Allocation of Berth Position and Quay Cranes to Container Vessels

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Master's Thesis

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MASTER PROJECT

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Subject

We consider a cluster of inter-acting container terminals, on which a set of vessels runs a regular service. Dependent on the number and destination(s) of inbound and outbound containers, the vessels require a certain handling time and terminal to berth. In [Hendriks et al., 2008], a strategic allocation method has been developed to allocate a terminal and a service time window to each of the vessels in the set. Such a generated allocation can be compared to a bus book or a train time table. On an operational level however, the considered processes, e.g. arrival and departure time of vessels and quay crane rates, are stochastic. Due to these stochastic properties, the strategic allocation has to be continuously adapted. A tool is being developed to re-allocate the vessels on-the-fly to a terminal and a service window. However, the exact position and the specific quay cranes that have to process the vessel, are still to be determined.

Assignment

The objective of this assignment is three-fold:

1. Construct an algorithm, which i) finds out whether the generated, higher-level allocation is feasible and if so ii) optimizes the interdependent position and quay crane allocations.
2. Use a discrete event tool to model the relevant stochastic processes.
3. Implement the algorithm into the discrete event model.

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Preface

Finishing this thesis marks the end of my studies at the Eindhoven University of Technology. In the past 7 years the focus has not been solely on studying. Luckily, I managed to enrich my education with a lot of extra-curricular activities. Especially the full-time year in the board of study association W.S.V. Simon Stevin and other committee work within the same study association contributed to my development as a person and as an engineer. And let's not forget the fun side of it. The weekly social drink, the various trips and the friendships are maybe as important as those valuable experiences. Also my experiences within the University Racing Team Eindhoven were of great value and showed me a more practical side of Mechanical Engineering. From this place, I would like to thank all people who made all this possible.

After obtaining my Bachelor's degree, I joined the Systems Engineering Group. This group aims to develop methods and tools for the analysis and design of manufacturing and logistics processes. I consider my internship at the University of Michigan as one of the highlights of the master's phase. Being completely on your own for the first time, meeting a lot of people from different cultures and traveling through the USA was a great experience. I want to thank professor Rooda, Pascal Etman and Simon Tosserams for offering me this opportunity and their coaching.
The work as presented in this thesis is part of a large solution approach. The results are obtained in close cooperation with Maarten Hendriks and Maarten Vullings. Our pleasant collaboration and lively discussions surely contributed to the quality of this thesis. From this place, I would like to wish Maarten Hendriks the best of luck with obtaining his PhD degree. In addition, I would like to thank professor Udding and Erjen Lefeber for their coaching during the graduation project.

Furthermore, I want to thank my parents who supported me in all my decisions. They gave me the opportunity and freedom to do whatever I thought was best to do. And last but not least, I want to thank my girlfriend Natasja for her interest, support and her ability to show me that there is more in life besides graduating during the last months.

Guido Karsemakers
Eindhoven, July 2008

Summary

The international conveyance of sea freight using containers has grown rapidly the last decennia, and this growth is expected to continue for the next decennia. In order to be able to cope with these increasing amounts of cargo, port operators have to develop efficient logistics processes.
This report investigates the *berth allocation problem* (BAP) and the *quay crane allocation problem* (QCAP) embedded in the large multi-step approach of [Hen08] to solve the periodic multi-terminal BAP and QCAP. The overall solution approach consists of two steps. In the first step, i) a terminal, ii) a time window, and iii) a time variant quay crane capacity are allocated to the vessels in the vessel set. The exact berth position allocation and the integer-valued quay crane allocation for each individual terminal are still to be determined in the second step. In this report, for both problems in the second step, a separate MILP optimization problem is proposed and a case study is performed.

It is shown that as a result of the chosen solution approach the BAP is reduced to a one-dimensional packing problem. Only the position of the vessels along the terminal is to be determined. The proposed formulation minimizes the weighted deviation from the lowest-cost berthing position without vessels overlapping each other.
Another result of the chosen solution approach is that a first step allocation might turn out to be infeasible in the second step. However, experiments suggest that for typical terminal quay utilizations first step allocations are always feasible in the second step. Moreover, the same experiments suggest that the computational effort to solve the proposed MILP is very small.

An existing MILP formulation for the QCAP [Liu06] is continuous in time, which has some restrictions. As a result of these restrictions it might be that allocations can be constructed in which vessels are departing later than their desired departure time, and the crane rate has to be equal for each quay crane. To avoid these restrictions, an QCAP formulation, which is discrete in time is proposed.
Experiments suggest that the proposed discrete MILP formulation results in a reduction in the number of allocations in which vessels are departing later than their desired departure time. Moreover, the proposed formulation enables to incorporate quay cranes with different quay crane rates.

The objective of the discrete formulation is extended to minimize i) the departure time of the vessels, ii) the number of required quay cranes, iii) the movement of the quay cranes along the quay iv) the possibility of process interruptions, and v) the isolation of idle cranes between processing quay cranes.
The discrete QCAP formulation requires a lot of computational effort. Therefore, a heuristics procedure, which cuts the complete problem into smaller subproblems is proposed. Experiments suggest that the heuristics procedure performs on average 5% worse than the complete formulation with respect to the objective function value. However, the same experiments suggest that the performance with respect to the computational effort improves with on average 80%.

The proposed formulations for the BAP and the QCAP are used to solve the second step optimization problems for a representative first step allocation. The results suggest that with a reasonable amount of computational effort, it is possible to construct and optimize a second step allocation on the strategic level and implement it in an model-based predictive (MPC) approach [Vul08] for the operational level.

Samenvatting

De hoeveelheid overzees containertransport is sterk gestegen de afgelopen decennia en zal de komende decennia blijven stijgen. Om deze groei te kunnen verwerken, zullen de dienstverleners in de haven efficiënte logistieke processen moeten ontwikkelen.
In dit verslag worden het *berth allocation problem* (BAP) en het *quay crane allocation problem* (QCAP) onderzocht. Deze problemen zijn opgenomen in de stapsgewijze aanpak van [Hen08] om de periodieke multi-terminal BAP en QCAP op te lossen. De stapsgewijze aanpak bestaat uit twee stappen. In de eerste stap worden een terminal, een tijdsinterval en een tijdsafhankelijke kraancapaciteit toegewezen aan de schepen. De exacte aanmeerlocatie binnen de individuele terminals en de exacte toewijzing van kranen aan schepen moeten nog bepaald worden in de tweede stap. Voor beide problemen in de tweede stap wordt een apart MILP optimalisatie probleem voorgesteld in dit verslag. Daarnaast wordt er voor beiden een representatieve case uitgevoerd.

Er wordt aangetoond dat dankzij de gekozen stapsgewijze aanpak, de BAP tot een één-dimensionaal probleem gereduceerd kan worden. Alleen de positie van de schepen langs de kade moet dan nog bepaald worden. De voorgestelde formulering minimaliseert de gewogen afwijking van de optimale aanmeerlocatie zonder dat schepen elkaar overlappen.
Het feit dat terminal- en tijdstoewijzingen uit de eerste stap mogelijk niet feasible zijn in de tweede stap is een ander gevolg van de stapsgewijze aanpak. Experimenten suggereren echter dat, voor typische kade-utilisatie , de terminal- en tijdstoewijzingen uit de eerste stap altijd feasible zijn in de tweede stap. Daarnaast suggereren dezelfde experimenten dat de rekentijd van de voorgestelde MILP erg laag is.

Een bestaande MILP formulering voor de QCAP [Liu06] is continu in de tijd, wat een tweetal beperkingen met zich meebrengt. Ten gevolge van de eerste beperking kunnen er onnodig kraantoewijzingen gevormd worden, waarin schepen later vertrekken dan hun gewenste vertrektijd. Ten tweede moet de werksnelheid van iedere kraan gelijk verondersteld worden. Om deze beperkingen te voorkomen, wordt er een QCAP formulering voorgesteld, die discreet in de tijd is.
Experimenten suggereren dat de voorgestelde discrete MILP formulering minder vaak leidt tot kraantoewijzingen waarin schepen later vertrekken dan hun gewenste vertrek-

tijd. Daarnaast maakt de formulering het gebruik van verschillende werksnelheden voor kranen mogelijk.
De doelfunctie van de discrete formulering is dusdanig uitgebreid dat het mogelijk is om i) de vertrektijd van de schepen, ii) het aantal benodigde kranen, iii) de beweging van de kranen langs de kade, iv) het aantal werkonderbrekingen en v) het aantal idle kranen opgesloten tussen werkende kranen te minimaliseren.
De rekentijd voor de discrete QCAP formulering is relatief hoog. Daarom wordt er een heuristiek voorgesteld die het totale probleem opdeelt in kleinere problemen. Experimenten suggereren dat de heuristiek een doelfunctie waarde oplevert die gemiddeld 5% slechter is dan de doelfunctie waarde van de gehele formulering. Echter, de rekentijd neemt gemiddeld met 80% af.

De voorgestelde formuleringen voor de BAP en de QCAP zijn gebruikt om de tweede stap van de gekozen aanpak op te lossen voor een representatieve oplossing van de eerste stap. De resultaten suggereren dat het mogelijk is om de tweede stap op te lossen en te optimaliseren op zowel een strategisch als een operationeel niveau. Dit alles kan gerealiseerd worden binnen een dusdanige rekentijd, dat het mogelijk is om de problemen per uur online op te lossen.

Contents

Chapter 1

Introduction

The last decennia, the international conveyance of sea freight using containers has grown rapidly. This growth is expected to continue for the next decennia. Nowadays, mega-vessels are capable of carrying 15.000 TEU's[1] and large container ports are processing up to 27 million TEU's a year. In order to cope with the current amounts of cargo and to anticipate the growth of these amounts, port operators have to develop efficient logistics systems. In [Ste04, Vis03, Sta08] an overview of descriptions, classifications and solution methods for the main logistics processes in container ports is given. The *berth allocation problem* (BAP) has been identified as one of the key issues in these studies. There is an explicit difference between i) the single-terminal BAP, which is concerned with the allocation of a set of vessels to one terminal and ii) the multi-terminal BAP, which is concerned with the allocation of a set of vessels to a cluster of interrelated terminals. The *quay crane allocation problem* (QCAP) is another key issue in a container port. Each terminal has a fixed number of quay cranes, which have to be allocated to berthing vessels. The research discussed in this report is embedded in a larger multi-step solution approach for multi-terminal container operations. In the next section the multi-step solution approach is discussed in general, and the BAP and QCAP in particular.

1.1 Container Operations

As stated in [Ott06] hardly any research has been conducted towards the multi-terminal BAP. Only in [Hen08] a solution approach is proposed to solve the multi-terminal BAP. In this section the problem and the solution approach of [Hen08] are described.

[1] TEU: twenty-foot equivalent unit

Problem description

Present mega-ports often consist of a cluster of terminals as depicted in Figure 1.1. Each terminal has a restricted quay length and quay crane capacity. Containers are exported from the hinterland to one of the terminals and vice versa. Since the mega-ports also face a significant amount of transshipment containers and it is not always possible to allocate the vessels which carry containers for each other to the same terminal, there is inter-terminal transport established by trucks or barges. As a result of the inter-terminal transport the different terminals become interdependent and cannot be considered separately anymore.
This interdependency has to be incorporated in one model in order to derive the optimal allocation. In addition, the in- and outbound containers and their destinations have to be taken into consideration. For example, inbound containers from an arriving vessel could be partly destined for the hinterland and partly for another vessel. Allocating the two involved vessels to different terminals implies inter-terminal traffic and thus additional costs. However, due to other objectives and constraints this may still be the best or only solution. This kind of trade-offs have to be weighted in the multi-terminal BAP.
In practice most vessels run a regular service on their ports, for instance once a week, which turns the problem into a cyclic problem. Vessels can arrive at the end of the considered time period (cycle) and leave at the beginning of this time period (next cycle). An example of a cyclic schedule of one week for a terminal is given in Figure 1.2. Vessels 1 – 6 are indicated by rectangles with dimensions length and processing time of the vessel. Hence, the horizontal axis indicates the position of the vessels along the terminal quay, and the vertical axis indicates the arrival and departure times of the vessels. For example, vessel 1 is arriving on Tuesday and departs on Wednesday; its left-most side is positioned at 162.5 m. Vessel 6 is colored differently, since it arrives at the end of the cycle (i.e. on Sunday) and departs at the beginning of the next cycle (i.e. on Tuesday).
The desired arrival and departure times of vessels are agreed on between the port operator and the shipping lines. In this report, unless stated differently, those contractual arrival and departure times are referred to as desired berthing time intervals. The operator pays a fine when a vessel is leaving after its desired departure time.

Multi-step solution approach

In short, the problem consists of the allocation of i) a vessel to a terminal, ii) a time interval to a vessel, iii) a vessel to a berthing position, and iv) quay cranes to a vessel. Incorporating all these decisions in a single model would result in a complex model. A multi-step approach is proposed in [Hen08] in which the sequential steps represent a part of the complete multi-terminal BAP. The different steps are depicted in Figure 1.3; two steps (1 and 2) are indicated. In the first step the vessel is allocated to a terminal

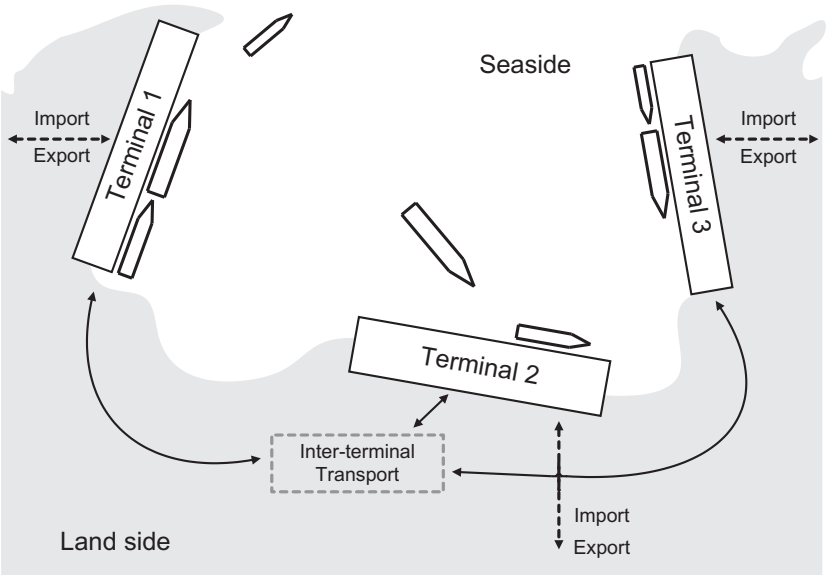


Figure 1.1: An example of a cluster of inter-related terminals [Hen08]

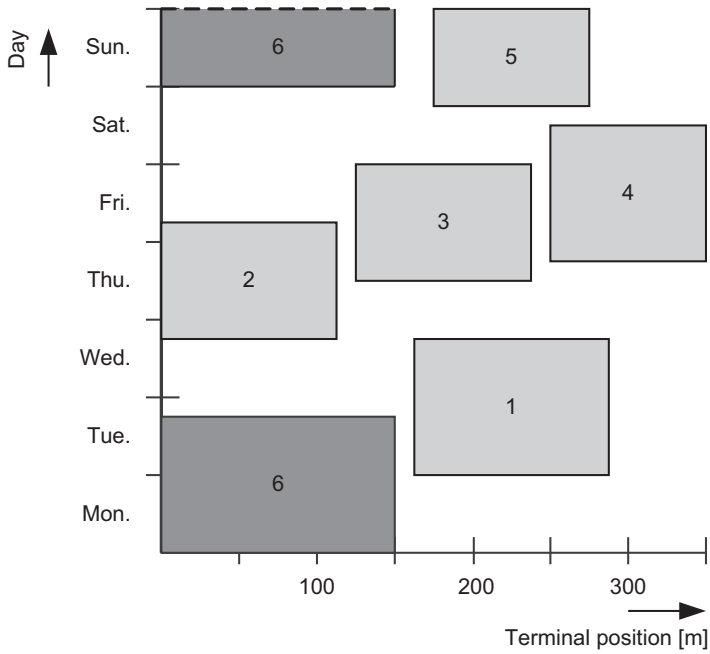


Figure 1.2: An example of a cyclic schedule

and a time window is allocated to the vessel. In the second step the vessel is allocated to an exact berthing position and quay cranes are allocated to the vessel. Hence, the second step introduces more detail to the allocation constructed in the first step. Below

the steps are described in more detail.

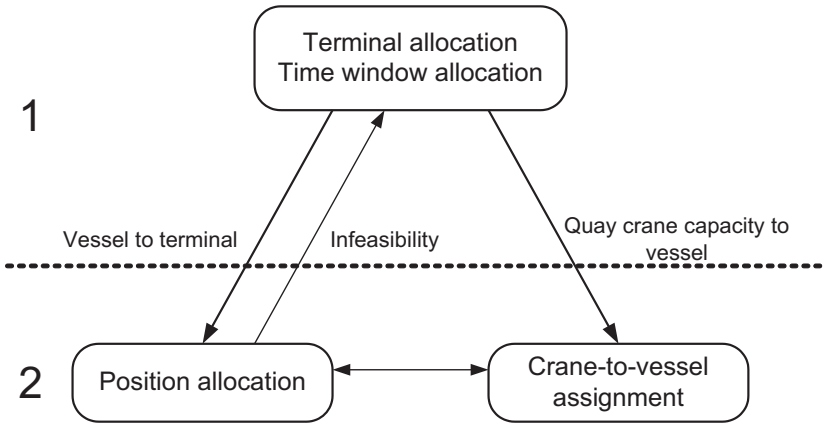


Figure 1.3: Multi-step solution strategy for the multi-terminal BAP

First step

In the first step, i) a terminal, ii) a time window, and iii) a time variant quay crane capacity are allocated to each vessel in the vessel set. The model of [Hen08] is discrete in time: For example, a week is divided in 168 time intervals of one hour. A vessel can be berthing during multiple of those time intervals. The problem is formulated as a multi-objective optimization problem. A straightforward and an alternative *Mixed Integer Linear Programming* (MILP) problem are proposed. There are four objectives: minimizing i) the amount of inter-terminal traffic, ii) the total weighted deviation from the desired berthing time intervals, iii) the amount of stored containers in each individual terminal, and iv) the required quay crane capacity in each individual terminal. Restricting properties are the terminal quay lengths and the available quay crane capacity in each individual terminal. In this model, it is guaranteed that the terminal quay length and the quay crane capacities are never exceeded. However, the exact position allocation and the exact quay crane allocation are still to be determined in the second step. In the first step an average quay crane rate is used, which means that each quay crane has the same average rate. In each time interval, quay crane capacity can be allocated to a vessel. The decision making in [Hen08] is of a strategic nature: The arrival and departure times of vessels are assumed to be deterministic. The formulation of the optimization problem is extended in [Vul08] to be able to use the method on an operational level, where disturbances are taken into consideration. Disturbances can be late arrival of a vessel or the breakdown of a quay crane. Model-based predictive control (MPC) is used to construct the operational allocation.

Second step

The exact berth allocation of the vessels and the exact quay crane allocation are determined in the second step. For both of them an optimization problem is formulated. In this report, these optimization problems are the subject of investigation.
After allocating vessels to terminals in the first step, the actual position allocation can be considered for each terminal separately. Since in the first step the arrival and departure times of the vessels have already been determined, the single-terminal BAP's are one-dimensional packing problems. Namely, only the position of the vessels along the terminal quay has to be determined. The most important constraint is that the rectangles (vessels) cannot overlap each other. Each vessel has its own lowest-cost berthing position within the terminal, which is related to issues as the distance to the stacking area, the capacity of the quay cranes at that quay location or the amount of traffic that is generated by the vessel. The objective of the single-terminal BAP is to minimize the weighted deviation from the lowest-cost berthing positions.
The first step terminal allocation may turn out to be infeasible in the second step allocation. In the first step it is only guaranteed that the sum of the lengths of the vessels, that are berthing simultaneously at one terminal, does never exceed the total terminal length. However, this is not a guarantee for a feasible 2D-packing. When the position allocation is infeasible, the first step problem has to be adapted and solved again to construct a new allocation. This is depicted in Figure 1.3.
In addition, quay cranes are allocated to vessels in the second step: The quay crane allocation problem (QCAP). As mentioned before, quay cranes can have different capacities and, moreover, they can have a different capacity on different vessels. In the first step allocation it is guaranteed that the total capacity in a terminal is never exceeded, however existence of a feasible integer quay crane allocation is not guaranteed. Nevertheless, since switching a quay crane from one vessel to another is possible, feasibility issues are not expected for the quay crane allocation. The problem can be formulated as a multi-objective optimization problem. The objectives can be the minimization of i) the departure time of each vessel, ii) the required quay cranes in an individual terminal, iii) the number of movements of the cranes along the terminal quay. In addition, the position of the quay cranes can be optimized. It is preferred to prevent that an idle quay crane is isolated between two processing quay cranes, since then the corresponding capacity cannot be used anywhere else along the terminal quay without moving the other quay cranes. The fact that the quay cranes cannot cross each other is the most important constraint of the optimization problem.
As shown in Figure 1.3, the two optimization problems in the second step are coupled. This enables a more accurate representation of reality. For example, when a vessel is processed by a number of quay cranes with a large capacity, the process time of this vessel decreases. This changes the dimension of the rectangle corresponding with the process time used in the one-dimensional packing problem: By solving the single-terminal BAP again, the allocation of some vessels may change due to this dimension change.

1.2 Contributions and Outline

In this report the problems in the second step are addressed. In this step the single-terminal BAP and the QCAP are still to be solved.
Firstly, a formulation for the BAP is proposed which fits in the considered multi-step solution approach. To achieve this, an extensive literature study has been performed and existing formulations are used to come up with a new one. The need for a new formulation is a result of the considered solution approach: As described before, the BAP in the solution approach is one-dimensional while in literature the formulations are two-dimensional. The proposed formulation is used to conduct a statistical study into the feasibility of the first step allocation in the second step. The performance of the formulation with respect to the computational effort is also investigated.
Secondly, for the QCAP a formulation that fits the multi-step solution approach is proposed. The formulation of [Liu06], which is continuous in time, fits (partly) within the solution approach. First of all, a few adjustments have to be made to able to solve cyclic systems with the formulation of [Liu06]. However, this formulation has a few restrictions. First of all, a quay crane is allocated to a vessel until it has completely been processed. In addition, the assumption of equal quay crane rates for each quay crane is made. Both restrictions are a result of the fact that the formulation is continuous in time. In this report, a different formulation for the QCAP is proposed. This formulation is discrete in time and does not require the above mentioned restrictions. However, this discrete formulation requires more computational effort. Therefore a heuristics procedure is proposed for the discrete QCAP formulation. The performance of both formulations and the heuristics procedure are compared in an extensive numerical study.
The above described formulations consider problems on a strategic level, in which all desired arrival and departure times of the vessels are deterministic. However, on an operational level, vessels are sometimes early or late. The obtained strategic allocations are not valid anymore then. Therefore, adjustments are made to the BAP and QCAP formulations in the second step to be able to solve operational problems. The results of the extended formulation for the first step problem of [Vul08] is then used as input for the second step.
This research is supported by the terminal operator PSA HNN, located in Antwerp, Belgium where they run a multi-terminal container operation.

