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Phase I Report

Adaptive a-CACC Controller for platoons with uncertain dynamics

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Author:
Kashish Singh Pilyal, 1485830

Supervisor:
Dr. Ir. Erjen Lefeber

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Table of contents

Nomenclature

Abbreviations

ISO: International Standards Organization
ADAS: Advanced Driver Assist Systems
CACC: Cooperative Adaptive Cruise Control
ACC: Adaptive Cruise Control
V2V: Vehicle to Vehicle
ISS: Input to State Stable
ODE: Ordinary Differential Equation

Symbols

$\mathcal{R}$: Set of all real numbers
q_i: Position of the i^{th} vehicle
v_i: Velocity of the i^{th} vehicle
a_i: Acceleration of the i^{th} vehicle
$\dot{a}_i$: Jerk of the i^{th} vehicle
h_i: Constant time headway of the i^{th} vehicle
τ_i: Time constant of the i^{th} vehicle
F_i: Driving force of the i^{th} vehicle
η_i: Desired input force of the i^{th} vehicle
m_i: Mass of the i^{th} vehicle
ρ_a: Density of air
c_d: Coefficient of aerodynamic drag
A_f: Frontal area of the vehicle
c_{i1}: Aerodynamic drag parameter of the i^{th} vehicle
μ: Friction coefficient between road and the vehicle
g: Acceleration due to gravity
θ: Slope of the road
c_{i2}: Rolling resistance parameter of the i^{th} vehicle
u_i: New input of the i^{th} vehicle
$\hat{c}_{i1}$: Input value of aerodynamic drag for the i^{th} vehicle
$\hat{c}_{i2}$: Input value of rolling friction for the i^{th} vehicle
$\hat{m}_i$: Input value of mass for the i^{th} vehicle
$\dot{e}_{i,1} = x_2$: Error of the i^{th} vehicle
$\dot{e}_{i,2} = x_3$: Derivative of the error of the i^{th} vehicle
q_{i-1}: Position of the $i-1$ vehicle
v_{i-1}: Velocity of the $i-1$ vehicle
a_{i-1}: Acceleration of the $i-1$ vehicle
u_{i-1}: New input of the $i-1$ vehicle
k_p: Proportional gain of the PID or PD or PDD controller
k_d: Derivative gain of the PID or PD or PDD controller
k_i: Integral gain of the PID controller
k_{dd}: Double derivative gain of the PDD controller
$\epsilon = x_4$: Difference in velocities of $i-1^{th}$ and i^{th} vehicle
e_{smi}: Distance error between i^{th} and $i-1^{th}$ vehicle
s_i: Sliding surface of the i^{th} vehicle
ζ: Positive design constant for sliding surface
w: Weighing factor for sliding mode control
p_{mi}: Mechanical drag of the i^{th} vehicle
a_{di}: Desired acceleration of the vehicle
$d_{r,i}$: Desired spacing between the vehicles
D: Desired safety inter-vehicle distance at standstill

T_i: Time gap of the i^{th} vehicle
Z: Upper-bound on acceleration
Γ: Update law gain

1 Introduction

1.1 Defining CACC

There has been a part in our lives when we have been stuck in a traffic jam. The reasons for this problem are numerous and we think there is nothing we can do to mitigate the problem. According to [1], if the leader of the platoon of the vehicles travelling in a line brakes and reduces the speed by 10 %, the tenth vehicle in platoon will reduce its speed by at least 20 %. This cumulative affect is one of the main causes for the congestion. While we may think that the jams are only irritating, it has also been found that they lead to an increase in carbon dioxide emissions and increase in exposure time to pollution for the passengers [2].

There have been efforts to subside the traffic congestion problem by improving infrastructure and enforcing laws but these methods do not reduce human error. The Advanced Driver Assist Systems (ADAS) can eliminate the human error and enhance the road travel by a great extent [3]. A type of ADAS, referred to as Adaptive Cruise Control (ACC) works by maintaining a proper following distance [4]. When Vehicle to Vehicle (V2V) communication is added to ACC, it takes the name of Cooperative Adaptive Cruise Control (CACC) which allows the cars to communicate with each other thereby resulting in better safety and cooperative movement of the vehicle platoon [4].

The CACC controllers follow a set of dynamics to function. We start by defining a system and the vehicle dynamics being applied to that system. The next step is to introduce an input to the system through the controller in order to make the system marginally stable. The next section gives an example of the dynamics used and how we decide on a controller input.

1.2 System dynamics

Consider a platoon of vehicles following each other along a straight line. We assume that the vehicles do not overtake each other and strictly follow each other. According to the ISO sign convention for vehicle dynamics [5], the direction of movement of the vehicles in our case is termed as longitudinal. The direction perpendicular to the longitudinal and parallel to the road is called lateral. Based on the assumption that the vehicles only travel in one dimension, we can deduce that the lateral dynamics of the vehicles can be neglected so that our problem can be simplified. We assume that there is no wind to simplify the aerodynamic drag parameter. We consider dry weather for the simplification of choosing the rolling resistance parameter which make our simulations in Chapter 5 easier.

The equations representing the longitudinal dynamics are:

$$\dot{q}_i = v_i, \tag{1.1a}$$
$$m_i\dot{v}_i = F_i - c_{i1}v_i^2 - c_{i2} \tag{1.1b}$$
$$\dot{F}_i = -\frac{1}{\tau_i}F_i + \frac{1}{\tau_i}\eta_i, \tag{1.1c}$$

where q_i, v_i, τ_i, and F_i represents position, velocity, time constant showing the first order drive-line dynamics, and the driving force of the i^{th} vehicle. The constants m_i, c_{i1} and c_{i2} show the values of mass, aerodynamic drag parameters and the rolling resistance of i^{th} vehicle of the platoon. The desired input force η_i as seen in the equations also influences the driving force.

We define a controller input transformation to cancel out the dynamics assuming we know the dynamics of the vehicle (which can be referred to in Appendix A) and to perform an input-output linearization:

$$\eta_i = c_{i1}v_i^2 + c_{i2} + 2c_{i1}v_i a_i \tau_i + m_i u_i. \tag{1.2}$$

The longitudinal dynamics when we know the dynamics can be rewritten as:

$$\dot{q}_i = v_i, \tag{1.3a}$$
$$\dot{v}_i = a_i, \tag{1.3b}$$
$$\dot{a}_i = -\frac{1}{\tau_i}a_i + \frac{1}{\tau_i}u_i. \tag{1.3c}$$

The parameter u_i is referred to as the new input.

1.3 Problem Formulation

In order to cancel out the dynamics of the vehicle, the controller has to assume some value for the unknown dynamics which may or may not match the actual value. The mass, aerodynamic drag, and the rolling resistance of a vehicle varies according to the situations it is put into use. For example, take the situations of a person driving to work and the person going on vacation with the family. We can see that the mass of the vehicle changes drastically in both cases. The same can be said for other unknown parameters. So, will the system be unstable when subject to different scenarios given we assume the wrong values for the dynamics? If the estimated values of the dynamics from the controller which can be written as $\hat{c}_1$, $\hat{m}_i$, and $\hat{c}_2$ (for aerodynamic drag, mass, and rolling resistance respectively) vary with time, can we find out update laws that could ensure stability or marginal stability for our system? Either way, in both of our cases, the desired input force in (1.2) is rewritten as:

$$\eta_i = \hat{c}_{i1} v_i^2 + \hat{c}_{i2} + 2\hat{c}_{i1} v_i a_i \tau_i + \hat{m}_i u_i, \tag{1.4}$$

which means that the dynamics is not linear for our vehicles.

1.4 Research Outline

The thesis is outlined as follows: Chapter 2 gives some insights about some definitions and lemmas to be used later in the thesis. The work done to tackle the problem as similar as ours is shown in Chapter 3. It also mentions the research gap which we need to cover. We have our system definition and mathematical proofs for our problem in Chapter 4. Then we present some simulations to cover some gaps which our proofs would not answer in Chapter 5. Finally, Chapter 6 gives us the conclusions and future work to be done regarding our research.

2 Preliminaries

Before we delve into the literature research to have a look at the methods taken to solve the problem in Section 1.3, we would need to familiarize ourselves with some aspects to deal with the work ahead.

2.1 Error dynamics

One approach to tackle the problem of platooning is by defining a new set of dynamics of the system called the error dynamics. Using this dynamics we can select a suitable controller input u_i so that the errors tend to zero or remain bounded. There are two types of controllers that use this method as of now.

2.1.1 u-CACC

This method was defined in [6]. It starts by defining the error equations:

$$e_{i,1} = q_{i-1} - q_i - h_i v_i, \tag{2.1a}$$

$$e_{i,2} = \dot{e}_{i,1} = v_{i-1} - v_i - h_i a_i, \tag{2.1b}$$

$$e_{i,3} = \dot{e}_{i,2} = a_{i-1} - \left(1 - \frac{h_i}{\tau_i}\right) a_i - \frac{h_i}{\tau_i} u_i, \tag{2.1c}$$

where h_i is the constant time headway of the i^{th} vehicle. The new input u_i is defined in [6] as:

$$\dot{u}_i = -\frac{1}{h_i} u_i + \frac{1}{h_i} \begin{bmatrix} k_p & k_d & k_{dd} \end{bmatrix} \begin{bmatrix} e_{i,1} \\ e_{i,2} \\ e_{i,3} \end{bmatrix} + \frac{1}{h_i} u_{i-1}. \tag{2.2}$$

The control law makes the closed loop platoon dynamics Input to State Stable (ISS) for the input to the closed loop u_i if $k_p > 0$, $k_{dd} > -1$, and $k_d > \frac{k_p \tau_i}{1+k_{dd}}$ which can be referred from [6].

2.1.2 a-CACC

It can be seen in (2.2), that the control law of a vehicle depends upon the control law of the predecessor. While this may not be a problem in a homogeneous platoon where all vehicles have same values for the parameters τ_i and h_i which the control law u_i requires from the predecessor vehicle as well, it does pose a problem in a heterogeneous platoon where the values for τ_i and h_i are different for every vehicle and some vehicle manufacturers are reluctant to share this confidential data. So, a new approach was required. As the acceleration of the predecessor can be measured from the on-board sensors, the new approach called a-CACC proposes a controller which only needs the acceleration of the preceding vehicle as input [8]. We started from the error equations again in [7]:

$$e_{i,1} = q_{i-1} - q_i - h_i v_i, \tag{2.3a}$$

$$e_{i,2} = \dot{e}_{i,1} = v_{i-1} - v_i - h_i a_i, \tag{2.3b}$$

$$\epsilon_i = v_{i-1} - v_i. \tag{2.3c}$$

The control law is chosen as:

$$u_i = \frac{\tau_i}{h_i} \begin{bmatrix} k_p & k_d \end{bmatrix} \begin{bmatrix} e_{i,1} \\ e_{i,2} \end{bmatrix} + \left(1 - \frac{\tau_i}{h_i}\right) a_i + \frac{\tau_i}{h_i} a_{i-1}. \tag{2.4}$$

The control law here only requires the acceleration of the predecessor a_{i-1}, which can be measured by the on-board sensors. As mentioned in [7], this controller performs well with the heterogeneous platoons.

2.2 Lemmas

Lemma 2.1 (Gronwall's Lemma [22]). *Let $\phi : [0,T] \rightarrow \mathcal{R}^+$ be a non- negative differential function for which there exists a constant α such that:*

$$0 \leq \phi'(t) \leq \alpha\phi(t), \quad \textit{for all } t \in [0,T] \tag{2.5}$$

then

$$\phi(t) \leq \phi(0)e^{\alpha t}. \tag{2.6}$$

Lemma 2.2 (Lyapunov stability for Time Invariant systems [18] [19]). *Let x=0 be an equilibrium point for $\dot{x} = f(x)$ and $D \subset R^n$ be the domain for f(x), where R represents all real numbers. Let V be a continuously differentiable function defined as $V : D \rightarrow R$ such that $V(0) = 0$ and $V(x) > 0$ for $x \in D \setminus \{0\}$. The derivative of $V(x)$ along the trajectories of $\dot{x} = f(x)$ is given by:*

$$\dot{V}(x) = \frac{\partial V}{\partial x} f(x). \tag{2.7}$$

If $\dot{V}(x) < 0$, in $D \setminus \{0\}$, then the solution x=0 is asymptotically stable.

