Feedback control of 2 queues server with setups and finite buffers

J.A.W.M. van Eekelen, E. Lefeber and J.E. Rooda
Technische Universiteit Eindhoven, Department of Mechanical Engineering
Systems Engineering Group, P.O.Box 513, 5600 MB Eindhoven, The Netherlands
{J.A.W.M.v.Eekelen, A.A.J.Lefeber, J.E.Rooda}@tue.nl

1 Introduction

Consider a server serving two job types arriving at constant rates $\lambda_1 > 0$ and $\lambda_2 > 0$ and non-zero setup times σ_{12} and σ_{21} involved (shown in Figure 1 left). The maximum serve rate for the two types are $\mu_1 > 0$ and $\mu_2 > 0$ respectively. An optimal process cycle is derived with respect to minimal time averaged wip level (work in process, i.e. the number of jobs in the system). A feedback law is proposed that steers the system to this optimal process cycle. The analysis has been done for finite buffer capacity. Although based on continuous models, the feedback law has been implemented successfully in a discrete event system.

2 Notation and optimal steady state cycle

The buffer levels are denoted by $x_1 \geq 0$ and $x_2 \geq 0$ and can not exceed the maximum capacity $x_1^{\max}$ and $x_2^{\max}$. Switching from serving type 1 to type 2 takes $\sigma_{12} > 0$ time units and $\sigma_{21} > 0$ time units the other way around. We define the utilization for type i as $\rho_i = \frac{\lambda_i}{\mu_i} < 1$. Serving jobs of type 1 or 2 is abbreviated as ① and ② respectively. Setting up for type 1 or 2 is abbreviated as ❶ and ❷. The optimal steady state process cycle with respect to wip-level is shown in Figure 1 (on the right, proof see [1, 2]). The points $(x_1^\sharp, 0)$ and $(0, x_2^\sharp)$ indicate the coordinates where setups ❶ and ❷ are started. Without loss of generality, we assume that $\lambda_1 \geq \lambda_2$. The vertical part of ① in this figure is the so called 'slow mode' (jobs are processed at arrival rate λ_1), which only occurs if $\lambda_1(\rho_1 + \rho_2) + (\lambda_2 - \lambda_1)(1 - \rho_2) < 0$ (proof see [1]).

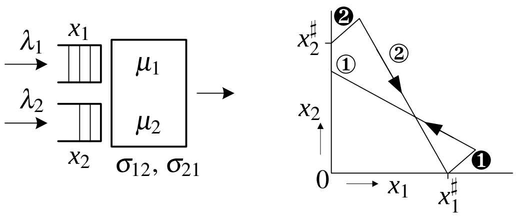

Figure 1: Server (left) and optimal steady state cycle (right).

3 Feedback control

A feedback control law which steers a process trajectory to the optimal process cycle from arbitrary feasible start point consists of 6 modes. Depending on the state of the system, the controller is in one of these modes, which follows trivially from the mode descriptions:

- Mode 1: ① at μ_1 as long as $x_1 > 0$ and $x_2 < x_2^{\max} - \lambda_2\sigma_{12}$, then go to Mode 2.
- Mode 2: ① at λ_1 as long as $x_2 < x_2^\sharp$, then go to Mode 3.
- Mode 3: perform ❷, after σ_{12} go to Mode 4.
- Mode 4: ② at μ_2 as long as $x_2 > 0$ and $x_1 < x_1^{\max} - \lambda_1\sigma_{21}$, then go to Mode 5.
- Mode 5: ② at λ_2 as long as $x_1 < x_1^\sharp$, then go to Mode 6.
- Mode 6: perform ❶, after σ_{21} go to Mode 1.

4 Discrete event example

The proposed control law (convergence is proven in [1]) has been implemented in a discrete event simulation (using χ [3]) with parameter settings as in Table 1. For these settings, $x_1^\sharp = 27$ (jobs) and $x_2^\sharp = 18$ (jobs). Simulation results are shown in Figure 2. The figure shows that convergence to the optimal process cycle is reached.

Table 1: Discrete event simulation settings.

λ_1:	9 jobs/hr.	σ_{12}:	2 hrs.	x_1^0:	36 jobs
λ_2:	3 jobs/hr.	σ_{21}:	2 hrs.	x_2^0:	24 jobs
μ_1:	24 jobs/hr.	$x_1^{\max}$:	60 jobs	initial:	②
μ_2:	27 jobs/hr.	$x_2^{\max}$:	30 jobs		

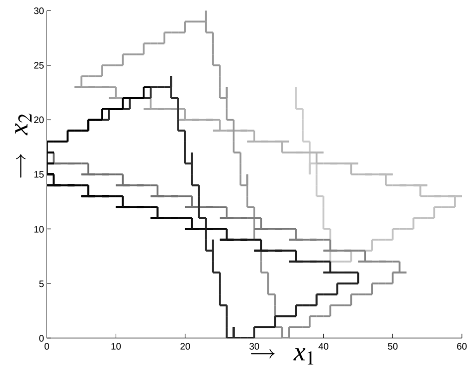

Figure 2: Simulation results trajectory.

References

[1] J.A.W.M. van Eekelen, E. Lefeber and J.E. Rooda, Feedback control of 2-product server with setups and bounded buffers, Accepted for the 2006 American Control Conference.

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[3] D.A. van Beek and J.E. Rooda, Languages and applications in hybrid modelling and simulation: Positioning of Chi, Control Engineering Practice, 2000.