This report is structured as follows: In Chapter 2, an extensive literature review for the BAP is given and it is clearly indicated why a new formulation is required in the considered solution approach. After that, the BAP is formally phrased and an example is used to explain how a first step allocation may turn out to be infeasible in the second step. Since the existence of feasible solutions in the second step depends on the utilization of a terminal, it is investigated what the performance of the formulation is for different terminal quay utilizations. In addition, a representative case is solved.
Chapter 3 describes both the continuous and discrete formulation for the QCAP. A

comparison is made between the proposed discrete formulation and the continuous formulation of [Liu06]. In addition, the performance of the discrete formulation is investigated when a heuristics procedure is used to solve the discrete QCAP.
In Chapter 4, a representative case is used to generate results on both a strategic level and an operational level. Based on data from terminal operator PSA HNN the strategic allocation is determined. The allocations from the first step are used as inputs for the BAP and QCAP in the second step. A simulation model is used to generate disturbances on the arrival times of the vessels. In that case, the model-based predictive approach of [Vul08] is used to construct the first step allocation.
Finally, conclusions are drawn and recommendations are given in Chapter 5.

Chapter 2

Berth Allocation Problem (BAP)

In this chapter, the BAP is explained in more detail. In Section 2.1, a literature review is given. In Section 2.2, it is explained why the considered multi-step solution approach requires an adapted formulation. An MILP for the BAP is formulated in Subsection 2.3.2. The possibility on infeasibility in the second step is explained in more detail in Section 2.4. In that section, a statistical study into the feasibility of the first step allocation in the second step is performed. In addition, a possible solution for when a first step allocation turns out be infeasible in the second step is described. The performance of the formulation with respect to computational effort is investigated in Subsection 2.4.4. In Section 2.5, a case study is performed to investigate how the formulation performs in a representative case.

2.1 Literature Review

Intensive research has been conducted on the single-terminal BAP. The single-terminal BAP consists of two interrelated allocation problems. A berthing position and a time interval of berthing have to be allocated to each vessel. Therefore the problem can be viewed as a two-dimensional packing problem, where each vessel is represented by a small rectangle. The process time of a vessel depends on both the amount of containers to be processed and the number of quay cranes allocated to it. In general, the objective of the single-terminal BAP is to minimize the total weighted processing time.
Either a static or dynamic, single-terminal BAP is considered. In the static case, it is assumed that the set of vessels is known before the construction of the allocation takes place. This turns the problem into an assignment problem and makes it solvable in polynomial time with the Hungarian method [Pap82]: Jobs are assigned to machines by sequentially computing shortest paths until each job is assigned to a machine. In the dynamic case, vessels arrive while work is in progress. In addition, a division is made between a discrete and continuous, single-terminal BAP. In the discrete case, the terminal is divided into a finite set of segments. The length of a vessel is not or partly

taken into consideration, since a vessel can only berth at one or more of these segments. Each vessel can then be modeled as a job and each berth as a machine. This turns the problem into a parallel machine scheduling problem [Pin95]. In the continuous case, vessels can berth anywhere along the terminal. Both the discrete and the continuous case have been proven to be NP-hard [Gar79].

In [Lim98] the continuous, single-terminal BAP is transformed into a two-dimensional packing problem. This problem is formulated as a graph-theoretical problem and solved with a heuristic. In [Moo06] the continuous, single-terminal BAP is viewed as a cyclic problem. The two-dimensional packing problem is therefore performed on a cylinder and solved with a sequence based simulated annealing method. [Dai04] formulates the continuous, static BAP as a two-dimensional packing problem with release time constraints. The problem is solved using a local search algorithm. In [Li98] the single-terminal BAP is transformed into a bin packing problem, which results in a discrete approach of the problem. Several variants of the problem are solved with the First Fit Decreasing Heuristic.

A nonlinear integer programming problem is formulated for the discrete, single-terminal BAP in [Nis01, Cor05]. Since such a nonlinear programming problem requires a lot of computational effort, heuristics procedures are proposed to solve the problem. [Nis01] proposes a heuristics procedure based on a genetic algorithm and [Cor05] uses tabu search heuristics to solve the single-terminal BAP.

In [Ima01, Par03, Han08], an MILP problem is formulated to solve the discrete, single-terminal BAP. Since the proposed MILP formulations require much computational effort when they are solved, different heuristics procedures are proposed to be able to solve problems of practical size within a reasonable amount of time. In [Ima01], the original problem is relaxed by a Lagrangian penalty function. A heuristics procedure based on a simulated annealing method is used to solve the single-terminal BAP in [Par03]. [Han08] presents a variable neighborhood search heuristic and shows that it outperforms three other heuristics.

The continuous, single-terminal BAP is also formulated as an MILP problem in [Kim03, Cor05, Ima05, Wan07]. To reduce the computational effort, again heuristics procedures are proposed to solve the MILP problems. In [Kim03] simulated annealing is used to solve the problem and it is shown that the solutions are close to optimal. A tabu search heuristics is proposed to solve the single-terminal BAP in [Cor05]. [Ima05] presents a heuristics procedure that solves the problem in two stages: In the first stage the algorithm for the discrete, single-terminal BAP identifies a solution given the number of partitioned berths, and in the second stage a procedure reallocates vessels that may overlap or are located far-away from each other. In [Wan07] a heuristics procedure based on a stochastic beam search is proposed to solve the problem.

In [Ima07] mega-ports with intended berths in which quay cranes can process a vessel from both sides are formulated as an MILP problem and solved with a genetic algorithm. Results suggest that while the intended berths serve mega-vessels faster, the total lead time of all the vessels at ports with intended berths is larger than the one at conventional ports.

As described above, all BAP formulations are two-dimensional; both the position and

the berthing time interval are determined. In all cases a heuristics procedure is necessary to be able to solve the BAP within a reasonable amount of time.

2.2 Problem Statement

The first step allocation is used as input for the BAP in the second step. Hence, the vessel set is known before the construction of the allocation, which means that a static BAP is solved. It is enabled that a vessel can berth anywhere along the terminal, since in that case the length of the vessels is completely taken into consideration. Hence, a static, continuous BAP is solved.
Since the first step allocation is used as input, both the arrival and the departure time of each vessel have already been determined. This means that the vessels have already been fixed in time, and only the position allocation has to be constructed. As a result the two-dimensional BAP, as presented in literature, can be reduced to a one-dimensional BAP.
Each vessel has a lowest-cost berthing position, which is related to issues as the distance to the stacking area, the quay crane rate, or the amount of traffic that is generated by the vessel. The objective is to minimize the deviation from the lowest-cost berthing position.

2.3 Mathematical Model

Firstly, the static, continuous BAP is described in more detail. Secondly, the BAP is formally phrased.

2.3.1 System Description

Unless stated differently, the following set of vessels is considered: $v \in \{1, ..., V\}$. Furthermore, it is assumed that vessels call cyclically, where each vessel in the set arrives exactly once each cycle. In general, the cycle length is in the order of one week for such a container operation.
As mentioned before, the arrival and departure times of each vessel (A_v and D_v respectively) have already been determined in the first step optimization. In the formulations of [Kim03, Cor05, Ima05, Wan07] for the continuous BAP, binary variables are used to indicate whether vessels are berthing at the same time. In addition, binary variables are used to indicate on which side a vessel is positioned with respect to another vessel. Hence, the first type of binary variables from those formulations can be eliminated, since it is already known which vessels are berthing simultaneously. Instead, a set $\mathcal{U}$ is defined, which contains the indices pairs of vessels that are berthing simultaneously during at least one time interval.

Table 2.1: Model parameters

Parameters	Definition
V	Number of vessels [-]
l_v	Length of vessel v [m]
L_t	Length of terminal t [m]
R_v	Lowest-cost berthing position of vessel v [m]
A_v	Actual arrival time of vessel v (start of processing of vessel v) [-]
D_v	Actual departure time of vessel v (end of processing of vessel v) [-]
C_v	Cost per unit distance introduced by vessel v
N	Sufficiently large positive number [-]

Terminal t has a restricted quay length L_t. Once berthing, vessel v requires a certain amount of quay meters l_v. A safety gap is added to each vessels' length.
Vessel v has its own lowest-cost berthing position R_v. The position of vessel v is represented by the position of its center. The origin for this position is the left-most boundary of the terminal quay. Hence, the value for the position of the center of vessel v is at least half of its length. For vessel v, costs are assigned for deviating from the lowest-cost berthing position, indicated by C_v. The goal is to minimize the costs of the system. Hence, the deviation from the lowest-cost berthing position is minimized for all vessels. In addition, a sufficiently large number N is introduced. The sets and parameters discussed above are conveniently arranged in Table 2.1. Below, the variables of the problem are stated first. After that, the constraints and the objective function are derived. Then, it becomes clear why the sufficiently large number N is needed.

2.3.2 MILP

Here, first the variables of the problem are defined. After that, the constraints and the objective function are derived.

Continuous variable

p_v : Position of the center of vessel v [m]. The origin for this position is the left-most boundary of the terminal quay.

Binary variable

$$s_{ij} = \begin{cases} 1 & \text{if vessel } i \text{ is positioned to the left of vessel } j \text{ (i.e. } p_i < p_j) \\ 0 & \text{otherwise.} \end{cases} \qquad (i,j) \in \mathcal{U},$$

In Figure 2.1, an illustration of notations for a berth-schedule on the (terminal,time)-space is given. Since vessel i and vessel j are berthing at the same time, this vessel pair (i, j) is included in set $\mathcal{U}$.

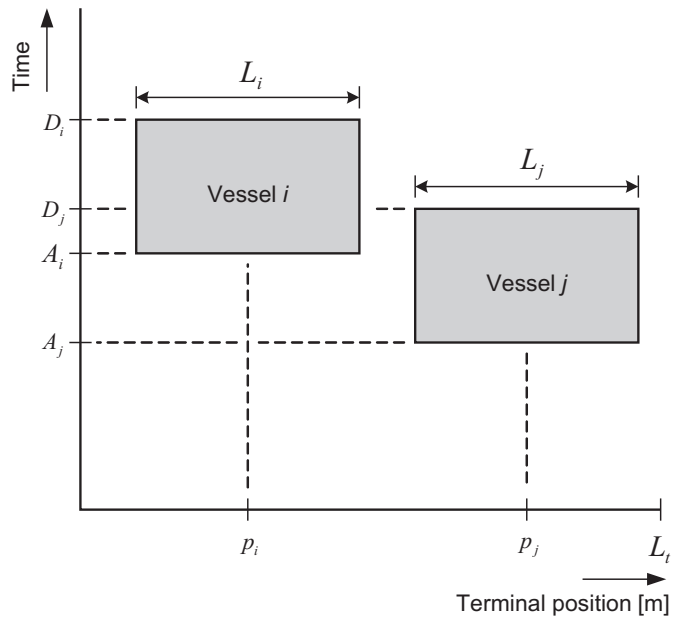


Figure 2.1: An illustration of a berth allocation of two vessels

Constraints

When berthing, vessel v should entirely be allocated in between the terminal ends:

$$\frac{l_v}{2} \leq p_v \leq L_t - \frac{l_v}{2} \qquad \forall v. \tag{2.1}$$

Two vessels, which are berthing simultaneously cannot overlap each other:

$$p_i - p_j \geq \frac{l_i + l_j}{2} - s_{ij} \cdot N \qquad \forall (i, j) \in \mathcal{U}, \tag{2.2}$$

and

$$p_j - p_i \geq \frac{l_i + l_j}{2} - \left(1 - s_{ij}\right) \cdot N \qquad \forall (i, j) \in \mathcal{U}. \tag{2.3}$$

Vessel i is always allocated to the right or left of vessel j. Suppose that vessel i is allocated to the right of vessel j. In that case, $p_i - p_j < 0$. Constraint (2.2) can only be fulfilled when $s_{ij} = 1$ since then the sufficiently large number N is deducted. Using $s_{ij} = 1$ in Constraint (2.3) ensures that the centers of vessels i and j are at least half

of the vessels' lengths away from each other. When the vessels would be allocated the other way around, the value of s_{ij} would be forced to 0 by Constraint 2.3, and the position of the vessels would be determined by Constraint 2.2. An appropriate choice for N is the terminal length L_t.

Objective function

In the objective the distance of each vessel to its own lowest-cost berthing position R_v is minimized:

$$\min \sum_{v=1}^{V} C_v \cdot |p_v - R_v|. \tag{2.4}$$

Since the problem is solved as a linear optimization problem, the absolute value has to be eliminated. Therefore, the auxiliary variable o_v is introduced. As a result, the objective is reformulated and two constraints are added:

$$\min \sum_{v=1}^{V} C_v \cdot o_v, \tag{2.5}$$

where

$$o_v \geq p_v - R_v \qquad \forall v, \tag{2.6}$$

and

$$o_v \geq R_v - p_v \qquad \forall v. \tag{2.7}$$

The additional constraints ensure that auxiliary variable o_v is always equal to or larger than 0.

2.4 Statistical Study

As mentioned in Section 1.1, the first step allocation may turn out to be infeasible in the second step. This is illustrated by a small example. Since it is expected that the percentage of feasible allocations depends on the utilization of the terminal quay, experiments are performed to investigate the influence of the terminal quay utilization on the feasibility.

2.4.1 Infeasibility

In Table 2.2 the length, arrival and departure times are given for a set of 5 vessels. This set represents a part of the output of the optimization problem in the first step: The allocated time variant crane capacity is left out, since this is not relevant for the BAP. The considered cycle is one week, consisting of 7 time intervals of one day. The seventh time interval ends at discrete time step 7, which is equal to the discrete time step 0. This means that vessel 5 berths at discrete time step 6 of the cycle and departs at discrete time step 2 of the next cycle. The terminal quay length is 350 m. The total terminal quay occupation is at most 300 m: There is no time interval were the combined lengths of the vessels which are berthing simultaneously is larger than 300 m. Hence, one would expect that it is possible to construct a feasible allocation in the second step. Nevertheless, this is impossible, since the time intervals and the dimensions of the vessels make a non-overlapping fit impossible. In Figure 2.2, it is depicted that the vessels do not fit in between the terminal ends. Since in this step it is impossible to change the arrival and departure times of the vessels, vessels 1 and 4 have to be positioned left or right to vessel 3. This also sets the position of vessel 5, which makes a feasible allocation of vessel 2 impossible due to the cyclic nature of vessel 5. There are more possibilities of arranging the vessels than depicted in Figure 2.2, however no feasible packing exists.

Table 2.2: Set of vessels

Vessel	l_v [m]	A_v [-]	D_v [-]
1	100	2	4
2	150	0	3
3	200	3	6
4	100	5	7
5	150	6	2

2.4.2 Feasibility Experiments

Looking at Figure 2.2, one can image that the terminal quay utilization has an influence on the probability of infeasibility in the second step. In a 'packed' terminal, it is more difficult to construct a non-overlapping fit. Hence, it is expected that for a larger terminal quay utilizations, it is more often not possible to construct a feasible allocation in the second step from a feasible first step allocation.
Herein, the terminal utilization is defined by:

$$u_t = \frac{\sum_{v=1}^{V} O_v}{A}, \tag{2.8}$$

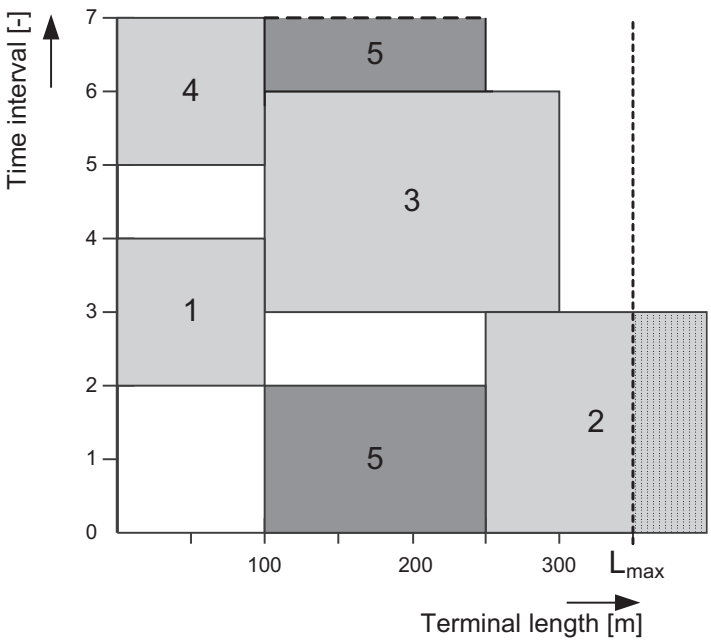


Figure 2.2: Allocation infeasibility in the second step

where
A : Product of the terminal length L_t and the number of time intervals K,
O_v : Product of the vessel length l_v and the processing time of vessel v.

For example, $A = 7 \cdot 350$ and $O_1 = 100 \cdot 2$ in Figure 2.2. When for all vessels O_v is determined, this results in a terminal utilization of 0.77.
In addition, it is expected that the mean length and the mean processing time of the vessels have an influence too. When both are decreased, the dimensions of the rectangles which represent a vessel become smaller (i.e. O_v is decreasing). Constructing a non-overlapping fit is expected to be easier with smaller rectangles.
Hypothesis: For larger terminal utilizations, it is more often not possible to construct a feasible allocation in the second step from a feasible first step allocation. Decreasing the average dimensions of vessels (rectangles) in time and space makes it easier to construct a feasible allocation in the second step from a first step allocation.

Experiments setup

Since the aim of the experiments is to check the feasibility of first step allocations in the second step, it is not necessary to include the objective function in the implementation of the formulation of Subsection 2.3.2. Hence, (2.4) – (2.7) are neglected in these experiments, and only a feasibility check is performed. Computation time is gained by doing this.

In the experiments, a cyclic period of one week is considered with time intervals of one hour, resulting in $K = 168$. The terminal quay length is chosen 1000 m (i.e. $L_t = 1000$ m). Vessel sets with different terminal utilization are generated randomly. The utilization is varied by generating sets with a different number of vessels. The mean processing time $\overline{P}_v$ and the mean length $\bar{l}_v$ of the vessels are not changed between the different sets. Vessel sets are generated for a terminal quay utilization up to 0.85. When the utilization is around 0.85, it is only possible to increase the utilization by introducing smaller vessels which are berthing for a shorter time interval since there has to be guaranteed that the terminal quay occupation is never larger than the terminal length. This would not be a correct representation of reality. The generated vessel sets have to represent first step allocations; meaning that the arrival and departure time of each vessel (A_v and D_v, respectively) have already been determined.
The arrival time is drawn from an uniform distribution between $k = 0$ and $k = K$. The processing time and the length of the vessels are determined with the help of representative data of container terminal operator PSA HNN. In Appendix A, it is described how the representative data is used to generate the processing times and lengths of the vessels for the vessel sets. The departure time is determined by the arrival time and the processing time. The length of the vessels l_v includes a safety gap of 50 m. Since the system is cyclic, a vessel can arrive at the end of the cycle and leave at the beginning of the next cycle.
Since the influence of the dimensions of the rectangles is also investigated, different experiments are performed. In each experiment, vessels in the generated vessel sets have a different mean length $\bar{l}_v$ and mean processing time $\overline{P}_v$. The parameters l_v and P_v are assumed to be related: Large vessels can carry more containers and require therefore more processing time. The different experiments are given in Table 2.3. The mean vessel length and mean processing time of the vessels in experiment 2 are based on representative averages of the vessels berthing at PSA HNN. The mean length and processing times of the vessels in the other experiments are scaled down or up with respect to the averages in experiment 2.
The generated vessel sets are used as input data, and together with the model of Subsection 2.3.2, fed into CPLEX. CPLEX indicates whether a feasible solution is found or not by giving a different solution code. In addition, the CPU time which is required to check whether a feasible solution exists, is monitored for each vessel set.

Table 2.3: Experiments setup

Experiment	$\bar{l}_v$ [m]	$\overline{P}_v$ [h]
1	200	14
2	250	18
3	300	22
4	350	26

Results

The results of the experiments are depicted in Figure 2.4. Figure 2.4a shows the percentage of feasible solutions dependent on the terminal quay utilization. In Figure 2.4b the mean CPU time and 95% confidence interval are depicted. The markers represent a collection of vessel sets of which the terminal quay utilization is within a certain range. For example, the first marker of all experiments represents the vessel sets with terminal quay utilization between 0 and 0.05, and the second marker represents the vessel sets with terminal quay utilization between 0.05 and 0.10, etc. Typically, the terminal quay utilization is on average between 0.2 and 0.5 in container terminals, which is indicated by the dashed vertical lines.
From Figure 2.4a, it can be concluded that for typical terminal quay utilizations it is not likely to encounter problems with infeasibility. For utilizations larger than 0.5 the the percentage of feasible allocations is decreasing gradually. From utilizations larger than 0.7 the percentage of feasible solutions decreases rapidly. These results are as expected. For larger utilizations the terminal is more 'packed', and the probability of violating the non-overlapping constraints increases with the terminal quay utilization. It has to be remarked that not all infeasible allocations are caused by the cut in the chosen multi-step solution approach. Suppose that the same vessel sets would be used, but that the vessels would not be fixed in time. In that case, a two-dimensional packing problem is solved. It is still possible that a non-overlapping allocation does not exist.
Although the sum of the areas of all small rectangles ($\sum_{v=1}^{V} O_v$) is less than the area of the large rectangle (A), it is still possible that the small rectangles are not fitting in the large rectangle. This is caused by the fixed dimensions of each small rectangle, resulting from the vessel length l_v and the processing time P_v. A very simple example of such a situation is a large rectangle with dimensions 2 by 2, and two smaller rectangles with dimensions 1.5 by 1 and 1 by 1.5, respectively. The area of the large rectangle is 4, and the sum of the areas of the two smaller rectangles is 3. Since it is not allowed to rotate the small rectangles, it is not possible to find a feasible fit. This example is depicted in Figure 2.3.