Lemma 2.3 (Quadratic Lyapunov Candidate for Time Invariant systems [18] [19]). *Consider a continuous time with $\dot{x} = f(x) = Ax$. Using a Lyapunov candidate, of the form $V(x) = x^T Px$ with a positive definite matrix P which satisfies the condition $P = P^T > 0$ such that $V(x) \geq 0$ and eigen values of P > 0. P must also satisfy the condition $A^T P + PA = -Q$, such that $\dot{V}(x) = -x^T Qx$ and $Q^T = Q > 0$. Then if real part of $eigs(A) < 0$, then it is possible to find a value of P for any given value of Q for which the system $\dot{x} = Ax$ is globally asymptotically stable.*

2.3 String Stability definition [8]

For a Linear time invariant system, if there is a disturbance in the beginning of the platoon of our defined system, it must be made sure that the disturbance does not propagate downstream. In other words, the defined system should be string stable. So the String Stability Complementary Sensitivity of a vehicle is defined as:

$$\Gamma_i(s) = \frac{A_i(s)}{A_{i-1}(s)}, \tag{2.8}$$

where $A_i(s)$ and $A_{i-1}(s)$ are the Laplace transforms of the accelerations a_i and a_{i-1} respectively. The condition for the disturbances to not be amplified downstream is:

$$||\Gamma_i(j\omega)||_{\mathcal{H}_\infty} \leq 1, \tag{2.9}$$

where $||\Gamma_i(j\omega)||_{\mathcal{H}_\infty}$ is the H-infinity norm of the SSCS. For further definition of the norms refer to Appendix B.

2.4 Propositions

Proposition 2.1. *Consider a system $\dot{x} = f(x)$. For a quadratic Lyapunov function of the form $V(x) = x^T Px$ as mentioned in Lemma 2.3 which satisfies the condition $\dot{V}(x) \leq -\frac{x^T x}{2} + C$ for the given system, where C is a constant term. The system dynamics x has a upper bound.*

Proof. The condition on the quadratic Lyapunov function can be represented as:

$$\lambda_{min}(P)||x||^2 \leq V(x) \leq \lambda_{max}(P)||x||^2, \tag{2.10}$$

where λ_{min} is the lowest eigen value of the matrix P. Multiplying the inequality by a minus and changing sign:

$$-\lambda_{min}(P)||x||^2 \geq -V(x) \geq -\lambda_{max}(P)||x||^2. \tag{2.11}$$

Now using the relation:

$$
\begin{aligned}
&-\lambda_{max}(P)||x||^2 \leq -V(x),\\
&-\lambda_{max}(P)\frac{||x||^2}{2} \leq -\frac{V(x)}{2},\\
&-\lambda_{max}(P)\frac{||x||^2}{2} + \lambda_{max}(P)C \leq -\frac{V(x)}{2} + \lambda_{max}(P)C,\\
&\lambda_{max}(P)\dot{V}(x) \leq -\frac{V(x)}{2} + \lambda_{max}(P)C,\\
&\dot{V}(x) \leq -\frac{V(x)}{2\lambda_{max}(P)} + C,\\
&\dot{V}(x) \leq -\alpha V(x) + \gamma,
\end{aligned}
\tag{2.12}
$$

where $\alpha = \frac{1}{2\lambda_{max}(P)}$ and $\gamma = C$. Defining a function W such that:

$$
W = V(x) - \frac{\gamma}{\alpha}, \tag{2.13}
$$

and after differentiating with respect to time on both sides:

$$
\begin{aligned}
&\dot{W} = \dot{V}(x),\\
&\dot{W} \leq -\alpha V(x) + \gamma,\\
&\dot{W} \leq -\alpha\left(W + \frac{\gamma}{\alpha}\right) + \gamma,\\
&\dot{W} \leq -\alpha W.
\end{aligned}
\tag{2.14}
$$

As, W is a non-negative differentiable function which has a starting point of 0 seconds to a finite amount of time T, by Lemma 2.1, we get the following:

$$
W(x) \leq W_\circ \cdot e^{-\alpha t}, \tag{2.15}
$$

where $W_\circ$ is the value of W at $t = 0$. Now substituting values:

$$
\begin{aligned}
V(x) - \frac{\gamma}{\alpha} &\leq (V_\circ - \frac{\gamma}{\alpha})e^{-\alpha t},\\
V(x) &\leq (V_\circ - \frac{\gamma}{\alpha})e^{-\alpha t} + \frac{\gamma}{\alpha},\\
V(x) &\leq V_\circ e^{-\alpha t} + \frac{\gamma}{\alpha}(1 - e^{-\alpha t}),\\
&\text{As the maximum value of } 1 - e^{-\alpha t} \text{ and } e^{-\alpha t} \text{ for time } [0,T] \text{ is 1, we can write:}\\
V(x) &\leq V_\circ + \frac{\gamma}{\alpha}.
\end{aligned}
\tag{2.16}
$$

Using (2.10), we get:

$$
\begin{aligned}
\lambda_{min}(P)||x||^2 &\leq V_\circ + \frac{\gamma}{\alpha}\\
||x||^2 &\leq \frac{V_\circ}{\lambda_{min}(P)} + \frac{\gamma}{\alpha\lambda_{min}(P)}
\end{aligned}
\tag{2.17}
$$

This proves that the system dynamics x have an upper-bound. □

Proposition 2.2. *Consider a system $\dot{x} = f(x)$. For a quadratic Lyapunov function of the form $V(x) = x^T Px$ as mentioned in Lemma 2.3 which satisfies the condition $\dot{V}(x) \leq -\frac{x^T x}{2} + K$, where K varies according to time. If $\lim_{t\to\infty} K(t) = 0$, then $\lim_{t\to\infty} V(t) = 0$*

Proof. The condition leads us to conclude that for every number $\epsilon > 0$, we have a value of time $t^* > 0$ such that $K < \epsilon$ whenever $t > t^*$. For a positive constant $\bar{\epsilon}$, let us assume:

$$
\epsilon = \frac{\bar{\epsilon}\alpha}{2}.
$$

Using the bound on Lyapunov function from Proposition 2.1, starting from $\dot{V}(x) \leq -\frac{x^T x}{2} + K$, we reach the condition similar to (2.12). The condition for this case can be written as:

$$\dot{V}(x) \leq -\alpha V(x) + K, \tag{2.18}$$

For a time instant of $t > t^*$, we have $K < \epsilon$, which gives us:

$$\begin{aligned} \dot{V}(x) &\leq -\alpha V(x) + \epsilon, \\ \dot{V}(x) &\leq -\alpha V(x) + \frac{\bar{\epsilon}\alpha}{2}, \\ \dot{V}(x) &\leq -\alpha V(x) + \frac{\bar{\epsilon}\alpha}{2}. \end{aligned} \tag{2.19}$$

Comparing (2.19) with (2.12), we get an equivalent result of (2.16) which can be written as:

$$V(x) \leq V_\circ e^{-\alpha t} + \frac{\bar{\epsilon}}{2}. \tag{2.20}$$

For a value of t, such that we have an upper bound on the term $V_\circ e^{-\alpha t}$:

$$\begin{aligned} &V_\circ e^{-\alpha t} \leq \frac{1}{2}\epsilon, \\ &\frac{2V_\circ}{\epsilon} \leq e^{\alpha t}, \\ &\alpha t \geq \ln \frac{2V_\circ}{\epsilon}, \\ &t \geq \frac{1}{\alpha} \ln \frac{2V_\circ}{\epsilon}. \end{aligned} \tag{2.21}$$

Using the solution of (2.21) in (2.20), we get:

$$\begin{aligned} V(x) &\leq V_\circ \cdot e^{-\alpha t} + \frac{1}{2}\bar{\epsilon}, \\ V(x) &\leq \frac{1}{2}\bar{\epsilon} + \frac{1}{2}\bar{\epsilon}, \\ V(x) &\leq \bar{\epsilon}. \end{aligned} \tag{2.22}$$

So, this proves that for time $t > t^*$, $V(x) < \bar{\epsilon}$. □

Proposition 2.3. *Consider a system $\dot{x} = f(x)$. For a quadratic Lyapunov function of the form $V(x) = x^T P x$ as mentioned in Lemma 2.3 which satisfies the condition $\dot{V}(x) \leq -\frac{x^T x}{2} + K + C$, where K is a variable that is dependent on time and C is a constant. If $\lim_{t\to\infty} K(t) = 0$, then $\lim_{t\to\infty} V(t) = 0$.*

Proof. We have the condition that for every number $\epsilon > 0$, we have a value of time $t^* > 0$ such that $K < \epsilon$ whenever $t > t^*$. We have a condition on the Lyapunov as:

$$\dot{V}(x) \leq -\frac{x^T x}{2} + K + C, \tag{2.23}$$

as mentioned in the proposition. Considering an upper-bound for K, we can write the equation as:

$$\dot{V}(x) \leq -\frac{x^T x}{2} + \epsilon + C. \tag{2.24}$$

Using the same steps in the proof for Proposition 2.2, we get the condition:

$$V(x) \leq \bar{\epsilon} + C, \tag{2.25}$$

for the $\bar{\epsilon}$ defined in the proof of the Proposition 2.2. As C is a constant, we can introduce a new constant $\hat{\epsilon} = \bar{\epsilon} + C$:

$$V(x) \leq \hat{\epsilon}. \tag{2.26}$$

So, for time $t > t^*$, we have $V(x) \leq \hat{\epsilon}$.

□

Proposition 2.4. *Given a system* $(\dot{x}, \dot{\tilde{\theta}}) = f(x, \tilde{\theta})$. *For a quadratic Lyapunov function of the form* $V(x, \tilde{\theta} = x^T Px + \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$, *where* $P^T = P > 0$ *and* Γ *is a positive constant which satisfies the condition* $\dot{V}_a(x) = \dot{V}(x, \tilde{\theta}) \leq -\frac{x^T x}{2} + C$ *where* C *is a constant and* $V_a = -\frac{x^T x}{2}$. *The dynamics of* x *and* $\tilde{\theta}$ *have an upper-bound.*

Proof. The Lyapunov in the proposition can be split into two different Lyapunov functions:

$$V_a(x) = x^T Px, \tag{2.27a}$$
$$V_b(\tilde{\theta}) = \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}. \tag{2.27b}$$

The conditions on the both the functions can be written as:

$$\lambda_{min}(P)||x||^2 \leq V_a(x) \leq \lambda_{max}(P)||x||^2 \tag{2.28a}$$
$$\lambda(\Gamma^{-1})||\tilde{\theta}||^2 = V_b(\tilde{\theta}), \tag{2.28b}$$

Using Proposition 2.1,we get that:

$$||x||^2 \leq \frac{V_{a\circ}}{\lambda_{min}(P)} + \frac{\gamma}{\alpha\lambda_{min}(P)}, \tag{2.29}$$

which clarifies that the dynamics of x are upper-bounded. From the statement of Proposition 2.4, we can deduce that:

$$\dot{V}_b = 0, \tag{2.30}$$

which means that the dynamics of $\tilde{\theta}$ are marginally stable. This means for the Lyapunov function in the statement of Proposition 2.4, the dynamics of x and $\tilde{\theta}$ have an upper-bound.

