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Design of Robust Topologies for Logistics Networks

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Abstract

We consider the distribution of goods from manufacturers to customers by a logistics provider, where manufacturers' supplies and customers' demands are given and cannot be controlled. The goods may temporarily be stored in warehouses to compensate for the stochastic behavior of the supplies and demands. Manufacturers, warehouses and customers are geographically connected by transportation links, e.g. roads, railways, waterways. The problem of a logistics provider is to determine which of these links to use and how much to ship through them, such that total costs are minimized and demands are met. This paper presents a method for designing close to optimal network topologies for this type of problem. We introduce a two-layer optimization procedure, which finds a cost-effective topology with a very limited number of links for a set of stochastic supplies and demands. In addition, we show that the obtained topology is only sensitive to changes in the first moments of supply and demand distributions. Hence, with only information about the individual means, a close to optimal topology can be determined based on a constructed set of stochastic time series of supplies and demands.

1 Introduction

In long-distance transportation networks, the distribution of goods is often performed by a third-party logistics provider. In general, such so-called logistics networks consist of manufacturers, warehouses and customers, which are all geographically connected by links, e.g. roads, railways, waterways. The logistics provider uses these links to ship goods from the manufacturers to the customers either directly or via warehouses for intermediate storage. In this setting, storage is the only way to compensate for the stochastic behavior of manufacturers' supplies and customers' demands. Namely, a logistics provider is not able to control or manipulate the supplies and demands. Their amounts are fixed and only known a few days in advance.

The tasks of a logistics provider can typically be classified into three levels: The strategic level, the tactical level and the operational level. The strategic level deals with decisions regarding the number, location and capacities of warehouses. These decisions have a long-lasting effect on the system's performance. The tactical level includes decisions on which transportation links to use, i.e. the design of the network topology. These decisions are updated somewhere between once a week, month or quarter. The operational level refers to day-to-day decisions such as scheduling and routing the shipments.

In this paper, we focus on the tactical and operational levels and leave the strategic level out of consideration. The number and locations of manufacturers, warehouses and customers are thus given a priori. Hence, the problem is restricted to deciding which transportation links to use and how much to ship through them, such that total costs are minimized and demands are met. We consider a network with only one type of product. This means that products of one or more manufacturers are delivered to one or more customers. Decisions on the tactical level result in a network topology. It is desirable that this topology is cost-effective in a wide range of situations, since stochastic properties of each individual supply and demand distribution may change in time.

A common setting for distribution problems can be found in [1], [2], [3], [4]. These studies investigate different policies for supply chain management from a suppliers' or customers' point of view. Orders are placed to manufacturers to keep inventory levels in warehouses at a desired level given the demand. However, a logistics provider is caught in the middle between supply and demand and is not able to control any of the other players in the logistics network. Manufacturers push their products into the network while customers have a demand for them.

To our knowledge, only a restricted number of studies [5], [6] have been done for distribution problems from the viewpoint of a logistics provider. These papers consider demands to be unknown in advance. They assume that demands are generated from given probability

distributions and base their daily decisions on the expected values of the demands for that particular day. This is categorized as stochastic optimization. In this paper, we also assume that supplies and demands are generated from given probability distributions. However, in addition, we assume that the exact supplies and demands are known a few days in advance, which makes sense from a practical point of view. This results in a number of deterministic optimization problems along the considered time period.

In [7], [8], [9], [10], [11], [12] networks are designed, which are similar to logistics networks. However, these studies only treat static cases without investigating performance dependency on changes in supplies and demands. On the other hand, in this paper we deal with the topology of optimal networks when supplies and demands vary in time.

The purpose of this paper is to design cost-effective network topologies for long-distance transportation networks, operated by logistics providers. In addition, these topologies should be cost-effective in a wide range of situations. We stipulate that a network topology is cost-effective when costs are close to minimal while the number of links is very limited. The reasons for this limited number of links are: i) A reduction in the number of links reduces the complexity of the network and with that the complexity of the organizational tasks of a logistics provider, ii) adding a link to an existing network may introduce fixed costs due to contracts, iii) we strive for thick flows in this study. Reducing the number of links leads to thicker flows per link. This results in a relatively large number of fully loaded transportation devices. In this way economy of scales is modeled here. This in contrast to [1], [2], [3], which consider thinner flows and model economy of scales by a stepwise cost-function dependent on the capacity of a transportation device. Such an approach reflects practice more accurately, however it demands much more computational effort. The approximation we make, can be justified, because we only consider the part of the logistics networks with large amounts of flows. Specifically we are not concerned with delivery of small amounts to individual customers (no "milkruns").

The paper is structured as follows: in Section 2, we introduce a general mathematical model, which describes the flows of products through a logistics network with an arbitrary topology. A two-layer optimization method is proposed to find a network topology, for which both the costs and the number of links are close to minimal. The first layer consists of a model predictive control with receding horizon (MPC) [13], which determines the optimal routing schedule for a certain time period, given a topology and a set of supplies and demands. For the second layer we propose two different heuristics, which both find a topology such that the costs as well as the number of links are close to minimal for the stochastic case. In Section 3, results for different network configurations show the validity of this optimization procedure. In Section 4, we show that topologies, found by the proposed optimization procedure, are only sensitive to the first moments of supplies' and demands' distributions. With reasonable forecasts about the individual means of the supplies and demands we can therefore a priori determine a close to optimal network based on a constructed set of stochastic time series of supplies and demands. This network is close to optimal for each stochastic case with these means.

