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On the benefit of modifying the strategic allocation of cyclically calling vessels for multi-terminal container operators

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Abstract

We present a case study based on a multi-terminal container operation in Antwerp, Belgium, where a set of cyclically calling container vessels is processed. The operator faces the problem of strategically allocating a terminal, a berthing interval, and a variable number of quay cranes to the vessels in the set. Restricting properties are terminal quay lengths, number of quay cranes and storage capacities. Currently, the operator's objective is to satisfy the preferences of the vessel lines, with respect to a terminal and berthing time, as much as possible. We are interested in the benefit of modifying a given allocation, i.e. the potential crane and inter-terminal costs savings if specific changes to a given allocation are allowed. An MILP is implemented in a two-step optimization, which enables us to efficiently investigate the benefit of modification. Experimental results suggest that small changes in a given allocation may lead to significant cost savings.

Introduction

In 1960 people started using containers for international transport of sea freight for the first time. Since then the containerization has grown rapidly. Nowadays, deep-sea vessels can carry up to 14 thousand TEU's (Twenty feet Equivalent Units) and mega container ports are processing up to 26 million TEU's a year. In order to cope with these tremendous amounts of cargo, port operators have to develop efficient logistics systems. Discussions on and classifications as well as solution methods for the main logistics processes in container ports can be found in [1], [2] and [3]. In the literature, the berth allocation problem (BAP) is seen as one of the key issues in a container port. We explicitly distinguish between i) the single-terminal BAP, which is concerned with the allocation of a set of vessels to one terminal and ii) the multi-terminal BAP, which is concerned with the allocation of a set of vessels to a cluster of interrelated terminals.

In the last two decades intensive research has been conducted on the single-terminal BAP. In this paper, however, we focus on the multi-terminal BAP. To our knowledge and as stated in [4], hardly any research has been conducted towards the multi-terminal BAP.

The studies concerning the single-terminal BAP all consider two interrelated allocation problems: allocate i) a berthing position at the terminal, ii) a time interval of berthing, and sometimes iii) a number of quay cranes to each vessel. The problem can be represented in a two-dimensional space, where each vessel is a rectangle, whose dimensions are the vessel's length and handling time. These rectangles have to be placed within a larger rectangle, with dimensions quay length and considered time horizon, such that the smaller rectangles are not overlapping and satisfy some additional, case-related constraints. A vessel's handling time depends on both the amount of containers to be loaded and unloaded and the number of quay cranes assigned to the vessel. The objective of the single-terminal BAP is usually to minimize the total weighted handling time. In this paper, however, we allocate i) a terminal, ii) a time interval of berthing, and iii) a time-slot dependent crane capacity to each vessel. Although we guarantee that the sum of the lengths of all vessels berthing at the same time instance in the same terminal does never exceed the total quay length, the exact berthing position at a terminal is not allocated yet. In the same vein, the total crane capacity of a terminal is never exceeded, but the exact integer-valued crane allocation is yet to be determined. This implies that we reduce the problem to a number of interrelated, one-dimensional packing problems, which allow capacitated parallel processing.

Both static and dynamic BAPs are considered in the literature. The static case assumes that all vessels can be allocated to the berth at each point in time. Different solution methods for the static single-terminal BAP are addressed in [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], and [16]. In the dynamic case, vessels arrive while work is in progress. Different solution methods for the dynamic single-terminal BAP are addressed in [17], [18], and [9]. In this paper, we consider a static multi-terminal allocation problem.

Furthermore, in the single-terminal BAP, both time and space dimensions can be modeled either discrete or continuous. If the space dimension is considered to be discrete, the terminal is divided into a finite set of segments. A vessel can only berth at one (or more) of these segments, which means that the length is only partially taken into consideration. Different solution methods for the discrete space BAP are addressed in [19], [20], [6], [17], [18],[14], [15], [7], [8], [9], and [16]. In the continuous space BAP, vessels can berth anywhere along the terminal ([10], [11], [12], [13], and [16]). Since we only allocate a terminal to each vessel, the allocation of a specific berth position within the terminal is not taken into consideration. This reduces the problem to a one-dimensional packing problem, which can be solved much faster. The exact position allocation within a terminal is solved in a different study. Preliminary findings suggest that in all practical cases this position allocation is feasible.

Most studies of the single-terminal BAP allocate a real-valued start and end time of berthing to each vessel ([19], [20], [17], [18],[14], [15], [7], [8], [9], [16], [10], [11], [12], [13], and [16]). A restricted number of studies of the single-terminal BAP considers time to be discrete, where a vessel is allocated to a berth position for a number of discrete time slots [6]. In this paper, we consider time to be discrete and allocate a vessel to a terminal for a number of discrete time slots.

Most of the studies of the single-terminal BAP assume a fixed processing time of each vessel and leave required quay crane capacities aside. The study in [6] considers the single-terminal BAP and allocates an integer number of quay cranes to each vessel during each time slot in a first step. In a second step, a detailed quay crane allocation is constructed. Both steps are solved using heuristic methods. In this paper, we allocate a terminal, a berthing interval, and a time-slot dependent real-valued quay crane capacity to each vessel. Assuming a continuous quay crane capacity does not introduce additional integer variables as in [6], which enables us to solve the formulated MILP relatively fast. Transforming the real-valued crane capacity allocations into integer-valued ones is solved in a different study. Also here, preliminary findings suggest that in all practical cases this integer-valued crane allocation is feasible.

Previous studies of the single-terminal BAP consider a set of vessels within a certain linear time horizon. The corresponding objective in these studies often reduces to fitting all vessels within a time horizon and minimizing the total weighted handling time for all vessels. However, in practice most vessels run a regular service on their ports, for instance once a week, which makes the horizon cyclic. Vessels can arrive at the end of the considered cycle and leave at the beginning of the next cycle. Relating this to the packing problem implies that rectangles (vessels) may need to be cut into two, where one piece is placed at the end and the other piece at the beginning of a time period, both at the same position in space. In this paper, this property is taken into consideration, while so far, to our knowledge, this has not yet been studied for the BAP.

As mentioned before, the multi-terminal BAP has not yet been considered. So far, studies present models and algorithms, which only solve some version of the single-terminal BAP. However, present ports often consist of a cluster of terminals (see Figure 1), where inter-terminal container transport is established by trucks and barges. Since most ports face a significant amount of transshipment traffic, the allocations of vessels to different terminals determine the amount of inter-terminal transport and these allocations should not be considered separately to obtain meaningful results.