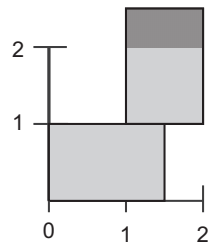
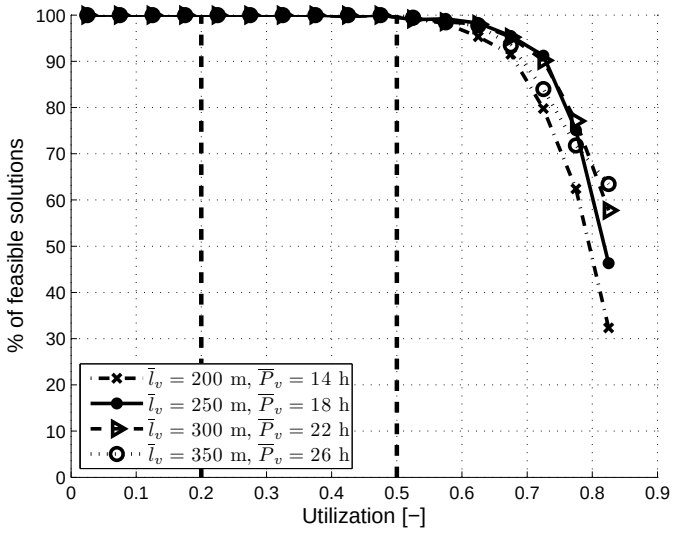
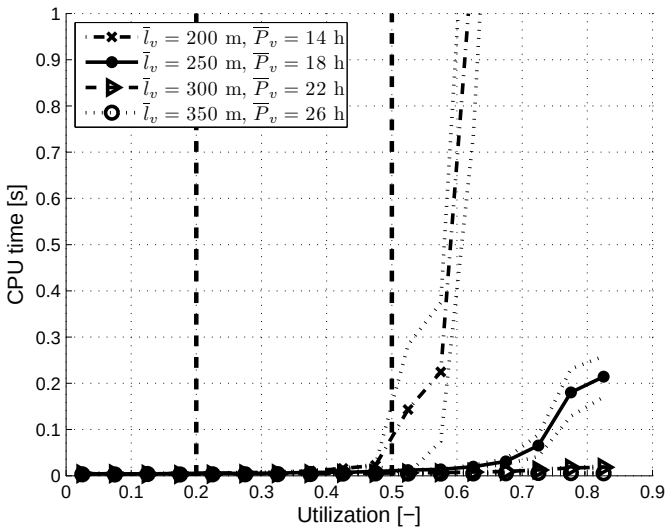


Figure 2.3: Example

In Figure 2.4a not all results are as expected. It is actually not the case that vessels



(a) Utilization and feasibility



(b) CPU time to check feasibility

Figure 2.4: Results for the feasibility experiments

that are on average smaller and are berthing on average for a shorter period of time result in less infeasible allocations. For the largest utilization range (i.e. 0.8 to 0.85) it is even the other way round. For terminal quay utilizations larger than 0.55, it is even the case that smaller vessels on average more often result in infeasible allocations than the larger ones. For terminal quay utilizations from 0.6 there is actually not a clear relationship between the mean vessel length and mean berthing time, and the percentage

of infeasible allocations. It is expected that two factors play an important role.
In the first place, the number of vessels that are berthing during the considered time period. When both the mean vessel length and mean berthing time decrease, the number of vessels increases quadratically. Both the mean length and mean width of the small rectangles decreases. The growth in vessels results also in an increase of non-overlapping constraints. The number of constraints increases with $\frac{1}{2}V(V-1)$, where V is the number of vessels. Hence, with the increase of the number of vessels, the number of constraints also increases, which makes it more difficult to construct a non-overlapping fit. In the second place, the mean length and mean berthing time of the vessels (rectangles) have an influence. Although the relation between vessel size and the percentage of infeasible allocations is not as expected, this has to play a role. As mentioned before, it is expected that smaller rectangles are easier to fit in the large rectangle. For example, for a terminal quay utilization between 0.6 and 0.75 the vessels with the largest mean length ($\bar{l}_v = 350$ m) and the largest mean berthing time ($\overline{P}_v = 26$ h) result in more infeasible allocations than the vessels with mean lengths of 250 and 300 m, respectively.
The above-mentioned factors are conflicting. For larger utilizations one would expect that it is easier to fit the smaller rectangles in the large rectangle. On the other hand, smaller rectangles result in more non-overlapping constraints, which makes it more difficult to construct a non-overlapping fit. The conflict between the two factors most probably causes the unexpected differences as depicted in Figure 2.4a.
In Figure 2.4b, it is shown that the required CPU time to find a feasible solution depends on the mean vessel length and mean berthing time. The smaller the mean vessel length and mean berthing time, the more CPU time is required. This is most probably caused by the fact that when the vessels are smaller and berthing for a shorter period of time, the number of vessels increases quadratically to reach the same terminal quay utilization. As described above, the number of non-overlapping constraints also increases. It is expected that when the number of vessels reaches a certain value, the required CPU time is increasing rapidly. In this case, that number is around 20 vessels. For the vessel sets with $\bar{l}_v = 200$ m and $P_v = 14$ h a terminal quay utilization of around 0.5 is reached with 20 vessels. That is also the utilization where the CPU time for those vessel sets starts increasing stronger. The mean maximum CPU time for those vessel sets is 23 s. Also for the vessel sets with $\bar{l}_v = 250$ m and $\overline{P}_v = 18$ h, the CPU time starts increasing in the utilization region (i.e. 0.70 to 0.85) where 20 vessels are berthing in the considered time period. For the two other experiments, less than 20 vessels are required to reach large quay utilizations. In those experiments no sharp increase of the required CPU time is observed, which confirms that when the problem size stays below a certain value no sharp increase in CPU time is observed.
From the experiments, it can be concluded that for typical terminal quay utilizations in real-life ports, the number of infeasible allocations is statistically negligible. The required CPU time to find a feasible allocation is also very small. When the utilization increases, not a lot of problems are to be expected with infeasibility issues and the required CPU time to find a feasible allocation.

2.4.3 Dealing with Infeasibility

As described in the previous section, for typical terminal quay utilizations there are no problems expected with infeasibility. However, for increasing terminal quay utilization the probability of infeasibility in the second step is also increasing. In such case, this has to be factored into the optimization problem in the first step as depicted in Figure 1.3. In the first step optimization problem, adjustments have to be made to prevent the allocation to be infeasible in the second step. An approach is to minimize the maximum quay occupation during the considered time period. The infeasibility is most probably caused by some consecutive time intervals in which the quay occupation is very large. By minimizing the maximum quay occupation, the vessels are balanced out over time, and the probability of infeasibility decreases. A variable for the maximum quay occupation has to be added to the allocation problem in the first step.
However, this proposed solution does not guarantee that infeasibility does not occur anymore in the second step optimization. Further research is required to investigate the impact of this proposed approach.

2.4.4 Parameter Sensitivity

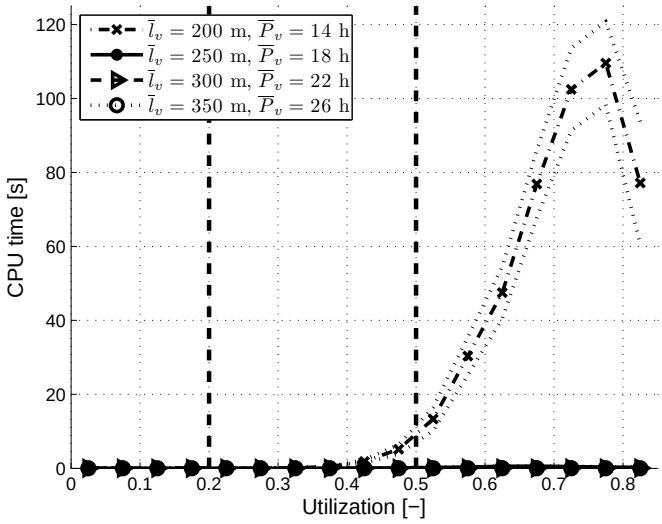
In the previous section no optimization is performed; there is only checked whether a feasible solution can be found for a certain vessel set. In this section, for the feasible data sets from Subsection 2.4.2 an optimization is performed. The required CPU time to perform the optimization is investigated dependent on the utilization of the terminal quay. It is expected that the required computational effort is small for all terminal utilizations since, as mentioned before, the considered problem is a one-dimensional problem. It is more easy to solve than the two-dimensional formulations in literature. Off course, the required CPU time is expected to increase with increasing terminal quay utilization.
In addition, the average distance of each vessel to its lowest-cost berthing position R_v, and the objective function value for the different experiments are monitored. It is expected that the average distance to the lowest-cost berthing position and the mean objective function value are dependent on both the terminal utilization and the vessels' length and berthing time. Most probably both values increase with the terminal utilization, since in a 'packed' terminal more vessels are berthing and more non-overlapping constraints have to be fulfilled. The possibility that one vessel is blocking the lowest-cost berthing position of another vessel is increasing with the terminal utilization. This possibility is also increasing when the average vessels' length and berthing time are decreasing, since then the number of vessels and non-overlapping constraints are increased as it has been observed in the previous section.
Hypothesis: Since a one-dimensional packing problem is solved, the required CPU time is expected to be small. The CPU time, the average distance to R_v and the average objective function value increase with the terminal quay utilization. They are also increasing when the average vessels' length and berthing time are decreased.

Experiments setup

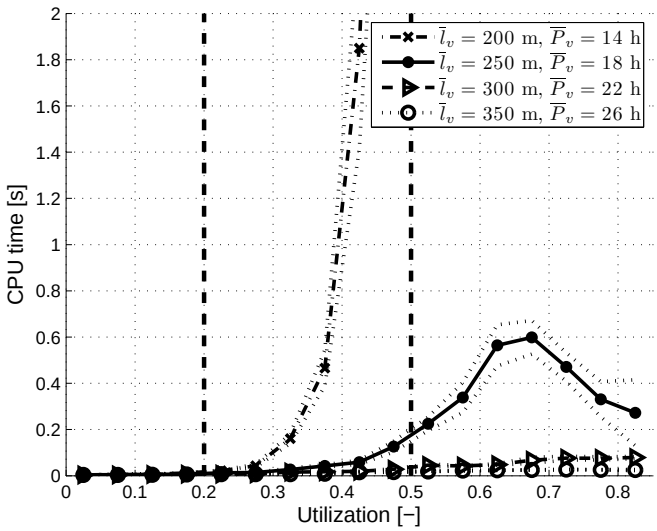
Since in these experiments an optimization is performed, the objective function (2.5) and the additional constraints (2.6) and (2.7) are added to the model. The lowest-cost berthing position R_v for each vessel is a randomly chosen position along the terminal quay. The cost per unit distance of a vessel C_v depends on the length of the vessel. In these experiments the large vessels introduce more costs when berthing on a non-optimal berthing position than small vessels since it is assumed that large vessels introduce more work than small vessels.
The vessel sets which result in a feasible allocation from Subsection 2.4.2 are used as input data. Together with the model, they are fed into CPLEX. The mixed integer optimization is terminated as soon as it has found an integer solution proven to be within 1% of optimal. The CPU time is monitored again. The objective function value and the mean distance to R_v of each vessel in each vessel set is stored.

Results

In Figures 2.5 and 2.6 the results of the experiments are depicted. The mean CPU time and 95% confidence intervals are depicted in Figure 2.5a. In Figure 2.5b the same results are depicted on a different time scale to show the results with a small required CPU time in more detail. In Figure 2.6a, the average distance of the vessels to R_v and the 95% confidence intervals dependent on the terminal quay utilization are depicted. The average objective function values and 95% confidence intervals of the different experiments are depicted in Figure 2.6b. The dashed vertical lines again indicate the typical terminal quay utilization.
As expected, for typical quay utilizations the MILP problem is solved very rapidly; for all experiments a solution is constructed within 10 seconds for typical terminal quay utilizations. For the vessel sets with $\bar{l}_v = 200$ m and $\overline{P}_v = 14$ h the overall largest CPU time is monitored at a terminal utilization between 0.75 and 0.8; it takes then up to two minutes to construct an optimal position allocation.
In Figure 2.5a, it is shown that for this vessel size the required CPU time is significantly larger than for the other vessel sizes. In Subsection 2.4.1 has already been observed that the problem size has most probably its influence on the required computational effort. As mentioned there, decreasing the mean length and the mean process time of the vessels in a vessel set results in a quadratic increase of the number of vessels to reach the same utilizations, and an increase of $\frac{1}{2}V(V-1)$ in non-overlapping constraints. In addition, more vessels in a vessel set means also more possibilities of allocating the vessels with respect to each other. Hence, there are more possible allocations which have to be compared with each other. The problem size and the number of allocation possibilities are most probably also the explanation for the relation between the different required CPU times in Figure 2.5b; the on average largest vessels require the least CPU time.



(a) CPU time to optimize



(b) Detail of CPU time to optimize

Figure 2.5: CPU time for the optimization experiments

For the vessel sets with $\bar{l}_v = 200$ m or $\bar{l}_v = 250$ m and $\overline{P}_v = 14$ h or $\overline{P}_v = 18$ h, the same behavior is observed. They both have a certain terminal quay utilization where the CPU time is at its peak, and decreases again for larger terminal quay utilizations. This is most probably caused by the fact that the number of possibilities to allocate the vessels with respect to each other only increases up to a certain terminal quay utilization. For the vessel sets with $\bar{l}_v = 200$ m and $\overline{P}_v = 14$ h this certain terminal

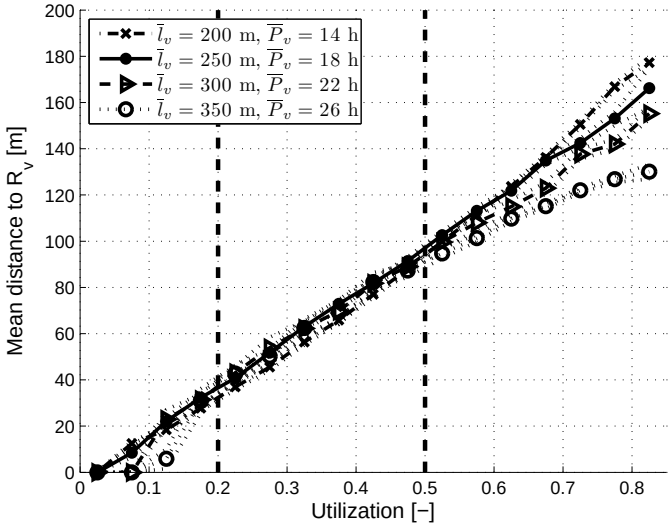
quay utilization is between 0.75 and 0.8, and for the vessel sets with $\bar{l}_v = 250$ m and $\overline{P}_v = 18$ h this utilization is between 0.65 and 0.7. For quay utilizations larger than this value, the number of possibilities to allocate the vessels with respect to each other most probably decreases. Because the terminal becomes more 'packed', there are less feasible allocations possible.
The results with respect to the average distance of the vessels to R_v are as expected. In Figure 2.6a, it is shown that from a terminal quay utilization of 0.5, the mean distance of each vessels to R_v is larger for vessel sets with vessels that are on average smaller and berthing for a shorter period of time. As mentioned before, this is most probably caused by the fact that the number of vessels and non-overlapping constraints increase with the decrease of the vessels' length and berthing time in the vessel sets. Hence, the possibility that a vessel is blocking the lowest-cost berthing position of another vessel is increased. The mean objective function value is also significantly larger for the vessel sets with vessels that are on average smaller, as depicted in Figure 2.6b. This is exactly as expected, since the average distance of each vessel to R_v is increased and the number of vessels in the vessel sets is larger.
From these experiments, it can be concluded that the BAP solves very fast for typical terminal quay utilizations. The required CPU time, the mean distance of the vessels to the lowest-cost berthing position and the mean objective function value most probably increase with the problem size. Hence, when the average length and berthing time of the vessels in the vessel set are small, the CPU time, the distance of each vessel to R_v and the objective function value are large in comparison with other vessel sets. However, in the performed experiments the maximum mean CPU time is not exceeding the 2 minutes.

2.5 Case Study

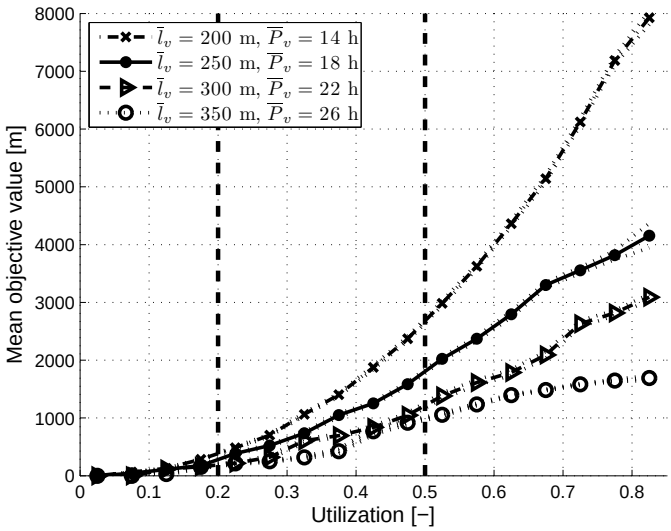
A case study is considered with representative data for the port of Antwerp. For the data, an allocation has been constructed manually by experienced planners of PSA HNN. Furthermore, an allocation has been constructed using the proposed formulation in Section 2.3.
The model parameter R_v, the lowest-cost berthing position, is defined in more detail in [Ove08] by using data about straddle carrier driving distances. The objective is to minimize the total amount of straddle carrier driving distance. The generated berth position allocation is compared with the manually constructed berth position allocation. Comparison suggests that a significant reduction in the driving distances can be achieved by using the proposed formulation.

2.5.1 Straddle Carrier Driving Distances

For each container, a straddle carrier has to cover a certain distance to put it from vessel to stack or vice versa. This distance depends on the position of the vessel at



(a) Mean distance of each vessel to R_v



(b) Mean objective function value

Figure 2.6: Results for the optimization experiments

the terminal quay and the position of the container in the stack. Different types of containers can be distinguished, of which some have a fixed position in the yard:

1. Empty containers: Stacked at two fixed positions in the yard; at the center of the terminal and at the right-most side of the terminal. For each vessel, it is known to which of the two empty-stacks the empty containers have to go.

2. Refrigerated containers: Stacked at one fixed position in the yard; at the rightmost side of the terminal.
3. Containers with dangerous goods: Stacked at three fixed positions in the yard; all three positions are located at the right side of the terminal.
4. Export containers: Stacked somewhere at the yard during the days before the arrival of the vessel. Most of the export containers are stacked in the center of the terminal. This is a result of the fact that the left side of the terminal is still under construction.
5. Import containers: Stacked somewhere in the yard nearby the berthing position of the vessel.
6. Transshipment containers: Temporarily stacked and then loaded onto another vessel.

The container types 2 and 3 have fixed positions in the stacking area due to regulations and facilities. There are strict regulations about stacking containers with dangerous goods and the refrigerated containers require for instance electricity connections. With container categories 4 to 6 full containers are meant, which are not falling in categories 2 or 3. Off course, the containers in categories 2 and 3 can also be imported, exported or transshipped. However, they always have to go to their fixed position in the yard. For example, consider a refrigerated, transshipment container. This container has to go temporarily to the fixed yard position for refrigerated containers, since it requires electricity to stay refrigerated. Category 1 is an exception, since an empty container is only moved from or to its fixed yard position when the container is exported or imported. When an empty container is transshipped, it falls under category 6 since it does not require special facilities.
In Figure 2.7a, representative straddle carrier driving distances for containers with dangerous goods are depicted as a function of the terminal position of a vessel. Since the three fixed stacking positions for the containers with dangerous goods are on the right side of the terminal, the driving distance for the straddle carriers is smallest at the right side of the terminal. An approximation of the measured driving distances is constructed by using piecewise linear functions, which are indicated by the dashed lines in the figure. The driving distance functions for container types 1 and 2 are approximated in the same way.
It is assumed that the stacking position of export containers for a certain vessel is fixed. With respect to container category 4, therefore an approximation of the straddle carrier driving distances has to be made for each vessel separately. In Figure 2.7b, representative driving distances dependent on the terminal position for one of the vessels are depicted. Apparently, most export containers for this vessel are stacked somewhere between position 700 and 1000 m. Namely, the mean driving distance for straddle carriers is smallest for this region. The driving distance of the straddle carriers is again approximated by piecewise linear functions. This is done for all vessels in the vessel set.

The driving distance for import containers is chosen independent of the berthing position of a vessel since the containers are stacked at the berthing position of that vessel. Finally, the driving distance of a straddle carrier for a transshipment container is equal to the distance between the berthing positions of the two involved vessels.
Most vessels ship containers from different categories. The total amount of straddle carrier distance of a vessel is assumed equal to the sum of the mean driving distances for the containers of each category multiplied by the number of containers.
As mentioned before, the MILP formulation of Subsection 2.3.2 is used to solve the case study. The objective is to minimize to total amount of straddle carrier distance. Hence, the objective function for this case study is given by:

$$\begin{aligned} \min \quad & \sum_{v=1}^{V} \big(N_v^{empt} \cdot D_{empt}(p_v) + N_v^{ref} \cdot D_{ref}(p_v) + N_v^{dang} \cdot D_{dang}(p_v) + N_v^{exp} \cdot D_{exp}(p_v) \\ & + N_v^{imp} \cdot D_{imp}(p_v) \big) + \sum_{x=1}^{V} \sum_{y=1}^{V} N_{xy}^{trans} \cdot |p_x - p_y|, \end{aligned} \tag{2.9}$$

where N_v^{empt}, N_v^{ref}, N_v^{dang}, N_v^{exp} and N_v^{imp} indicate the number of containers to be processed for vessel v of container categories 1 to 5. The parameter N_{xy}^{trans} gives the number of containers from vessel x that are destined for vessel y. The indices of vessel pairs that ship containers for each other are placed in set $\mathcal{TS}$. The functions $D_{empty}(p_v)$, $D_{ref}(p_v)$, $D_{dang}(p_v)$, $D_{exp}(p_v)$ and $D_{imp}(p_v)$ are the approximations for the straddle carrier distances of the different container categories as derived in [Ove08].
Similar to (2.4), the absolute values are eliminated by introducing auxiliary variable ts_{xy} and two additional constraints:

$$\begin{aligned} \min \quad & \sum_{v=1}^{V} \big(N_v^{empt} \cdot D_{empt}(p_v) + N_v^{ref} \cdot D_{ref}(p_v) + N_v^{dang} \cdot D_{dang}(p_v) + N_v^{exp} \cdot D_{exp}(p_v) \\ & + N_v^{imp} \cdot D_{imp}(p_v) \big) + \sum_{x=1}^{V} \sum_{y=1}^{V} N_{xy}^{trans} \cdot ts_{xy}, \end{aligned} \tag{2.10}$$

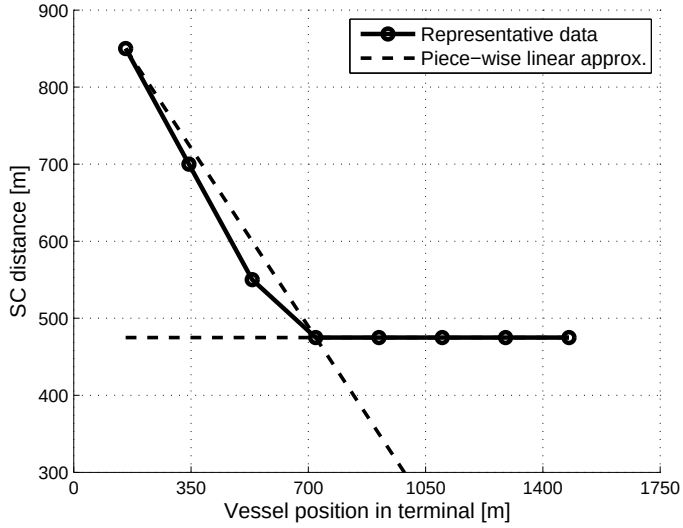
where

$$ts_{xy} \geq p_x - p_y, \qquad \forall (x,y) \in \mathcal{TS}, \tag{2.11}$$

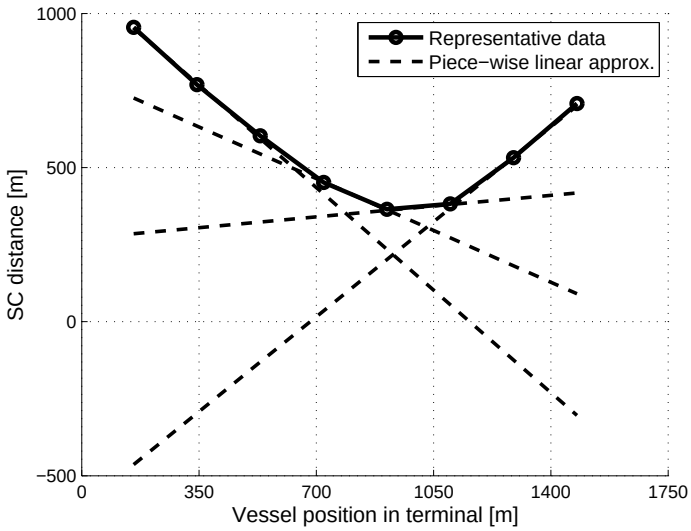
and

$$ts_{xy} \geq p_y - p_x, \qquad \forall (x,y) \in \mathcal{TS}, \tag{2.12}$$

A vessel set of 38 vessels (i.e. $V = 38$) is considered during a time period of 251 time intervals of one hour (i.e. $K = 251$). The terminal length L_t is 1750 m. Of each vessel the arrival time, departure time and length are known (A_v, D_v and l_v respectively) in advance.