2 Methodology

In this section we describe the procedure to determine a close to optimal network topology. To this end, we first derive a general mathematical model, which mimics the flows through an arbitrary logistics network. Then, we construct a suitable objective function, which assigns costs to different parts of the system. Model predictive control is used to determine the optimal routing schedule for a particular topology based on limited future information about the supplies and demands. Finally, two heuristics are proposed, which both find a close to optimal network topology among all possible ones.

The directed graph depicted in Figure 1 serves as an illustration of the properties of the type of logistics network, considered in this paper. Manufacturers supply goods and push them into the network while customers have a demand for them. Furthermore, goods can be either sent

directly from manufacturer(s) to customer(s) or stored temporarily in warehouses. Finally, shipments from one warehouse to another are not possible.

The problem tackled in this section is defined as follows: Given m manufacturers, n warehouses, c customers and their geographical orientation, and given the stochastic time series of manufacturers' supplies and customers' demands over a time period T , $T \in \mathbb{N}$, determine a network topology such that both the costs and the number of links are minimal. In addition, we require that there is an equilibrium between total supply and total demand. To this end, the total supply is assumed to be equal to the total demand, i.e. the total amount of material flowing into the network is equal to the total amount flowing out of it within time period T . We also assume that future supplies and demands are only known over a time horizon ω , where $\omega \in \mathbb{N}$ and $\omega \ll T$. This assumption follows from practice.

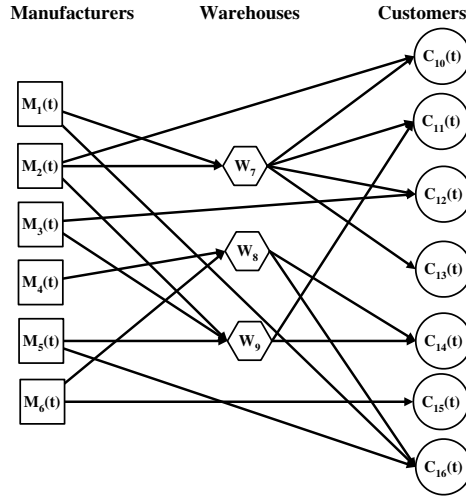


Figure 1: Simple example of a logistics network.

In order to solve the stated problem we first introduce a general mathematical model, which describes the flows through a logistics network with a certain topology. The decision variables of this model are the flows through each of the links of that topology. The modeling is in discrete time and is based on the following conservation laws: i) The supply of a certain manufacturer at a certain time instance should be distributed among the links, which are connected to that particular manufacturer, ii) the storage level in a warehouse is updated at each time instance, dependent on incoming and outgoing flows. In practice, several processes between arriving at and storing in a warehouse introduce delay. The total delay is captured in a constant τ , where $\tau \leq \omega$ and $\tau \in \mathbb{N}$. Products arriving in a warehouse at a certain time instance can thus at earliest be shipped out τ time units later.

An objective function is constructed by assigning costs per unit of goods per day to transport, storage and not delivering in time. The amount of goods, which are not delivered to a certain customer at a certain time instance, is added to the original demand of that particular customer for the next time instance. Furthermore, the derived conservation laws serve as equality constraints for this optimization problem. Since forecasts of supplies and demands are available over the horizon ω , routing decisions for a certain time instance can be based on this information. To this end, a model predictive control (MPC) scheme with receding horizon ω is constructed. This MPC scheme has to be executed for each time instance in period T to determine the most cost-effective routing schedule over this period for a certain network topology.

The MPC along period T , as discussed above, should be applied to all possible topologies to find a topology with close to minimal costs and a very limited number of links. However,

the number of possible topologies grows very large when realistic values for m , n and c are considered. It will then take an enormous computational effort to solve the problem. Instead of evaluating all possible topologies, we therefore propose two heuristics. Both heuristics evaluate a restricted number of topologies to determine a close to optimal solution.

2.1 Conservation laws

In this section we derive a mathematical model for the considered type of logistics network. We firstly introduce a discrete time variable $k = 0, 1, 2, 3, \dots$, representing time in days. Furthermore, we consider stochastic time series of supplies and demands over a certain time period T . Once a logistics provider has decided on the topology, he has to decide on how much to send through each of the links of that topology every time instance k . For this, suitable continuous variables are introduced. We define $u_{ij}(k)$ to be the amount of goods transported from node i to node j on day k , and $w_p(k)$ to be the storage level in warehouse p on day k . Moreover, we require that $u_{ij}(k) \geq 0$ and $w_p(k) \geq 0$. Although the total supply on day k can differ from the total demand on day k , we require that the following conservation law holds for period T .

$$\sum_{k=0}^T \sum_{i=1}^m M_i(k) = \sum_{k=0}^T \sum_{j=1}^c C_j(k), \quad (1)$$

where $M_i(k)$ is the supply of manufacturer i on day k and $C_j(k)$ the demand of customer j on day k . This assumption implies an equilibrium between the total supply and the total demand over time period T . In this way, the number of goods supplied in time period T satisfies the number of goods demanded in time period T .

