the target number of lots processed or released during the k'th sample interval. Realized production $p(k, \tau)$ denotes the number of lots processed or released during the first τ units time of the k'th

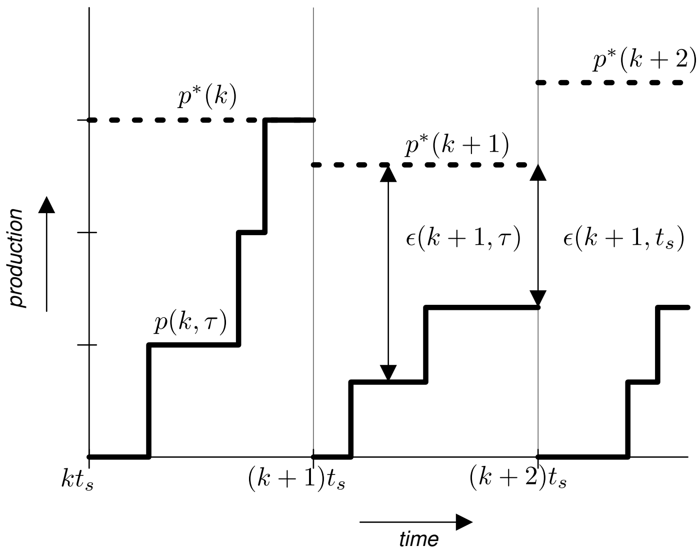

Figure 4: An example of p and p^* during time

sample interval. Define backlog error $\epsilon(k,\tau)$ as the difference between the target production and the realized production:

$$\epsilon(k,\tau) = p^*(k) - p(k,\tau). \tag{6}$$

Amongst the product types available in the upstream buffer, the product type with the largest conversion backlog $\epsilon(k,\tau)$ is selected to be processed. In case $\epsilon(k,\tau)$ is less than a certain limit (e.g. 0), the demand is assumed to be met. The actual production $p(k,\tau)$ is updated after finishing a lot. The target production is updated at each sample instant:

$$p^*(k) = \gamma_c(k) \cdot t_s + \alpha \cdot \epsilon(k-1,t_s) \quad \text{with } 0 \leq \alpha \leq 1. \tag{7}$$

The second term of (7) transfers a fraction of the remaining backlog error to the next sample interval. Keep in mind that it is not necessary to transfer the entire backlog to the next period. Even a value of zero is allowed for α, since the controller reacts to a deviation from the reference signal.

Figure 4 shows an example of the target production p^* being updated each sample time, and the realized production p increasing during the sample interval.

Conversion B

For conversion B, the continuous time DEM output signal is sampled. In Figure 5, the $(\mathbb{R}^+ \rightarrow \mathbb{N}^k)$-signal is represented by a line, and the sampled conversion output by dots. Both conversions and the controller use the same sample interval t_s. More advanced methods, for example explicitly filtering the integer character of the signal, may be used as well.

Now that a simple implementation is available for all components of the framework shown in Figure 3, experiments may be performed with the closed loop system. These experiments and the results are described in the next section.

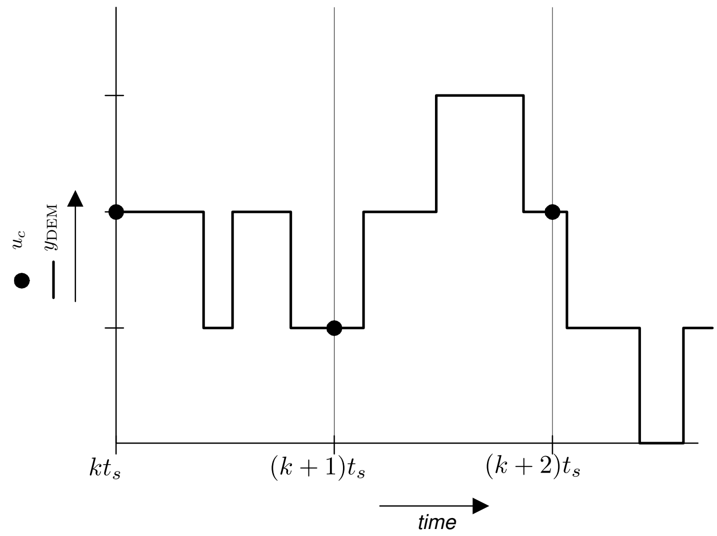

Figure 5: Sampling of the $\mathbb{R}^+ \rightarrow \mathbb{N}^k$ signal

6 Experiments

In the previous sections, a framework is introduced for designing a feedback controller for a Discrete Event Model (DEM). The framework consists of an approximation model for the DEM, a feedback controller and two signal conversions, enabling the interconnection of the DEM and the controller. A DEM of a three machine flow line is introduced as a test case. In Section 5, simple suggestions for the components of the framework have been made: a linear state space model is used for designing an MPC controller.

This section describes experiments performed with the MPC controlled DEM. The first experiments serve to test the framework and determine whether the system is able to follow a certain reference signal. After that, the reference signal is further discussed.