(a) SC distances for containers with dangerous goods



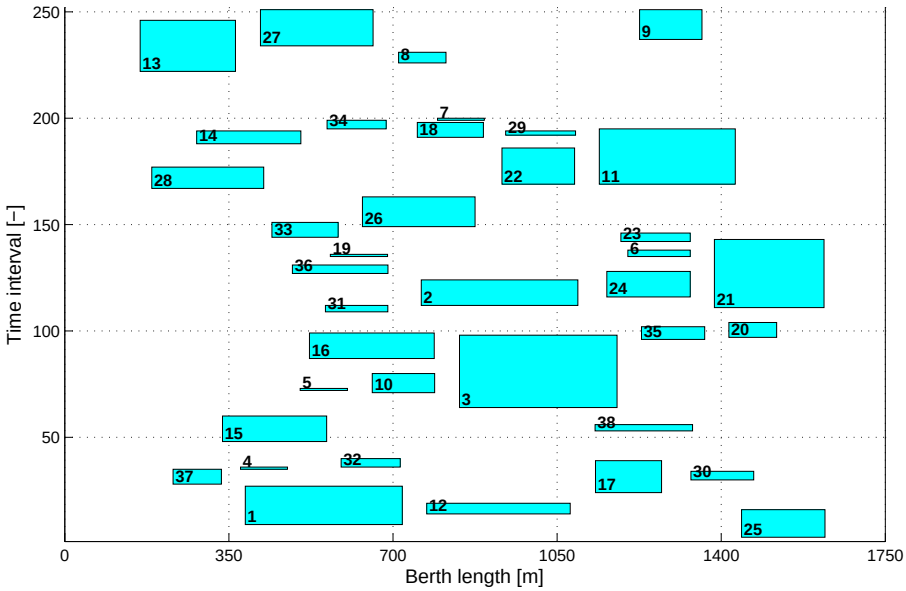
(b) SC distances for export containers for one of the vessels

Figure 2.7: Mean straddle carrier distances as a function of the berth position of a vessel

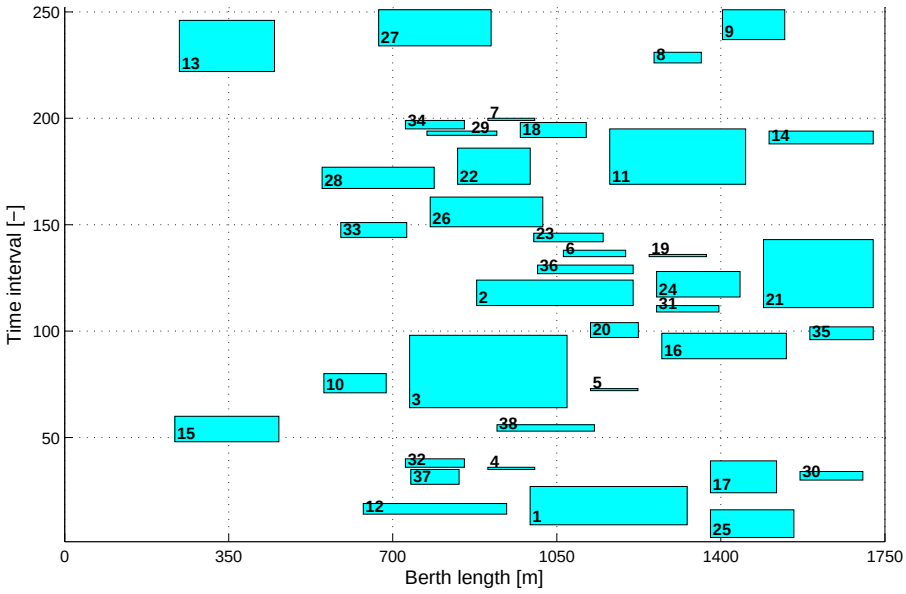
The allocation from the proposed MILP is compared to the manually constructed allocation in means of straddle carrier distance. The total driving distance for the manually constructed allocation is also determined from the piecewise-linear approximations. In this way, a fair comparison can be made.

2.5.2 Results

In Figure 2.8, the results of the case study are depicted. The manually constructed berth position allocation is depicted in Figure 2.8a, and the generated berth position allocation is depicted in Figure 2.8b. The manually constructed allocation results in a total straddle carrier driving distance of 8595 km, and the generated allocation results in a total straddle carrier driving distance of 7842 km. Hence, the generated allocation leads to an improvement of 8.8%. The MILP is solved in 1.63 s.
When the allocations are compared, it is noticed that in the generated allocation more vessels are allocated to the center and the right side of the terminal than in the manually constructed allocation. This has been expected since most containers have to be stacked at the right side of the terminal.
The decrease of the driving distance is mainly caused by a combination of i) a small reallocation of a vessel that loads and/or unloads a lot of containers, and ii) a large reallocation of a vessel that loads and/or unloads a small amount of containers. An example of the first situation is vessel 2. This vessel is only reallocated over 118.5 m, but this results in an improvement of 70 km (= 9.3% of the total improvement) in straddle carrier driving distance. This is caused by the fact that vessel 2 loads and/or unloads 10% (= 2025 containers) of all the processed containers during the considered time period. An example of the second situation is vessel 14. This vessel is reallocated over 1221.5 m, and results in an improvement of 71 km. This improvement is completely caused by 10 refrigerated containers, 10 containers with dangerous goods and 53 empty containers. All these containers are stacked at the right side of the terminal. In addition, the vessel imports 498 containers, however those do not influence the position allocation of the vessel. Hence, although the number of containers is small for vessel 14, it is beneficial to reallocate it for such a large distance.
The results of the case study are very convincing: They suggest that with little computational effort an improved berth position allocation can be constructed. A significant decrease in straddle carrier driving distance is observed for the generated allocation. It has to be criticized that the generated allocation is constructed with full knowledge on all vessels' load in advance. In practice however, the composition of the vessels' load and the transhipment amounts (i.e. N_{xy}^{trans}) are sometimes only available 8 hours before the arrival of the vessel. A solution could be that the BAP is solved more often a week, which is possible because the MILP requires not much computational effort. Then, historical data on load composition could be used for the vessels of which no information is available yet, and this data can be updated while the vessel approaches the port and more information becomes available. This would fit very well in the MPC approach of [Vul08].



(a) Manually constructed berth position allocation



(b) Generated berth position allocation

Figure 2.8: Berth position allocation

Chapter 3

Quay Crane Allocation Problem (QCAP)

The QCAP is the problem in the second step of the solution approach in which quay cranes are allocated to vessels. First, a short literature review about the QCAP is given. From literature, the formulation of [Liu06] is presented and adapted in Section 3.3 to be able to solve the cyclic QCAP in the second step. This formulation is continuous in time, which requires a few assumptions. These assumptions can be eliminated by choosing a formulation, which is discrete in time. Therefore, a discrete QCAP formulation is proposed in Section 3.4. However, the discrete formulation requires more (integer) variables than the continuous formulation, which increases the required computational effort. Therefore, a heuristics procedure for the discrete QCAP is proposed in Section 3.4. In Section 3.5, a comparison is made between the continuous en discrete formulation with respect to the objective function value and the computational effort. In Section 3.6, it is presented how the objective function of the discrete formulation can be extended to incorporate the minimization of i) the departure time of the vessels, ii) the number of required quay cranes, iii) the movement of the quay cranes along the quay iv) the possibility of process interruptions, and v) the isolation of idle cranes between processing quay cranes.

3.1 Literature Review

In each terminal a restricted number of quay cranes is available to process the vessels. The quay cranes are able to move along the terminal and thus along the vessels. Since they are all situated on the same track, it is not possible for the quay cranes to cross each other.
In [Par03], the QCAP is incorporated in the solution procedure for the single-terminal BAP. This problem is solved in two phases: In the first phase, the position allocation

of the vessels is constructed as well as the number of quay cranes allocated to each vessel. In the second phase, a simple heuristics procedure is proposed to construct a more detailed allocation for each individual quay crane. Such a procedure implies that each quay crane has the same capacity since in the first phase only the number quay cranes is determined for each individual vessel.
[Dag89] considers the static QCAP for a set of vessels, which are all available at time 0. The berth length limit and crane traveling time are ignored. The objective is to minimize the total weighted completion time.
In [Liu06], the QCAP is formulated as an MILP problem to minimize the maximum relative tardiness of the vessel departures. The result of the single-terminal BAP is used as an input for the QCAP. The authors assume that a quay crane, allocated to a certain vessel, is occupied until that vessel has completely been processed. A vessel is divided in several bays, each having their own number of containers; each bay can be processed by exactly one quay crane. To reduce the computational effort a heuristics decomposition procedure is proposed to breakdown the model into two smaller, linked models. In the first model, the optimal process time for each vessel is determined, using different numbers of allocated quay cranes. In the second model, specific quay cranes are allocated to each vessel. Both the MILP problem and the heuristics procedure require the assumption of equal crane rates for each quay crane. Furthermore, the formulation of [Liu06] is not suitable to solve cyclic systems.
Since [Liu06] solves to a great extent the same problem as the QCAP that is solved in the second step of the solution approach, the formulation is adapted en presented in the next sections.

3.2 Problem Statement

The adapted, continuous formulation of [Liu06] and the proposed discrete formulation are presented in this chapter. Both QCAP's take the result of the BAP, as presented in Chapter 2, as their input. Hence, the position, the scheduled arrival time A_v and the desired departure time D_v have already been determined for each vessel. Let d_v be the processing completion time of the vessel in the QCAP; the processing completion time d_v can be earlier or later than the desired departure time D_v. As mentioned before, both QCAP's have the objective to minimize the maximum relative tardiness of a set of vessels. The relative tardiness of a vessel v is defined as:

$$\frac{\max(0, d_v - D_v)}{D_v - A_v}. \tag{3.1}$$

The min-max criterion is chosen rather than the total-tardiness criterion because the former ensures that the tardiness is spread evenly over the vessels. It avoids scenarios where a few vessels are delayed a lot, while others are leaving on time or earlier. Such sharing fits the application: It is better to delay all vessels a little than delaying a few vessels a lot [Liu06].

3.3 Continuous QCAP

The continuous formulation is an adaptation of the formulation of [Liu06]. The main difference is that [Liu06] adds more detail to the problem; a vessel is divided into bays to which a single quay crane can be assigned. Since in the second step of the solution approach it is only determined which quay cranes are allocated to a certain vessel v, this level of detail is eliminated from the formulation. In addition, the formulation of [Liu06] is discrete in the position of the vessels and the position of the quay cranes. Since quay cranes are not allocated to the bays of the vessels anymore, it is possible to present a formulation which is continuous in the position of the vessels and the position of the quay cranes. Those two adjustments probably have a positive effect on the computational effort that is required by the formulation since the problem becomes smaller in size, and the number of integer variables is decreased. Finally, constraints are added to be able to consider a cyclic system. For the completeness of this report the adapted formulation of [Liu06] is given in Appendix B.
The formulation requires two assumptions as a result of the fact that the formulation is continuous in time:

- Once a quay crane starts processing a vessel, it is allocated to that vessel until the processing of the vessel is completed. The processing start and end time are determined per vessel. As a result, each quay crane allocated to a certain vessel has the same processing time on that particular vessel.
- The capacity of each quay crane is the same. The processing times of a vessel are calculated for different numbers of allocated quay cranes in advance. These processing times are considered as parameters in the continuous formulation. Since it is not known in advance which quay cranes and which number of quay cranes are allocated to a vessel, it is necessary to assume that the processing rates of the quay cranes are identical.

3.4 Discrete QCAP

In this section, a discrete formulation for the QCAP is formally phrased. Experimental results show that the computational effort is large for realistic sized problems. Therefore, a heuristics procedure is proposed to decrease the required computational effort.

3.4.1 System Description

In the discrete formulation, unless stated differently, the following sets are considered: $i \in \{1, ..., M\}$, the set of quay cranes, $v \in \{1, ..., V\}$, the set of vessels, and $k \in \{1, ..., K\}$, the set of discrete time intervals in de considered time period.

By using discrete time intervals instead of continuous time, it is possible to allocate each quay crane i to vessel v during time interval $[k, k+1\rangle$. Hence, a quay crane can change vessels from one time interval to another. For example, a quay crane can start on vessel 1, move to vessel 2, and after a while move back to vessel 1. The processing rate λ_{iv} may differ per quay crane i and vessel v. Hence, this formulation enables to incorporate quay cranes with different quay crane rates. This suits the application: The quay cranes in container terminals do not have equal process rates due to usage of twin loads. In such a twin load the quay crane is able to pick up two twenty feet containers at once. In addition, the quay cranes at the boundaries of the terminal often operate at a smaller process rate. This is a result of the fact that they are more difficult to reach with the straddle carriers than the middle ones. The process rate of a quay crane also varies with the vessel length; a large vessel is processed at a larger rate than a small vessel. Each vessel v requires a certain amount of quay crane capacity to be processed, indicated by Q_v. This value is determined in the optimization of the first step. When, for example, 3.4 quay cranes are allocated to a certain vessel in the first step allocation, Q_v is simply the product of 3.4 and the average quay crane rate. Each vessel v can be processed simultaneously by a maximum number of quay cranes S_v.
The position of each vessel v along the quay has already been determined in the BAP. The left-most position is indicated by X_v^l and the right-most position is indicated by X_v^r. The origin for the left- and right-most positions of vessel v is the left-most boundary of the terminal quay. Quay cranes have to be positioned along the vessel which they are processing. The terminal quay has length L_t. Between two neighboring quay cranes there has to be a minimal gap G.
In the first step allocation, the desired arrival and departure time (A_v and D_v respectively) have already been determined. The processing of a vessel cannot start before A_v since the vessel is not present in the terminal yet. Since the maximum relative tardiness is minimized, it is preferred that the actual departure of vessel v is before or equal to D_v.
With respect to the cyclic property of the considered system, a remark has to be made: Both $A_v \geq D_v$ and $A_v < D_v$ are possible. Therefore, the auxiliary parameter E_v is introduced, which explicitly distinguishes between both cases:

$$E_v = \begin{cases} 1 & \text{if} A_v \geq D_v \\ 0 & \text{if} A_v < D_v. \end{cases} \qquad \forall v,$$

The sets and parameters discussed above are conveniently arranged in Table 3.1. Below, the variables of the problem are stated first. After that, the constraints and the objective function are derived. Then, it becomes clear why the auxiliary parameter E_v is needed.

3.4.2 MILP

Firstly, the variables of the problem are stated. Then, the constraints and the objective function are derived.

Table 3.1: Model parameters

Parameter	Definition
M	Number of quay cranes [-]
V	Number of vessels [-]
K	Number of discrete time intervals in the considered time cycle [-]
λ_{iv}	Processing rate of quay crane i on vessel v [containers/time interval]
Q_v	Required quay crane capacity for vessel v [containers]
S_v	Maximum number of quay cranes, which can process vessel v simultaneously [-]
X_v^l	Left-most position of vessel v [m]
X_v^r	Right-most position of vessel v [m]
L_t	Length of terminal t [m]
G	Minimal gap between to neighboring quay cranes [m]
A_v	Desired berth time of vessel v (start of processing of vessel v) [-]
D_v	Desired departure time of vessel v (end of processing of vessel v) [-]

Continuous variables

$l_i(k)$: Position of quay crane i during time interval $[k, k+1\rangle$ [m].
t : The maximum relative tardiness of all vessels [-].

Integer variable

a_v : The service start time of vessel v [-].
d_v : The service completion time of vessel v [-].
Δ_v^d : Number of time intervals that vessel v departs too late or too early [-].

Binary variables

$$x_{iv}(k) = \begin{cases} 1 & \text{if crane } i \text{ is allocated to vessel } v \text{ during time interval } [k, k+1\rangle, \\ 0 & \text{otherwise.} \end{cases}$$

$$b_v(k) = \begin{cases} 1 & \text{if vessel } v \text{ is berthing during time interval } [k, k+1\rangle, \\ 0 & \text{otherwise.} \end{cases}$$

$$e_v = \begin{cases} 1 & \text{if } a_v > d_v, \\ 0 & \text{if } a_v < d_v, \\ 1 & \text{if } a_v = d_v \text{ and vessel } v \text{ is continuously berthing,} \\ 0 & \text{if } a_v = d_v \text{ and vessel } v \text{ does not berth at all.} \end{cases}$$

$$e_v^a = \begin{cases} 1 & \text{if } a_v < A_v, \\ 0 & \text{if } a_v > A_v. \end{cases}$$

Constraints

Vessel v berths between its arrival and departure time (a_v and d_v respectively). Generic constraints [Hen08] are required to relate a_v and d_v to $b_v(k)$ as well as $b_v(k)$ to a_v and d_v for the cases where $a_v < d_v$, $a_v = d_v$ and $a_v > d_v$. The latter case follows from the cyclic nature of the problem. The auxiliary variable e_v ensures that the three different cases are captured in the constraints (3.2), (3.3), and (3.4):

$$\sum_{k=1}^{K} \big(b_v(k) - e_v\big) = d_v - a_v \qquad \forall v, \tag{3.2}$$

and

$$1 - a_v \leq k \cdot \big(b_v(k) - e_v\big) \leq d_v - 1 \qquad \forall k, v, \tag{3.3}$$

and

$$d_v - K \leq \big(K - k\big) \cdot \big(b_v(k) - e_v\big) \leq K - a_v \qquad \forall k, v. \tag{3.4}$$

Constraint (3.2) sets variable e_v to one, when a vessel v is arriving at the end of the cycle and leaves at the beginning of the next cycle. In that case, $d_v - a_v < 0$; the left term can only be negative when $e_v = 1$. Constraints (3.3) and (3.4) ensure that $b_v(k)$ is only set to 1 between a_v and d_v. The binary variable e_v ensures that $b_v(k)$ is set to 1 between a_v and K and between 0 and d_v for a vessel that arrives at the end of the cycle and leaves at the beginning of the next cycle.
Each quay crane i is allocated to maximally one vessel v during time interval $[k, k+1\rangle$:

$$\sum_{v=1}^{V} x_{iv}(k) \leq 1 \qquad \forall i, k. \tag{3.5}$$

The total number of quay cranes allocated to vessel v has to be less or equal to the maximal number of quay cranes, which can process vessel v simultaneously. In addition, quay cranes can only be allocated to vessel v when it is actual berthing during time interval $[k, k+1\rangle$:

$$\sum_{i=1}^{M} x_{iv}(k) \leq S_v \cdot b_v(k) \qquad \forall k, v. \tag{3.6}$$

The quay crane capacity allocated to vessel v has to be larger or equal to the required quay crane capacity of vessel v:

$$\sum_{k=1}^{K}\sum_{i=1}^{M}\lambda_{iv}x_{iv}(k) \geq Q_v \qquad \forall v. \tag{3.7}$$

When quay crane i is allocated to vessel v during time interval $[k, k+1\rangle$, it has to be positioned along vessel v. When quay crane i is idle, it has to be positioned somewhere along the terminal quay:

$$x_{iv}(k) \cdot \; X_v^l \leq l_i(k) \leq X_v^r \cdot x_{iv}(k) + \big(1 - x_{iv}(k)\big) \cdot L_t \qquad \forall i, k, v. \tag{3.8}$$

Quay crane i cannot cross quay crane $i+1$, and there has to be a gap G between quay crane i and quay crane $i+1$:

$$l_i(k) \leq l_{i+1}(k) - G \qquad \forall i < M, k. \tag{3.9}$$

The actual processing start time of vessel v has to be larger or equal to the desired arrival time. Due to the cyclic property of the considered system, the auxiliary variable e_v^a is required to indicate whether the actual arrival a_v takes place in the same cycle as the desired arrival time A_v:

$$a_v \geq A_v - K \cdot e_v^a \qquad \forall v. \tag{3.10}$$

With respect to departing too early or too late there are four possible permutations of d_v and D_v. Each vessel can depart earlier or later than D_v. However, since a cyclic system is considered the actual departure time d_v can shift between cycles. A vessel of which the desired departure time D_v is at the end of a cycle can also leave at the beginning of the next cycle or a vessel of which the desired departure time D_v is at the beginning of a cycle can also leave at the end of the previous cycle. With the help of the introduced auxiliary variables e_v and e_v^a and the auxiliary parameter E_v, it is possible to construct appropriate constraints for Δ_v^d to satisfy each of those four cases:

$$\Delta_v^d \geq -\big((D_v - d_v) - K \cdot e_v^a + K \cdot E_v - K \cdot e_v\big) \qquad \forall v. \tag{3.11}$$

The maximum relative tardiness of the complete vessel set is larger or equal to the relative tardiness of the individual vessels. Again, the auxiliary parameter E_v is required to indicate whether the desired berthing times of a vessel are cyclic:

$$t \geq \frac{\Delta_v^d}{D_v - A_v + K \cdot E_v} \qquad \forall v. \tag{3.12}$$

Objective

The objective of the problem is to minimize the maximum relative tardiness of the complete vessel set:

$$\min t. \tag{3.13}$$

3.4.3 Heuristics Procedure

Since the discrete QCAP formulation requires a lot of computational effort, a heuristics procedure is proposed to reduce the number of variables. In the heuristics procedure, the QCAP is not solved at once. The complete QCAP is cut into smaller subproblems, which are linked with each other. Instead of solving the QCAP for the complete considered time period, multiple subproblems are solved sequentially. Each subproblem represents only a part of the considered time period, reducing the number of vessels and time intervals that are considered at once. The discrete QCAP formulation of Subsection 3.4.1 is used to solve the subproblems. Below, it is explained which adjustments have to be made to that formulation to enable the heuristics procedure.