In addition, two other conservation laws are derived, which constrain the problem. i) The supply of a certain manufacturer i on day k is distributed among all the links connected to that particular manufacturer on the same day as described in

$$M_i(k) = \sum_{j \in S_i} u_{ij}(k), \quad (2)$$

where the set S_i contains the indices of all the nodes, to which manufacturer i is connected, ii) a conservation law is valid with respect to each warehouse. Products which arrive in a warehouse undergo delay between unloading and storing. We define τ to be the total delay time between unloading and storing in a warehouse. Taking this into consideration, the storage level in warehouse p on day k is updated by

$$w_p(k) = w_p(k-1) + \sum_{i \in I_p} u_{i,p}(k-\tau_p) - \sum_{j \in O_p} u_{p,j}(k), \quad (3)$$

where the set I_p contains the indices of all the nodes which are sources of warehouse p , whereas the set O_p contains the indices of all the nodes which are destinations of warehouse p . The conservation laws in (2) and (3) are the equality constraints of the optimization problem we propose in the next subsection.

2.2 Model Predictive Control

We assign costs to transport and storage on day k according to

$$\sum_{i,j \in U} a_{ij} u_{ij}(k) + \sum_{p \in W} s_p w_p(k), \quad (4)$$

where a_{ij} represents the linear cost factor for transportation from node i to node j , and s_p represents the linear cost factor for storage in warehouse p . The set U contains the index pairs of all the links of the considered topology, whereas the set W contains the indices of all the warehouses of the considered topology. The reason for the linear approximation of the transportation costs is the following: when the number of goods per link becomes large, costs per product converge to a linear proportion. Since we strive for thick flows and therefore aim to minimize the number of links, we assume a relatively large number of goods through each present link every day k and hence linear transportation costs are justifiable.

5 Methodology

Besides transportation and storage costs, we assign a penalty associated with not delivering in time. We introduce the backlog $B_j(k)$ for customer j on day k according to

$$B_j(k) = D_j(k) - \sum_{i \in Q_j} u_{ij}(k), \quad (5)$$

where the set Q_j contains the indices of all the nodes which are connected to customer j . $D_j(k)$ is defined to be the updated demand of customer j on day k , according to

$$D_j(k+1) = C_j(k+1) + B_j(k), \quad (6)$$

where $C_j(k+1)$ is the original demand of customer j on day $k+1$, and $B_j(0) = 0$. The combination of (5) and (6) implies that the backlog $B_j(k)$ of customer j on day k is added to the original demand of customer j on day $k+1$. Costs for backlog are chosen to be quadratic according to

$$\sum_{j=1}^c b_j B_j^2(k), \quad (7)$$

where b_j represents the cost factor of customer j . In that way we are penalizing early as well as late delivery. The reasons for not using $|B_j(k)|$ are twofold: i) it introduces additional complexity in programming, ii) backlog costs involve intangibles like customer satisfaction and lost business, which arguable are not linear in $B_j(k)$.

We have defined U as the set of index pairs of the topology under consideration and W as the set of warehouses. Moreover, $u_{ij}(k)$ is the amount of flow through the link from node i to node j on day k , for $(i,j) \in U$. For the remainder of this section we consider U and W to be fixed, and let $\mathbf{u}(k)$ be the vector representing the amounts of flow $u_{ij}(k)$, for $(i,j) \in U$. The total costs $\alpha(\mathbf{u})(k)$ on day k are now obtained by adding (4) and (7), which results in

$$\alpha(\mathbf{u})(k) = \sum_{j=1}^c b_j B_j^2(k) + \sum_{(i,j) \in U} a_{ij} u_{ij}(k) + \sum_{p \in W} s_p w_p(k). \quad (8)$$

The total costs of (8) only take the costs on day k into account. However, forecasts of supplies and demands are available over a time horizon of length ω . Since decisions made on day k influence the decision space for future days, it makes sense to base decisions on day k on the supplies and the demands for days k through $k + \omega - 1$. To this end, we make use of model predictive control with receding horizon (MPC), which is often used in such kind of problems. The principle of MPC is that besides costs on the current time instance, also costs on future time instances are incorporated in the objective function. For our specific problem, the resulting MPC scheme on day k is then given by

$$\begin{aligned} \min_{\mathbf{u}(k), \dots, \mathbf{u}(k+\omega-1)} \quad & \sum_{q=0}^{\omega-1} \alpha(\mathbf{u})(k+q) \\ \text{subject to:} \quad & M_i(k+q) = \sum_{j \in S_i} u_{ij}(k+q), \quad \text{for } 1 \leq i \leq m \text{ and } 0 \leq q \leq \omega-1, \\ & w_p(k+q) = w_p(k+q-1) + \sum_{i \in I_p} u_{ip}(k+q-\tau_p) - \sum_{j \in O_p} u_{pj}(k+q), \\ & \quad \text{for } 1 \leq p \leq n \text{ and } 0 \leq q \leq \omega-1, \\ & u_{ij}(k+q) \geq 0, \quad \text{for } i, j \in U \text{ and } 0 \leq q \leq \omega-1, \\ & w_p(k+q) \geq 0, \quad \text{for } i, j \in U \text{ and } 0 \leq q \leq \omega-1. \end{aligned} \quad (9)$$