Hence, in order to derive an optimal allocation, it is necessary to incorporate the total of interrelated terminals and their inter-terminal container flow in one model: this is called the multi-terminal BAP. Therefore, this model should also take into account the amount of inbound and outbound containers and their corresponding destinations. Inbound containers of an arriving vessel for instance could be partly destined for the hinterland and partly for another vessel. Hence, allocation of the two involved vessels to different terminals implies inter-terminal traffic and thus additional costs. However, due to other objectives and constraints this may still be the best or only solution.

In [21], two MILP formulations have been proposed to model and optimize this allocation problem in a multi-terminal port, where a set of vessels arrives cyclically. Experiments showed that a straightforward approach is outperformed by an alternative formulation from a computational point of view. Furthermore, these experiments suggested that with the alternative formulation, real-life problems can be solved within a couple of hours. In this paper we confirm this suggestion, and show how we can substantially further reduce the solution time for practical cases.

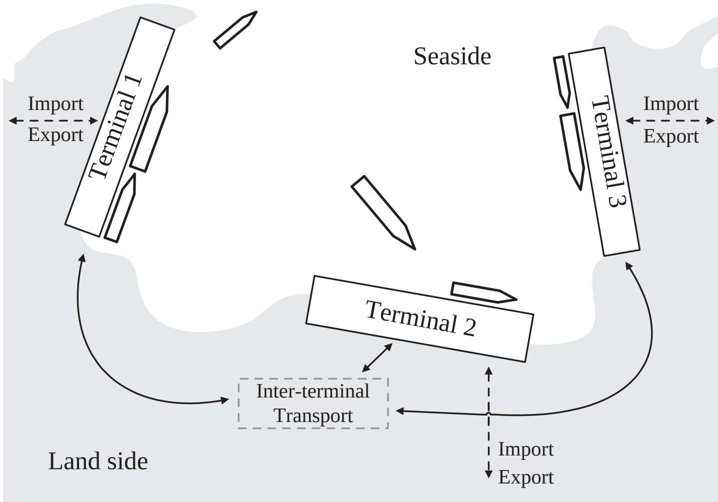


Figure 1: A cluster of three interrelated terminals.

Contributions and Outline

In this paper, the MILP formulation of the cyclic multi-terminal BAP, addressed in [21], is firstly considered. In the MILP formulation, a terminal, a berthing interval, and a variable quay crane capacity is allocated to each vessel. The study aims for a strategic time table and hence leaves system disturbances (e.g. vessel delays) aside. Although the constraints guarantee that terminal quay lengths as well as quay crane capacities are never exceeded, the exact berthing positions and the integer-valued quay crane allocation within a terminal are still to be determined at a tactical level. Basically, we consider a number of interrelated, one-dimensional packing problems, each of which allows capacitated parallel processing. The actual assignment of berths and cranes is left to a tactical level. Separating strategic and tactical levels (see Figure 2), we are able to construct an accurate allocation for real-life instances (in the order of 3 terminals, 20 quay cranes and 40 vessels) at a strategic level rather fast. Moreover, given such vessel allocations, fixing some where others are still to be determined, as is often the case in practice, leaves an optimization problem that is solved in a matter of seconds. This would also allow it to be applied at an operational level, e.g. in case of delays. Work in progress suggests that for real-life data, feasible position and quay crane allocations on a tactical level exist.

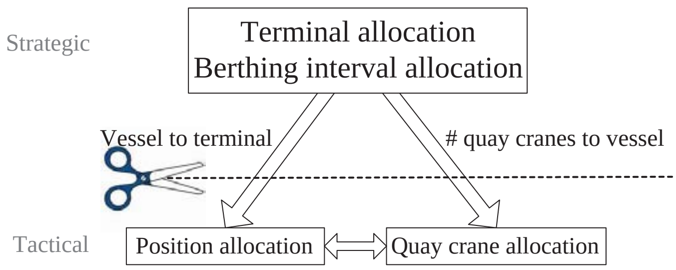


Figure 2: Schematic illustration of the chosen cut.

In this paper, we investigate the benefit of modifying given allocations for a cyclic multi-terminal container operation. The research is supported by the terminal operator PSA Hesse-Noord Natie, located in Antwerp, Belgium.

A two-step optimization is introduced to efficiently investigate the benefit of modification. In the first step, we adapt the alternative MILP formulation in [21] by allowing bounded flexibility in arrival and departure times. For an arbitrarily selected set of vessels the terminal allocation and the time position of the berthing interval can now be adapted to reduce the costs for quay cranes and inter-terminal transport. The resulting allocation is generated on an 8-hours time slot grid within minutes of computation time. In the second step of the optimization, a similar MILP as in the first step is proposed to refine the obtained allocation to a one-hour time slot grid within seconds. We perform two experiments, which suggest that with a small modification significant cost reductions can be achieved. Whether such a reallocation is practically and commercially feasible is beyond the scope of this paper.

The outline of the paper is as follows: in Section 3, we formally phrase the problem of the cyclic multi-terminal BAP. Then in Section 4, the alternative MILP formulation of [21] is used in a case study: first, we expand the MILP formulation with additional constraints to model the bounded flexibility. CPLEX is able to solve the MILP on an eight-hours time grid within minutes for cyclic problems with 3 terminals, 20 cranes, and 40 vessels. Next, we show that a similar model can be applied to refine the constructed allocation per terminal to a one-hour time grid within less than a tenth of a second. With this approach we can choose any desired level of flexibility and quickly determine the potential cost savings. In a first experiment, a given terminal and time allocation is optimized by allowing a bounded time flexibility and a different terminal allocation for only a couple of vessels. By varying the allowed deviations in arrival times (the amount of flexibility), we construct Pareto frontiers of the number of required quay cranes versus the inter-terminal transport costs. Results suggest that slightly adapting the current allocation already leads to a saving of about 25% of the number of quay cranes and at the same time a saving of over 3% of the costs for inter-terminal transport. In a second experiment, the terminal allocation of all vessels is allowed to be adapted while the time allocation is fixed. Results suggest that, with the current number of required quay cranes but a different terminal allocation, about 40% of inter-terminal costs can be saved. Next, we confirm that for all generated allocations, feasible position and quay crane allocations on a tactical level exist. Finally, we draw conclusions and discuss ongoing research projects.

Mathematical Model

In this section, the multi-terminal allocation problem is formally defined. The accompanying MILP formulation (without costs for deviating from the current schedule), described as the alternative approach in [21], is presented in the Appendix.