In Figure 3.1, an illustration of the heuristics procedure is given; subproblems I and II are partly depicted. This figure shows that the subproblems are linked, since vessels can be present in more than one subproblem, and it is possible that a vessels' departure time exceeds the subproblems' end time. A vessel that is present in multiple subproblems is divided into parts. Vessel 2 consists of parts $2A$ and $2B$, for instance. In Algorithm 1, the heuristics procedure is formulated globally. With the help of the example in Figure 3.1 and Algorithm 1, the heuristics procedure is explained in more detail.
Although vessel 1 is berthing in the first subproblem, it is possible that its processing is finished later than its desired departure time (i.e. $D_1 \geq 7$). Then, tardiness is introduced. In that case there are two possibilities: i) The actual departure time is before or at the end of the considered time period of the first subproblem (i.e. $d_1 \leq 8$), or ii) The actual departure time is after the end of the considered time period of the first subproblem (i.e. $d_1 > 8$). These two possibilities are clearly indicated in Algorithm 1. For the first possibility nothing different from the complete QCAP occurs in the heuristics step; vessel 1 simply introduces tardiness.
To enable the second possibility, it is necessary to solve the first subproblem for more time intervals than the subproblem actually consists of. This means that each subproblem is solved for twice the number of time intervals that it consists of to make the late departure possible. When vessel 1 departs later than the end of the first subproblem (i.e. $d_1 > 8$), this has to be taken into account in the second subproblem. The new, delayed departure time and the number of containers that are not processed before discrete time step 8 of vessel 1 are transferred to the second subproblem. Hence, when part $2B$ is processed in the second subproblem it is taken into consideration that vessel 1 still requires quay crane capacity to be finished. It is assumed that it is always possible to finish such a delayed vessel in the next subproblem.
Vessel 2 and 3 are berthing in both subproblems. Hence, they are split into multiple parts. Each part has its own desired arrival and departure time. The desired arrival time for vessel part $2A$ is equal to the desired arrival time of vessel 2 (i.e. $A_{2A} = 5$), but the desired departure time of part $2A$ is equal to the end of the considered time period of the first subproblem (i.e. $D_{2A} = 8$). For part $2B$ it is exactly the other way around,

the desired arrival time is equal to the end of the first subproblem (i.e. $A_{2B} = 8$) and the desired departure of part $2B$ is equal to the desired departure time of vessel 2 (i.e. $D_{2B} = 11$). The desired arrival and departure times of vessel 3 are determined in the same way.

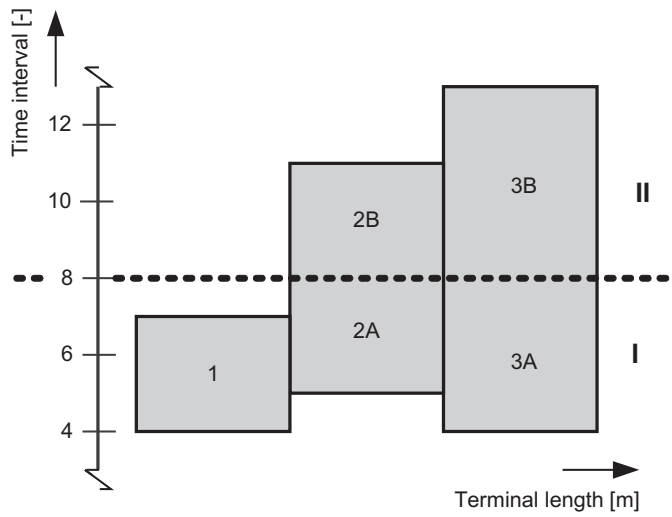


Figure 3.1: Illustration of the heuristics procedure

The first time that a vessel is present in a subproblem (i.e. part A) the total required quay crane capacity Q_v is allocated to it. Hence, in the first subproblem Q_2 is assigned to part $2A$, and Q_3 is assigned to part $3A$. This is possible since there is solved for more time intervals than the number of time intervals of which the subproblems consist.
In the case of part $2A$, there are two possibilities for its departure: i) Part $2A$ is completed before or at the end of the considered time period of the first subproblem (i.e. $d_{2A} \leq 8$), or ii) Part $2A$ is completed after the end of the considered time period of the first subproblem (i.e. $d_{2A} > 8$). The first possibility yields that vessel 2 is processed completely in the first subproblem. In that case, vessel 2 is not present anymore in the second subproblem. This is also the reason for assigning the total required quay crane capacity Q_v to the vessel part in the first subproblem where the vessel appears in. By minimizing the tardiness, it is enforced that in the first subproblem as much as possible of the required capacity is processed. In this approach it is necessary to minimize the tardiness of each individual vessel, and weigh the importance of minimizing the tardiness of the vessels with respect to each other.
For the second possibility the capacity that remains after the end time of the first subproblem is transferred to the second subproblem. Hence, part $2B$ appears then in the second subproblem. The two possibilities are equal to the two possibilities for vessel 1 and are again clearly indicated in Algorithm 1: Whether a vessel is processed completely in subproblem s or a vessel is present in the next subproblem since its processing is not

completed. Vessel 3 is treated the same way as vessel 2.
To make the interdependency between the subproblems stronger, information from the first step allocation is used. In the first step, the required quay crane capacity is determined per time interval, as mentioned in Chapter 1. For all vessel parts, it is attempted to process at least the number of containers that is allocated to the vessel during the corresponding time intervals in the first step allocation. This required capacity for a subproblem is denoted by Q_v^*. The actual amount of containers that is processed in a subproblem is denoted by q_v. By minimizing the positive difference between Q_v^* and q_v, it is ensured that as much as possible of Q_v^* is processed in each subproblem. Hence, the following constraint is added:

$$q_v - \sum_{k=1}^{K^*}\sum_{i=1}^{M}\lambda_{iv}x_{iv}(k) \geq Q_v^* \qquad \forall v, \tag{3.14}$$

where K^* is the number of time intervals in a subproblem. The variable q_v is added to the objective function, since it has to be minimized.
Since the first step allocations guarantee that in each time interval sufficient quay crane capacity is present, the likelihood on discrepancies between different vessel parts is minimized. For example, when in part $2A$ at least the containers that are allocated to vessel 2 in the first step optimization between discrete time steps 5 and 8 are processed, it is most likely that it is possible to process the remainder of the required capacity in part $2B$ before the desired departure time.
This is not a hard constraint, since the size of the time intervals also has an important role. For example, when for part $3A$ 4.2 quay cranes are necessary to process Q_v^* in the first subproblem, but only 4 quay cranes are simultaneously available to do this job, there are two options. First, the required capacity of 0.2 is transferred to part $3B$. And second, the time intervals can be chosen smaller which makes it possible for another quay crane to switch from another vessel. The first option is kept possible, since the value for Q_v^* can also be very objectionable when using the second possibility. For example, when $Q_{3A}^* = 4.01$, the original time intervals should be divided by at least a factor 100 to be able to meet Q_v^*.
In case of a vessel arriving at the end of the considered cycle and leaving at the beginning of the next cycle, part A is present in the last subproblem. When not all containers are processed in part A in the last subproblem, all subproblems are solved again. The number of containers that have not been processed in part A are assigned to a part B of the vessel in the first subproblem. For part A of such a vessel also Constraint (3.14) holds. The same situation occurs when a vessel in the last subproblem is not processed completely before the end of the last subproblem. Then all subproblems are solved again, and in the first subproblem a vessel part is created for the vessel that cannot be processed completely in the last subproblem. In Algorithm 1, this is indicated by the variable *loop*. When in the last subproblem (i.e. $s = NS$) a vessel is not processed completely, $loop = 0$, which results in solving all subproblems again.

A few adjustments are made to the discrete QCAP formulation of Subsection 3.4.1

Algorithm 1 Heuristics procedure for discrete QCAP

```
Require: loop = 0
  while loop = 0 do
    loop = 1
    for s = 1 : NS do
      if s ≠ NS then
        Solve the subproblem
        for v = 1 : V do
          if Σ_{i=1}^{m} Σ_{k=1}^{K*} x_iv λ_iv ≥ Q_v then
            Processing of vessel v is completed
          else
            Vessel v has to be added partly to the next subproblem
          end if
        end for
      else if s = NS then
        Solve the subproblem
        for v = 1 : V do
          if Σ_{i=1}^{m} Σ_{k=1}^{K*} x_iv λ_iv ≥ Q_v then
            Processing of vessel v is completed
          else
            Vessel v has to be added partly to the next subproblem
            loop = 0
          end if
        end for
      end if
    end for
  end while
```

where

NS	:	Number of subproblems
$s \in \{1, ..., NS\}$	:	Set of subproblems
K^*	:	Number of time intervals in a subproblem
$loop$	:	Indicator whether all subproblems have to be solved again

to make it suitable to solve the subproblems. The parameter E_v and the variables e_v and e_v^a are not necessary anymore, since within the same subproblem none of the vessels arrive at the end of the subproblem and leave at the beginning of it, assuming that the end and beginning of the cycle coincides with a subproblem boundary. By eliminating the parameter and the variables which are introduced due to the cyclic nature of the problem, a few constraints are adjusted. Parameter K denotes the number of considered time intervals in one subproblem in the heuristics procedure, and K^* is the number of

time intervals before the subproblem end time. It should be noted that $K > K^*$. In addition, each vessel part is considered as a vessel with its own starting and end time, as described above.
Constraints (3.2 – 3.4) can be reformulated, since the variable e_v is eliminated:

$$\sum_{k=1}^{K} b_v(k) = d_v - a_v \qquad \forall v, \tag{3.15}$$

and

$$k \cdot b_v(k) \leq d_v - 1 \qquad \forall k, v, \tag{3.16}$$

and

$$(K - k) \cdot b_v(k) \leq K - a_v \qquad \forall k, v. \tag{3.17}$$

Constraint (3.10) can be reformulated, since variable e_v^a is eliminated:

$$a_v \geq A_v \qquad \forall v. \tag{3.18}$$

Constraints (3.11) and (3.12) are also reformulated due to the elimination of the parameter and the variables:

$$\Delta_v^d \geq (D_v - d_v) \qquad \forall v, \tag{3.19}$$

and

$$t_v \geq \frac{\Delta_v^d}{D_v - A_v} \qquad \forall v. \tag{3.20}$$

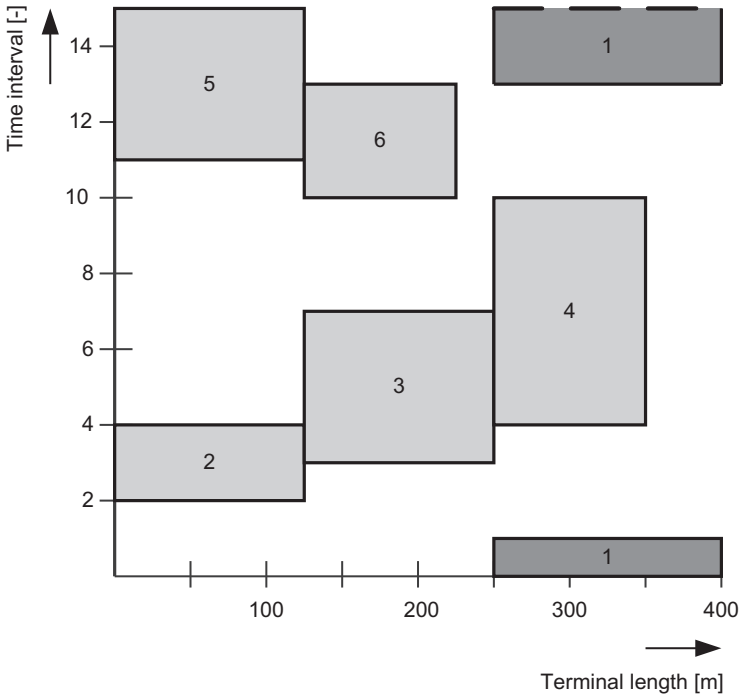
The tardiness in (3.20) is now calculated per vessel. As mentioned before, the importance of minimizing the tardiness of vessels is weighted with respect to each other. The objective of each subproblem s is formulated as:

$$\min \sum_{v=1}^{V} C_v^t \cdot t_v + \sum_{v=1}^{V} C_q^a \cdot q_v, \tag{3.21}$$

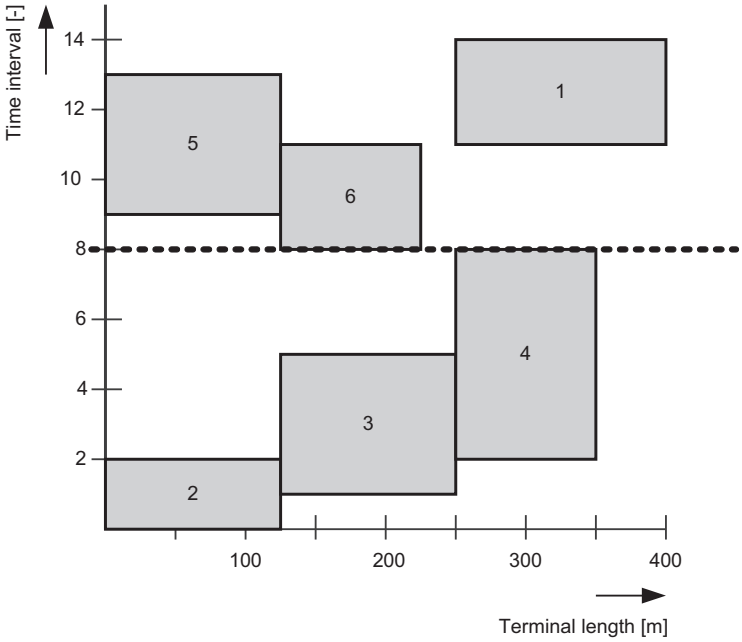
where C_v^t represents the cost for tardiness introduced by vessel part v, and C_q^a represents the cost for not meeting Q_v^* by vessel part v.
This change in objective is introduced to be able to give priority to certain vessel parts. For example, consider vessel 1 and vessel part $2A$ in the first optimization problem in Figure 3.1. When vessel 1 departs later than its desired departure time, this introduces tardiness in the first subproblem and in the complete problem. When vessel part $2A$ departs later than its desired departure time, this only introduces tardiness in the first subproblem. However, the containers that could not be processed in the first subproblem are processed in the second subproblem. Then, it is still possible that vessel 2 meets its overall desired departure time. Hence, it is preferred that vessel 1 meets its desired

departure time over vessel part $2A$ in the first subproblem.
This is enforced by introducing more costs for vessel 1 than for vessel part $2A$ (i.e. $C_1^t > C_{2A}^t$, and $C_1^q > C_{2A}^q$). Vessel parts $2A$ and $3A$ are also weighted with respect to each other. The relation between those two weighting factors is dependent on how long they are berthing in the next subproblem(s) and how large the total required capacity Q_v is. However, this does not guarantee that vessel parts are weighted correctly in all possible combinations of vessels. The situation exists that too few containers of a part A are processed in a subproblem, resulting in a late departure of that particular vessel in the next subproblem.

For the above mentioned reasons, the subproblems have to be defined carefully. Two issues have to be regarded with respect to defining the subproblems: It is beneficial i) to eliminate the cyclic nature of the problem when possible, and ii) to avoid that a vessel is present in more than one subproblem. Eliminating the cyclic nature decreases the possibility that all subproblems have to be solved twice. And avoiding that a vessel appears in two subproblems prevents difficulties in weighting the importance of two vessel parts with respect to each other.
How to choose time intervals for the subproblems is explained in more detail with the help of the example in Figure 3.2. In Figure 3.2a, the typical output of a BAP is depicted. This result could be used as input for QCAP. However, in the heuristics procedure the output of the BAP is processed a little bit.
First, it is tried to eliminate the cyclic nature of the problem. By repositioning the vessels in time, it is possible to produce an input for the QCAP without vessels that are arriving at the end of the cycle and leaving at the beginning of the next cycle. In the example, the arrival and departure times are all decreased with two time intervals, resulting in the berth position allocation in Figure 3.2b. This allocation is completely the same as the original one; only the starting point is changed. For example, when discrete time step 0 in Figure 3.2a represents Monday morning, the same time interval represents Tuesday morning in Figure 3.2b. Repositioning the vessels in time is only possible when there is at least one point in time where no vessels are processed. Hence, in Figure 3.2a examples of such points in time are discrete time steps 1, 2 and 10.
After eliminating the cyclic nature of the problem, an appropriate choice has to be made about the start and end times of the subproblems. As mentioned before, it is beneficial to avoid that a vessel is present in more than one subproblem. Therefore, the first subproblem is ended at discrete time step 8 in Figure 3.2b. By choosing that point in time, none of the vessels is present in more than one subproblem. Again, the possibility to avoid the presence of a vessel in multiple subproblems is dependent on whether a point in time can be found where none of the vessels is processed.
To make sure that the size of the subproblems is not becoming too small or too large, a minimal number of vessels V_{min}, and a maximum number of vessels V_{max} in a subproblem is defined. When V_{max} is exceeded between two points in time where no vessels are processed, a standard subproblem size TI is chosen. Hence, when there is no point in time where no vessel is processed, all subproblems have the standard size TI. When



(a) Original output of the BAP



(b) Adjusting the result of the BAP

Figure 3.2: Example of cutting the QCAP

the number of vessels that are berthing in the considered time period is less than V_{min} the original, cyclic discrete QCAP is solved.

3.5 Comparison of the Discrete and Continuous QCAP

In Section 3.3, two required assumptions for the continuous QCAP are described, which are not necessary in the discrete QCAP. In Subsection 3.5.1 a situation is sketched in which it is a disadvantage that in the continuous formulation a quay crane remains allocated to a vessel until the complete vessel is processed. In Subsection 3.5.2, it is investigated whether this example situation has a large influence on the performance of the continuous formulation in comparison with the discrete formulation and its heuristics procedure.

3.5.1 Example Situation

In this section, a terminal where $K = 7$ is considered. Three identical quay cranes with process rate $\lambda_{iv} = 25\frac{\text{containers}}{\text{time interval}}$ are present in the terminal. Two vessels are berthing in the considered time period. The desired arrival and departure times of the vessels (A_v and D_v, respectively) have already been determined in the first step allocation and the position of the vessels within the terminal has already been determined in the BAP (i.e. X_v^l and X_v^r are known). Both vessels require a certain quay crane capacity Q_v and can be processed by a maximum number of quay cranes S_v at the same time. These parameters are given in Table 3.2.

Table 3.2: Parameters for vessels 1 and 2

Parameter	vessel 1	vessel 2
A_v [-]	1	2
D_v [-]	5	4
l_v [m]	200	125
X_v^l [m]	25	225
X_v^r [m]	225	350
S_v [-]	3	2
Q_v [containers]	250	50

In Figure 3.3, the results of both formulations for this example are depicted. As the results in Figure 3.3a show, vessel 1 departs later than its desired departure time for the continuous formulation. This results in a maximum relative tardiness of 0.25. Vessel 1 departs too late since only two quay cranes are allocated to it while three quay cranes are necessary. When three quay cranes would be allocated to vessel 1, vessel 2 would be departing later than its desired departure time. In that case, vessel 1 would be processed completely at 5.33, resulting in a departure time of 6.33 for vessel 2, and a

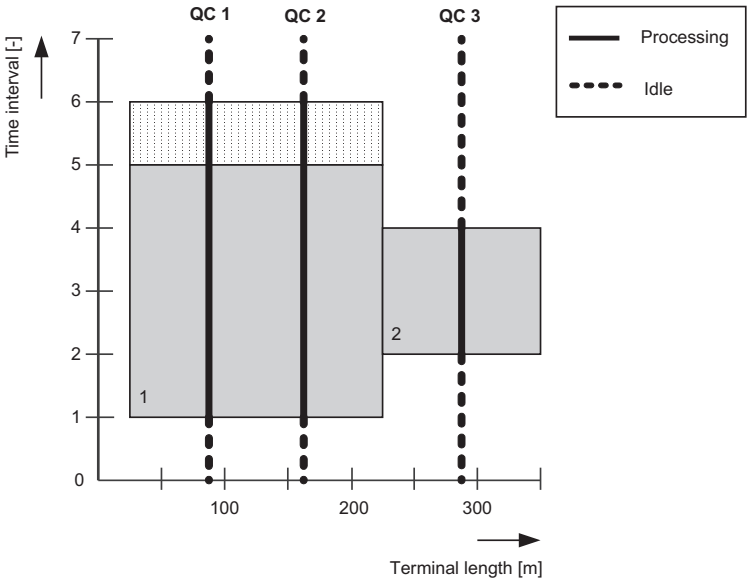
maximum relative tardiness of 0.66. This is a consequence of the fact that in that case the processing of vessel 2 can only start after the completion of vessel 1.
For the discrete formulation both vessels depart at their desired departure time, as shown in Figure 3.3b. This is possible, since a quay crane can be allocated to a certain vessel in one time interval and to another vessel in the next time interval, and then back to the first vessel again. Where in the continuous case the capacity of quay crane 3 cannot be used during time interval 1 and time interval 4, capacity is optimally used during the berthing times of the vessels in the discrete case. Hence, this example illustrates one advantage of the discrete formulation. It has to be remarked that this advantage only comes into play when cranes are used against the total capacity.
Although in the discrete case both vessels are departing in time, the depicted solution is not the most preferred one, since both quay crane 2 and quay crane 3 are moved to process vessel 2. It makes sense to move only quay crane 3. In Section 3.6 the objective of the discrete QCAP formulation is extended in such a way that this allocation can be enforced.

3.5.2 Experiments

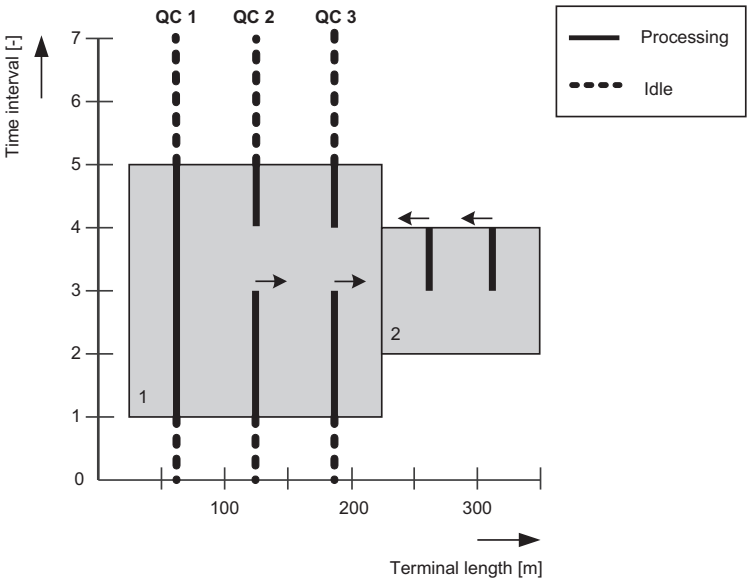
In this section, the two QCAP formulations are compared for many different vessel sets. Two performance indicators are used. The first performance indicator is the percentage of allocations without tardiness. This is an important performance indicator, because it is profitable for the container port operator that vessels depart before or at their desired departure time. The second performance indicator is the computational time which is required to solve the QCAP. As one can image both are most probably a function of the quay crane utilization.
Each vessel set has its own quay crane utilization. For all vessel sets in the experiments the same number of quay cranes M is available. Those quay cranes are all available for the entire cycle, and all have the same process rate λ_{iv} since the continuous QCAP is not suitable for different quay crane rates. Each vessel requires a certain amount of quay crane capacity Q_v. Hence, the quay crane utilization in a terminal for a certain time cycle is given by:

$$u_{QC} = \frac{\sum_{v=1}^{V} Q_v}{N_t \cdot T_{end} \cdot \overline{\lambda}_{iv}}. \tag{3.22}$$

It is expected that for a large quay crane utilization the continuous QCAP results more often in allocations with tardiness than the discrete QCAP. This tardiness is introduced by the situation as described in Subsection 3.5.1. However, it has to be remarked that the performance of the discrete QCAP is highly dependent on the size of the time intervals. For example, when a week is divided in time intervals of 8 hours the probability on solutions with tardiness is much larger than when is chosen for time intervals of 4 hours.