The above MPC determines the optimal routing schedule for day k , i.e. the optimal values $\mathbf{u}^*(k)$. As a next step, the MPC for day $k+1$ is executed. The most cost-effective routing schedule for the time period T can thus be determined by solving the optimization problem (9) T

times. The total costs Ω for the routing schedule over period T are then given by

$$\Omega = \sum_{k=1}^T \alpha(u^*)(k). \quad (10)$$

2.3 Heuristics

The MPC scheme along period T , determines the optimal routing schedule for only one particular topology. In this subsection we aim to find one topology amongst all possible ones, such that the number of links is minimized, while the routing schedule is still cost-effective. In order to find the topology with both minimal costs and minimal number of links, the total costs Ω , given in (10), should be determined for every possible network topology. We can determine a lower bound on the number of possible network topologies dependent on the number of manufacturers m , warehouses n and customers c . In such a kind of network, the number of possible links β is given by

$$\beta = m \cdot n + m \cdot c + n \cdot c. \quad (11)$$

When the number of links is β , then we have a fully connected network, i.e. a network in which each manufacturer is connected to each warehouse and each customer, and each warehouse is connected to each customer. Furthermore, for each set of m , n and c there exists a unique number of links θ , such that the network is minimally connected. A minimally connected network is a network with the minimal number of links, such that all nodes of that network are connected. This means that each manufacturer has at least one outgoing link, each customer at least one incoming link and each warehouse at least one incoming and one outgoing link. In general, a logistics network has less manufacturers than customers and only a couple of warehouses. Without loss of generality we therefore consider only networks with $c > m > n$ in this paper. In this case it holds that

$$\theta = n + c. \quad (12)$$

Although the number θ is unique for each set of m , n and c , there is no unique minimally connected network topology. A number of network topologies can be found for each specific set of m , n and c , all having the same number of links θ . We define κ to be the number of links, which can be deleted from the fully connected network to obtain an arbitrary minimally connected network. From (11) and (12) it follows that

$$\kappa = \beta - \theta. \quad (13)$$

The number of possible topologies γ between the fully connected network and that particular minimally connected network is then given by

$$\gamma = \sum_{r=0}^{\kappa} \binom{\kappa}{r} = 2^{\kappa}. \quad (14)$$

Since more than one minimally connected network exist, γ reflects a lower bound of all feasible topologies. This lower bound already shows that a large number of possible topologies has to be considered to determine the global minimum for a network configuration. Since this demands an enormous computational effort, a heuristic is proposed, which only evaluates κ possible topologies to find a close to optimal solution.

Heuristic 1:

1. Start with the fully connected network and run the MPC for that topology for each day of period T . Compute the accompanying costs Ω .
2. Until the network is minimally connected: Delete a link that is least used over the considered time period T and run the MPC for the newly obtained network for each day in T . Compute the accompanying costs Ω .
3. Plot costs Ω against the number of deleted links.

In the next section, we show results of the above proposed heuristic for four different network configurations and discuss a second heuristic based on adding links to a minimally connected network at random.

3 Results

In the previous section, we proposed a heuristic method to determine a cost-effective network topology for an arbitrary logistics network configuration. To test the heuristic, we generate stochastic time series of m supplies and c demands from uniform distributions with random means and variances for a time interval $T = 100$. We assume that supplies and demands are only known 2 days in advance, so the window size ω is equal to 3. In this paper, we consider networks, of which the size is relatively small from a geographical point of view. Therefore, it is assumed that goods can be transported from any arbitrary node to another within one day. In addition, we assume that the delay before storage in each warehouse p is equal to one day, which implies that $\tau_p = 1$.

The proportion between the different costs constants in (8) is chosen to be roughly according to $b_j : a_{i,j} : s_p = 1 : 0.01 : 0.001$, i.e. delivery on time is of paramount importance (consider JIT production). Costs per unit per day for transportation and storage are assigned randomly to two types of links, direct and indirect links, with the following constraints: i) the maximal ratio of costs per unit of two links of one type is 3, ii) the costs per unit of shipping via a warehouse are always less than the costs per unit of shipping directly. In this way, flows are mostly forced to merge into warehouses, which mimics the economies of scale principle. The only reason for shipping directly would be when the demand of a certain customer exceeds the total amount of goods in the warehouses, connected to that customer. Since shipping through a warehouse introduces delay, shipping directly is in that case more profitable.

3.1 Heuristic 1

Figure (2) depicts the results of Heuristic 1 for four different network configurations, i.e. four different sets of m , n and c . Costs are plotted on a logarithmic scale against the number of deleted links. In these plots, zero deleted links corresponds to the fully connected network, whereas the maximal number of deleted links corresponds to a minimally connected network.