System Description

For all of this paper the following holds, unless stated differently: $t \in \{1, 2, \ldots, T\}$, the cluster of terminals, $v \in \{1, 2, \ldots, V\}$, the set of vessels, $z \in \{0, 1, 2, \ldots, V\}$, the set of container destinations. Furthermore, we assume vessels to call cyclically, where each vessel in the set arrives exactly once a week. We consider discrete time k and unless stated differently, $k \in \{1, 2, \ldots, K\}$, is the set of discrete time slots within the cycle.

In the cluster of terminals, the set of container vessels has to be unloaded and loaded. Vessel v imports a pre-determined number of inbound containers $I_{vz} \in \mathbb{N}$ with destination(s) z, where $v \neq z$. In this context, $z = 0$ means that containers are destined for the hinterland, whereas

$z = 1, 2, \ldots, V$ means that containers are destined for vessels $v = 1, 2, \ldots, V$ respectively. Besides import containers brought in by vessels, a certain amount of containers H_v with destination v is imported from the hinterland by trucks and trains. These containers have to be distributed among the different terminals dependent on their destinations. Furthermore, each vessel v exports a number of outbound containers $O_v \in \mathbb{N}$. Container transport between the cluster of terminals is established by trucks.

Terminal t has a restricted quay length $L_t \in \mathbb{R}^+$ and a number of quay cranes $N_t \in \mathbb{N}$ to unload and load the vessels. Once berthing, vessel v requires a certain amount of quay meters l_v. In addition, this length l_v determines the maximum number of quay cranes $S_v \in \mathbb{N}$ processing vessel v and the efficiency $\eta_v \in [0, 1]$ of the quay cranes on vessel v. In practice, quay cranes with different processing rates are present in the terminals. We do not take the specific allocation of quay cranes to vessels into account, but we use the average processing rate $\bar{\lambda}_t \in \mathbb{N}$ of all quay cranes in terminal t. Then the handling time of vessel v in terminal t depends on i) the mean processing rate $\bar{\lambda}_t$ in terminal t, ii) the efficiency η_v of quay cranes operating vessel v, iii) the number of quay cranes processing vessel v and iv) the number of inbound and outbound containers I_{vz} and O_v of vessel v. We assume the processing time of vessel v to be inversely proportional to the first three of these items and proportional to the latter. Furthermore, the number of quay cranes processing vessel v may change from one time slot to another.

After the unloading and before the loading, containers can temporarily be stored in the yard of terminal t up to the yard's capacity W_t. The number of time slots it takes to transport containers from terminal p to terminal r is defined as $\Delta_{pr} \in \mathbb{N}$, where $p \in \{1, 2, \ldots, T\}$, $r \in \{1, 2, \ldots, T\}$ and $p \neq r$. Furthermore, we assume that the total number of time slots vessel v is actually berthing, is less than the number of time slots K in the cycle. In addition, we assume that vessels arrive at the beginning of a time slot and depart at the end of a time slot.

Our goal is to minimize the quay crane and inter-terminal costs when modifications to an existing allocation are allowed. Hence, the current terminal allocation as well as the current arrival and departure time (A_v and D_v, respectively) are taken as a starting point. Next, we allow some modifications to the allocations of some arbitrarily chosen vessels. With respect to the terminal allocation, this modification can be an allocation to a different terminal. With respect to the modifications in the time position of the berthing interval, we define a flexibility gap: although we demand that the length of the current berthing interval P_v of vessel v is retained, the time position of this interval can be placed maximally G_v time slots earlier or G_v time slots later than in the current allocation. The resulting lower and upper bounds for the arrival and departure time are defined as A_v^l and D_v^u, respectively. Consequently, the costs for arriving and departing too late in [21] are left out. Two conflicting objectives remain: first of all, costs are associated with the number of quay cranes required to perform the proposed schedule. We define c_t to be the costs of a quay crane in terminal t. Second of all, a fixed amount of money c_{pr} has to be paid for each container that is transported from terminal p to terminal r, where $p \in \{1, 2, \ldots, T\}$, $r \in \{1, 2, \ldots, T\}$ and $p \neq r$.

Due to the cyclic property of the considered system, we require conservation with respect to the arriving and departing containers:

$$\sum_{i=1}^{V} I_{iv} + H_v = O_v \qquad \forall v. \tag{1}$$

The sets and parameters discussed above are conveniently arranged in Table 1.

Parameter	Definition
T	Number of terminals in the cluster
V	Number of vessels in the set
K	Number of discrete time slots within the cycle
L_t	Quay length [m]
l_v	Quay length required for vessel [m]
I_{vz}	# inbound containers to be unloaded from vessel v with destination z and $v \neq z$
O_v	# outbound containers to be loaded onto vessel v
H_v	# containers with destination v arriving from the hinterland during the cycle
A_v	Current arrival time of vessel v
D_v	Current departure time of vessel v
G_v	Flexibility gap for time allocation of vessel v [hrs]
A_v^l	Earliest arrival time of vessel v
D_v^u	Latest departure time of vessel v
P_v	Contractual berthing/ process time of vessel v
E_v	Parameter to distinguish between the cases $A_v < D_v$ and $A_v \geq D_v$
N_t	# quay cranes available in terminal t
S_v	Maximum # quay cranes, which can process vessel v
$\bar{\lambda}_t$	Mean processing rate of quay cranes in terminal t [containers/time slot]
η_v	Vessel efficiency with respect to quay crane rate [-]
Δ_{pr}	# time slots needed to transport containers from terminal p to r
W_t	# containers that can be stored in terminal t
c_{pr}	Costs for transportating a container from terminal p to r [euro/ container]
c_t	Costs per quay crane in terminal t [euro/ quay crane]

Table 1: Model parameters

Case Study

In practice, a vessel line owns a number of (identical) container vessels, which follow the same route, leading along several ports all over the world. This route may take the vessels a couple of weeks or even months. We assume that the number and the time phasing of the vessels of a line is such that exactly once a week, one vessel of this line calls at a port at a fixed time. Additionally, we assume that all vessels of a line have the same length and carry the same number of inbound and outbound containers. It therefore suffices to construct a one-week allocation, which is repeated over and over again.

We consider three interacting terminals ($T = 3$) in the port of Antwerp, where thirty-seven vessel lines have one of their vessels processed exactly once a week ($V = 37$). Furthermore, we assume that each vessel line has a preferred terminal and a preferred arriving and departure time, which fit best to their schedule. The current policy of PSA HNN is commercially driven and aims to satisfy the preferred allocation as good as possible. The costs for the corresponding number of quay cranes and inter-terminal traffic are of lower priority in this policy.

We think significant cost savings that can be obtained if these two objectives are of higher importance. The quay cranes for instances are the most expensive devices in a container port. A small reduction in the number of quay cranes would therefore already lead to substantial cost savings.