□

Proposition 2.5. *Given a system* $(\dot{x}, \dot{\tilde{\theta}}) = f(x, \tilde{\theta})$. *For a quadratic Lyapunov function of the form* $V(x, \tilde{\theta} = x^T Px + \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$, *where* $P^T = P > 0$ *and* Γ *is a positive constant which satisfies the condition* $\dot{V}_a(x) = \dot{V}(x, \tilde{\theta}) \leq -\frac{x^T x}{2} + K$ *where* K *varies with time and* $V_a = -\frac{x^T x}{2}$. *If* $\lim_{t\to\infty} K(t) = 0$, *then* $\lim_{t\to\infty} V(t) = 0$.

Proof. In a similar fashion to Proposition 2.4, we split the Lyapunov into two parts as shown in (2.27a) and (2.27b). Using Proposition 2.2, we get:

$$V_a(x) \leq \bar{\epsilon}, \tag{2.31}$$

for a time $t > t^*$. From Proposition 2.4, we get that $V_b(\tilde{\theta}$ is a constant so let us assume that at time $t > t^*$, we have $V_b(\tilde{\theta}) = \epsilon_1$. So we can show that:

$$\begin{aligned} V_a(x) + V_b(\tilde{\theta}) &\leq \bar{\epsilon} + \epsilon_1 \\ V(x, \tilde{\theta}) &\leq \epsilon_2, \end{aligned} \tag{2.32}$$

where $\epsilon_2 = \bar{\epsilon} + \epsilon_1$. This proves that if $\lim_{t\to\infty} K(t) = 0$, then $\lim_{t\to\infty} V(t) = 0$

□

2.5 Summary

We have gone through lemmas, propositions, and some definitions in this chapter. These preliminaries are used later in the thesis.

3 Literature Research

The use of string stability and u-CACC which were defined in Chapter 2 is seen in almost all the researches whose problem definition somewhat aligns with our own. Although not mentioned here, these methods make use of Lemma 2.2 to prove stability. Upon taking a look at the steps taken by others, we discuss in Section 3.5 about the gap in the previous research.

3.1 Mass adaptation law and continuous sliding mode control

The method mentioned in [9] uses some adaptation laws to estimate mass and combines it with sliding mode control method. This approach does give some promising results. It is started off as giving the expression for error for distance error between two vehicles which is expressed as:

$$e_{smi} = (q_{i-1} - q_i) - q_i^d, \quad (\text{for i} = 1, \dots, n.) \tag{3.1}$$

where q_i^d is the desired inter-vehicle distance. Following the error expression, the proposed sliding surface as cited in [9] is can be derived and is written as:

$$s_i = \dot{e}_{smi} + \zeta e_{smi}, \tag{3.2}$$

where ζ is a positive design constant. The equation (3.2) however does not guarantee string stability, so a new sliding surface which requires information from both preceding and following vehicles is designed in [9] as:

$$S_i = \begin{cases} ws_{smi} - s_{smi+1}, & (i = 1, ..., n-1) \\ ws_{smi}, & (i = n) \end{cases} \tag{3.3}$$

where the parameter $w > 0$ is a weighting factor. To guarantee string stability of the platoon, a control law is used in [9] is expressed as:

$$u_i = \hat{c_{i1}}\dot{q_i}^2 + \hat{c_{i2}} + \hat{D}_i \operatorname{sgn}(S_i) + \frac{\hat{m}_i}{w+1}T_i + \frac{k}{w+1}S_i + \frac{\bar{k}}{w+1}\operatorname{sgn}(S_i), \tag{3.4}$$

where $\bar{k}$ and k are positive constants, $\hat{c_{i1}}$, $\hat{c_{i2}}$, $\hat{m}_i$ are estimates of vehicle's unknown parameters c_{i1} (aerodynamic drag), c_{i2} (rolling resistance), m_i (mass of vehicle) and $T_i = w\ddot{q}_{i-1} + \ddot{q}_{i+1} + \zeta(w\dot{e}_{smi} - \dot{e}_{smi+1})$. The term D_i is bounded by the condition $|m_i\delta_i| \leq D_i$ whose estimate is defined as $\hat{D}_i$. In the control law, the unknown parameters are estimated with adaptation laws and are mentioned in [9] as follows:

$$\dot{\hat{c_{i1}}} = \gamma_i^{ci1}(w+1)S_i\dot{q}_i^2, \tag{3.5}$$

$$\dot{\hat{c_{i2}}} = \gamma_i^{ci2}(w+1)S_i, \tag{3.6}$$

$$\dot{\hat{D}}_i = \gamma_i^D(w+1)|S_i|, \tag{3.7}$$

$$\dot{\hat{m}}_i = \gamma_i^m T_i S_i, \tag{3.8}$$

where γ_i^{ci1}, γ_i^{ci2}, γ_i^D and γ_i^m are positive adaptation gains for $i = 1, \dots, n$. Using this method we get that $\lim_{t\to\infty} e_{smi} = 0$.

3.2 Combining constant time headway and predecessor following communication topology

The approach of combining constant time headway strategy and predecessor following communication topology to model a robust time-delay feedback control is discussed in [11]. By using some linear matrix inequalities conditions, the platoon formation and its stability is guaranteed. If we refer the position, velocity, and acceleration of the i^{th} vehicle is given by symbols q_i, v_i, and a_i, then the longitudinal dynamics of the vehicles in [11] is given by:

$$\dot{q}_i(t) = v_i(t), \tag{3.9}$$

$$\dot{v}_i(t) = a_i(t), \tag{3.10}$$

$$\dot{a}_i(t) = f_i(v_i(t), a_i(t)) + g_i(v_i(t)\nu_i(t)), \tag{3.11}$$

where $\nu_i(t)$ is the engine input of the ith vehicle at time $t \geq 0$. The functions f_i and g_i are given as:

$$f_i(v_i, a_i) = -\frac{1}{\tau_i}\left(a_i + \frac{\rho_a A_f c_d}{2m_i} v_i^2 + \frac{p_{mi}}{m_i}\right) - \frac{\rho_a A_f c_d v_i a_i}{2m_i}, \tag{3.12}$$

$$g_i(v_i) = \frac{1}{\tau_i m_i}, \tag{3.13}$$

where τ_i is the internal actuator dynamics of the vehicle in tracking any desired acceleration command, ρ_a is the air density, and A_f, c_d, p_{mi} and m_i are the cross sectional area, drag coefficient, mechanical drag and mass of the i^{th} vehicle respectively. The value of the term ν_i used to linearize the equation in (3.11) is:

$$\nu_i = a_{di} m_i + \frac{1}{2}\rho_a A_f c_d v_i^2 + p_{mi} + \tau_i \rho_a A_f c_d v_i a_i, \tag{3.14}$$

where a_{di} is the desired acceleration of the vehicle. Upon substituting (3.14) in (3.11):

$$\dot{q}_i(t) = \dot{v}_i(t), \tag{3.15a}$$

$$\dot{v}_i(t) = \dot{a}_i(t), \tag{3.15b}$$

$$\dot{a}_i(t) = -\frac{1}{\tau_i} a_i + \frac{1}{\tau_i} a_{di}. \tag{3.15c}$$

The safe spacing policy is given as:

$$d_{r,i}(t) = D_{r,i} + T_i v_i(t), \tag{3.16}$$

where $d_{r,i}$ is the desired spacing between vehicles, $D_{r,i}$ is the desired safety inter-vehicle distance at standstill, v_i is the velocity and T_i is the time gap of the i^{th} vehicle. The car following error in [11] is defined as $e_{fi} = d_i - d_{r,1} = q_{i-1} - q_i - L_i - d_{r,1}$. The error state vector of the ith vehicle is selected as $y_i = [e_{fi} \quad \Delta v_i \quad a_i]^T$, where Δv_i is the relative velocity between the ith-pair adjacent vehicles. The dynamics of the error variables for vehicle in [11] can be represented as:

$$\dot{y}_i(t) = (G_i + \Delta G_i(t)) y_i(t) + (E_i + \Delta E_i(t)) a_{di}(t) + H_i y_{i-1}(t), \tag{3.17}$$

where matrices

$$G_i = \begin{bmatrix} 0 & 1 & -T_i \\ 0 & 0 & -1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}, \; E_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix}, \; H_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \tag{3.18}$$

for $i = 1, ..., N$ and uncertainties ΔG_i and ΔE_i are unknown parameter perturbations of the model. The perturbations are caused by uncertain internal dynamics of vehicles and uncertain conditions. The uncertainties are described by a set of unknown matrix functions. This method guarantees that the system is robust string stable for the given unknown parameter perturbations ΔG_i and ΔE_i.

3.3 Using predecessor-following topology to make data-driven optimal CACC

The approach taken in section 3.2 is used to some extent to get the dynamics equations (3.9), (3.10) and (3.15c). The state representation of the vehicle in [12] however is given as:

$$\dot{x}_i(t) = G_i x_i(t) + E_i a_{di}(t) + H_i a_{i-1}(t), \tag{3.19}$$

with matrices

$$G_i = \begin{bmatrix} 0 & 1 & -h_i \\ 0 & 0 & -1 + \frac{h_i}{\tau_i} \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}, \; E_i = \begin{bmatrix} 0 \\ -\frac{h_i}{\tau_i} \\ \frac{1}{\tau_i} \end{bmatrix}, \; H_i = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \tag{3.20}$$

where h_i is the constant time headway and τ_i is the time constant of the i^{th} vehicle. The CACC controller input is defined as $u_i = -k_i x_i$ with the gain $k_i = [k_{i1}, k_{i2}, k_{i3}]$. The gains $k_i, i = 1, ..., n$ are computed so that the system is stable for unknown dynamical parameters. Also, instead of using matrix inequalities, this approach in [12] deviates and uses a Riccati equation to minimize a cost function which is defined as:

$$J_i(x_i(t)) = \int_t^\infty [x_i^T(s) Q_i x_i(s) + a_{di}^2(s)] ds, \tag{3.21}$$

where $x_i(t)$ is the state of the system at time $t \geq 0$ and weighted matrix $Q_i = Q_i^T > 0$. The Riccati equation which is given as:

$$G_i^T P_i + P_i G_i^T + Q_i - P_i B_i B_i^T P_i = 0, \tag{3.22}$$

and the solution P_i is used to obtain the optimal feedback gain $k_i^* = E_i^T P_i^*$. Since (3.22) is nonlinear some methods are used to get the value of P_i. The optimal controller input used is then used in the input as $u_i = -k_i^* x_i$. If $k_{0,i} \in \mathcal{R}^{1x3}$ is the stabilizing controller gain and $P_{l,i} \in \mathcal{R}^{3x3}$ is the positive symmetric definite solution to the Lyapunov equation:

$$(G_i - E_i k_{k,i})^T P_{l,i} + P_{l,i}(G_i - E_i k_{l,i}) + Q_i + k_{l,i}^T k_{l,i} = 0, \tag{3.23}$$

where l is the iteration number and feedback gain:

$$k_{l,i} = E_i^T P_{l-1,i}, \quad l = 1, 2, ... \tag{3.24}$$

Then the conditions $(G_i - B_i k_{l,i})$ is Hurwitz and $P_i^* \leq P_{l+1,i} \leq P_{l.i}$ hold for any iteration of $l \geq 1$. Moreover, $k_{l.i} = k_i^*$ and $P_{l,i} = P_i^*$ when $l \to \infty$. The optimal solutions show that the system is stable to the origin according to [13] although [13] has no mention of a cost function or the stability but instead uses an estimated Kalman filter.