All these curves exhibit the same typical characteristics from the left to the right: at first, a large number of links is deleted without affecting the costs at all. Then, at about 70-90% of the total number of links that can be deleted, costs start increasing slightly with the number of deleted links. Finally, at about 95% of the total number of links that can be deleted, costs start increasing dramatically. Obviously, a topology with only a small number of links suffices to gain close to minimal total costs. In [12], similar characteristics are presented for a manufacturing system with deterministic supplies and demands. Figure (3) shows the found network topology with only this restricted number of links for the configuration with $m = 6$, $n = 3$ and $c = 7$. Furthermore, it should be noticed that the curves in Figure (2) are all monotonically increasing, however not convex. The reason for the several horizontal plateaus is that sometimes not one, but a group of links is needed to affect the costs.

3.2 Heuristic 2

In this section we show results of a second heuristic. Instead of deleting links, this heuristic starts adding links at random to a minimally connected network, which is chosen at random. For each newly obtained topology the accompanying costs Ω are computed. The links that cause a costs decrease are collected. Starting from the same minimally connected network again, the links from this collection are then added first. The rest of the links are added at random afterwards. Again the costs Ω are computed for the topologies concerned. Simulations show that the collecting and adding has to be repeated four times at most to obtain a cost-effective network, which is equivalent to the one found by Heuristic 1. The second heuristic thus needs to evaluate between κ and 4κ topologies to find a close to optimal one.

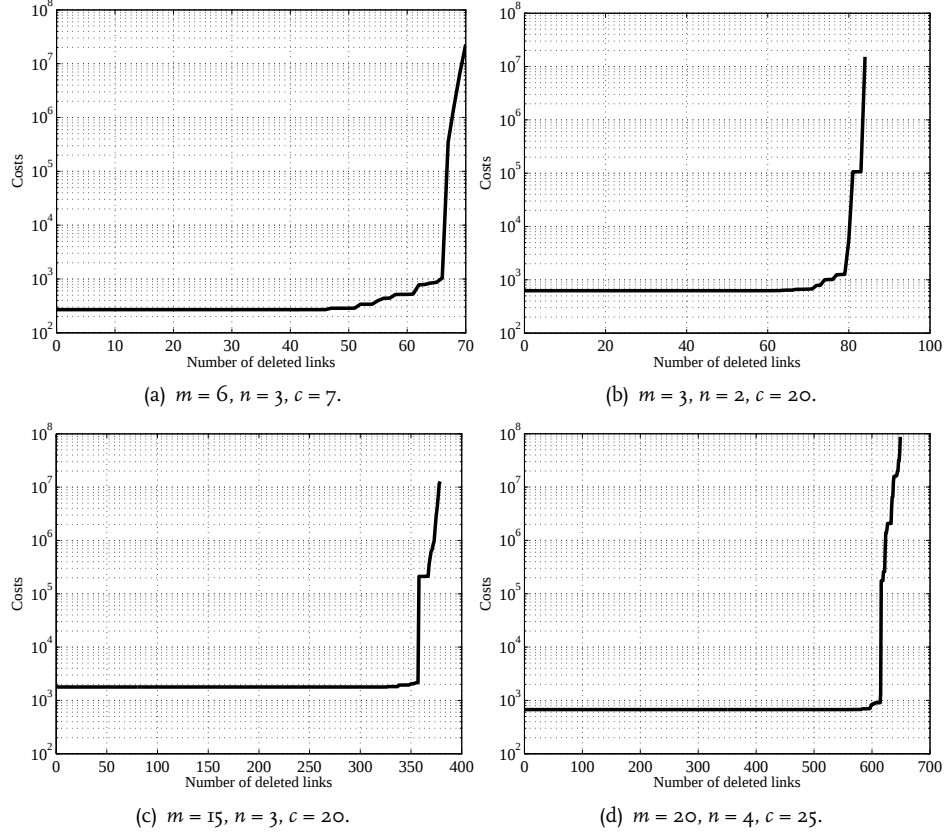


Figure 2: Results Heuristic 1 for four different network configurations, $T=100$, $\tau_p=1$, $\omega=3$.

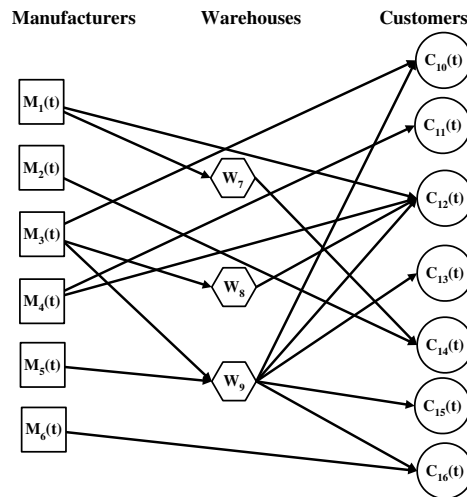


Figure 3: Close to optimal network topology found by Heuristic 1, $m = 6, n = 3, c = 7$.