As an illustration, Figure 3 depicts the scaled number of used quay crane capacity in one of the terminals for each hour in a one week cycle ($K = 168$) according to the allocation as currently applied in Antwerp. The black line represents the scaled mean crane capacity usage per hour. From the high fluctuations in crane usage, the following can be concluded: i) during a couple of time slots, a certain amount of quay cranes is needed and ii) during a lot of time slots a large percentage of this amount is simply not used.

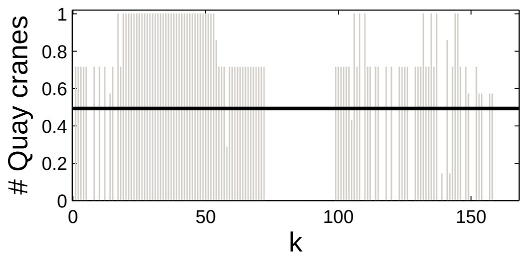


Figure 3: Current quay crane usage in terminal 1 during the hours of a one-week cycle.

We are interested in the benefit of modifying the current allocation, i.e. the potential reduction of the required number of quay cranes and the costs for inter-terminal transport when both the current terminal and time allocation are adapted. We expand the existing MILP of [21], such that for an arbitrarily chosen level of flexibility, the cost reduction is maximized.

Nowadays, the allocations in ports are constructed on a one or two hour(s) time grid. Since these sizes of time slots lead to large computation times when solving the MILP, the optimization is performed in two steps: First, the MILP is solved for time slots of eight hours. For a weekly cycle this means $K = 21$. The model can then be solved within minutes while allocations are generated quite accurately, since port employees work in shifts and vessels commonly berth during multiples of a shift. In a second step, a similar MILP is built to refine the constructed allocation per terminal to a one-hour time grid. Solving this MILP takes less than a tenth of a second.

The resulting two-step optimization approach enables us to efficiently investigate the dependency of the cost savings on the level of flexibility: a number of vessels and their level of flexibility can arbitrarily be selected. Next, these settings can easily be implemented in the model to determine possible savings in quay crane and inter-terminal transport costs. In Section 4.1, we discuss the two-step optimization in detail and apply it in two experiments presented in Sections 4.2 and 4.3.

In the first experiment, our hypothesis is that a slight adaptation of the allocation of only a couple of vessels already leads to large cost reductions. Hence, the terminal allocation of a small part of the vessels is chosen to be flexible and the time position of the berthing interval can be shifted between some pre-specified bounds. For different values of these bounds, we construct Pareto frontiers of the number of required cranes versus the inter-terminal transport costs, which confirm the hypothesis. One of the points from one of the Pareto-frontiers is highlighted to further illuminate the hypothesis. This point represents the allocation after selecting less than one third of the vessels and allowing i) a change in their terminal allocation and ii) a shift in the time position of the berthing interval of these vessels of maximal 24 hours ($G_v = 24$). The results of the two-step optimization suggest that about 25% of the number of quay cranes can be saved while at the same time the inter-terminal transportation costs are reduced by 3%.

In the second experiment, we assume that all vessel lines prefer to stay with their current time position of the berthing interval ($G_v = 0$), while they allow a different terminal allocation. Consequently, the terminal allocation of all vessels is chosen to be flexible while the time positioning of the berthing interval is fixed to the existing one. Results suggest that with the same number of quay cranes as in the current allocation, 40% of the costs for inter-terminal transport can be saved by adapting the terminal allocation.

Two-step optimization

Step 1

We want to minimize the number of quay cranes required for the current throughput, and at the same time reduce the costs for inter-terminal transport by adapting the current terminal and time allocation. We expect that changing the current terminal allocation of a couple of vessels and slightly shifting their current berthing interval in time already leads to a significant reduction in both these objectives. Hence, we select 11 vessels (out of the total set) from the busy peaks arbitrarily and define two sets: the set $\mathcal{S}$ to be the set of 11 vessels, which current terminal and berthing allocation can be adapted, and the set $\mathcal{F}_{ij}$ to be the set of index pairs of all the other vessels together with their current terminal. We have to specify additional constraints for the vessels in the different sets. First of all, for the vessels in $\mathcal{F}_{ij}$, the time-position of the berthing interval should be fixed:

$$a_i = A_i, \qquad i \in \mathcal{F}_{ij} \tag{2}$$

$$d_i = D_i, \qquad i \in \mathcal{F}_{ij} \tag{3}$$

Furthermore, the current terminal allocation for the vessels in $\mathcal{F}_{ij}$ should be fixed:

$$x_{ij} = 1, \qquad \forall i,j \in \mathcal{F}_{ij}, \tag{4}$$

The vessels in set $\mathcal{S}$ on the other hand are free to be allocated to any of the terminals according to (16). Additionally, we allow some freedom in the time allocation of their berthing interval: although the length of the berthing interval should be equal to P_v according to:

$$\sum_{k=1}^{K} b_v(k) = P_v, \qquad \forall v, \tag{5}$$

where $b_v(k)$ is 1 iff vessel v berths during time interval $[k, k+1\rangle$ and 0 otherwise. Furthermore, the time position of the berthing interval of the vessels in $\mathcal{S}$ can be placed maximally G_v time slots earlier or G_v time slots later with respect to its current time position as shown in Figure 4.

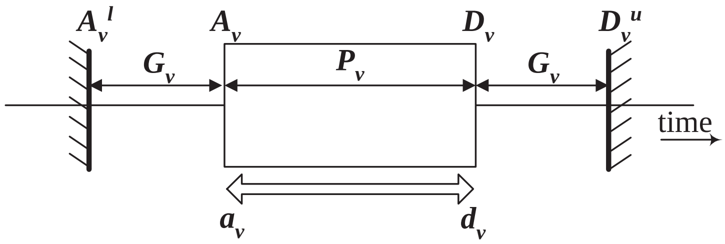


Figure 4: Flexibility in the time-position of the berthing interval P_v of vessel $v \in \mathcal{S}$.