3.4 Using the dwell time switching approach and adaptation laws

We came across a dwell time switching approach in [14]. The procedure uses two different controllers, one for CACC and one for ACC. It switches to ACC when there is communication loss. The uncertainty is introduced in the drive-line dynamics or the time constant which we refer to as τ_i. The time constant of i^{th} vehicle is given as:

$$\tau_i = \tau_\circ + \Delta\tau_i, \tag{3.25}$$

where $\tau_\circ$ is the time constant of the leader and the term $\Delta\tau_i$ is the unknown uncertainty in the drive-line dynamics which can not be determined. The system used draws inspiration from [6]. The research shows asymptotic stability around the equilibrium point using the method.

3.5 Research gap

The sliding mode control shows a lot of promise on paper but it is not viable practically. In theory, we make our system follow a defined path called the sliding surface regardless of the starting point but in practical the system is prone to chattering effect.

As for the approach taken in Section 3.2 and Section 3.3, the paper assumes that they have desired performance when they cancel out the vehicle dynamics in (3.15c) and make the longitudinal dynamics linear. The question arises about the performance if the dynamics of the vehicle is non-linear as mentioned in Section 1.3 which is not being considered here.

The adaptation laws in Section 3.4 seem promising but it can be a problem if the leader decides not to share the value of its time constant $\tau_\circ$. For that case, the research shows that it could switch the ACC controller or back. The dwell time switching only works effectively of the rules of switching are followed [15]. For non-linear systems, some uniformity assumptions have to be satisfied for the method to work [16].

3.6 Summary

This chapter starts with a method which made use of sliding mode control to solve the problem. After this we saw the use of using unknown perturbations and cost function which assumed that aerodynamic drag, rolling resistance and the unknown mass had already been satisfied. The use of dwell time was also seen which made use of switching between different controllers to work. As mentioned in the start of the chapter, the last three approaches made use of u-CACC which is something we are going to avoid for our work. Finally we discussed the research gap in Section 3.5 which we have to fill in our work.

4 Stability of a system with unknown parameters

The previous chapter got us acquainted with some of the methods used to deal with the problem defined in Section 1.3. Contrary to the other approaches, we do not linearize the system by cancelling out all the dynamics. As mentioned in Section 2.1.2, we use the a-CACC controller to work out our problem due to the inherent advantages it has over u-CACC. We use the same system as defined in Section 1.2. We also add the integral action to our PD controller as to improve the controller from [7]. Following the same steps as a-CACC in Section 2.1.2, we start by defining the error:

$$x_2 = e_{i,1} = q_{i-1} - q_i - h_i v_i, \tag{4.1}$$

the derivative of the error:

$$x_3 = \dot{e}_{i,1} = e_{i,2} = v_{i-1} - v_i - h_i a_i, \tag{4.2}$$

the integral of the error:

$$x_1 = \int e_{i,1} = x_1 = (\int q_{i-1} dt - q_i t - L_i t) - (r_i t + h_i q_i), \tag{4.3}$$

and the difference between the velocities:

$$x_4 = \epsilon = v_{i-1} - v_i, \tag{4.4}$$

as our error coordinates for the system. When the dynamics are not known, the equation of jerk from (A.10) based on the input defined in (1.4) changes to:

$$\begin{aligned} \dot{a}_i = &-\frac{1}{\tau_i} a_i + \frac{1}{\tau_i m_i} [(\hat{c}_{i1} - c_{i1})(v_i^2 + 2 v_i a_i \tau_i) + (\hat{c}_{i2} - c_{i2}) + \begin{bmatrix} k_i & k_p & k_d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &+ \hat{m}_i \left(1 - \frac{\tau_i}{h_i}\right) a_i + \frac{\tau_i}{h_i} a_{i-1}] \end{aligned} \tag{4.5}$$

Before we try to check the stability of the system with unknown parameters, we need to make sure that the system would be stable or marginally stable when all the dynamics are known. In (4.5) when the parameters are known, the equation is written as:

$$\dot{a}_i = -\frac{1}{\tau_i} a_i + \frac{1}{\tau_i m_i} [\begin{bmatrix} k_i & k_p & k_d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \left(1 - \frac{\tau_i}{h_i}\right) a_i + \frac{1}{m_i h_i} a_{i-1}]. \tag{4.6}$$

We define the system state vector as $x = [x_1,\ x_2,\ x_3,\ x_4]$. The closed loop error dynamics can be written as:

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\bar{k}_i & -\bar{k}_p & -\bar{k}_d & 0 \\ 0 & 0 & \frac{1}{h_i} & -\frac{1}{h_i} \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 - \frac{1}{m_i} \\ 1 \end{bmatrix}}_{B} a_{i-1}, \tag{4.7}$$

where $\bar{k}_p = \frac{h_i k_p}{\tau_i m_i}$ and $\bar{k}_d = \frac{h_i k_d}{\tau_i m_i}$. To check whether the system is bounded or not, we start by defining a quadratic Lyapunov function as defined in Lemma 2.2:

$$V_1 = x^T P x, \tag{4.8}$$

where $P = P^T$ is a constant matrix such that $A^T P + PA = -I$, where I is an identity matrix and the condition for $eigs(A) < 0$ holds when $\bar{k}_p > 0$, $\bar{k}_d > 0$, $\bar{k}_i > 0$, $\bar{k}_i < \bar{k}_p \bar{k}_d$ and $h_i > 0$. Now, for the next step we differentiate the Lyapunov function:

$$
\begin{aligned}
\dot{V}_1 &= \frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial x}\dot{x}, \\
&= 0 + \frac{\partial x^T P x}{\partial x}\dot{x}, \\
&= \frac{\partial \sum_{n=1}^{i}\sum_{n=1}^{j} P_{ij}x_i x_j}{\partial x}\dot{x}, \\
&\text{For a kth element of x we have,} \\
&= \sum_{n=1}^{k} P_{ij}x_j \frac{\partial x_k}{\partial x_k}\dot{x}_k + \sum_{n=1}^{k} P_{ij}x_i \frac{\partial x_k}{\partial x_k}\dot{x}_k, \\
&= \dot{x}^T P x + x^T P \dot{x}, \\
&= ((Ax + Ba_{i-1})^T P x + x^T P(Ax + Ba_{i-1})), \\
&= x^T(A^T P + PA)x + a_{i-1}B^T P x + x^T P B a_{i-1}, \\
&= -x^T x + a_{i-1}(B^T P x + x^T P B), \\
&= -\frac{x^T x}{2} - \left(\frac{x}{\sqrt{2}} - \frac{PBa_{i-1}}{\sqrt{2}}\right)^T \left(\frac{x}{\sqrt{2}} - \frac{PBa_{i-1}}{\sqrt{2}}\right) + \frac{(PB)^T(PB)(a_{i-1})^2}{2}, \\
&\leq -\frac{x^T x}{2} + \frac{(PB)^T(PB)(a_{i-1})^2}{2}, \\
&\leq -\frac{x^T x}{2} + \frac{\beta(a_{i-1})^2}{2}.
\end{aligned} \tag{4.9}
$$

If the acceleration of the predecessor is 0, the derivative of the Lyapunov function is negative at all times suggesting that the function keeps on decreasing and the system is asymptotically stable according to Lemma 2.2. If the acceleration has an upper-bound Z then the system can be written in the form $\dot{V}_1(x) \leq -\frac{x^T x}{2} + C_1$, where $C_1 = \frac{\beta Z^2}{2}$. According to Proposition 2.1, the error dynamics is bounded and the system is marginally stable. Also if $\lim_{t\to\infty} a_{i-1}^2 = 0$, then the system can be written in the form $\dot{V}_1(x) \leq -\frac{x^T x}{2} + K_1$ and according to Proposition 2.2, $\lim_{t\to\infty} V_1(x) = 0$.

We first check if the system is marginally stable or bounded when we assume a wrong value for the unknown parameter in the a-CACC controller. Then we look at the methods to correct this by using some update laws to stabilize the system to make a adaptive controller.

4.1 Incorrect assumption of the unknown parameters by a-CACC controller

We start by assuming a value for the unknown parameters which is different than the actual value. In this case, the values of $\hat{c}_{i1}$, $\hat{c}_{i2}$, and $\hat{m}_i$ are constants.

4.1.1 When the rolling resistance is unknown

The closed loop error dynamics is represented as:

$$
\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\bar{k}_i & -\bar{k}_p & -\bar{k}_d & 0 \\ 0 & 0 & \frac{1}{h_i} & -\frac{1}{h_i} \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 - \frac{1}{m_i} \\ 1 \end{bmatrix}}_{B} a_{i-1} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{G_1} \underbrace{\left(\frac{h_i c_{i2}}{m_i \tau_i} - \frac{h_i \hat{c}_{i2}}{m_i \tau_i}\right)}_{\tilde{\theta}_1}. \tag{4.10}
$$

The value of $\tilde{\theta}$ is a constant here as we are assuming an arbitrary value from the controller. We use the Lyapunov function:

$$
V_2 = x^T P x + \frac{1}{2}\tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1, \tag{4.11}
$$

where $P^T = P > 0$ and $A^T P + PA = -I$. Differentiate both sides of the equation. Since the part $\frac{1}{2}\tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1$ is constant, it becomes 0. Following the same steps in (4.9) until we get:

$$\begin{aligned}\dot{V}_2 &\leq -\frac{x^T x}{2} + \frac{\beta(a_{i-1})^2}{2} + (G_1^T P x + x^T P G_1)\tilde{\theta}_1 \\ &\leq -\frac{x^T x}{2} + \frac{\beta(a_{i-1})^2}{2} - \left(\frac{x}{2} - \frac{PG_1\tilde{\theta}_1}{2}\right)^T \left(\frac{x}{2} - \frac{PG_1\tilde{\theta}_1}{2}\right) + \frac{(PG_1)^T (PG_1)\tilde{\theta}_1^2}{4} - \frac{x^T x}{4} \\ &\leq -\frac{x^T x}{4} + \frac{\beta(a_{i-1})^2}{2} + \frac{(PG_1)^T (PG_1)\tilde{\theta}_1^2}{4}\end{aligned} \tag{4.12}$$

If we give the acceleration of the predecessor an upper-bound Z like before, we can upper-bound the terms $\frac{\beta(Z)^2}{2} + \frac{(PG_1)^T (PG_1)\tilde{\theta}_1^2}{4}$ as a constant F_r. The system takes the form $\dot{V}_2(x) \leq -\frac{x^T x}{4} + F_r$ and according to the Proposition 2.1, the error dynamics remain bounded. It can also be shown that if $\lim_{t\to\infty} a_{i-1}^2 = 0$, then according to Proposition 2.3, $\lim_{t\to\infty} V_2(x) = 0$.

4.1.2 When the mass is unknown

The derivative of the $e_{i,2}$ is now:

$$\dot{e}_{i,2} = \bar{k}_i x_1 - \bar{k}_p x_2 - \bar{k}_d x_3 - \left(1 - \frac{\hat{m}_i}{m_i}\right)\left(\frac{1}{\tau_i} - \frac{1}{h_i}\right) x_3 + \left(1 - \frac{\hat{m}_i}{m_i}\right)\left(\frac{1}{\tau_i} - \frac{1}{h_i}\right) x_4 + \left(1 - \frac{1}{m_i}\right) a_{i-1} \tag{4.13}$$

The closed loop error dynamics comes out to be:

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\bar{k}_i & -\bar{k}_p & -\bar{k}_d - \left(1 - \frac{\hat{m}_i}{m_i}\right)\left(\frac{1}{\tau_i} - \frac{1}{h_i}\right) & \left(1 - \frac{\hat{m}_i}{m_i}\right)\left(\frac{1}{\tau_i} - \frac{1}{h_i}\right) \\ 0 & 0 & \frac{1}{h_i} & -\frac{1}{h_i} \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 - \frac{1}{m_i} \\ 1 \end{bmatrix}}_{B} a_{i-1}, \tag{4.14}$$

where $h_i > 0$, $\bar{k}_d > 0$, $\bar{k}_p > 0$, $\left(1 - \frac{\hat{m}_i}{m_i}\right)\left(\frac{1}{\tau_i} - \frac{1}{h_i}\right) > 0$, $\bar{k}_i > 0$ and $\left(\bar{k}_d \left(1 + \frac{1}{h}\right) + \bar{k}_p\right) > 0$ so that the matrix A has all eigen values in the left half plane. We define a quadratic Lyapunov function:

$$V_3 = x^T P x, \tag{4.15}$$

where $P^T = P > 0$ and $A^T P + PA = -I$. Following the same steps in (4.9), we get the condition:

$$\dot{V}_3 \leq -\frac{x^T x}{2} + \frac{\beta(a_{i-1})^2}{2}. \tag{4.16}$$

Similar to V_1, we can prove that the system is bounded if a_{i-1} is bounded and if $\lim_{t\to\infty} a_{i-1}^2 = 0$ then $\lim_{t\to\infty} V_3(x) = 0$ using the Proposition 2.1 and Proposition 2.2.