The black curve in Figure (4a) shows the result of Heuristic 2 for the configuration with $m = 6, n = 3$ and $c = 7$. The costs are plotted on a logarithmic scale against the number

of added links. In this plot, zero added links corresponds to the randomly chosen minimally connected network and the maximal number of added links corresponds to the fully connected network. Furthermore, each grey curve in Figure (4a) represents the costs for a different random sequence of added links. Studying the black curve we observe: adding links causes a slight decrease of costs initially. At a certain number of added links, the costs drop dramatically. Then, costs start decreasing slightly again with the number of added links. Finally, adding links does not affect the costs at all. Again only a restricted number of links is needed to drop the costs close to the minimum. Furthermore, one can see that the black line is monotonically decreasing, however not convex. Likewise the results for Heuristic 1, horizontal plateaus are present here as well. The fact that the black curve is a left border of all the grey curves, shows that Heuristic 2 finds a satisfactory solution.

3.3 Heuristic 1 vs. Heuristic 2

In this subsection we compare the heuristics proposed in the previous subsections. The results for both the heuristics, where $m = 6$, $n = 3$, $c = 7$ are plotted together in Figure (4b). Costs are plotted against the number of links in the network. For this configuration, the number of links in the fully connected network is $\beta = 81$, according to (11). The number of links in a minimally connected network is $\theta = 10$, according to (12). This figure shows that in both cases about 5% of all the links that can be added to a minimally connected network is needed to result in a cost-effective topology. The close to optimal network topology for Heuristic 1 is depicted in Figure (3). The equivalent topology for the result of Heuristic 2 is not depicted here. However, it turns out that Heuristic 2 finds a topology with several different links with respect to the topology found by Heuristic 1. This is due to the fact that transportation and storage costs are relatively small with respect to backlog costs in this example and therefore two links with almost identical cost can be interchanged, without affecting the costs considerably. However, a cost-effective network topology is in both cases obtained with about 5% of all the possible extra links. Since this transition point is of the most interest here, we can conclude that both heuristics are able to find a close to optimal solution to the posed problem. Since the second heuristic demands more computational effort, the first one is used through the rest of the paper.

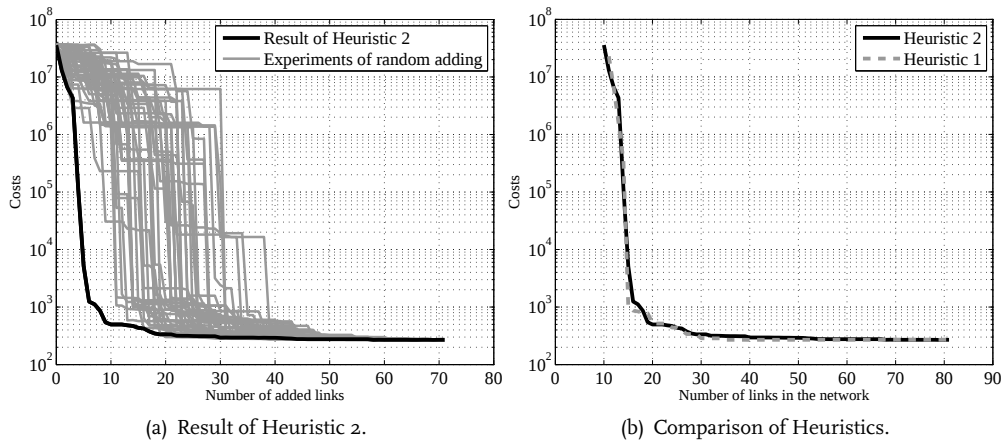


Figure 4: Results of Heuristic 2 and comparison between Heuristics, $m = 6$, $n = 3$, $c = 7$, $T=100$, $\tau_p=1$, $\omega=3$.

4 Robustness

In the previous section we showed that both the proposed heuristics find a close to optimal network topology. However, this optimization concerns only one specific set of supplies' and demands' time series. In the present section, we first show that the found topologies are insensitive to a wide range of changes in the stochastic distributions of supplies and demands. In fact, it turns out that only the means of supplies and demands determine the close to optimal network topology. Next, we reason about the question whether solving the static case will result in a cost-effective topology for the stochastic case. From this, it is concluded that a specific property of stochastic behavior has to be incorporated in the time series of supplies and demands in order to find a satisfactory topology. Thus, if information about the individual means of supplies and demands is available a priori, we only need to generate stochastic time series with these means and the specific property incorporated, to find a cost-effective topology for any future stochastic case with these means. Finally, as we optimize over a finite period of time T we have to address the influence of the initial inventory. We show that the steady state value of the costs converges to the minimal transportation plus storage costs, as initial inventory increases.

4.1 Sensitivity

The optimization procedure for the case with $m = 6$, $n = 3$ and $c = 7$ considers a specific set of stochastic supplies and demands over time period T . We define this original set to be $S_o(t)$. The result of this optimization is the sequence of deleted links, such that close to minimal costs are obtained for a topology with a very limited number of links. Each time series in $S_o(t)$ was generated out of a uniform distribution with a specific mean and variance. In practice, properties of supplies' and demands' distributions may change for the next time period of length T . The question rises whether the originally found topology is still cost-effective. To verify this, the originally found order of deleted links is used for five cases with different sets of times series generated out of different distributions. We consider the following cases: generate a new sample from

1. The same distribution with the same parameter values.
2. Different distribution with the same means and variances.
3. A sinusoidal shaped time series with the same means and variances.
4. The same distribution with the same means but different variances.
5. The same distribution with different means but the same variances.