(a) Solution for the continuous formulation



(b) Solution for the discrete formulation

Figure 3.3: Results for the example situation

In the case of smaller time intervals more flexibility is introduced to deal with the example situation as described in Subsection 3.5.1. In addition, when the time intervals are too large it is more difficult to deal with the fact that in the optimization in the first step of the solution approach a continuous number of quay cranes is allocated to a vessel. This problem is also described in Subsection 3.4.3. For example, 4.2 quay cranes

are allocated to a vessel in the first step allocation, and there are only 4 of 5 quay cranes continuously available. The other required capacity has to be compensated by another quay crane, which switches from another vessel. When the time intervals are too large, there might be not enough freedom to make this switch possible. When for example, the time intervals make an accuracy of 0.5 possible, at least 4.5 quay cranes have to be allocated to the vessel. When another vessel requires 0.6 quay cranes, this is not possible. This subject is most probably mainly an issue when the quay crane utilization is large.
Since in the discrete QCAP more (integer) variables are required than in the continuous QCAP, it is expected that much more computational effort is required to solve the discrete QCAP. However, experiments suggest that the heuristics procedure as described in Subsection 3.4.3 requires much less computational effort. In addition, it is not expected that the heuristics procedure does not generate significantly fewer solutions without tardiness as the complete discrete QCAP.
Hypothesis: For large quay crane utilizations the continuous QCAP results more often in allocations with tardiness than the discrete QCAP. However, the discrete QCAP requires more computational effort than the continuous QCAP. The proposed heuristics procedure for the discrete QCAP decreases the computational effort significantly.

Experiments setup

In the experiments, a cycle of 15 time intervals of 8 hours are considered. In the discrete QCAP formulation this means that $K = 15$. In the continuous formulation the time is continuous between 0 and 15, since the last time interval of the discrete formulation ends at discrete time step 15, as depicted for the example in Figure 3.2. The following parameters are set: $L_t = 1000$ m, $M = 6$, and $\lambda_{iv} = 27 \frac{\text{containers}}{\text{time interval}}$.
Vessel sets are generated in the same way as the vessel sets for the experiments in Subsection 2.4.2. Hence, vessel sets of which each vessel has its own arrival and departure time and length are generated to represent a first step allocation. Each vessel v is allocated a certain required capacity Q_v. Initially, a number of containers is allocated per time interval, since in the optimization problem in the first step it is guaranteed that in each time interval sufficient quay crane capacity is available to process the vessels that are berthing during that time interval. The number of containers allocated to a vessel in a certain time interval is determined in the same way as the length and the process time of a vessel in Subsection 2.4.2. From representative data from terminal operator PSA HNN a histogram is constructed in which is depicted which percentage of the maximum available capacity for that vessel is used. In Appendix A, it is described how the histogram is used to generate a number of containers for each vessel in each time interval. The sum of the assigned containers in the time intervals is the total required capacity Q_v for a vessel.
For these sets the BAP, as described in Section 2.3 is solved. Each vessel in each vessel set is now fixed in time and space. These results are used as input for the different formulations of the QCAP: the continuous QCAP, the discrete QCAP, and the heuristics

procedure for the discrete QCAP. For the discrete formulations, the size of the time intervals is varied. Experiments are conducted in which the time intervals are not divided into smaller time intervals, and experiments in which the time intervals are divided into two smaller time intervals (i.e. $K = 30$).
As described in Subsection 3.4.3, the size of the subproblems of the heuristics procedure depends on the minimum and maximum number of vessels in a subproblem. In the experiments, the minimum number of vessels $V_{min} = 3$, and the maximum number of vessels $V_{max} = 6$. The weight factors C_v^t and C_q^a are chosen to be equal. They are set to 1 in a subproblem which includes the desired departure time D_v of a vessel v. In the other cases, they are given by:

$$C = \frac{1}{K_v^n} \cdot Q_v, \tag{3.23}$$

where K_v^n is the number of time intervals that the vessel is berthing in the next subproblem, and Q_v is scaled to $\max(Q_v)$. Hence, the weight factors for vessels that are not departing in a subproblem are always less than 1.
The mixed integer optimization is stopped as soon as it has found an integer solution proven to be within 5% of optimal. For each vessel set, it is monitored if the solution includes tardiness. In addition, the CPU time, which is required to find a solution is monitored.

Results

The results of the experiments are depicted in Figure 3.4. Figure 3.4a shows the percentage of allocations without tardiness dependent on the quay crane utilization for each QCAP formulation. In Figure 3.4b, the mean CPU time and 95% confidence interval dependent on the quay crane utilization are depicted for each QCAP formulation. For each QCAP formulation, the markers represent a collection of vessel sets of which the quay crane utilization is within a certain range. For example, the first marker of each QCAP formulation includes all vessel sets with a quay crane utilization between 0 and 0.05, and the second marker includes all vessel sets with a quay crane utilization between 0.05 and 0.10, etc. The vertical dashed lines represent the range of the typical quay crane utilizations at container ports; the typical quay crane utilizations vary between 0.45 and 0.7.
The results in Figure 3.4a are as expected: Especially for larger quay crane utilizations, the continuous formulation constructs more allocations with tardiness than the discrete formulation. The discrete formulation constructs in at most 0.3% of the experiments an allocation with tardiness, while the continuous formulation constructs allocations with tardiness for up to 8% of the experiments with the same quay crane utilization. This difference is the result of the occurrence of a situation like the one described in Subsection 3.5.1. In most cases, the discrete formulation is able to deal with this situation, where the continuous formulation is not. The performance of the continuous formulation is already decreasing at a quay crane utilization of 0.25. This is caused by

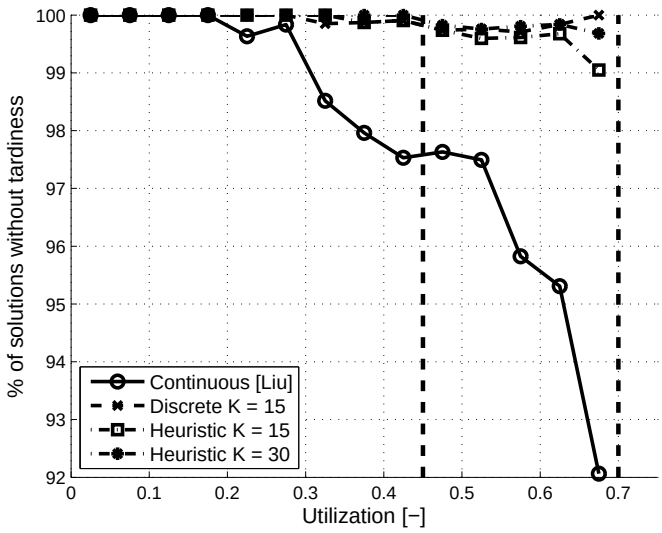
the fact that the described situation can also occur at smaller quay crane utilizations. In those cases most of the vessels from the vessel set are berthing in the same time intervals, which causes a large quay crane utilization during a restricted number of time intervals.

The tardiness in allocations of the discrete formulation is caused by the time interval size being too large. When the discrete formulation is solved for $K = 15$, sometimes not enough flexibility is introduced to deal with the described situation or with the fact that in the first step of the solution approach a real number of quay cranes is allocated to vessels, as described before in this section. Decreasing the size of the time intervals is a possible solution to prevent tardiness. However, a large number of experiments for the discrete formulation with $K = 30$ did not solve within a reasonable amount of time. Solution times of more than 4 hours have been observed. This might be due to the large number of integer variables. For example, the number of binary variables $b_v(k)$ and $x_{iv}(k)$ gets twice as large when the time intervals are divided into two time intervals. Therefore, the results for the discrete formulation with $K = 30$ are not depicted in Figure 3.4.

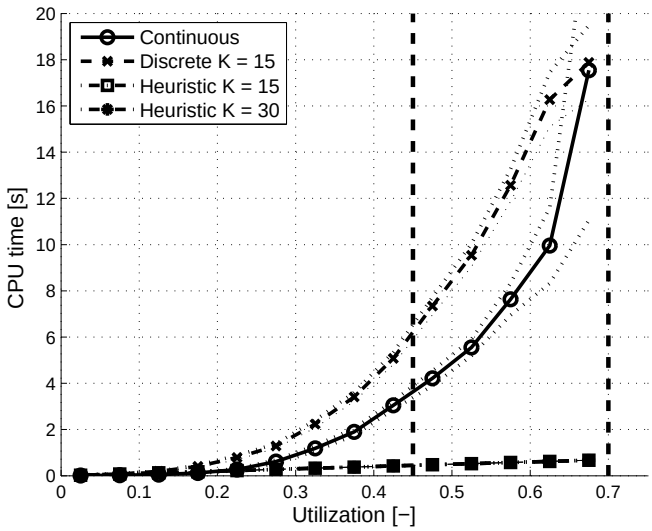
An import remark is that the continuous formulation results on average in a smaller tardiness than the discrete formulation. This is caused by the fixed time interval size of the discrete formulation. When a vessel departs later than its desired departure time the delay is at least one time interval, which in this case is 8 hours. In these experiments, for all allocations of the discrete formulation with tardiness, the departure of a vessel is at most delayed with one time interval, resulting in a average tardiness of 0.37 In the continuous case, the average tardiness over all allocations with tardiness is 0.1. In these experiments, this means that a delayed vessel departs an average 2.16 hours after its desired departure time.

The heuristics procedure for the discrete formulation is solved for $K = 15$ and $K = 30$. As expected, the required CPU time is significantly less than for the discrete formulation itself, as shown in Figure 3.4b. The percentage of allocations without tardiness for the heuristics procedure is not more than 1% less than for the discrete formulation. The difference is caused by vessels that are present in multiple subproblems. When multiple vessels are present in more than one subproblem, the importance of the vessel parts have to be weighted with respect to each other. In some cases, the preference to process one vessel over another vessel is not strong enough, resulting in tardiness for that particular vessel in the next subproblem. In that case, too few containers are processed in the first subproblem. As shown in Figure 3.4a, this situation also occurs for $K = 30$. In those cases choosing even smaller time intervals (e.g. $K = 45$) results in allocations without tardiness, since again more flexibility is added. However, using $K = 45$ still results in allocations with tardiness for other vessel sets.

For the heuristics procedure also holds the remark that the tardiness of the heuristics procedure for the discrete formulation is on average larger than the tardiness of the continuous formulation. However, since it is possible to construct allocations for $K = 30$ and $K = 45$, the tardiness becomes smaller. In these experiments, for all allocations of the heuristics procedure with tardiness the departure of a vessel is at most delayed by one time interval. For $K = 30$, this results in an average tardiness of 0.19. Hence, the



(a) Percentage of solutions without tardiness



(b) Required CPU time

Figure 3.4: Results for the comparison

possibility of using smaller time intervals reduces the average tardiness of the discrete formulation.

Another important remark is that especially in the case of the discrete formulation and the heuristics procedure for the discrete formulation less preferred allocations are constructed. Namely, in time intervals where the quay crane utilization is small, the processing of vessels is interrupted, unnecessary quay crane switches are performed and

idle quay cranes are isolated between two processing cranes. The possibility of allocating a quay crane to another vessel in each time interval causes these situations. In Section 3.6, examples of those situations are given, and it is shown how the formulation is changed to prevent the construction of such allocations.
These experiments suggest that the discrete formulation outperforms the continuous formulation with respect to constructing allocations without tardiness. However, when tardiness is introduced in an allocation of the discrete formulation, this tardiness is on average larger than in the continuous formulation. It is shown that the heuristics procedure does not construct significantly fewer allocations without tardiness than the complete discrete formulation. In addition, since in the heuristics procedure smaller time intervals can be used, the average tardiness is smaller for the heuristics procedure. The heuristics procedure requires significantly less computational effort than the complete discrete formulation and the continuous formulation. However, decreasing the time interval size in the heuristics procedure does not always prevent the construction of allocations with tardiness.

3.6 Extensions to the Discrete QCAP

As mentioned in Section 3.5, for the discrete QCAP less preferred allocations are constructed. In this section, it is first explained which particular allocations are excluded. Both the discrete QCAP and the heuristics procedure for the discrete QCAP are adjusted in such a way that the described situations are (partly) excluded. In addition, two other features are added to the discrete formulation. In the first place, the possibility of vessels departing after their desired departure time is excluded, since in the previous section it is shown that in most cases it is possible to generate allocations without tardiness with the heuristics procedure for the discrete QCAP. It becomes also possible to minimize the processing time of the vessels. Secondly, it becomes possible to minimize the number of required quay cranes. Experiments are conducted to investigate the performance of the adjusted discrete QCAP and its heuristics procedure.

3.6.1 Example Quay Crane Allocations

In this subsection a terminal is considered, where $K = 5$. Four identical quay cranes with process rate $\lambda_{iv} = 25 \frac{\text{containers}}{\text{time interval}}$ are present in the terminal. Two vessels are berthing in the considered time period. The desired arrival and departure times of the vessels have already been determined in the first step optimization and the position of the vessels within the terminal has already been determined in the BAP. Both vessels require a certain quay crane capacity Q_v and can be processed by a maximum number of quay cranes S_v. These parameters are given in Table 3.3.
In Figure 3.5a, a possible allocation of the discrete QCAP, as described in Section 3.4, is depicted. Although the allocation is without any delay, some less preferred allocations

are noticed and indicated by the rectangles. Quay crane 1 is processing vessel 1 with an interruption. During this interruption, capacity cannot be used anywhere else along the quay without moving all other quay cranes. The same holds for quay crane 2 during the 4th time interval; quay crane 2 is idle, but the capacity of it cannot be used anywhere else without moving quay cranes 3 and 4. Both quay crane 3 and quay crane 4 are switching between vessel 1 and vessel 2. It is not really optimal to move two quay cranes to another vessel, while interrupting the processing of another vessel. Movement of cranes is not preferred since during movement also capacity is wasted.

Table 3.3: Parameters for vessels 1 and 2

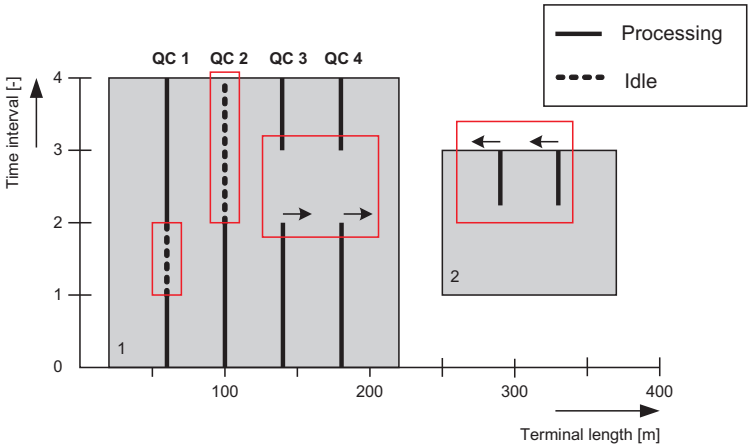
Parameter	vessel 1	vessel 2
A_v [-]	0	1
D_v [-]	4	3
l_v [m]	200	120
X_v^l [m]	20	250
X_v^r [m]	220	370
S_v [-]	4	2
Q_v [containers]	275	50

In Figure 3.5b, a preferred allocation is depicted. Quay cranes 1 – 3 process vessel 1 continuously and vessel 2 is only processed by quay crane 4. The position of the quay cranes when they are idle is also more preferred. They are positioned alongside or near empty quay lengths, where it is possible to use them for unexpected events or for processing small barges.

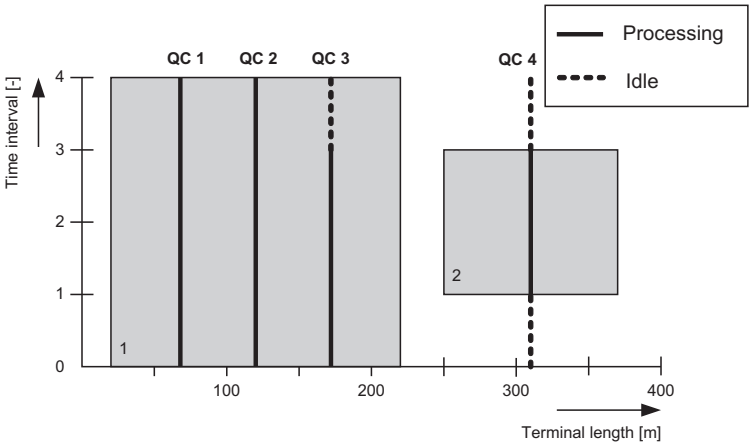
3.6.2 Adjustment of the Discrete QCAP

Since it is enforced that vessels depart before or at their desired departure time, an upper bound of 0 is set to variables Δ_v^d. Then, it is only possible to leave at or before the desired departure time (i.e. $d_v \leq D_v$). This automatically means that Constraint (3.12) is not needed anymore, because tardiness is not possible anymore. The variables Δ_v^d are added to the objective function, which enforces that the processing time of the vessels is minimized.
To be able to minimize the maximum number of quay cranes that are used simultaneously in the terminal, an additional auxiliary integer variable is introduced:

n_t : The maximum number of quay cranes that is used simultaneously in terminal t [-].



(a) Possible current quay crane allocation



(b) Preferred quay crane allocation

Figure 3.5: Quay crane allocation

The variable n_t is actually a soft upper bound on the required number of quay cranes in the terminal. This variable is added to the objective function:

$$\sum_{i=1}^{M}\sum_{v=1}^{V} x_{iv}(k) \leq n_t \qquad \forall k. \tag{3.24}$$

Up till now, it is possible to minimize the processing time of the vessels, and to minimize the required number of quay cranes in the terminal. By adding more additional terms to the objective, it is possible to exclude less preferred allocations of the quay cranes.

The objective function of the discrete QCAP formulation is then given by:

$$\begin{aligned}\min \quad & \sum_{v=1}^{V} C_v^a \cdot \Delta_v^d + C_q^b \cdot n_t + \sum_{k=1}^{K}\sum_{i=1}^{M} C_v^b \cdot |l_i(k+1) - l_i(k)| \\ & + \sum_{k=1}^{K}\sum_{i=1}^{M}\sum_{v=1}^{V} C_v^c \cdot |x_{iv}(k+1) - x_{iv}(k)| \\ & + \sum_{k=1}^{K}\sum_{i=1}^{M-1} C_v^d \cdot |\sum_{v=1}^{V}(x_{(i+1)v}(k) - x_{iv}(k))|.\end{aligned} \tag{3.25}$$

The first term maximizes the early departure of all the vessels in the vessel set. The parameter C_v^a represents the benefits per time interval that a vessel is departing before its desired departure time [-].
The second term minimizes the number of quay cranes that are used simultaneously. The parameter C_q^b represents the cost introduced per quay crane [-].
The third term minimizes the movement of all the quay cranes in the terminal over all time intervals. The difference between the position of a quay crane between two consecutive time intervals is minimized. By minimizing the movements of the quay cranes in Figure 3.5, it is enforced that not both quay crane 3 and 4 are moving to vessel 2, but that vessel 2 is only processed by quay crane 4. The parameter C_v^b represents the cost introduced by the movement of the quay cranes per meter [$\frac{1}{m}$]. It has to be mentioned that if $k = K$, then the difference $|l_i(0) - l_i(K)|$ is evaluated due to the cyclic nature of the problem.
The fourth term minimizes the difference in quay crane activity between consecutive time intervals for each vessel v. The activity of a quay crane is indicated by the variable $x_{iv}(k)$, which can be 0 ('idle') or 1 ('processing'). Minimizing this difference in activity enforces that once a quay crane started processing a vessel, it keeps processing that vessel as long as possible. This soft constraint is added to prevent useless process interruptions of a vessel by a quay crane, as depicted in Figure 3.5a for quay crane 1. The parameter C_v^c represents the cost for a change in activity between two consecutive time intervals of a quay crane processing vessel v. Again, it has to be mentioned that if $k = K$, then the difference $|x_{iv}(0) - x_{iv}(K)|$ is evaluated due to the cyclic nature of the problem.
The fifth term minimizes the difference in quay crane activity between two consecutive quay cranes in the same time interval. The activity of the quay cranes is again indicated by the variable $x_{iv}(k)$. This soft constraint enforces that active quay cranes are grouped as much as possible; it is prevented that an idle crane is isolated between processing quay cranes. For example, in Figure 3.5b quay cranes 3 and 4 are both idle, instead of that quay crane 2 is isolated between quay cranes 1 and 3. The parameter C_v^d represents the cost for a change in activity between two consecutive quay cranes.

It is important to notice that in such a multi-objective optimization problem, the weight factors C_q^b, and C_v^a to C_v^d determine which component of the objective is decisive. For example, when $C_q^b >> C_v^a$ the emphasis is on minimizing the number of required quay cranes in the terminal, meaning that the process times of the vessels may be rela-

tively large. When, as another example, $C_v^a >> C_v^b$ the emphasis is on minimizing the process times of the vessels, it might be beneficial for the processing time of a vessel to make a large move with a quay crane. In that particular case, it is possible that in Figure 3.5 still two cranes are moved from vessel 1 to vessel 2.

Similar to the objective of the BAP in Section 2.3 the absolute values in (3.25) have to be eliminated. Therefore, the auxiliary variables $a_i(k)$, $b_{iv}(k)$, and $c_i(k)$ are introduced. Consequently, the objective is reformulated and six additional constraints are added:

$$\begin{aligned} \min \quad & \sum_{v=1}^{V} C_v^a \cdot \Delta_v^d + C_q^b \cdot n_t + \sum_{k=1}^{K-1} \sum_{i=1}^{M} C_v^b \cdot a_i(k) + \sum_{k=1}^{K} \sum_{i=1}^{M} \sum_{v=1}^{V} C_v^c \cdot b_{iv}(k) \\ & + \sum_{k=1}^{K} \sum_{i=1}^{M-1} C_v^d \cdot \sum_{v=1}^{V} c_i(k), \end{aligned} \tag{3.26}$$

where

$$a_i(k) \geq l_i(k+1) - l_i(k) \qquad \forall i, k, \tag{3.27}$$

$$a_i(k) \geq l_i(k) - l_i(k+1) \qquad \forall i, k, \tag{3.28}$$

$$b_{iv}(k) \geq x_{iv}(k+1) - x_{iv}(k) \qquad \forall k, v, \tag{3.29}$$

$$b_{iv}(k) \geq x_{iv}(k) - x_{iv}(k+1) \qquad \forall k, v, \tag{3.30}$$

$$c_i(k) \geq \sum_{v=1}^{V} (x_{(i+1)v}(k) - x_{iv}(k)) \qquad \forall i < M, k, v, \tag{3.31}$$

and

$$c_i(k) \geq \sum_{v=1}^{V} (x_{iv}(k) - x_{(i+1)v}(k)) \qquad \forall i < M, k, v. \tag{3.32}$$

The additional constraints ensure that the auxiliary variables are always equal to or larger than 0. Also in (3.27) – (3.30) holds that, when $k = K$, the variable values are compared with the variable values in the first time interval of the problem due to the cyclic nature of the problem.