4.1.3 When the aerodynamic drag is unknown

Following the equation for jerk in (4.5) and assuming that all the other dynamics apart from the aerodynamic drag are satisfied, we get:

$$\begin{aligned}\dot{a}_i = -\frac{1}{\tau_i} a_i + \frac{1}{\tau_i m_i}\left[(\hat{c}_{i1} - c_{i1})(v_i^2 + 2 v_i a_i \tau_i) + \begin{bmatrix} k_i & k_p & k_d \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right] \\ + \frac{1}{\tau_i}\left(1 - \frac{\tau_i}{h_i}\right) a_i + \frac{1}{h_i m_i} a_{i-1}.\end{aligned} \tag{4.17}$$

The change in the derivative of the derivative of the error is:

$$\dot{e}_{i,2} = -\bar{k}_i x_1 - \bar{k}_p x_2 - \bar{k}_d x_3 - \frac{h_i}{\tau_i m_i}(\hat{c}_{i1} - c_{i1})(v_i^2 + 2v_i a_i \tau_i). \tag{4.18}$$

We can see that (4.18) has nonlinear terms which complicates things. Proving marginal stability for this case was tough. Therefore, in Section 5.2, we use simulations to find out "if the system is marginally stable when the aerodynamic drag parameters are incorrectly estimated?"

4.2 Adaptive controller using update laws

We have seen that the system stays bounded when we assume certain values for the unknown parameters but the question arises can we correct them? Can we find a suitable law which can allow our system to be stable or marginally stable? We take a look at each cases one at a time. In this section we introduce an update law gain Γ which is different for every case of unknown dynamics.

4.2.1 When the rolling resistance is unknown

So, the dynamics with the input η_i is written as:

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\bar{k}_i & -\bar{k}_p & -\bar{k}_d & 0 \\ 0 & 0 & \frac{1}{h_i} & -\frac{1}{h_i} \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 - \frac{1}{m_i} \\ 1 \end{bmatrix}}_{B} a_{i-1} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{G_1} \underbrace{\left(\frac{h_i c_{i2}}{m_i \tau_i} - \frac{h_i \hat{c}_{i2}}{m_i \tau_i} \right)}_{\tilde{\theta}_1}. \tag{4.19}$$

For the sake of simplicity, we can represent $Ax + Ba_{i-1}$ as $f(x,t)$. We have already proven that the value of $f(x,t)$ is bounded for an upper-bound to the acceleration of the preceding vehicle. We start by again defining a different Lyapunov function than before:

$$V_4 = x^T P x + \frac{1}{2}\tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1, \tag{4.20}$$

where $\Gamma_1 > 0$ is the rolling resistance gain constant. Then we differentiate the function and it can be written as:

$$\begin{aligned} \dot{V}_4 &= \frac{\partial V_4}{\partial t} + \frac{\partial V_4}{\partial x}\dot{x} + \frac{\partial V_4}{\partial \tilde{\theta}_1}\dot{\tilde{\theta}}_1, \\ &= 0 + \frac{\partial (x^T P x + \frac{1}{2}\tilde{\theta}_1^T \Gamma_1^{-1}\tilde{\theta}_1)}{\partial x}\dot{x} + \frac{\partial (x^T P x + \frac{1}{2}\tilde{\theta}_1^T \Gamma_1^{-1}\tilde{\theta}_1)}{\partial \tilde{\theta}_1}\dot{\tilde{\theta}}_1, \\ &= \frac{\partial x^T P x}{\partial x}\dot{x} + \frac{1}{2}\frac{\partial \tilde{\theta}_1^T \Gamma_1 \tilde{\theta}_1}{\partial \tilde{\theta}_1}\dot{\tilde{\theta}}_1, \\ &= \dot{x}^T P x + x^T P \dot{x} + \frac{1}{2}\dot{\tilde{\theta}}_1^T \Gamma_1^{-1}\tilde{\theta}_1 + \frac{1}{2}\tilde{\theta}_1^T \Gamma_1^{-1}\dot{\tilde{\theta}}_1. \end{aligned} \tag{4.21}$$

The value of $\dot{\tilde{\theta}}_1^T \Gamma_1^{-1}\tilde{\theta}_1$ is a 1 by 1 matrix and Γ_1^{-1} is symmetric. This means that $\dot{\tilde{\theta}}_1^T \Gamma_1^{-1}\tilde{\theta}_1 = \tilde{\theta}_1^T \Gamma_1^{-1}\dot{\tilde{\theta}}_1$. Using this it follows:

$$\begin{aligned} \dot{V}_4 &= \dot{x}^T P x + x^T P \dot{x} + \dot{\tilde{\theta}}_1^T \Gamma_1^{-1}\tilde{\theta}_1, \\ &= (f(x,t) + G_1\tilde{\theta}_1)^T P x + x^T P (f(x,t) + G_1\tilde{\theta}_1) + \dot{\tilde{\theta}}_1^T \Gamma_1^{-1}\tilde{\theta}_1, \\ &= (f(x,t))^T P x + x^T P (f(x,t)) + (G_1\tilde{\theta}_1)^T P x + \\ &\quad x^T P (G_1\tilde{\theta}_1) + \dot{\tilde{\theta}}_1^T \Gamma_1^{-1}\tilde{\theta}_1. \end{aligned} \tag{4.22}$$

The matrix P is a scalar (adaptive law gain) and the value of $(G_1\tilde{\theta}_1)^T P x$ is a 1 by 1 matrix and so $(G_1\tilde{\theta}_1)^T P x = x^T P (G_1\tilde{\theta}_1)$. This leads us to:

$$\begin{aligned} \dot{V}_4 &= -x^T x + a_{i-1}(B^T P x + x^T P B) + 2x^T P (G_1\tilde{\theta}_1) + \dot{\tilde{\theta}}_1^T \Gamma_1^{-1}\tilde{\theta}_1, \\ &= -x^T x + a_{i-1}(B^T P x + x^T P B) + (2x^T P G_1 + \dot{\tilde{\theta}}_1^T \Gamma_1^{-1})\tilde{\theta}_1. \end{aligned} \tag{4.23}$$

As we know that V_1 is already bounded so for V_4 to be bounded we need an update law that fulfills the condition $2x^T PG_1 + \dot{\tilde{\theta}}_1^T \Gamma_1^{-1} = 0$. So we can start by:

$$\begin{aligned}
-2x^T PG_1 &= \dot{\tilde{\theta}}_1^T \Gamma_1^{-1}, \\
-2x^T PG_1\Gamma_1 &= \dot{\tilde{\theta}}_1^T, \\
-2(x^T PG_1\Gamma_1)^T &= \dot{\tilde{\theta}}_1, \\
-2\Gamma_1 G_1^T Px &= \dot{\tilde{\theta}}_1.
\end{aligned} \tag{4.24}$$

The (4.24) is an update law which can guarantee boundedness for the system. As, by using the update law, we get the relation:

$$\dot{V}_4 = -x^T x + a_{i-1}(B^T Px + x^T PB). \tag{4.25}$$

Following the same steps as in (4.9), we get the condition:

$$\dot{V}_4 \le -\frac{x^T x}{2} + \frac{\beta(a_{i-1})^2}{2}. \tag{4.26}$$

Using Proposition 2.4, we can prove that the errors remain bounded if the value a_{i-1} has an upper bound and when $\lim_{t\to\infty} a_{i-1}^2 = 0$ then $\lim_{t\to\infty} V_4(x) = 0$ using the Proposition 2.5.

4.2.2 When the aerodynamic drag is unknown

For this case, we are assuming no wind conditions and no fluctuations in temperature so that there is no change in the aerodynamic drag parameter c_a. The closed loop error dynamics are:

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\bar{k}_i & -\bar{k}_p & -\bar{k}_d & 0 \\ 0 & 0 & \frac{1}{h_i} & -\frac{1}{h_i} \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1-\frac{1}{m_i} \\ 1 \end{bmatrix}}_{B} a_{i-1} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{2v_i a_i \tau_i + v_i^2}{\tau_i m_i} \\ 0 \end{bmatrix}}_{G_2(v_i,a_i)} \underbrace{(c_a - \hat{c}_a)}_{\tilde{\theta}_2}. \tag{4.27}$$

Using the Lyapunov function:

$$V_5 = x^T Px + \tilde{\theta}_2^T \Gamma_2^{-1} \tilde{\theta}_2, \tag{4.28}$$

where $\Gamma_2 > 0$ is a scalar (aerodynamic drag gain constant). We are able to reach the condition in the same way as we did when the rolling resistance was unknown. The final equation can be written as:

$$\dot{V}_5 = -x^T x + a_{i-1}(B^T Px + x^T PB) + (2x^T PG_2(v_i, a_i) + \dot{\tilde{\theta}}_2^T \Gamma_2^{-1})\tilde{\theta}_2. \tag{4.29}$$

The update law for this case can be obtained by:

$$\begin{aligned}
-2x^T PG_2(v_i, a_i) &= \dot{\tilde{\theta}}_2^T \Gamma_2^{-1}, \\
-2x^T PG_2(v_i, a_i)\Gamma_1 &= \dot{\tilde{\theta}}_2^T, \\
-2(x^T PG_2(v_i, a_i)\Gamma_2)^T &= \dot{\tilde{\theta}}_2, \\
-2\Gamma_2 G_2(v_i, a_i)^T Px &= \dot{\tilde{\theta}}_2.
\end{aligned} \tag{4.30}$$

Using the update law in (4.30) and the steps in (4.9), we reach the condition:

$$\dot{V}_5 \le -\frac{x^T x}{2} + \frac{\beta(a_{i-1})^2}{2}. \tag{4.31}$$

The system errors are bounded if a_{i-1} has an upper-bound which we can prove using Proposition 2.4 and by Proposition 2.5 it can be proven that when $\lim_{t\to\infty} a_{i-1}^2 = 0$ then $\lim_{t\to\infty} V_5(x) = 0$.

4.2.3 When only the mass is unknown

So, the error dynamics would be:

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\bar{k}_i & -\bar{k}_p & -\bar{k}_d & 0 \\ 0 & 0 & \frac{1}{h_i} & -\frac{1}{h_i} \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1-\frac{1}{m_i} \\ 1 \end{bmatrix}}_{B} a_{i-1} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \left(\frac{h_i}{\tau_i}-1\right) a_i \\ 0 \end{bmatrix}}_{G_3(a_i)} \underbrace{\left(1-\frac{\hat{m}_i}{m_i}\right)}_{\tilde{\theta}_3}. \tag{4.32}$$

The Lyapunov function for this case is:

$$V_6 = x^T P x + \tilde{\theta}_3^T \Gamma_3^{-1} \tilde{\theta}_3, \tag{4.33}$$

where $\Gamma_3 > 0$ is a scalar (unknown mass gain constant). Following the same steps we can reach the condition of the update law:

$$-2\Gamma_3 G_3(a_i)^T P x = \dot{\tilde{\theta}}_3. \tag{4.34}$$

Using similar steps in Section 4.2.2 and the update law in (4.34), we are able to get the condition:

$$\dot{V}_6 \leq -\frac{x^T x}{2} + \frac{\beta (a_{i-1})^2}{2}. \tag{4.35}$$

If we provide an upper-bound to a_{i-1}, then using Proposition 2.4, we can show that the errors of the system are bounded. Moreover, using Proposition 2.5 we can prove that when $\lim_{t\to\infty} a_{i-1}^2 = 0$ then $\lim_{t\to\infty} V_5(x) = 0$.