The corresponding lower and upper bounds for berthing of vessel $v \in \mathcal{S}$, A_v^l and D_v^u respectively, are then given by:

$$A_v^l = \begin{cases} A_v - G_v & \text{if } A_v - G_v \geq 1, \\ A_v - G_v + K & \text{otherwise} \end{cases} \qquad \forall v \in \mathcal{S}, \tag{6}$$

$$D_v^u = \begin{cases} D_v + G_v & \text{if } D_v + G_v \leq K, \\ D_v + G_v - K & \text{otherwise} \end{cases} \qquad \forall v \in \mathcal{S}. \tag{7}$$

Due to the cyclic property of the system, we distinguish two cases and derive appropriate constraints for the vessels in $\mathcal{S}$ accordingly:

1. $A_v^l < D_v^u$:

$$e_v = 0, \tag{8}$$

$$a_v \geq A_v^l, \tag{9}$$

$$d_v \leq D_v^u. \tag{10}$$

2. $D_v^u < A_v^l$ (additional binary variables e_v^a and e_v^d are introduced):

$$K \cdot e_v^a \geq A_v^l - a_v \tag{11}$$

$$-K \cdot (1 - e_v^a) \leq A_v^l - a_v \tag{12}$$

$$K \cdot e_v^d \geq D_u^l - d_v \tag{13}$$

$$-K \cdot (1 - e_v^d) \leq D_u^l - d_v \tag{14}$$

$$d_v \geq a_v + K \cdot e_v^a - K \cdot e_v^d. \tag{15}$$

Experiments show that, with the resulting MILP, allocations on a grid of eight-hours time slots are generated within minutes. Further decreasing the width of the time slots turns out to lead to large computation times. Hence, as a first step, the model is built of eight hours time slots. The actual berthing time is therefore rounded up to a multiple of a shift to express the parameter P_v of vessel v in the model.

Step 2

Since in today's ports most allocations are constructed on a two-hours or even on a one-hours grid, we introduce a second step to refine the constructed allocation per terminal from eight-hours time slots into one-hour time slots. We define p_v^n to be the berthing time of vessel v on the refined time grid and thus $p_v^n \leq P_v$. Additionally, we require that the refined berthing time interval is positioned between the allocated arrival and departure time (optimal values $a_v^\star$ and $d_v^\star$ of the first step optimization) on the coarse time grid. Hence, we introduce the variables a_v^n and d_v^n to be the arrival and departure time of vessel v on the refined time grid, respectively. Hence in the second step, we build an MILP with similar constraints as given in (8) through (15), where A_v^l is substituted by $a_v^\star$, D_v^u by $d_v^\star$, a_v by a_v^n, d_v by d_v^n, and e_v^a and e_v^d by e_v^{na} and e_v^{nd}, respectively. The computation time turns out to be less then a second per terminal.

Experiment 1

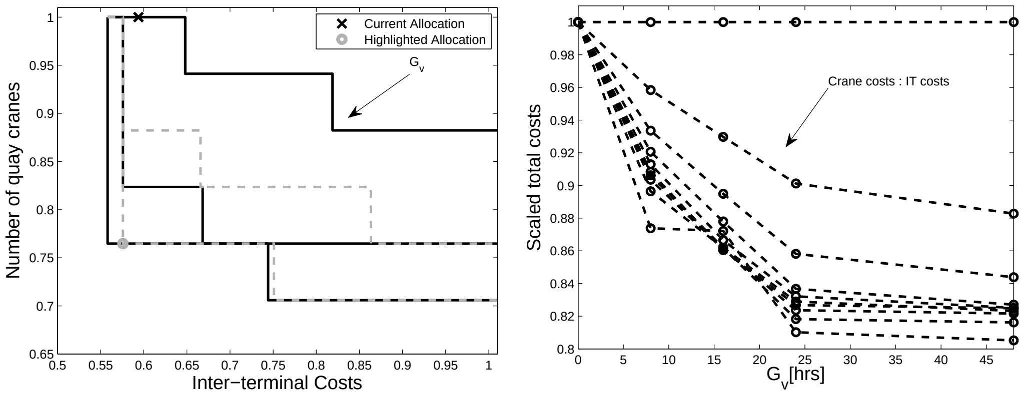
For each value of $G_v \in \{0, 8, 16, 24, 48\}$ (in hours) for the vessels in $\mathcal{S}$, a Pareto frontier of the total number of required quay cranes versus the costs for inter-terminal transport is constructed. Each point in a frontier results from a single two-step optimization with a specific ratio between costs for quay cranes and costs for inter-terminal transport. The results are depicted in Figure 5a. The cross represents the state of the allocation currently applied in Antwerp. From Figure 5a the following can be concluded:

- For $G_v = 0$ for $v \in \mathcal{S}$ yet the costs for inter-terminal transport or the number of required quay cranes can be reduced. Apparently, an adaptation in the terminal allocation of the vessels in $\mathcal{S}$ suffices to achieve this.
- For $G_v = 0$ for $v \in \mathcal{S}$ yet a reduction in the number of quay cranes is possible at the expense of higher inter-terminal costs.
- The improvements going from $G_v = 0$ to $G_v = 8$ are relatively large, whereas the improvements going from $G_v = 24$ to $G_v = 48$ are approximately zero.
- All fronts intersect (the upper left point) where the crane costs are zero. Apparently, the inter-terminal costs cannot be further reduced even if G_v grows and the maximum number of cranes is used.
- The grey bullet suggests that if $G_v = 24$ for $v \in \mathcal{S}$ the number of required quay cranes can be reduced by almost 25% and the costs for inter-terminal by about 3%. This means that besides a possible change in terminal allocation, the time allocation of only 11 vessels has to be shifted one day maximally to gain significant improvements.

The quay crane usage of the allocation, represented by the grey bullet in Figure 5a, is depicted in Figures 6b, 6d and 6f. The results are scaled to the quay crane usage in the current allocation as shown in Figures 6a, 6c and 6e. The black lines represent the mean quay crane usage per hour in the different terminals. If we compare Figures 6a, 6c, 6e with Figures 6b, 6d, 6f, the following can be noticed:

- The workload in the generated allocation is better balanced than in the current allocation. This results in the previously mentioned reduction of almost 25% of the required quay cranes.
- At some points in time still some quay cranes are not working in the generated allocation. Introducing either a higher level of flexibility (by increasing G_v, $v \in \mathcal{S}$) or including more vessels into the set $\mathcal{S}$ would probably fill up these gaps and lead to an even better workload balance and a smaller number of required cranes.
- The mean quay crane usage in a specific terminal can differ for the current allocation and the generated allocation. This can be explained by a difference in terminal allocation of the vessels in $\mathcal{S}$. The total quay crane usage however is equal for both allocations.

Additionally, we depict the benefit of modification in a different way. For a constant ratio of quay crane costs and inter-terminal costs, the scaled total costs are plotted versus the level of flexibility G_v. Figure 5b presents the results for 10 ratios ($\frac{c_t}{c_{pr}} \in \{0, 20, 40, 60, 80, 100, 120, 140, 160, \infty\}$). From this figure, the following is noticed:



(a) Pareto frontiers..

(b) Costs vs flexibility level G_v for different cost ratios.