Reference tracking

An objective of a feedback controller is to make the outputs of a system follow a certain reference trajectory as good as possible. Since the DEM output signal consists of WIP levels and cumulative production, the reference signal should also consist of reference WIP signals and reference production signals. Experiments have been performed to determine whether the reference signal is followed.

Table 1 shows the controller design and conversion parameters.

H_u	control horizon	3 (samples)
H_p	prediction horizon	3 (samples)
H_w	penalty start	1 (sample)
$Q : R$	weighing ratio	1:8
t_s	sample time	10 (units time)
α	filtering factor for conversion A	0.3

Table 1: Parameter settings

Figure 6 and 7 show examples of the two system outputs versus time: the inventory of one product type in the last buffer (B3 in Figure 2) and the amount of finished goods for the same product type. The dashed lines represent the imposed reference levels.

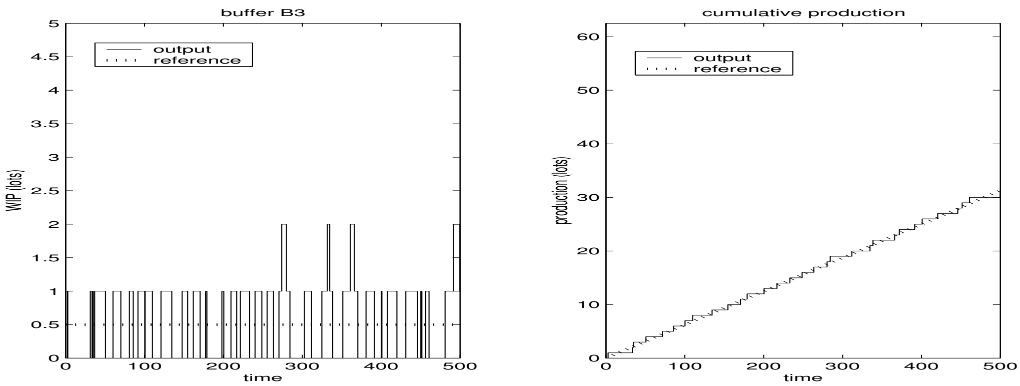

Figure 6: Reference tracking at 50% utilization

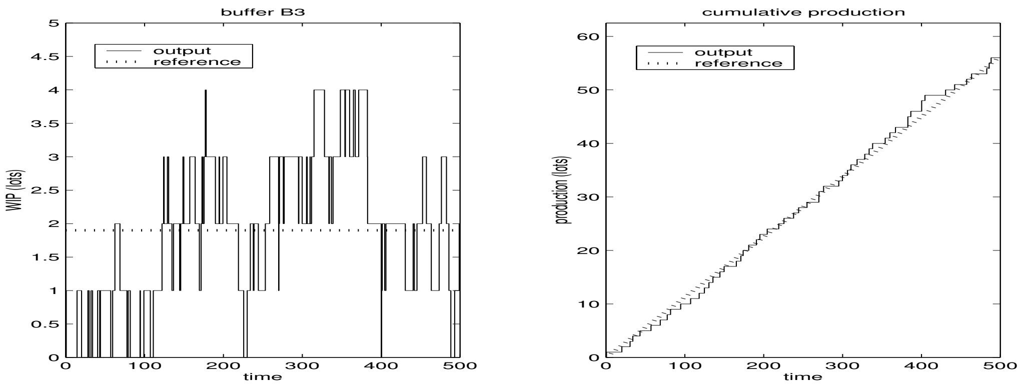

Figure 7: Reference tracking at 90% utilization

The figures show that the reference signals are correctly tracked. Causes for the deviations from the reference signal are the integer character of the output signals and the stochastic process times.

Clearly, the imposed reference signal should be feasible. For example, a decreasing cumulative production trajectory, or a trajectory demanding a utilization of more than 100% can not be followed correctly. Furthermore, because a throughput requires a certain WIP level, a too low reference WIP level cannot be tracked. Appendix A describes experiments performed to determine the effects of the reference WIP level.

7 Conclusions

This report describes a framework to apply standard methods from control theory to complex models such as Discrete Event Models (DEMs). The complex model is described by an approximation model. This approximation model is used in feedback controller design. After the feedback controller has been tuned using the approximation model, it can be interconnected to the DEM. The incompatibility of signals require signal conversions.

To illustrate the concept, simple suggestions are provided for the implementation of the framework's components. A linear state space model is used as an approximation model for the DEM, and basic suggestions are provided for both signal conversions. Model Predictive Control (MPC) is used for controller design. These suggested implementations are applied to a DEM of a manufacturing system: a three machine multi product flow line.

Experiments are performed to determine whether the framework functions. The results show that proper reference signals can be correctly tracked. The selection of a proper reference signal is an important issue, because it has large effects on the behavior of the system.

With this basic implementation and for the simplified case, the framework has shown to work. More sophisticated solutions can be developed for the approximation model, the controller and the conversion algorithms, further improving the performance of the controlled system.