4.3 Summary

When presented with the problem presented in Section 1.3, we checked and verified that the system is still marginally stable if we are to assume a wrong value for the unknown parameter. Then we were able to get some update laws which when introduced to the system would make the system marginally stable. The results were verified mathematically.

5 Simulations

We proved the stability for most of the unknown dynamics in Chapter 4 but is the a-CACC controller stable when aerodynamic drag is assumed incorrectly in Section 4.1.3? How do the incorrectly estimated a-CACC controller and adaptive controller hold against each other in face of unknown dynamics? Do we need the integral action of the error in either of our controllers

5.1 Simulation Environment

For the simulations we make use of ode45 solver in MATLAB. We simulate a homogeneous platoon of five Toyota Prius cars. The values of mass ($m = 1380\ kg$ [23]), aerodynamic drag ($c_d = 0.24$ [24]), frontal area ($A_f = 2.22\ m^2$ [25]), time constant($\tau = 0.1$ [7]), friction coefficient ($\mu = 0.7$ [26]) , and constant time headway ($h = 0.5$ [7]) is the same for every vehicle. The values of gains used are $k_{pm} = \frac{k_p}{m_i} = 0.2$ [7], $k_{dm} = \frac{k_d}{m_i} = 0.68$ [7], and $k_{im} = \frac{k_i}{m_i} = 10^{-5}$. The value of k_{im} is chosen close to 0 due to the reason we see in Section 5.4. We make use of a rectangular pulse similar to [8] in the upcoming sections. The input for the leader is kept at 0 for 5 seconds, then an acceleration of 1 m/s^2 is applied for 5 seconds and for the remaining time the acceleration of the leader is put to 0. In all simulations, the initial velocity, position and acceleration of the followers is set to 0.

5.2 Unknown aerodynamic drag

We make use of (4.17) in the ode45 solver. We assume that the wind speed does not change and take into account the change in temperature in the environment. We take the assumed density of air to be 1.225 kg/m^3 at 15° Celsius and refer to the value from [21]. We take the actual temperature to be about 30° Celsius and the air density to be 1.164 kg/m^3. The rest of the parameters used are listed in Section 5.1. As mentioned in Section 4.1.3, we do not know if the system is stable if we estimated the wrong value for the a-CACC controller. Just like the cases when only the mass or the rolling resistance is wrongly estimated, we expect that the system is stable. We subject the leader vehicle to a constant acceleration of 1 m/s^2 for 50 seconds, so that it simulates a step input for the first follower as seen in Figure 5.1. The a-CACC controller settles down to the value of 1 m/s^2 when starting from an initial acceleration of 0 m/s^2 in approximately 11 seconds.

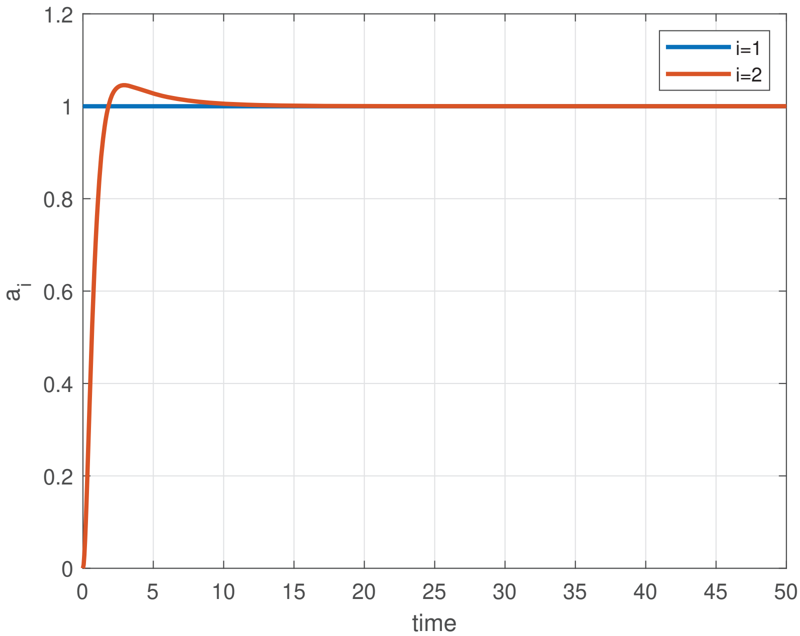
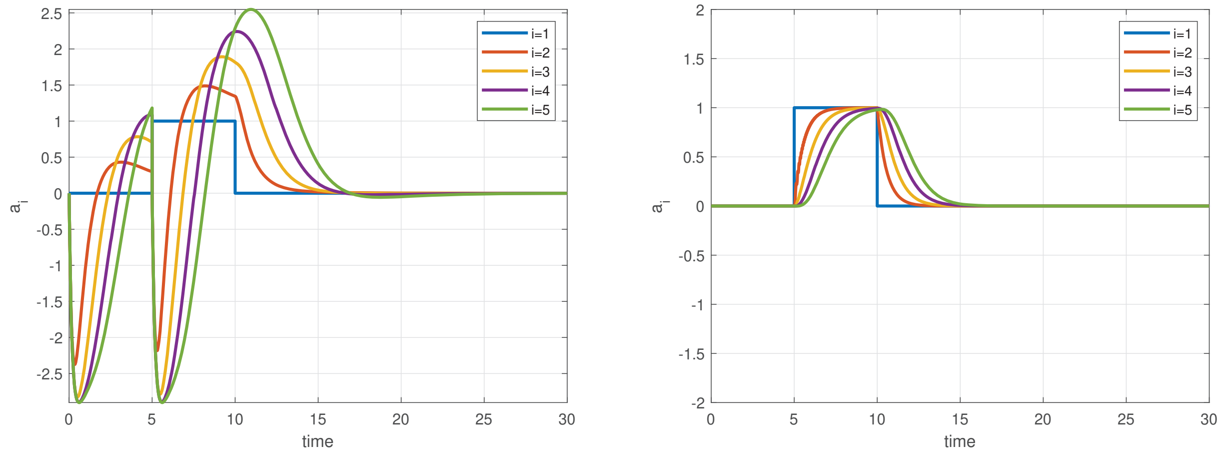


Figure 5.1: a-CACC controller with estimated aerodynamic drag subject to a step input



(a) a-CACC controller with estimated values *(b) Adaptive controller with* $\Gamma = 1000$

Figure 5.2: Response of the adaptive and the a-CACC controller to a rectangular pulse for unknown dynamics

5.3 Comparison between estimated a-CACC and adaptive a-CACC

When the dynamics are unknown, the adaptive controller must outperform the a-CACC controller which is estimating incorrect values of the unknown dynamics. The adaptive laws as seen in Section 4.2.1, Section 4.2.2, and Section 4.2.3 make use of Γ_1, Γ_2, and Γ_3 which are the update law gains. We estimated that the higher values of $\Gamma = \Gamma_1 = \Gamma_2 = \Gamma_3$ would give better performance as we proposed that higher the gains, faster the update law works to correct the uncertainty in the dynamics. We also assumed that there should be a maximum value for the gains as there is a possibility of overshoots when the convergence to the response is faster. As seen in Appendix C, the higher values of Γ have lesser settling times and lower overshoots. We also see that for lower values of Γ, there are oscillations in the followers which dissipate as we increase the value of the update law gain. We make a comparison of both the controllers when subject to a rectangular pulse mentioned in 5.1 when the dynamics are unknown. It can be seen in Figure 5.2 that the adaptive controller has lesser overshoots and undershoots than the a-CACC controller with incorrect estimated values of the unknown dynamics.

5.4 Use of the Integral action

One question still arises that although there was no integral action in the controller used in [7], we have included the integral action to the a-CACC controller and the adaptive controller but do we still need it? We make use of a step input from the leader of 1 m/s^2 for 50 seconds. We start by simulating the response of the a-CACC controller when it estimates wrong values to different values of k_{im}. As mentioned in Chapter 4, the condition $0 < k_{im} < k_{dm}k_{pm}$ must be satisfied for stability, we get the bounds $0 < k_{im} < 0.136$ according to the values of k_{dm} and kpm mentioned in Section 5.1. The resulting figures as seen in Appendix D show that the integral action for the estimated a-CACC controller is not required which is more clear when we compare the parameters for different k_{im} values in Table 5.1. We simulate the adaptive controller using the same step input for different k_{im} values. As seen in Appendix E, the graphs look the same so the Table 5.2 gives us an insight that there is a significant change to the parameters of the graphs in Appendix E. We estimated the system to be unstable when the bounds on k_{im} are not respected. The Figure D.5 confirms that the system indeed becomes stable but the Figure E.5 shows a different story. Although not preferable, the adaptive controller still stabilizes with a higher value of k_{im}

k_{im}	Settling time (secs)	Overshoot	Undershoot	Peak
0	10	48	218	2.18
0.02	10.28	59	218	2.18
0.03	19.03	65	218	2.18
0.13	26.53	113	218	2.18

Table 5.1: Characteristics of the first follower using the estimated a-CACC controller when subjected to a step input for different values of k_{im}

k_{im}	Settling time (secs)	Overshoot	Undershoot	Peak
0	2.17	1.3 x 10^{-8}	0	1
0.02	2.17	1.36 x 10^{-5}	0	1
0.03	2.17	2.12 x 10^{-5}	0	1
0.13	2.17	9.8 x 10^{-5}	0	1
10	2.16	0.1108	0	1.011

Table 5.2: Characteristics of the first follower using the adaptive controller when subjected to a step input for different values of k_{im}

5.5 Summary

We get our answer to the question in Section 4.1.3 in Section 5.2 that the a-CACC controller is stable when the aerodynamic drag is estimated incorrectly. We compared our a-CACC (which has incorrectly assumed dynamics) to adaptive CACC controller in Section 5.3. We checked if the integral action is necessary in Section 5.4. The next chapter focuses on conclusions.

6 Conclusions and Future work

We present conclusions to the simulations in the Chapter 5 in Section 6.1 and then we present some future work that can be done regarding the research.

6.1 Conclusions

Starting off with the results in Section 5.2, we see that as expected the a-CACC controller with incorrect estimate of the aerodynamic drag settles down to the value of the step input of 1 which shows that the a-CACC controller for the case presented in Section 4.1.3 is stable.

Comparing the graphs in Figure 5.2, we can conclude that the adaptive controller works better than the a-CACC controller whose dynamics have been incorrectly estimated. The large undershoot and overshoot causes the vehicle to experience high jerks which would cause discomfort for the passengers. Moreover, the settling time for the a-CACC controller with unknown dynamics is quite large compared to the adaptive controller as it can be seen in the Figure 5.2.

In Section 5.4, we simulated the a-CACC with incorrect dynamics and the adaptive controller with different values of k_{im} and found out that it does not contribute much to the system in both cases. Based on these findings we concluded that the integral action for the system is not required for either of our controllers. The case when the adaptive controller stabilizes at $k_{im} = 10$ shows that the adaptive controller outperforms the estimated a-CACC controller when parameters are unknown.

6.2 Future work

As mentioned in Section 5.3 and shown in Appendix C, the higher values of the update law gain Γ which we assumed as $\Gamma = \Gamma_1 = \Gamma_2 = \Gamma_3$ prove better performance. We saw lesser overshoots which were negligible as we kept on increasing the gain. The value of gains required are needed to be verified experimentally as there is bound to be an upper limit to these gains.