3.6.3 Adjustment of the Heuristics Procedure

The heuristics procedure for the discrete QCAP is adjusted in the same way as the discrete QCAP itself. Hence, variables Δ_v^d get an upper bound of 0, and constraint (3.20) is eliminated from the constraint set. The auxiliary variables n_t, $a_i(k)$, $b_{iv}(k)$ and $c_i(k)$ are added to the variable set. The Constraint (3.24) is also necessary in the heuristics procedure.
However, more adjustments are required. Two terms in (3.25) are minimizing the value

between two consecutive time intervals: Minimizing i) the difference in position of a quay crane between two consecutive time intervals, and ii) the activity of quay cranes between two consecutive time intervals. These soft constraints are also valid between consecutive subproblems in the heuristics procedure. Hence, additional terms have to be added to the objective function to ensure that the soft constraints are also imposed between the last time interval of one subproblem and the first time interval of the next subproblem.
The position of a quay crane in the last time interval of a subproblem is indicated by L_i^{s-1}, and the activity of quay crane i on vessel v in the last time interval of a subproblem is indicated by X_{iv}^{s-1}.
The adjusted objective for subproblem s is given by:

$$\begin{aligned}\min \quad & \sum_{v=1}^{V} C_v^a \cdot \Delta_v^d + C_q^a \cdot q_v + C_q^b \cdot n_t + \sum_{k=1}^{K-1}\sum_{i=1}^{M} C_v^b \cdot |l_i(k+1) - l_i(k)| \\ & + \sum_{k=1}^{K}\sum_{i=1}^{M}\sum_{v=1}^{V} C_v^c \cdot |x_{iv}(k+1) - x_{iv}(k)| \\ & + \sum_{k=1}^{K}\sum_{i=1}^{M-1} C_v^d \cdot |\sum_{v=1}^{V}(x_{(i+1)v}(k) - x_{iv}(k))| + \sum_{i=1}^{M} C_v^b \cdot |L_i^{s-1} - l_i(k_0^s)| \\ & \sum_{i=1}^{M}\sum_{v=1}^{V} C_v^c \cdot |X_{iv}^{s-1} - x_{iv}(k_0^s)|,\end{aligned} \tag{3.33}$$

where k_0^s is the initial time interval of subproblem s. The seventh and eight term are added to link the subproblems, as described above. These are also used to ensure the cyclic nature of the problem. In the last subproblem is also counted for $|X_{iv}^0 - x_{iv}(K)|$ and $|L_i^0 - l_i(K)|$, where X_{iv}^0 and L_i^0 are the quay crane activity and the position of the quay cranes at $k = 0$, respectively.

Again, the absolute values have to be eliminated. Therefore, the auxiliary variables d_i and e_{iv} are introduced. As a result, the objective function is reformulated and four additional constraints are added:

$$\begin{aligned}\min \quad & \sum_{v=1}^{V} C_v^a \cdot \Delta_v^d + C_q^a \cdot q_v + C_q^b \cdot n_t + \sum_{k=1}^{K-1}\sum_{i=1}^{M} C_v^b \cdot a_i(k) + \sum_{k=1}^{K}\sum_{i=1}^{M}\sum_{v=1}^{V} C_v^c \cdot b_{iv}(k) \\ & + \sum_{k=1}^{K}\sum_{i=1}^{M-1} C_v^d \cdot \sum_{v=1}^{V} c_i(k) + \sum_{i=1}^{M} C_v^b \cdot d_i + \sum_{i=1}^{M}\sum_{v=1}^{V} C_v^c \cdot e_{iv},\end{aligned} \tag{3.34}$$

where

$$d_i \geq L_i^{s-1} - l_i(k_0^s) \qquad \forall i, \tag{3.35}$$

$$d_i \geq l_i(k_0^s) - L_i^{s-1} \qquad \forall i, \tag{3.36}$$

$$e_{iv} \geq X_{iv}^{s-1} - x_{iv}(k_0^s) \qquad \forall i, v, \tag{3.37}$$

$$e_{iv} \geq x_{iv}(k_0^s) - X_{iv}^{s-1} \qquad \forall i, v. \tag{3.38}$$

These additional constraints ensure that the auxiliary variables are always equal to or larger than 0.

3.6.4 Experiments

In this section, the results of the heuristics procedure for the discrete QCAP are compared to the results of the complete discrete formulation. Both objective function value and required CPU time are compared. In addition, the influence of the size of the subproblems in the heuristics procedure is investigated.
It is expected that the heuristics procedure results in allocations with a larger objective function value. One reason is that each subproblem only considerers restricted information on the data. A decision in subproblem s might be bad for the allocation in subproblem $s + 1$, but this cannot be detected in subproblem s yet. In addition, when vessels are present in multiple subproblems, the importance of processing them is weighted with respect to each other as described in Subsection 3.4.3. There are cases where this weighting of importance results in larger departure times than in the complete formulation. Finally, in the subproblems it is not known how many quay cranes are used in the other subproblems. For example, when in the first subproblem already the maximum number of quay cranes is used, it is still possible that in the second subproblem the number of quay cranes is minimized resulting in larger departure times.
The performance of the heuristics procedure with respect to the objective function value is expected to be dependent on the size of the subproblems. The smaller the subproblems are chosen, the less information is considered at once. Hence, the heuristics procedure results most probably in a larger objective function value than the complete formulation when the subproblems are chosen small.
Although the complete discrete formulation finds an optimal allocation and the heuristics procedure finds only this optimal allocation in the best case, it is most probably still beneficial to use the heuristics procedure. As shown in Subsection 3.5.2, the heuristics procedure requires significantly less computational effort. In addition, it is expected that choosing appropriate cost factors in the objective function results in a better performance of the heuristics procedure.
Hypothesis: The objective function value of the heuristics procedure is equal to or larger than the objective function value for the complete discrete formulation, since in the heuristics procedure the problem is not considered at once. Therefore, it is expected that when the subproblems in the heuristics procedure are chosen large, the performance of the procedure increases. In addition, the heuristics procedure is expected to solve significantly faster than the complete discrete formulation.

Experiments setup

The same cycle and terminal as in Subsection 3.5.2 are considered. Hence, the following parameters are set: $K = 15$, $L_t = 1000$ m and $M = 6$. The quay crane rates λ_{iv} however are dependent on the quay crane and the vessel length. Each quay crane has its own maximum quay crane rate as given in Table 3.4. This quay crane rate decreases with the decrease of the vessel length.
Sets with 7 vessels are generated, exactly in the same way as in Subsection 3.5.2. For

each experiment, 100 instances are solved.
The cost factors for the multi-objective are given in Table 3.5. As one can see, the emphasis is on the early departure of vessels and not on the minimization of the number of quay cranes. This suits the application, since in the first step optimization the minimum number of quay cranes has already been determined. This number of quay cranes is set as an upper bound in the second step of the solution approach. The values for C_q^a and C_v^b seem to be large. However, the values for the variables q_v and $l_i(k)$ are scaled to $\max(Q_v)$ and L_t, respectively.
As described in Subsection 3.4.3, the size of the subproblems of the heuristics procedure depends on the minimum and maximum number of vessels in a subproblem. This minimum and maximum number of vessels is varied between the different experiments, and are given in Table 3.6.
The mixed integer optimization is stopped as soon as it has found an integer solution proven to be within 1% of optimal. For each vessel set, the objective function value and the required CPU time are monitored.

Table 3.4: Quay crane rates

Quay crane	1	2	3	4	5	6
Rate $\left[\frac{\text{containers}}{\text{time interval}}\right]$	25	26	27	27	26	25

Table 3.5: Cost factors

Cost factor	value
Benefit for early departure (C_v^a)	1.0
Cost for not meeting Q_v^* (C_q^a)[1]	1.0
Cost per quay crane (C_q^b)	0.4
Cost per meter movement (C_v^b)	0.1
Cost for interrupting processing (C_v^c)	0.1
Cost for closing in idle cranes (C_v^d)	0.3

Table 3.6: Subproblem size in the different experiments

Experiment	V_{min}	V_{max}
1	2	4
2	3	5
3	3	6

[1] Only for the heuristics procedure

Results and discussion

The results of the experiments are given in Table 3.7. The results are as expected, the heuristics procedure results on average in a larger objective function value than the complete discrete formulation. The mean difference is at most 7.1% in the disadvantage of the heuristics procedure. On the other hand, the heuristics procedure requires significantly less computational effort. On average the gain in computation time is up to 80% of the required computation time of the complete discrete formulation.
The difference in objective function values is mainly caused by the movement of the quay cranes and the difference in departure time. As mentioned before, this is probably caused by the fact that in the heuristics procedure only a part of the problem is considered. The allocation of the vessels in the next subproblems is not considered, which sometimes results in a non-optimal allocation of the quay cranes. In addition, the weighting of vessels which are present in multiple subproblems sometimes results in a later departure time than it is the case in the complete discrete formulation.
As shown in Table 3.7, the performance of the heuristics procedure decreases when the subproblem size decreases. This is most probably caused by the fact that in small subproblems less information of the complete problem is considered than in large subproblems. However, when a careful look is taken at the standard deviations on the difference in objective function values, it can be concluded that the differences in these experiments are not significant. The same holds for the small decrease in required computational effort. The standard deviation on the difference in computational effort is large, which makes the improvement not significant. In spite of this, it is advised to see the decrease in performance of the heuristics procedure with respect to the objective function value as a trend. Hence, it is preferred to keep the subproblems as large as possible.
A critical remark about the results should be made. The accuracy of the optimization has an important role in the presented results. The average objective function value of the complete formulation is in the order of magnitude of 2.0, and the average objective function value of the subproblems in experiment 3 is in the order of magnitude of 2.5. With an accuracy of 1% this means that contributions to the objective function value of respectively 0.02 and 0.025 are taken into consideration. Since the cost on the movement of quay cranes is set to 0.1, this means that movements are minimized up to an accuracy of 0.2, corresponding to 200 m due to scaling, in the complete formulation. This distance is on average 250 m for the subproblems in the heuristics procedure. Additional experiments where the cost factor C_v^b is larger are also performed. In those experiments the difference in objective function value between the heuristics procedure and the complete formulation was much larger, which is most probably caused by the difference in accuracy with respect to the different objectives between the subproblems and the complete formulation. Hence, this suggests that it is not preferred to use the same cost factors in the subproblems of the heuristics procedure as in the complete formulation. A more extensive numerical study into the influence of the cost factors on the performance of both the heuristics procedure and the complete formulation would therefore be an interesting future study.

Table 3.7: Results

Experiment	Objective function value difference [-]	%	Standard deviation	CPU time difference [s]	%	Standard deviation
1	0.15	7.1%	0.24	120	80%	230
2	0.13	6.1%	0.24	120	80%	230
3	0.094	4.5%	0.18	115	76%	230

These experiments suggest that with respect to computational effort it is beneficial to use the heuristics procedure to solve the QCAP. The objective function value increase is within an acceptable order of magnitude, however more insights on the influence of the cost factors on the performance of the heuristics procedure are required to give a better advice.

Chapter 4

Case Study

In this chapter, the optimization problems in the second step of the solution approach are solved for a representative first step allocation on both the strategic and the operational level. It is shown that the solution approach, as depicted in Figure 1.3, can be solved sufficiently fast in both the strategic and the operational case.

4.1 Strategic Allocation

In this section, it is explained how the strategic second step allocation is constructed. The strategic allocation is constructed somewhere between once a month and once a year. This weekly planning is then used as a reference allocation for a large time period.

4.1.1 Description

A cluster of 3 terminals where 38 vessels are berthing once each week is considered. A time interval size of 1 hour is chosen. The formulation of [Hen08] is used to construct a strategic first step allocation, where all desired arrival and departure times are considered deterministic.
As depicted in Figure 1.3, in the first step optimization i) a terminal, ii) a time window, and iii) a time variant quay crane capacity are allocated to each vessel. There are four objectives: Minimizing i) the amount of inter-terminal traffic, ii) the total weighted deviation from the desired berthing time intervals, iii) the amount of stored containers in each individual terminal, and iv) the number of required quay crane capacity in each individual terminal.
To illustrate the second step allocation, one terminal to which 15 vessels are allocated is considered. Furthermore, in the first step of the solution approach, it is determined that at most 5 quay cranes are necessary to process the 15 vessels. This number is used in the QCAP in the second step, although more quay cranes are available in the

terminal.
The implementation of the second step of the solution approach for this strategic part of the case study and its results are shortly discussed below.

4.1.2 Berth Allocation Problem

The BAP is solved with the formulation of Section 2.3. In this case study, the center of the terminal is chosen as the lowest-cost berthing position for all vessels. Costs are proportional to the deviation from the optimal position, where the cost factor increases with the vessel length.
The terminal quay utilization u_1 as defined in (2.8) is 0.26. Hence, looking at the results of the feasibility experiments in Subsection 2.4.2 and the results of the parameter sensitivity experiments in Subsection 2.4.4, no problems are expected with infeasibility and/or the required computational effort in the second step BAP in this case study.

Indeed, a feasible berth position allocation is constructed within 0.08 s. As expected, the proposed BAP formulation is very suitable to construct a strategic berth position allocation. However, the chosen lowest-cost berthing position is rather random. To determine a more realistic berth position allocation the formulation of Section 2.5, where the total straddle carrier distance is minimized, should be used. Then, historical data about the stacking positions of different types of containers in the yard and the vessels' load composition are taken into consideration to construct the strategic berth position allocation.

4.1.3 Quay Crane Allocation Problem

The QCAP is solved with the heuristics procedure for the discrete QCAP of Chapter 3. The heuristics procedure is used since the problem size is large, and a large computational effort is expected. Since the strategic allocation is only constructed a few times a year, a computation time in the order of magnitude of one hour is considered acceptable. The complete problem is divided into 3 subproblems of respectively 44, 44 and 80 time intervals of 1 hour. One vessel is present in more than one subproblem. In the subproblems respectively 4, 6 and 6 vessels are berthing.
Different quay crane rates are used in the different steps of the solution approach. In the first step a quay crane rate of 70% of the maximum quay crane rate is used, and in the second step a quay crane rate of 85% of the maximum quay crane rate is used. This is necessary to be able to convert the real number of quay cranes allocated in the first step into an integer-valued quay crane allocation in the second step. For example, two vessels are berthing simultaneously and in each time interval respectively 2.3 and 2.7 quay cranes are allocated to them in the first step allocation. When in the second step the same process rate is used as in the first step, a feasible second step quay crane allocation can only be constructed when the time intervals are divided by 10 since the

quay cranes should be able to switch between the two vessels. By using a larger quay crane rate in the second step this is not necessary since more containers can be processed in each time interval than it was accounted for in the first step. Then, the need for switching a quay crane between vessels is decreased.
The aim of the optimization is minimizing i) the processing time of the vessels, ii) the possibility for process interruptions, and iii) the isolation of an idle quay crane between processing quay cranes. The mixed integer optimization of the subproblems is terminated as soon as the solution is proven to be 1% of optimal.
The quay crane utilization u_{QC} as defined in (3.22) is 0.59. Hence, looking at the results of the experiments in Section 3.5, it is expected that it is possible to construct a quay crane allocation without any vessels that are departing later than their desired departure time.

The quay crane allocation is constructed in 996 s. Hence, when the quay crane utilization in the other terminals is in the same order of magnitude as in the considered terminal, it is expected that the construction of quay crane allocations for three terminals requires about one hour.
Since the aim of the optimization has been the minimization of the process time, all vessels are processed during only a part of their berthing time. The end of the processing time of the vessels is between 1 and 10 hours before their desired departure time. The processing time of two vessels starts respectively 1 and 2 hours after the actual arrival time of those vessels. The fact that the processing times are shorter than it is determined in the first step of the solution approach is most probably a result of using a larger quay crane rate in the second step than in the first step.
It is now possible to calculate the average utilization u_{QC}^{mean} of the quay cranes. This utilization is defined as:

$$u_{QC}^{mean} = \frac{\sum_{v=1}^{V} Q_v}{\sum_{v=1}^{V} \sum_{k=1}^{K} M_v^{QC}(k) \cdot \lambda_{iv}^{max}}, \tag{4.1}$$

where $M_v^{QC}(k)$ is the number of quay cranes that is processing vessel v in time interval $[k, k+1\rangle$, and λ_{iv}^{max} is the maximum quay crane rate of quay crane i on vessel v. Hence, it is the total number of processed containers divided by the number of containers that the allocated quay cranes could have processed when they would have been used for 100%. For the constructed allocation, $u_{QC}^{mean} = 0.82$. This is less than the chosen 0.85. This is most probably caused by the fact that quay cranes are allocated to a vessel for at least the time interval size. For example, a vessel has to be processed for another 0.5 hour by one quay crane. However, the quay crane is allocated for one hour to the vessel, since the time interval size is one hour.
The quay crane allocation is checked for less preferred quay crane allocations, as described in Subsection 3.6.1. An idle quay crane is never isolated between processing quay cranes, and the processing of a vessel is never interrupted unnecessarily.

These results suggest that the discrete QCAP formulation is suitable to construct a strategic quay crane allocation. When the heuristics procedure is used, the computational effort to solve the QCAP is within a reasonable amount of time. It is expected that it is possible construct a quay crane allocation for three terminals within one hour.

4.2 Operational Allocation

In this section, it is explained how the second step allocation is constructed at the operational level. The operational allocation is constructed using the model-based predictive control (MPC) approach of [Vul08]. This means that the second step operational allocation is constructed multiple times a day.

4.2.1 Description

The same terminal and the same vessel set as in the strategic case are considered. However, the number of quay cranes that is used in the second step QCAP is increased to 7. In Section 4.1, the arrival and departure times are considered to be deterministic. In practice however, vessels have stochastic arrivals (e.g. due to storm, break-downs, etc). To cope with a part of these disturbances, terminal operators and shipping lines agree on so-called arrival windows. In this case study, that means that a vessel is allowed to arrive 4 hours before or after its desired arrival time.
When a vessel arrives within its arrival window, a certain maximal processing time is guaranteed by the terminal operator. This maximal processing time is equal to the processing time of the vessel on the strategic level. Hence, the first step allocation of [Hen08] is used as the reference allocation on the operational level. It is determined that in the worst case scenario 7 quay cranes are required to meet those processing times. For the vessels that are arriving outside their arrival window, there are no guarantees about their maximum processing time.
In this case only disturbances on the arrival times of vessels are taken into consideration. The disturbances on the arrival times can be within the arrival windows, but also out of the arrival windows. A simulation model is used to generate the disturbances on the arrival times of the vessels. These disturbances are generated with respect to the deterministic arrival times of the vessels on the strategic level. In total a period of 10 weeks is simulated.
As mentioned before, the MPC approach of [Vul08] is used to reallocate the first step allocation under the described disturbances. In each iteration step the first step allocation is reallocated for the next 48 hours with the latest available information on the arrival times of the vessels. For this case study, each hour an iteration step is performed in which the next 48 hours are taken into consideration. The objective of the MPC approach is to minimize the departure times of the vessels.
The implementation of the second step of the solution approach into this MPC approach and its results are described below.

4.2.2 Berth Allocation Problem

The BAP is solved after each reallocation of the first step allocation with the formulation of Section 2.3 for the complete horizon of 48 hours. One adaptation to the formulation is necessary. Vessels that are already berthing at the beginning of the considered horizon cannot change position anymore since they are already being processed. Hence, the position of these vessels is not variable anymore:

$$p_v = P_v^b \qquad v \in \mathcal{V}_\mathrm{b}(k_c), \tag{4.2}$$

where P_v^b is the actual berthing position of vessel v, and k_c is the first time interval of the considered horizon. The set $\mathcal{V}_\mathrm{b}(k_c)$ includes all vessels that are already berthing at the beginning of the considered horizon.
The lowest-cost berthing position is chosen to be equal to the position to which the vessel has been allocated in the strategic part of this case. Large vessels introduce more cost when berthing on a non-optimal berthing position than small vessels.

The average computation time to construct the berth position allocation after each first step iteration is 0.0044 s. The maximum computation time to construct the berth position allocation was 0.02 s. No issues with respect to feasibility of the berth position allocation in the second step are encountered.
These results suggest that the proposed BAP formulation is also suitable to construct an operational berth position allocation within the MPC approach of [Vul08]. The same remark as in the strategic case has to be made: The usage of the formulation in Section 2.5 would result in a more realistic berth position allocation. In each iteration the current forecast on the vessels' load composition can be taken into consideration to construct the operational berth position allocation. Then, the actual vessels' load composition would determine the berthing position of the vessels.

4.2.3 Quay Crane Allocation Problem

The QCAP is solved after each reallocation of the first step and berth position allocation with the formulation of Section 3.6. Since the QCAP has to be solved each hour, it is important that the QCAP is solved within a reasonable amount of time. Therefore, it is decided to solve the QCAP only for the first 16 hours of the considered horizon. In addition, the heuristics procedure of Subsection 3.4.3 is used when more than 4 vessels are berthing in the considered horizon of 16 hours.
Since the quay crane allocation is constructed each hour it is preferred that quay cranes that have been processing vessel v during the previous hour continue processing vessel v in the first hour of the considered horizon. Otherwise, the possibility for unnecessary quay crane switches exists. Therefore, the term $C_v^c \cdot g_{iv}$ and two additional constraints are added to the objective function (3.34):

$$g_{iv} \geq x_{iv}(k_c) - X_{iv} \qquad \forall i, v, \tag{4.3}$$

and

$$g_{iv} \geq X_{iv} - x_{iv}(k_c) \qquad \forall i, v, \tag{4.4}$$

where X_{iv} is the activity of quay crane i on vessel v during time interval $k_c - 1$.
Similar to the strategic part of this case study, the process rate of the quay cranes is taken 85% of the maximum process rate in the second step of the solution approach. In the first step again a process rate of 70% of the maximum process rate is chosen.
The aim of the optimization is to minimize i) the possibility for useless processing interruptions, and ii) isolation of an idle crane between two processing quay cranes. Both the number of quay cranes and the departure time have already been minimized in the first step of the solution approach and its reallocation. The mixed integer optimization is terminated as soon as the solution is proven to be within 1% of optimal.

The average computation time to construct the quay crane allocation is 22 s. However, the maximum computation time to construct the quay crane allocation is 325 s. In only 38 of the 1680 iterations, it has been necessary to use the heuristics procedure to solve the QCAP since the number of vessels in the considered horizon was larger than 4. In those iteration steps, at most 2 subproblems have been formulated. The average quay crane utilization as defined in (4.1) is equal to 0.83. Again, this is less than the chosen 0.85 since quay cranes are allocated to a vessel for at least the time interval size. The total quay crane allocation is checked for less preferred quay crane allocations, as described in Subsection 3.6.1. An idle crane is never isolated between two processing cranes. However, there are two situations where an unnecessary processing interruption is noticed. When those particular iteration steps are conducted again with a larger value for C_v^c, the unnecessary process interruptions are not noticed anymore. However, if the value for C_v^c is set to that value for all iterations, it is more often observed that an idle quay crane is isolated between two processing quay cranes. This confirms the remark in Subsection 3.6.4 that the performance of the discrete QCAP is dependent on the value of the cost factors. Up till now the value of the cost factors is set independent on the problem size (e.g. the number of vessels in the vessel set or the number of time intervals). It might be beneficial for the performance of the discrete QCAP when the problem size is taken into consideration when assigning the cost factors.
These results suggest that the discrete QCAP is also suitable to construct the operational quay crane allocation. On average the computational effort is small, and therefore it is possible to solve the QCAP each hour. However, as already remarked in the previous chapter, improvement can be made by assigning more appropriate cost factors.

Chapter 5

Conclusions and Recommendations

Conclusions

This report investigates the berth allocation problem (BAP) and quay crane allocation problem (QCAP) embedded in the large multi-step solution approach of [Hen08] to solve the multi-terminal BAP and the QCAP. The overall solution approach consists of two steps in which different optimization problems are formulated. In the first step, i) a terminal, ii) a time window, and iii) a time variant quay crane capacity are allocated to each vessel in the set. In [Hen08], a mixed integer linear programming (MILP) problem is proposed to solve this first step allocation problem. Restricting properties are the terminal quay lengths and the available quay crane capacity in each individual terminal. The objectives are to minimize i) the amount of inter-terminal traffic, ii) the total weighted deviation from the desired berthing time intervals iii) the amount of stored containers in each individual terminal, and iv) the maximum required quay crane capacity in each individual terminal.
In the second step, the exact berth position allocation of the vessels and the exact quay crane allocations for each individual terminal are still to be determined. In this report, the problems in the second step of the solution approach are addressed. For both problems, a separate MILP optimization problem is proposed and a case study is performed.