Figure 5: Improvements of introducing a flexibility level.

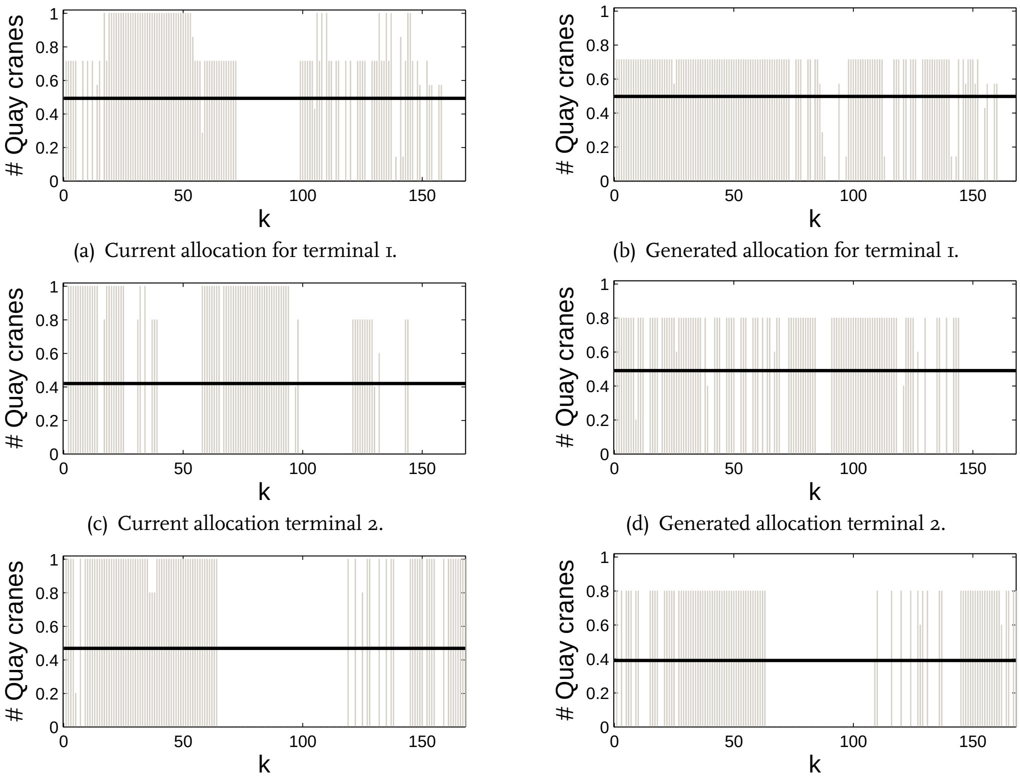
- For the ratio equal to 0 (no costs for quay cranes), the total costs (for this ratio only inter-terminal costs) are not affected as the level of flexibility increases. Apparently, the inter-terminal costs are not affected by G_v, because the terminal length and crane capacity are not binding at $G_v = 0$, so that the 'best' (lowest inter-terminal traffic) solution is already obtained.
- For all ratios larger than zero the total costs decrease as the level of flexibility increases.
- As the ratio grows between zero and forty, the total costs-decrease becomes relatively large. Apparently, these are the sensitive ratios, where a slide increase of the crane costs already leads to a significant reduction in quay cranes. This suggests that the current number of quay cranes is far from minimal.
- After the cost ratio of forty, the scaled cost curve approximately stays in steady-state. For these ratios, the relative quay crane costs are that large, that further increasing them does not significantly affect the costs.

Experiment 2

In this experiment we assume that none of the vessel lines is prepared to change their berthing times in Antwerp, however a change in terminal allocation is allowed by all lines. We are interested in decreasing the current costs for inter-terminal transport, while the current berthing times are remained. Additionally, we require that the number of quay cranes needed is at most equal to the number of quay cranes required for the current allocation. Hence, we allow a terminal adaptation for each vessel and fix its berthing interval in time to the current allocation ($G_v = 0$). Figure 7 shows the cumulative costs for inter-terminal transport for the current allocation and the generated allocation for each hour in the weekly cycle. The costs are scaled to the total costs in the current allocation. These results suggest that, with the same number of quay cranes, about 40% of the costs for inter-terminal transport can be saved.

Feasibility

Although the proposed method allocates a terminal, a time interval of berthing and an time varying number of quay cranes to a vessel, the actual position within that terminal as well



(a) Current allocation for terminal 1.

(b) Generated allocation for terminal 1.

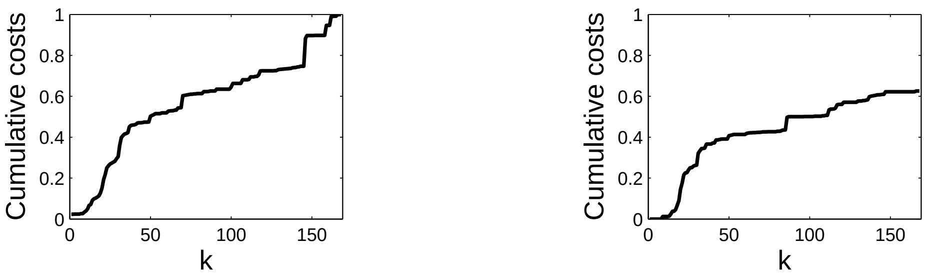
(c) Current allocation terminal 2.

(d) Generated allocation terminal 2.

(e) Current allocation for terminal 3.

(f) Generated allocation for terminal 3.

Figure 6: Current and generated quay crane usage during the hours of a one-week cycle.



(a) Inter-terminal costs current allocation.

(b) Inter-terminal costs generated allocation.

Figure 7: Current and generated cumulative inter-terminal costs for a one-week cycle.

as the actual quay cranes processing that vessel are not specifically generated (see also Figure 2). The omission of these allocations allows to solve the model relatively fast, at the expense of possible infeasibility of the found solution on a tactical level. If the terminals would continuously be utilized against their quay lengths capacities, this could lead to a situation where (24) is fulfilled, but a feasible two-dimensional packing solution does not exist. In practice however, ports require a significant utilization margin to compensate for disturbances (e.g. early/late arrivals and departures) on an operational level. In a related study, we are currently developing an algorithm to optimize the position and quay crane allocation after the generation of the strategic allocation. With the current status of these algorithms, we are already able to construct a feasible two-dimensional packing solution and a feasible crane allocation for the strategic allocations generated in the experiments shown in this paper. Since we focus on an allocation at a strategic scale, an (arbitrary) feasible solution on an operational level is satisfactory at this point in time. The ongoing study can possibly i) optimize the position and quay crane allocation at a second level or ii) incorporate the position and quay crane allocation in the top-level optimization.