Although the use of integral action is shown to not be needed in both of our controllers, in an experiment the integral action may be used to offset unwanted disturbances. The case for the aerodynamic drag can be expanded to include wind speed into account because as mentioned in [20] the wind speed does have a large influence on the aerodynamic drag coefficient c_d. The string stability of both the controllers needs to be verified. The values of the gains k_{pm} and k_{dm} were chosen from the research mentioned in [7] which were tuned experimentally for a homogeneous platoon. In our simulations these depend on the mass of the vehicle which will vary according to the mass and the gains would be needed to be adjusted accordingly. The effect of the change in mass would also affect the time gap h_i of every vehicle which can be explored further.

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A Longitudinal dynamics

From the longitudinal dynamics defined in Section 1.2, We assume that aerodynamic drag, rolling resistance and the mass of the vehicle affect our vehicle. The aerodynamic drag equation is written as:

$$F_{aerodrag} = \frac{1}{2}\rho_a c_d A_f v_i^2, \tag{A.1}$$

in which the parameters ρ_a, c_d and A_f show the values of density of air, coefficient of aerodynamic drag (varies from vehicle to vehicle) and the frontal area of the vehicle (in m^2). We have multiplied the constants from (A.1) such that we get the term $c_{i1}v_i^2$ in (1.1b). The term c_{i2} represents the rolling resistance which can be defined as:

$$c_{i2} = F_{rollingresistance} = \mu m_i g \cos\theta + m_i g \sin\theta, \tag{A.2}$$

where g, μ, and θ are defined as acceleration due to gravity, the friction coefficient between the road and the tyres of the vehicle, and the slope of the road on which the vehicle is driving respectively. In the Netherlands, we have a low amount of roads with any slope so we can assume here $\theta = 0$ which leads to the rolling resistance as $\mu m_i g$.

The derivative of velocity is termed as acceleration which we can represent here as a_i for the i^{th} vehicle, so in (1.1b), we can replace $\dot{v}_i$ with a_i and rewrite the equation as:

$$m_i a_i = F_i - c_{i1}v_i^2 - c_{i2}. \tag{A.3}$$

Differentiating both side of the equation, we get:

$$m_i \dot{a}_i = \dot{F}_i - 2c_{i1}v_i\dot{v}_i = \dot{F}_i - 2c_{i1}v_i a_i. \tag{A.4}$$

Substitute the value of $\dot{F}_i$ from (1.1c):

$$m_i \dot{a}_i = -\frac{1}{\tau_i}F_i + \frac{1}{\tau_i}\eta_i - 2c_{i1}v_i a_i. \tag{A.5}$$

We rearrange (A.3) for a proper value of F_i to get:

$$F_i = m_i a_i + c_{i1}v_i^2 + c_{i2}. \tag{A.6}$$

We substitute this value in (A.5) to achieve the result:

$$m_i \dot{a}_i = -\frac{1}{\tau_i}(m_i a_i + c_{i1}v_i^2 + c_{i2}) + \frac{1}{\tau_i}\eta_i - 2c_{i1}v_i a_i. \tag{A.7}$$

Dividing both sides of the equation by m_i and simplifying the equation:

$$\begin{aligned} \frac{m_i\dot{a}_i}{m_i} &= -\frac{1}{\tau_i m_i}(m_i a_i + c_{i1}v_i^2 + c_{i2}) + \frac{1}{\tau_i m_i}\eta_i - \frac{2c_{i1}v_i a_i}{m_i}, \\ \dot{a}_i &= -\frac{1}{\tau_i}a_i - \frac{1}{\tau_i m_i}(c_{i1}v_i^2 + c_{i2}) + \frac{1}{\tau_i m_i}\eta_i - \frac{2c_{i1}v_i a_i}{m_i}, \\ \dot{a}_i &= -\frac{1}{\tau_i}a_i - \frac{1}{\tau_i m_i}(c_{i1}v_i^2 + c_{i2} + 2c_{i1}v_i a_i \tau_i - \eta_i). \end{aligned} \tag{A.8}$$

The linearizing of the dynamics is done by using the controller input:

$$\eta_i = c_{i1}v_i^2 + c_{i2} + 2c_{i1}v_i a_i \tau_i + m_i u_i. \tag{A.9}$$

The linearization makes the system simpler to solve. Substitute the value in (A.8) to get the value of jerk as:

$$\dot{a}_i = -\frac{1}{\tau_i}a_i + \frac{1}{\tau_i}u_i. \tag{A.10}$$

B Norms

The definitions of the norms is taken from [17]

Definition B.1 ($\mathcal{L}_p$ signal norm [17])**.** *Let u(t) be a time-dependent signal according to u(t)=($u_1(t)$ $u_2(t)$ ... $u_n(t)$)^T, then the signal p-norm, or $\mathcal{L}_p$ norm, of u(t) is defined as:*

$$||u(t)||_p := \left(\int_{-\infty}^{\infty} \sum_i |u_i(t)|^p dt \right)^{\frac{1}{p}}, \tag{B.1}$$

where $||u(t)||_p$ denotes the vector p-norm.

Definition B.2 ($\mathcal{H}_\infty$ system norm [17])**.** *Consider a transfer function $G(s)$ of a linear system, then the $\mathcal{H}_\infty$ norm is defined as:*

$$||G(j\omega)||_{\mathcal{H}_\infty} = \sup_{\omega} \max_{\hat{u} \neq 0} \frac{||G(j\omega)\hat{u}(j\omega)||_2}{||\hat{u}(j\omega)||_2}, \tag{B.2}$$

where $\hat{u}(j\omega)$ is the input signal to the system.

C Tuning of the gain parameter Γ for the adaptive controller

The use of (4.5) is made for the adaptive controller. The parameter Γ is kept the same for all the update laws. The use of Γ is made in Section 5.3. We use the values given in Section 5.1. We apply a constant acceleration of 1 m/s^2 to the leader for 50 seconds and calculate the characteristics as seen in Table C.1. The response to a rectangular pulse to the system defined in Section 5.1 to different values of Γ is shown in Figures C.1, C.2, C.3, C.4, C.5 and C.6.

Gamma	Settling time (secs)	Overshoot	Undershoot	Peak
1	2.6	11.2	0	1.11
5	2.35	1.92	0	1.01
10	2.41	0.08	0	1.0008
50	2.22	1.11 x 10^{-5}	0	1
100	2.17	1.32 x 10^{-7}	0	1
1000	2.17	1.3 x 10^{-8}	0	1

Table C.1: Characteristics of the first follower using the adaptive controller when subjected to a step input