The results of these five experiments are depicted in Figure (5). As one can see, the original topology is insensitive to the changes introduced in the experiments 1 through 4. Namely, in these experiments, the same number of links as in the original case are needed to obtain a cost-effective network. However, experiment 5 shows that more links are needed to obtain a cost-effective network. Apparently, the close to optimal topology is only sensitive to the means of the supplies and demands. The reason for the differences in steady-state costs for experiment 1 through 4 is discussed in section 4.3.

4.2 Static vs. Stochastic case

From the previous subsection it can be concluded that only the means determine the optimal network topology. This suggests that applying one of the heuristics to the static problem may lead to a network, which is also cost-effective for stochastic cases. To study this question, we perform the following experiment: We consider a time period T , where supply and demand are equal each day, and where hence (1) is valid. We start with a fully connected network and store just enough initial inventory in the warehouses to meet the demand on the first day. Then, the lowest costs are obtained by using only indirect links, because there is never any backlog and the indirect links are the cheapest. Since each supply and each demand is equal

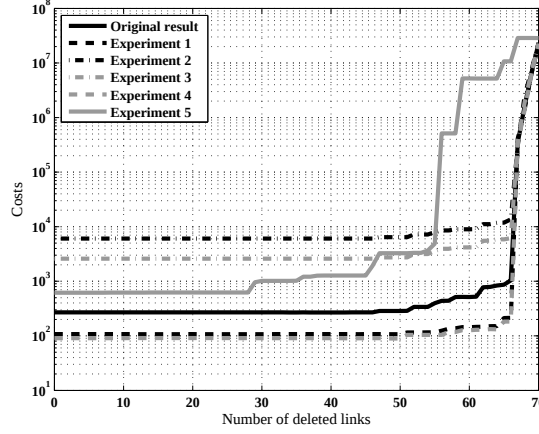


Figure 5: Sensitivity of originally found topology to changes in the moments of supplies' and demands' distributions.

each day k , the decisions made by the MPC will also be the same for each day k . If we now apply Heuristic 1, we end up with a minimally connected network with only indirect links. However, Figure (3) shows, besides indirect links, several direct links in the close to optimal topology for the stochastic case. Apparently, the direct links are needed to compensate for the backlog, resulting from typical phenomena in the stochastic time series of supplies and demands. Since the static case does not incorporate this typical information, the desired close to optimal network will not be found by solving the static case.

Obviously, some kind of stochastic property should be incorporated in the set of time series to find a close to optimal network, which is valid in a wide range of stochastic supplies and demands. Numerous simulations show that there is only one condition with respect to the time series that has to be fulfilled: the system should be forced into backlog. Based on performed simulations, we propose the following: suppose we have a reasonable forecast about the means of each supply and each demand for a future time period. If we now add a short time stochastic behavior by having a large peak in total demand on the beginning of time period T , we can construct a cost-effective network topology for all stochastic cases with the same means.

4.3 Inventory flexibility

While the network topology determines the number of links and the costs associated with an almost optimal network, it cannot affect minimal costs for a fully connected network. However, experiments 1 through 4 in Figure (5) show rather different minimal costs. It should be noticed that for all these experiments (1) is valid and the length of the considered time period is equal to T . Moreover, the means of each specific supply and demand are the same through the considered experiments. This implies that over period T the same total amount of goods is transported through the network in each of these experiments. Therefore, the deviations in steady-state costs cannot be caused by differences in transportation or storage costs, but only by backlog costs. Dependent on the time series in a specific set, there will be more or less backlog during period T . This results in different steady state costs for different experiments.

To confirm this, we perform two other experiments, where the original set of time series $S_o(t)$ is used each time. The only parameter we vary is the initial inventory in the warehouses. The results of these experiments are plotted in Figure (6). As one can see, the steady-state costs converge to the minimal amount of transportation plus storage costs when initial inventory increases.

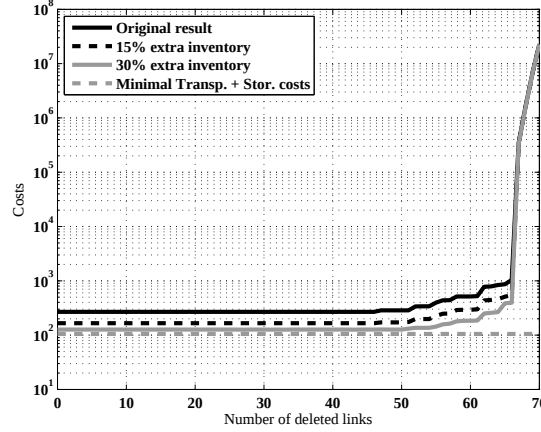


Figure 6: Influence of initial inventory on steady-state costs.