Conclusions and Future Work

We have considered a cluster of interrelated terminals, at which a set of container vessels calls cyclically. By abstracting from specific position and quay crane allocations, an MILP formulation enables us to strategically allocate a terminal, a berthing interval, and a time-slot dependent real-valued crane capacity to each of the vessels in the cycle. A two-step optimization approach was introduced adapting the MILP to perform a case study, based on a multi-terminal port in Antwerp. This approach enabled us to efficiently investigate the benefit of modifying an existing allocation, i.e. the potential crane and inter-terminal transport cost savings if the existing terminal and time allocations were to be adapted. Pareto frontiers were presented to give insights in the possible reduction of quay cranes at the expense of higher inter-terminal transportation costs, and vice versa. Results suggest that a small adaptation of an existing allocation suffices to gain significant improvements: a reduction of almost 25% of the number of cranes and at the same time a reduction of more than 3% of the inter-terminal costs. Furthermore, if the current terminal allocation of all vessels is allowed to be adapted while the current time allocation is fixed, costs for inter-terminal transport can be reduced by 40%. Whether such a reallocation is practically and commercially feasible is beyond the scope of this paper. Finally, work in progress confirms that the generated strategic allocations are feasible on an operational level meaning that actual position and quay crane allocations exist.

We make the following remarks with respect to current and future research:

- Usually a contract between a terminal operator and a shipping line stipulates that the terminal operator has to process a vessel within a certain time, provided that the vessel arrives within a certain agreed time window around the scheduled arrival time. We have recently developed a model to analyze the sensitivity of a strategic allocation to these time windows. This means that we can determine the maximum number of quay cranes needed in the worst-case scenario of all possible arrival patterns for a strategic allocation. Additionally, we have expanded this model to minimize this worst-case scenario, if adaptations to this strategic allocation are allowed.
- As mentioned before, in an ongoing study algorithms are being developed to show the feasibility of the strategic allocation at a tactical level, and to further optimize these position and quay crane allocations.
- The current strategic allocation assumes all parameters to be deterministic. At an operational level however, the considered processes, e.g. arrival of vessels and quay crane productivity, are stochastic. Due to these stochastic properties, the strategic allocation has to be continuously adapted. We are currently developing a method to quickly adjust a given allocation, if operational disturbances occur.
- In the current paper, we assumed all vessels to arrive exactly once during the cycle, implying that all vessels have the same period. However, in practice it can happen that some vessels have different cycle lengths. An extension of the model, which incorporates this phenomenon, is an interesting future study.

- In today's ports, inter-terminal transport is not only established by trucks, but also by barges. In the current approach we model the resource utilization of the barges by simply reducing the quay lengths by 200 meters and dedicating one quay crane in each terminal to barge operations. It is worth investigating the trade-off between the amount of inter-terminal transport by trucks and barges and therefore worthwhile extending the current model with the actual loading and unloading of barges.

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Appendix I: MILP formulation

Integer decision variables

$$x_{tv} = \begin{cases} 1 & \text{if in terminal } t \text{ vessel } v \text{ berths,} \\ 0 & \text{otherwise.} \end{cases}$$

Continuous decision variables

a_v = Actual berth time slot of vessel v (start of processing vessel v)
d_v = Actual departure time slot of vessel v
$q_{tv}(k)$ = Amount of quay cranes processing vessel v in terminal t during time slot $[k, k+1\rangle$

Integer auxiliary variables

$$b_v(k) = \begin{cases} 1 & \text{if vessel } v \text{ is berthing during time slot } [k, k+1\rangle, \\ 0 & \text{otherwise.} \end{cases}$$

$$e_v = \begin{cases} 1 & \text{if } a_v > d_v, \\ 0 & \text{if } a_v < d_v, \\ 1 & \text{if } a_v = d_v \text{ and vessel } v \text{ is continuously berthing,} \\ 0 & \text{if } a_v = d_v \text{ and vessel } v \text{ does not berth at all.} \end{cases}$$

$$e_v^a = \begin{cases} 1 & \text{if } a_v < A_v^l, \\ 0 & \text{if } a_v \geq A_v^l. \end{cases}$$

$$e_v^d = \begin{cases} 1 & \text{if } d_v < D_v^u, \\ 0 & \text{if } d_v \geq D_v^u. \end{cases}$$

n_t = Number of quay cranes required in terminal t

Continuous auxiliary variables

$m_{tv}(k)$ = Amount of quay meters consumed in terminal t by vessel v during time slot $[k, k+1\rangle$ [m]
$q_{tv}(k)$ = Amount of quay cranes processing vessel v in terminal t during time slot $[k, k+1\rangle$
$h_{tv}(k)$ = Amount of containers from hinterland transported into terminal t with destination v during time slot $[k, k+1\rangle$ [containers/ time slot]
$f_{prv}(k)$ = Amount of containers transported from terminal p to terminal r with destination v during time slot $[k, k+1\rangle$ [containers/ time slot], p /= r
$w_{tv}(k)$ = WIP in terminal t with destination v at time k
Δ_v^a = Number of time slots vessel v berths too late
Δ_v^c = Number of time slots vessel v departs too early
Δ_v^d = Number of time slots vessel v departs too late

Constraints

Vessel v berths at only one terminal t:

$$\sum_{t=1}^{T} x_{tv} = 1 \qquad \forall v. \tag{16}$$

The arrival and departure times (a_v and d_v respectively) of vessel v are within the cycle:

$$1 \le a_v \le K \qquad \forall v \tag{17}$$

and

$$1 \le d_v \le K \qquad \forall v. \tag{18}$$

Vessel v berths between its arrival and departure time, a_v and d_v respectively. We need generic constraints, which relate a_v and d_v to $b_v(k)$ as well as $b_v(k)$ to a_v and d_v for the cases where $a_v < d_v$, $a_v = d_v$ and $a_v > d_v$. To incorporate the latter case, which follows from the cyclic property of the system, we introduce an auxiliary binary variable e_v:

$$\sum_{k=1}^{K} \left(b_v(k) - e_v\right) = d_v - a_v \qquad \forall v \tag{19}$$

and

$$1 - a_v \le k \cdot \left(b_v(k) - e_v\right) \le d_v - 1 \qquad \forall v, k \tag{20}$$

and

$$d_v - K \le \left(K - k\right) \cdot \left(b_v(k) - e_v\right) \le K - a_v \qquad \forall v, k. \tag{21}$$

The two possible scenarios resulting from the cyclic system property are depicted in Figure 8. Vessel v requires an amount of quay meters l_v at a terminal t during time slot $[k, k+1\rangle$, iff

(a) $e_v = 0$. (b) $e_v = 1$.