Since in the first step of the solution approach the vessels have already been allocated to a terminal, the actual position allocation can be determined for each terminal separately. The single terminal BAP's in this report can be considered as one-dimensional packing problems since in the first step the arrival and departure times of the vessels have already been determined. The positions of the vessels along the terminal quay are still to be determined. In the proposed formulation, a lowest-cost berthing position is defined for all vessels in the vessel set. The objective is to minimize the weighted

deviation from the lowest-cost berthing position without vessels overlapping each other. Due to the specific cut in the solution approach, an allocation constructed in the first step might turn out to be infeasible in the second step. In the first step it is only guaranteed that the sum of the lengths of the vessels, that are berthing simultaneously at one terminal, never exceeds the total terminal length. However, this is a necessary but not sufficient constraint for a feasible 2D-packing.
Experiments suggest that for typical terminal quay utilizations, first step allocations are always feasible in the second step. In addition, the experiments suggest that the computational effort to solve the proposed MILP is very small. This is most probably a result of the fact that the packing problem is one-dimensional.
In a representative case, a berth position allocation constructed from the proposed MILP is compared to a berth position allocation constructed manually. The results suggest that with little computational effort an improved berth position allocation can be constructed with respect to total driving distance of the straddle carriers.

The first step crane capacity allocation and the result of the single terminal BAP are used as input for the QCAP. Enough quay cranes have to be allocated to a vessel to be able to process all its containers in a timely fashion. An important constraint is that the quay cranes cannot cross each other since they are situated on the same track. An existing QCAP formulation [Liu06] is continuous in time, which has two restrictions: i) A quay crane is allocated to a vessel until that vessel has completely been processed, and ii) the quay crane rates are equal for each quay crane. The first restriction can result in allocations in which vessels are departing later than their desired departure time. Both restrictions are a result of the fact that the formulation is continuous in time. Therefore, an QCAP formulation, which is discrete in time is proposed. Experiments suggest that the proposed discrete formulation results in a reduction in the number of allocations in which vessels are departing later than their desired departure time. Moreover, it is possible to take quay cranes with different quay crane rates into consideration with the discrete formulation.
The objective of the discrete formulation is extended to enable minimization of i) the departure time of the vessels, ii) the number of required quay cranes, iii) the movement along the quay of the quay cranes, iv) the possibility for process interruptions, and v) the isolation of idle cranes between processing quay cranes. The different terms are weighted by different cost factors.
Since the discrete formulation requires a lot of computational effort, a heuristics procedure is proposed. The complete problem is intelligently divided into smaller subproblems, which significantly reduces the number of variables and constraints that are considered at once. Experiments suggest that the heuristics procedure performs on average 5% worse than the complete formulation with respect to the objective function value. However, the performance with respect to the computational effort improves with on average 80%. The same experiments also suggest that the performance of both the complete formulation and the heuristics procedure is highly dependent on the chosen values for the cost factors.

The proposed formulations for the BAP and the QCAP are used to solve the second step optimization problems for a representative first step allocation on both the strategic and operational level. On the strategic level, the first step allocation of [Hen08] is used as the input for the second step. The arrival and departure times of all vessels are considered to be deterministic. A strategic allocation is used as a reference allocation for a longer period of time. The strategic case has been solved for a cyclic period of one week.
On the operational level, the first step allocation of [Hen08] is used as the reference allocation. A model-based predictive control (MPC) approach [Vul08] reallocates the first step allocation under disturbances. Both the BAP and the QCAP are solved each time interval representing for a horizon of 48 and 16 hours during a time period of 10 weeks. The results suggest that with a reasonable amount of computational effort it is possible to construct and optimize a second step allocation on both the strategic and operational level.

Recommendations

Although experiments suggest that for typical quay utilizations feasibility issues in the second step of the solution approach are not expected, it might be that for increasing quay utilization, a first step allocation results in an infeasible second step allocation. In that case, the first step problem has to be adapted and solved again. Since the international conveyance of sea freight using containers is still growing rapidly, it may be a useful future study how to adapt the first step problem.
For the representative case in Chapter 2, the berth position allocation is constructed with the vessels' load composition given in advance. In practice, this information is sometimes only available 8 hours before arrival of a vessel. Implementing the BAP formulation, which minimizes the total straddle carrier distance in the MPC approach of [Vul08] would make it possible to take current forecasts on load compositions into consideration while constructing the berth position allocation at an operational level.
Experiments suggest that the performance with respect to the objective function value of the heuristics procedure for the discrete QCAP is highly dependent on the values of the cost factors. Currently, the cost factors are assigned independent of the subproblem sizes. It may be beneficial to investigate whether the performance of the heuristics procedure improves when the size of the subproblems is taken into consideration for assigning the cost factors. The information that is taken into consideration in a subproblem also has its influence on the performance of the heuristics procedure. One can think of adding information to a subproblem about the position of the vessels in the other subproblems and/or the number of quay cranes that is used in previous subproblem(s).
Up till now the QCAP uses the results of the BAP as an input. However, the results of the QCAP can also influence the BAP. The use of different quay crane rates introduces the possibility that a vessel can leave before its desired departure time. In that case, the dimension of the rectangle, representing the vessels' processing time in the BAP,

has become smaller. Thus, solving the BAP again could result in a different, more optimal berth position allocation, which for its part could result in a different quay crane allocation, etc. Hence, it would be interesting to investigate whether iterating between the BAP and the QCAP results in improved second step allocations.

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Appendix A

Vessel Set Generation

In this appendix is described how representative data from terminal operator PSA HNN Antwerp can be used to generate vessel sets to experiment with. The vessel sets have to serve as input for the single-terminal BAP and the different formulations of the QCAP. To this end, the processing time P_v, the length l_v and the required capacity Q_v of the vessels in the set have to be generated. In this appendix, the procedure is only explained for generating the process times of the vessels since the procedure for generating the lengths and the required quay crane capacity of the vessels is exactly the same.

Representative data

In Figure A.1a, a histogram is depicted for the representative process times for a set of 37 vessels. Each beam indicates how often the process time of a vessel is in between a certain range. For example, the first beam in Figure A.1a indicates that 6 vessels have a process time between 8.0 and 10.8 hours.

Generated data

The information from the histogram in Figure A.1a is used to generate process times for vessels, which are distributed in the same way as in the representative vessel set. A set of beams: $b \in \{1, ..., B\}$, and a set of vessels: $v \in \{1, ..., V\}$ are considered. In addition, the following parameters are defined:

V	:	The number of vessels in the set [-].
B	:	The number of beams in the histogram [-].
V_s	:	The number of vessels in the representative data set of PSA HNN [-].
P_{ub}	:	The upper bound of beam b for the processing time [h].
MP_b	:	The mean process time of beam b [h].

LB_b : The lower bound of beam b on the number of vessels [-].
UB_b : The upper bound of beam b on the number of vessels [-].

The parameters LB_b and UB_b are cumulative with respect to the lower and upper bound of the preceding beam. The relationship is given by $LB_{b+1} = UB_b$, where $1 < b \leq B$. For example, in Figure A.1a the lower bound of the first beam is 0 (i.e. $LB_1 = 0$) and its upper bound is 6 (i.e. $UB_1 = 6$). For the second beam holds that $LB_2 = 6$ and $UB_2 = 9$.

The variables are given by:

x : A real number between 0 and V [-].
p_v : The process time of vessel v [h].

The following algorithm is used to determine the process time for the vessels in the set:

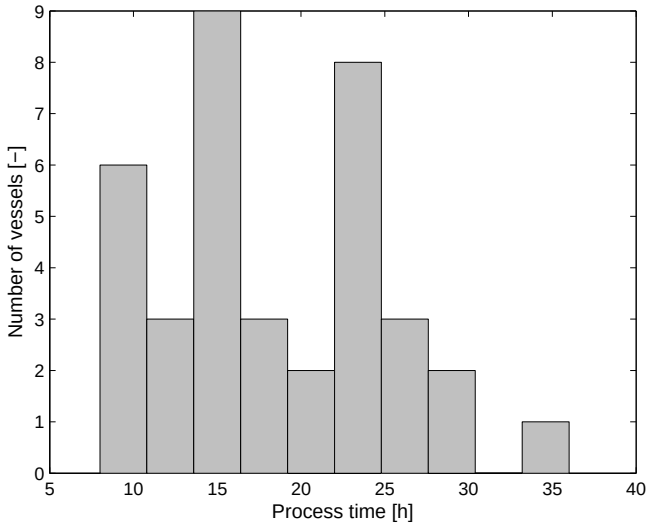
Algorithm 2 Algorithm to determine the process time p_v

for $v = 1 : V$ **do**
  $x = V \cdot \text{rand}$
  if $\frac{LB_b}{V_s} \cdot V < x \leq \frac{UB_b}{V_s} \cdot V$ **then**
    $p_v = MP_b \pm \text{rand} \cdot (P_{ub} - MP_b)$;
  else if $\frac{LB_{b+1}}{V_s} \cdot V < x \leq \frac{UB_{b+1}}{V_s} \cdot V$ **then**
    $p_v = MP_{b+1} \pm \text{rand} \cdot (P_{u(b+1)} - MP_{b+1})$;
    $\vdots$
  else if $\frac{LB_B}{V_s} \cdot V < x \leq \frac{UB_B}{V_s} \cdot V$ **then**
    $p_v = MP_B \pm \text{rand} \cdot (P_{ub} - MP_B)$;
  end if
end for

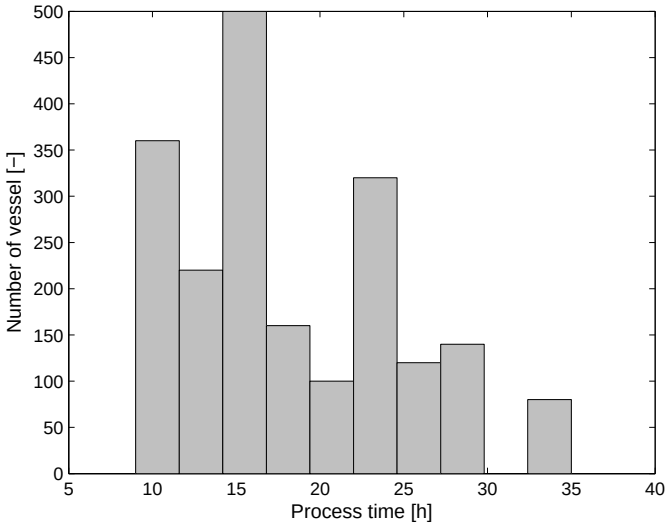
where the function `rand` draws a random value between 0 and 1 from a uniform distribution.
For each vessel a uniform distributed value is drawn and it is determined in which beam of the histogram this value is captured. Then, a uniform distributed processing time is determined between the lower and upper bound for the process time of that particular beam of the histogram. The determined variable p_v can be used as the parameter P_v in the experiments.
In Figure A.1b the histogram is depicted for the generated process times of 100 vessel sets, each consisting of 20 vessels. The distribution of the generated process time is very similar to the distribution of the representative process times.

For the vessel length exactly the same approach is followed. Hence, first a histogram is made of the representative data of PSA HNN. Then the same algorithm is used to determine l_v instead of P_v.

For the required quay crane capacity Q_v a histogram is plotted of how often a certain percentage of the maximum quay crane capacity for a vessel per time interval is used from the representative data of PSA HNN. The maximum quay crane capacity for a vessel per time interval is given by the product of the maximum number of quay cranes that can process a vessel at once S_v and the mean quay crane rate $\overline{\lambda}_{iv}$. Then again Algorithm 2 is used to determine Q_v for each vessel.



(a) Representative process times for a vessel set of 37 vessels



(b) Generated process times for 100 vessel sets of 20 vessels

Figure A.1: Histograms of the representative process times and the generated process times

Appendix B

QCAP Formulation of [Liu06]

In this appendix, the adapted QCAP formulation of [Liu06] is presented. Contrary to the original formulation, the position of the vessels and the quay cranes is continuous and the adapted formulation is suitable for cyclic systems. The notation is equal to the notation of the discrete QCAP in Chapter 3 to make a comparison possible. For the original formulation and a more detailed description of the formulation the reader is referred to [Liu06].

B.1 System Description

Unless stated differently, the following sets are considered: $i \in \{1, ..., M\}$, the set of quay cranes, $v \in \{1, ..., V\}$, the set of vessels. The set of vessels is also denoted by $\mathcal{VS}$. The end of the considered time cycle is denoted by T_{end}. In the first step optimization the desired arrival and departure time (A_v and D_v respectively) have already been determined. The processing of a vessel cannot start before A_v since the vessel is not present in the terminal yet. Since the maximum relative tardiness is minimized, it is preferred when the actual departure of vessel v is before or equal to D_v. Each vessel v has a processing time f_{vl} when l quay cranes are allocated to that vessel. Each vessel v can be processed by a maximum number of quay cranes S_v.
The position along the quay of each vessel v has already been determined in the BAP. The left-most position is indicated by X_v^l and the right-most position is indicated by X_v^r. The origin for the left- and right-most position of vessel v is the left-most boundary of the terminal quay. Between two neighboring quay cranes there has to be a minimal gap G.
With respect to the cyclic property of the considered system, a remark has to be made: Both $A_v \geq D_v$ and $A_v < D_v$ are possible. Therefore, auxiliary parameter E_v is intro-

duced, which explicitly distinguishes between both cases:

$$E_v = \begin{cases} 1 & \text{if } A_v \geq D_v \qquad \forall v, \\ 0 & \text{if } A_v < D_v. \end{cases}$$

The sets and parameters discussed above are conveniently arranged in Table B.1. In the next section, first the variables of the problem are stated. After that, the constraints and the objective function are derived. It becomes also clear why the auxiliary parameter E_v is needed.

Table B.1: Model parameters

Parameter	Definition
M	Number of quay cranes [-]
V	Number of vessels [-]
T_{end}	End of the considered time cycle [h]
S_v	Maximum number of quay cranes, which can process vessel v simultaneously [-]
X_v^l	Left-most position of vessel v [m]
X_v^r	Right-most position of vessel v [m]
G	Minimum gap between two neighboring quay cranes [m]
A_v	Desired berth time of vessel v (start of processing of vessel v) [-]
D_v	Desired departure time of vessel v (end of processing of vessel v) [-]
f_{vl}	The processing time of vessel v when l quay cranes are allocated to it [h]
N	Significant large positive number [-]

B.2 MILP

Continuous variables

a_v : The service start time of vessel v [h].
d_v : The service completion time of vessel v [h].
l_{iv}^s : The position of quay crane i at the start time of vessel v [m].
l_{iv}^c : The position of quay crane i at the completion time of vessel v [m].
t : The maximum relative tardiness of all vessels [-].
Δ_v^d : Time that vessel v departs too late or too early [h].

Binary variables

$$x_{iv} = \begin{cases} 1 & \text{if quay crane } i \text{ is allocated to vessel } v, \\ 0 & \text{otherwise.} \end{cases}$$

$$\phi_{vl} = \begin{cases} 1 & \text{if } l \text{ quay cranes are allocated to vessel } v, \\ 0 & \text{otherwise.} \end{cases}$$

$$e_v = \begin{cases} 1 & \text{if } a_v > d_v, \\ 0 & \text{if } a_v < d_v, \\ 1 & \text{if } a_v = d_v \text{ and vessel } v \text{ is continuously berthing,} \\ 0 & \text{if } a_v = d_v \text{ and vessel } v \text{ does not berth at all.} \end{cases}$$

$$e_v^a = \begin{cases} 1 & \text{if } a_v < A_v, \\ 0 & \text{if } a_v > A_v. \end{cases}$$

$$y_{qr}^{ss} = \begin{cases} 1 & \text{if } a_q \text{ is before } a_r \qquad (q,r) \in \mathcal{VS}, q \neq r, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{qr}^{sc} = \begin{cases} 1 & \text{if } a_q \text{ is before } d_r \qquad (q,r) \in \mathcal{VS}, q \neq r, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{qr}^{cs} = \begin{cases} 1 & \text{if } d_q \text{ is before } a_r \qquad (q,r) \in \mathcal{VS}, q \neq r, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{qr}^{cc} = \begin{cases} 1 & \text{if } d_q \text{ is before } d_r \qquad (q,r) \in \mathcal{VS}, q \neq r, \\ 0 & \text{otherwise.} \end{cases}$$

Constraints

$$1 < \sum_{i=1}^{M} x_{iv} \leq S_v \qquad \forall v, \tag{B.1}$$

$$\sum_{l=1}^{Sv} l\phi_{vl} = \sum_{i=1}^{M} x_{iv} \qquad \forall v, \tag{B.2}$$

$$a_r \geq a_q + N\big(y_{qr}^{ss} - 1\big) \qquad (q,r) \in \mathcal{VS}, q \neq r, \tag{B.3}$$

$$d_r \geq a_q + N\big(y_{qr}^{sc} - 1\big) \qquad (q,r) \in \mathcal{VS}, q \neq r, \tag{B.4}$$

$$a_r \geq d_q + N\big(y_{qr}^{cs} - 1\big) \qquad (q,r) \in \mathcal{VS}, q \neq r, \tag{B.5}$$

$$d_r \geq d_q + N\big(y_{qr}^{cc} - 1\big) \qquad (q,r) \in \mathcal{VS}, q \neq r, \tag{B.6}$$

$$y_{qr}^{ss} + y_{rq}^{ss} = 1 \qquad (q,r) \in \mathcal{VS}, q \neq r, \tag{B.7}$$

$$y_{qr}^{sc} + y_{rq}^{cs} = 1 \qquad (q,r) \in \mathcal{VS}, q \neq r, \tag{B.8}$$

$$y_{qr}^{cs} + y_{rq}^{sc} = 1 \qquad (q,r) \in \mathcal{VS}, q \neq r, \tag{B.9}$$

$$y_{qr}^{cc} + y_{rq}^{cc} = 1 \qquad (q,r) \in \mathcal{VS}, q \neq r, \tag{B.10}$$

$$l_{iv}^{s} \geq X_v^l + G + N\big(x_{iv} - 1\big) \qquad \forall i, v, \tag{B.11}$$

$$l_{iv}^{s} \leq X_v^r - G + N\big(1 - x_{iv}\big) \qquad \forall i, v, \tag{B.12}$$

$$l_{iv}^{c} \geq X_v^l + G + N\big(x_{iv} - 1\big) \qquad \forall i, v, \tag{B.13}$$

$$l_{iv}^{c} \leq X_v^r - G + N\big(1 - x_{iv}\big) \qquad \forall i, v, \tag{B.14}$$

$$l_{ir}^{s} \geq X_q^l + G + N\big(x_{iq} + y_{qr}^{ss} + y_{rq}^{sc} - 3\big) \qquad \forall i, (q,r) \in VS, q \neq r, \tag{B.15}$$

$$l_{ir}^{s} \leq X_q^r - G + N\big(3 - x_{iq} - y_{qr}^{ss} - y_{rq}^{sc}\big) \qquad \forall i, (q,r) \in VS, q \neq r, \tag{B.16}$$

$$l_{ir}^{c} \geq X_q^l + G + N\big(x_{iq} + y_{qr}^{sc} + y_{rq}^{cc} - 3\big) \qquad \forall i, (q,r) \in VS, q \neq r, \tag{B.17}$$

$$l_{ir}^{c} \leq X_q^r - G + N\big(3 - x_{iq} - y_{qr}^{sc} - y_{rq}^{cc}\big) \qquad \forall i, (q,r) \in VS, q \neq r, \tag{B.18}$$

$$l_{iv}^{s} - l_{(i-1)v}^{s} \geq G \qquad \forall i > 1, v, \tag{B.19}$$

$$l_{iv}^{c} - l_{(i-1)v}^{c} \geq G \qquad \forall i > 1, v, \tag{B.20}$$

$$a_v \geq A_v - T_{end} \cdot e_v^a \qquad \forall v, \tag{B.21}$$

$$d_v = a_v + \sum_{l=1}^{S_v} f_{vl} \cdot \phi_{vl} + T_{end} \cdot e_v \qquad \forall v, \tag{B.22}$$

$$\Delta_v^d \geq (D_v - d_v) - T_{end} \cdot e_v^a + T_{end} \cdot E_v - T_{end} \cdot e_v \qquad \forall v, \tag{B.23}$$

$$t \geq \frac{\Delta_v^d}{d_v - A_v} \qquad \forall v. \tag{B.24}$$

Constraints (B.1) define the range for the number of quay cranes that can be allocated to vessel v. Constraints (B.2) determine the number of quay cranes allocated to vessel v. Constraints (B.3) and (B.7) ensure that $y_{qr}^{ss} = 1$ iff $a_r \geq a_q$. From (B.3) for each pair (q,r) there are two related constraints: i) $a_r \geq a_q + M\big(y_{qr}^{ss} - 1\big)$, and ii) $a_q \geq a_r + M\big(y_{rq}^{ss} - 1\big)$. From (B.7) follows that only one of y_{qr}^{ss} and y_{rq}^{ss} is one. For example, if $y_{qr}^{ss} = 1$, i) becomes $a_r \geq a_q$, and ii) is redundant due to the significant large number M. The constraint couples $\big((B.4),(B.8)\big)$, $\big((B.5),(B.9)\big)$, and $\big((B.6),(B.10)\big)$ work the same way to match with the definitions of y_{qr}^{sc}, y_{qr}^{cs} and y_{qr}^{cc}, respectively.
Constraints (B.11) – (B.14) ensure that quay crane i is positioned alongside vessel v at the service start and completion times of the vessel, if quay crane i is allocated to the vessel (i.e., $x_{iv} = 1$). These constraints are void if quay crane i is not allocated to the vessel (i.e., $x_{iv} \neq 1$). Constraints (B.15) – (B.18) ensure that, if quay crane i is allocated to vessel v (i.e., $x_{iv} = 1$), quay crane i stays alongside vessel v at all event time points between the service start and completion times of this vessel v. Collectively,

constraints (B.11) – (B.18) make sure that a quay crane can only work on one vessel at any time, and once it starts processing a vessel it stays there until the processing of the vessel is completed.
Constraints (B.19) and (B.20) ensure that quay cranes cannot cross each other.
The processing start time of a vessel cannot be earlier than the actual berthing time of that vessel, which is ensured by constraint (B.21). The binary variable e_v^a ensures the cyclic nature of the system. Constraints (B.22) make sure that a vessel is processed for the required duration with respect to the number of quay cranes allocated to it. In this constraint, binary variable e_v ensures the cyclic nature. With respect to departing too early or too late there are four possible permutations of d_v and D_v. With the help of the introduced auxiliary variables e_v and e_v^a, and the auxiliary parameter E_v, it is possible to construct appropriate constraints for Δ_v^d to satisfy each of the four cases; as formulated in Constraint (B.23).
The maximum relative tardiness of the complete vessel set is larger or equal to the relative tardiness of the individual vessels, which is captured in Constraints (B.24).

Objective

The objective of the problem is to minimize the maximum relative tardiness of the complete vessel set:

$$\min t. \tag{B.25}$$