5 Conclusions

In this paper we considered long-distance transportation networks, where a logistics provider deals with the problem of distributing goods from manufacturers to customers via transportation links. Warehouses between the manufacturers and customers may be used to compensate for the stochastic behavior of manufacturers' supplies and customers' demands. The amounts of these supplies and demands are assumed to be given and assumed to be uncontrollable for the logistics provider. With only information about the supplies and demands a few days in advance, the logistics provider has to decide on which transportation links to use and how much products to ship through them. This should be done in the most cost-effective way. In addition, the determined network topology should be cost-effective in a wide range of situations, since supplies' and demands' distributions may change in time.

We presented a method to find cost-effective network topologies by deriving a general mathematical model, which describes the flows through such a network. A model predictive control with a receding horizon (MPC) was used to determine the optimal routing schedule for a given topology along a certain time period. Two heuristics were proposed, which find equivalent cost-effective network topologies with a rather small number of links: the number of links needed to construct a minimally connected network plus about 5% of all other links.

We showed that the resulting topology is only sensitive to the means of the individual supplies and demands. Information about the means of supplies and demands over a certain time period therefore suffices to determine a close to optimal network topology based on a constructed set of stochastic time series of supplies and demands. This topology is close to optimal for each set of stochastic supply and demand distributions with these individual means.

6 Outlook: Backlog vs. Transportation costs

The proposed method can now be used to run through different scenarios, which arise in these kinds of networks. It is for example very interesting to investigate whether the type of product affects the structure of the close to optimal topology. To this end, we vary the proportion between the backlog costs and the transportation and storage costs in the model and execute the proposed optimization method for these two extreme cases.

Figure (7) shows the resulting close to optimal network topologies for $m = 20$, $n = 4$, $c = 25$. Two differences between the structures are very obvious: i) the close to optimal topology for the product with relatively low backlog costs contains far fewer links (52) than the topology for the JIT-product (78), ii) the close to optimal topology for the product with relatively low

backlog costs (e.g. toilet paper) consists of only indirect links, while a lot of direct links are present in the close to optimal topology for the JIT-product (e.g. pharmaceuticals). Apparently, direct links are only crucial, when products need to be delivered just in time. In such a case, backlog costs exceed the transportation costs of small amounts of products along a direct link. As a future study, it is interesting to see how the typical structure properties change in between these two extreme cases, dependent on the ratios between the various associated costs.

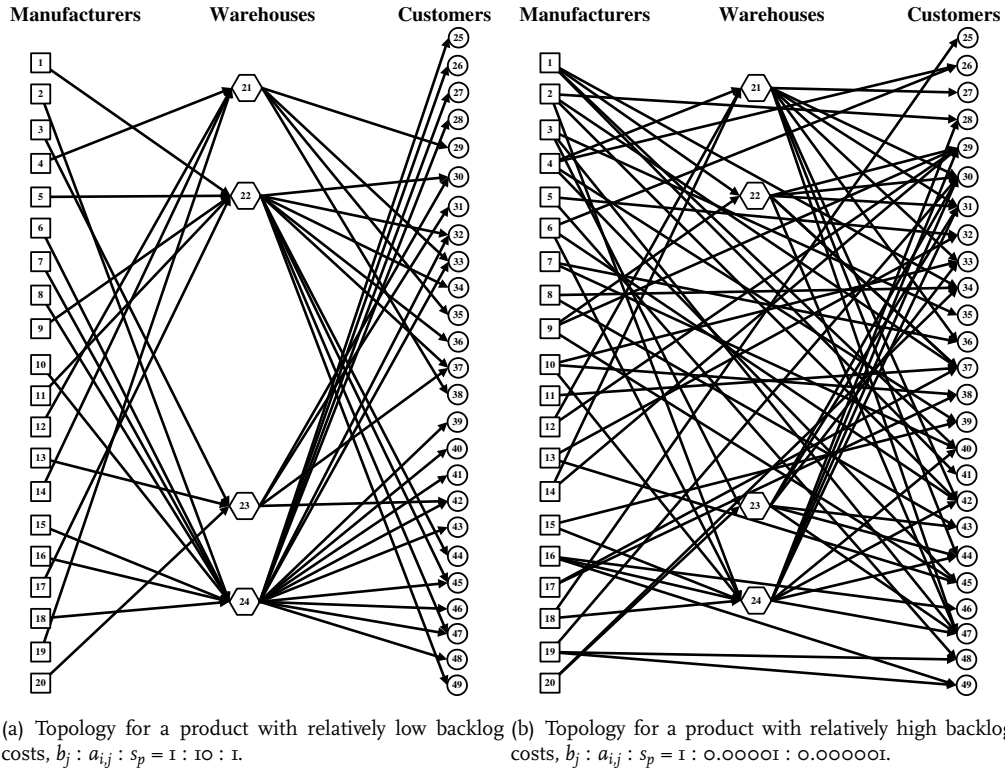


Figure 7: Optimal network structure dependent on the type of product, $m = 20$, $n = 4$, $c = 25$, $T=100$, $\tau_p=1$, $\omega=3$.

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