Figure 8: Possible arrival A_v and departure D_v scenarios of vessel v in a weekly cycle.

the vessel is actually berthing at that particular terminal t and during that particular time slot $[k, k+1\rangle$.

$$m_{tv}(k) \le l_v \cdot x_{tv} \qquad \forall t, v, k \tag{22}$$

and

$$\sum_{t=1}^{T} m_{tv}(k) = l_v \cdot b_v(k) \qquad \forall v, k. \tag{23}$$

Furthermore, the sum of lengths of all vessels berthing at terminal t during time slot $[k, k+1\rangle$ should be less than or equal to the total quay length of terminal t:

$$\sum_{v=1}^{V} m_{tv}(k) \le L_t \qquad \forall t, k. \tag{24}$$

Vessel v can only be operated in terminal t iff vessel v is berthing in terminal t. Furthermore, a maximum number of quay cranes S_v can be assigned to vessel v:

$$q_{tv}(k) \le S_v \cdot x_{tv} \qquad \forall t, v, k \tag{25}$$

and

$$q_{tv}(k) \leq S_v \cdot b_v(k) \qquad \forall t, v, k. \tag{26}$$

Vessel v has to be fully processed during the cycle:

$$\sum_{t=1}^{T} \sum_{k=1}^{K} \eta_v \bar{\lambda}_t \cdot q_{tv}(k) = \sum_{z=0}^{Z} I_{vz} + O_v \qquad \forall v. \tag{27}$$

We want to minimize the maximum number of quay cranes in terminal t ever required during the cycle. Therefore, we introduce an auxiliary variable n_t, which is a soft upper bound on the number of quay cranes in terminal t. This variable n_t is present in the objective function:

$$\sum_{v=1}^{V} q_{tv}(k) \leq n_t \qquad \forall t, k. \tag{28}$$

The maximum number of quay cranes ever required in terminal t during the cycle cannot be larger than the number of quay cranes actually available in terminal t:

$$n_t \leq N_t \qquad \forall t. \tag{29}$$

The sum over the cycle's time slots of the number of containers with destination v, transported from the hinterland into the different terminals, should be equal to the total number of containers with destination v arriving from the hinterland during the cycle.

$$\sum_{k=1}^{K} \sum_{t=1}^{T} h_{tv}(k) = H_v \qquad \forall v. \tag{30}$$

Since the system is cyclic, the storage level in the terminals and the inter-terminal transport during time slot $[k, k+1\rangle$ should equal the storage level in the terminals and the inter-terminal transport during time slot $\langle k-1+\alpha K, k+\alpha K]$, where $\alpha \in \mathbb{N}$:

$$w_{tv}(k) = w_{tv}(k+\alpha K) \qquad \forall t, v, k \tag{31}$$

and

$$f_{prv}(k) = f_{prv}(k+\alpha K) \qquad \forall p, r, v, k. \tag{32}$$

We assume that inbound containers with destination 0 ("hinterland") are transported into the hinterland directly after they arrive in the terminal and are not counted as stack.

The amount of containers in terminal t with destination v during time slot $[k, k+1\rangle$ is equal to the amount of containers in terminal t with destination v during time slot $[k, k+1\rangle$ plus all incoming flows (inbound containers from vessels, containers from other terminals and containers from the hinterland) minus all outgoing flows (outbound containers to vessels and containers to other terminals). We assume that loading and unloading of containers from vessel v with different destinations is divided proportionally among the time slots vessel v is actually berthing. To this, we first define the constants $\beta_{vz} = \frac{I_{vz}}{\sum_{z=0}^{Z} I_{vz}+O_v}$ and $\gamma_v = \frac{O_v}{\sum_{z=0}^{Z} I_{vz}+O_v}$, and derive appropriate constraints:

$$\begin{aligned} w_{tv}(k) \quad = \quad & w_{tv}(k-1) + \sum_{i=1}^{V} \beta_{iv}\eta_i\bar{\lambda}_t \cdot q_{ti}(k) - \gamma_v\eta_v\bar{\lambda}_t \cdot q_{tv}(k) + h_{tv}(k) + \\ & \sum_{r=1}^{T} f_{rtv}(k-\Delta_{pr}) - \sum_{r=1}^{T} f_{trv}(k) \qquad \forall t, v, k. \end{aligned} \tag{33}$$

If we start with bringing containers into the yard during time slot $[k, k+1\rangle$, the following constraint has to be satisfied:

$$\sum_{v=1}^{V} \left(w_{tv}(k-1) + \sum_{i=1}^{V} \beta_{iv}\eta_i\bar{\lambda}_t \cdot q_{ti}(k) + h_{tv}(k) + \sum_{r=1}^{T} f_{rtv}(k-\Delta_{pr}) \right) \quad \leq \quad W_t \qquad \forall t, k. \tag{34}$$

If we start with taking away containers from the yard during time slot $[k, k+1\rangle$, the following constraint has to be satisfied:

$$w_{tv}(k-1) - \gamma_v \eta_v \bar{\lambda}_t \cdot q_{tv}(k) - \sum_{r=1}^{T} f_{trv}(k) \quad \geq \quad 0 \qquad \forall t, v, k. \tag{35}$$

Whatever order is applied during the cycle, (34) and (35) guarantee that never too much and never too less (negative amount of) containers are in the yard.

Finally, we have some additional non-negativity constraints:

$$\begin{aligned} q_{tv}(k) &\geq 0 & (36)\\ h_{tv}(k) &\geq 0 & (37)\\ f_{prz}(k) &\geq 0. & (38) \end{aligned}$$

Objective function

A linear unit penalty cost is assigned when containers are transported from one terminal to another (c_{pr}). Furthermore, linear costs are assigned to the number of required quay cranes in terminal t (c^t). The decision variables are represented in a vector $u(k) = [x_{tv}, a_v, d_v, h_{tv}(k), q_{tv}(k), f_{prz}(k)]^T$ and the objective function is formulated as follows:

$$\min_{u(1),\ldots,u(K)} \qquad \sum_{v=1}^{V} \sum_{k=1}^{K} \sum_{p=1}^{T} \sum_{r=1}^{T} \sum_{z=1}^{Z} c_{pr} f_{prz}(k) + \sum_{t=1}^{T} c_t n_t \tag{39}$$

Remark: In the solution of this MILP it could be that an arbitrary amount of containers is stored in a certain terminal during the entire cycle. This could be prevented by assigning a small cost for each stored container