Bibliography

[1] W.J. Hopp and M.L. Spearman. *Factory Physics: foundations of manufacturing management.* McGraw-Hill, 2000.

[2] J. Kimemia and S.B. Gershwin. An algorithm for the computer control of a flexible manufacturing system. *IIE Transactions*, 1983.

[3] A. Krishnamurthy and R. Suri. Re-examining the performance of push, pull and hybrid material control strategies for multi-product flexible manufacturing systems. *International Journal of Flexible Manufacturing Systems*, 2000.

[4] J.M. Maciejowski. *Predictive Control with Constraints.* Prentice Hall, 2002.

[5] Y. Monden. *Toyota Production System.* Industrial Engineering and Management Press, 1983.

A The reference signal

As mentioned in Section 6, only feasible reference signals can be correctly tracked. This appendix shows the results of gradually descreasing the initially feasible reference WIP levels.

The relation between the reference WIP level and the resulting mean WIP level has been determined experimentally at 80% utilization. For a number of reference WIP levels, experiments have been performed to determine the resulting mean WIP level. The reference WIP was chosen equal for all workstations and product types. Each experiment has a duration of 20000 units time. To avoid the influence of transient responses, the data of the first 5000 units time is not taken into account. The mean WIP w is computed using $\bar{\varphi}$, δ and (3). For each reference WIP level, 30 experiments have been performed and the mean of the results are used. Figure 8 shows the resulting mean WIP level as a function of the reference WIP level. The imposed reference throughput is met for all reference WIP levels.

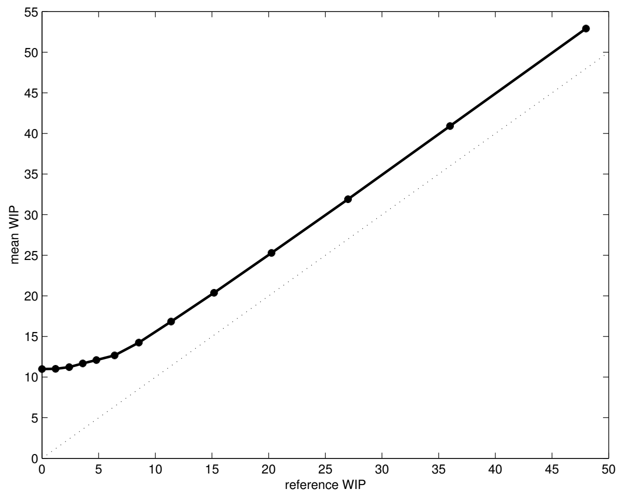

Figure 8: Mean w as a function of reference w

As expected, Figure 8 shows that high reference WIP levels can be followed correctly. The constant difference between the reference WIP and the realized WIP is caused by the discrete sampling of the output signal. From a certain reference WIP level, a decrease of the reference WIP level does not lead to the demanded corresponding decrease in realized mean WIP. Even with a reference WIP level of zero, the imposed throughput is only realized with a certain positive amount of WIP.

Be aware that an unrealizably low reference WIP signal may cause an accumulation of work in conversion A. The value of backlog error ϵ in conversion A (see Equation (6)) at the end of the sample intervals can be regarded as a measure for the difficulty to track a reference signal: when the reference signal can be easily tracked, ϵ is close to zero at the end of a sample interval. When the reference cannot be followed, the mean ϵ is not even expected to reach equilibrium. Therefore, the experiments described in this subsection also include measuring the mean remaining backlog, $\bar{\epsilon}$.

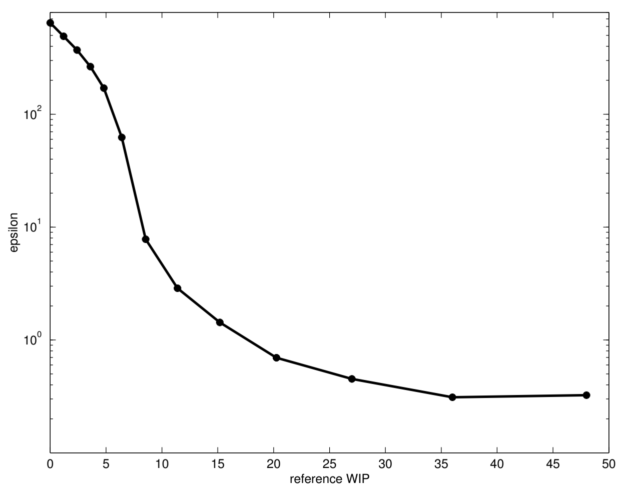

Figure 9: Conversion backlog ϵ for a product type in buffer B3

Figure 9 shows the mean remaining conversion backlog $\bar{\epsilon}$ resulting from the experiments. The figure shows that the mean remaining backlogs $\bar{\epsilon}$ are relatively small for trackable reference WIP levels. For lower reference WIP levels, ϵ increases. Note that Figure 9 only represents the mean ϵ over a finite interval of time, so equilibrium has not necessarily been reached. A very large remaining ϵ may even disable the closed loop of the framework, because it causes the DEM input to merely react to the controller output.