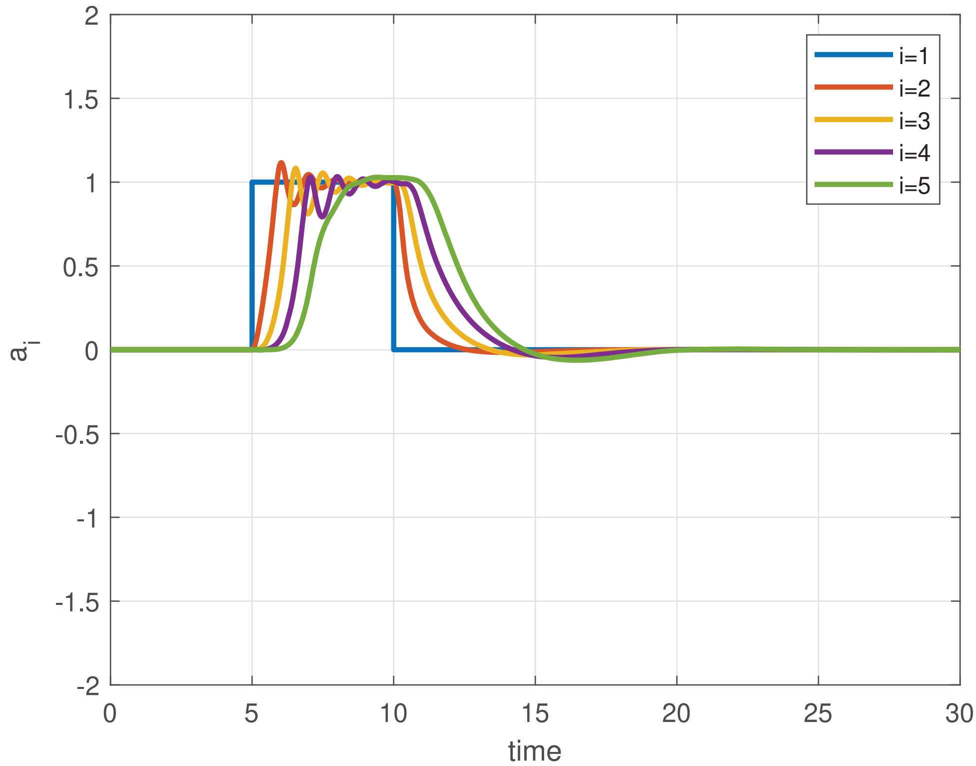

Figure C.1: Adaptive Controller with $\Gamma = 1$

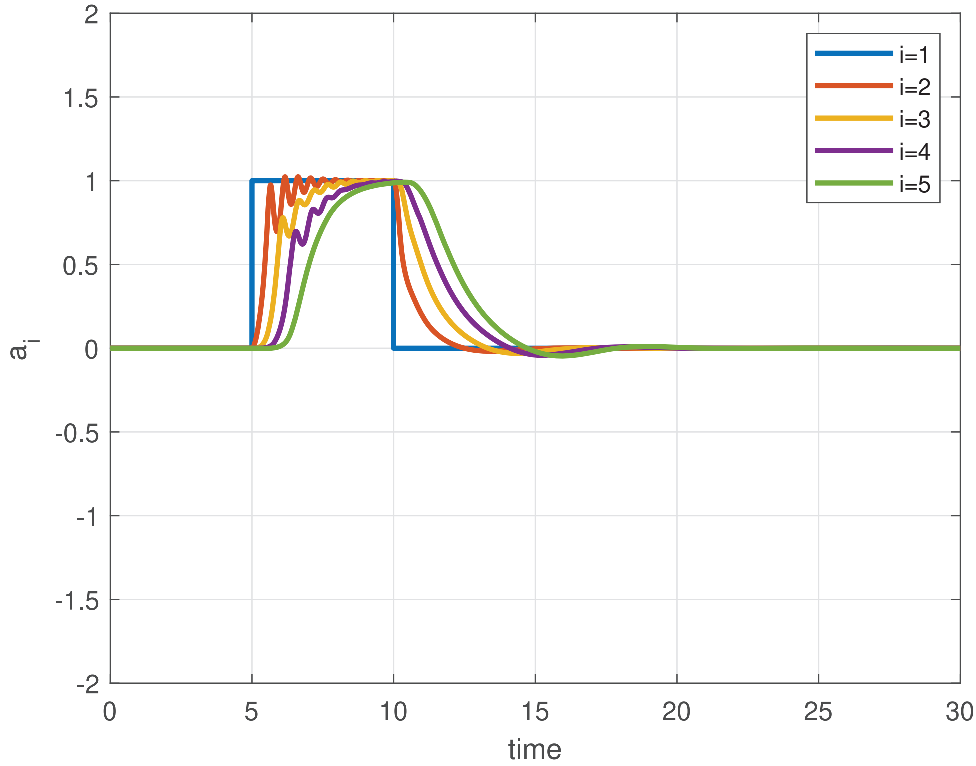

Figure C.2: Adaptive Controller with $\Gamma = 5$

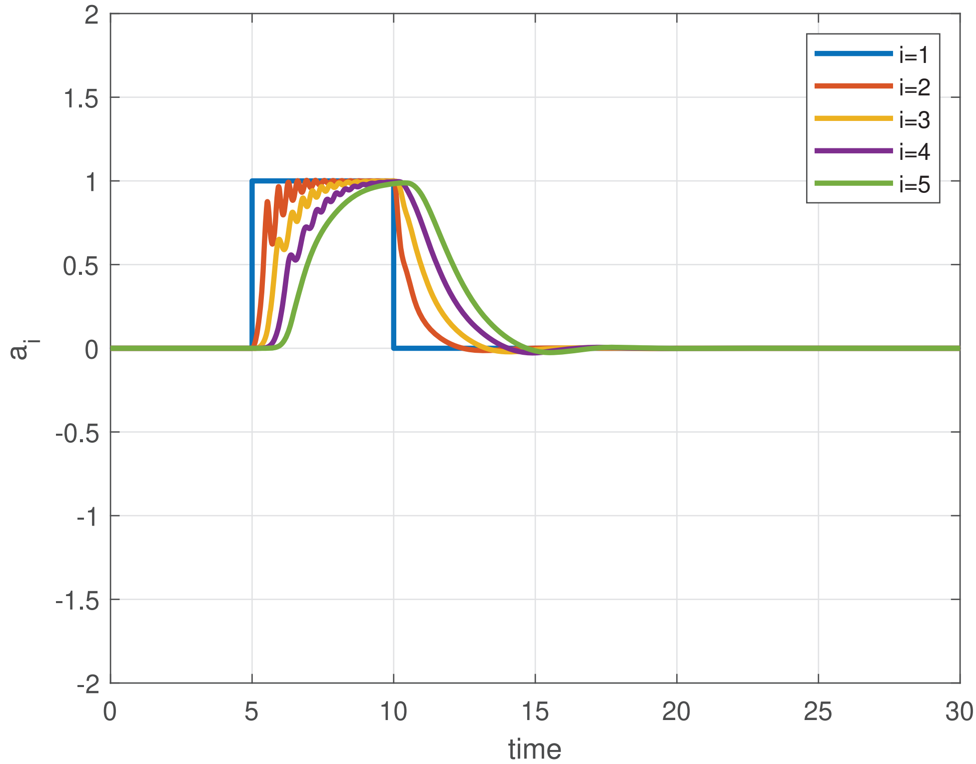

Figure C.3: Adaptive Controller with $\Gamma = 10$

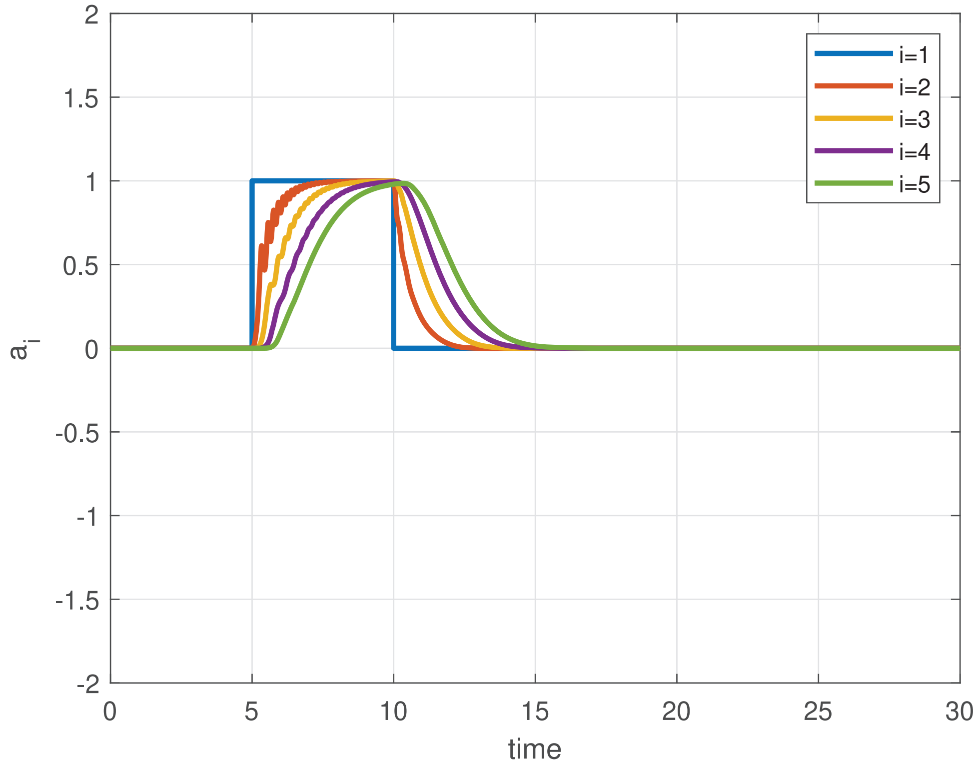

Figure C.4: Adaptive Controller with $\Gamma = 50$

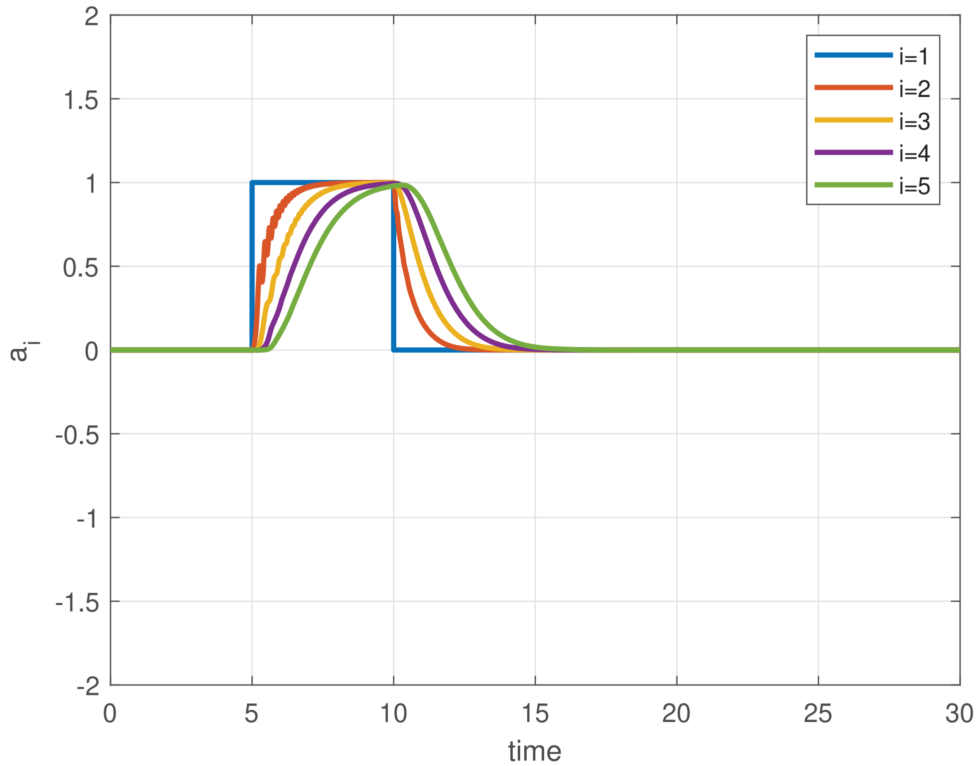

Figure C.5: Adaptive Controller with $\Gamma = 100$

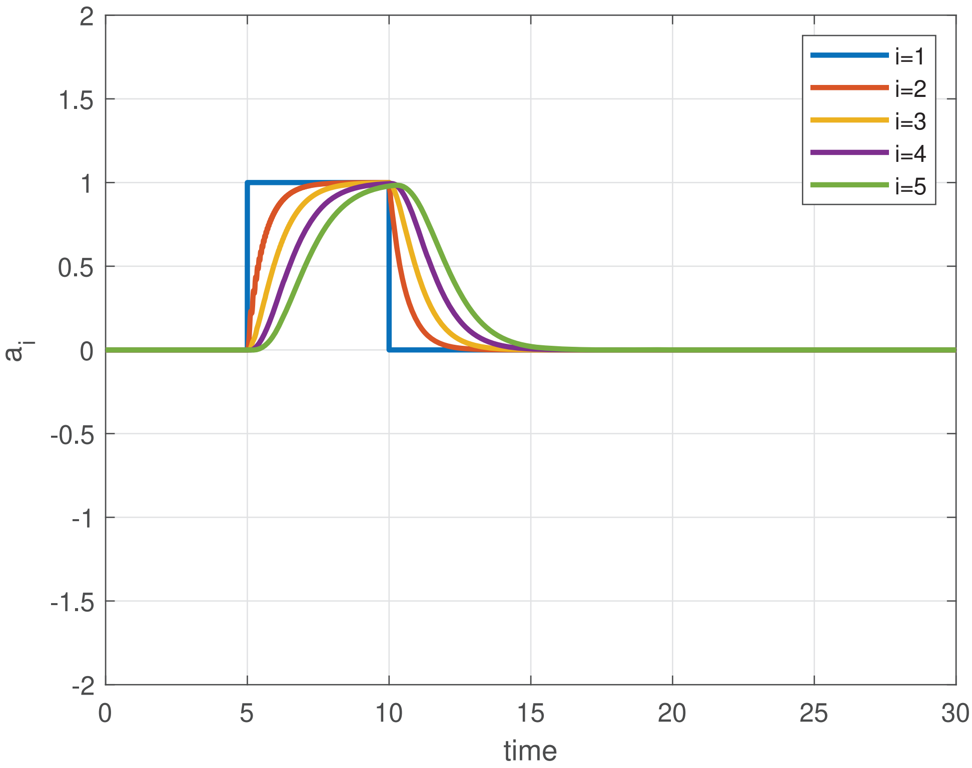

Figure C.6: Adaptive Controller with $\Gamma = 1000$

D Tuning of the gain for the integral action for the estimated a-CACC controller

We make use of the input mentioned in Section 5.4, we make use of the dynamics in (4.5) to verify if the integral action is needed for the controller. For the parameters of the signal seen in Table 5.1, it becomes clear that the integral action seems unnecessary as seen that it has little effect on stability of the system.

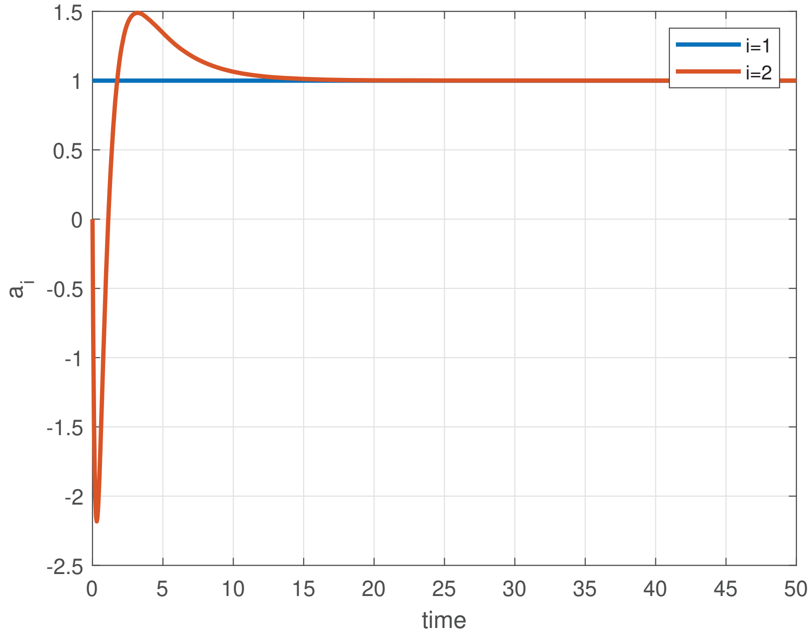

Figure D.1: Estimated a-CACC controller with $k_{im} = 0$

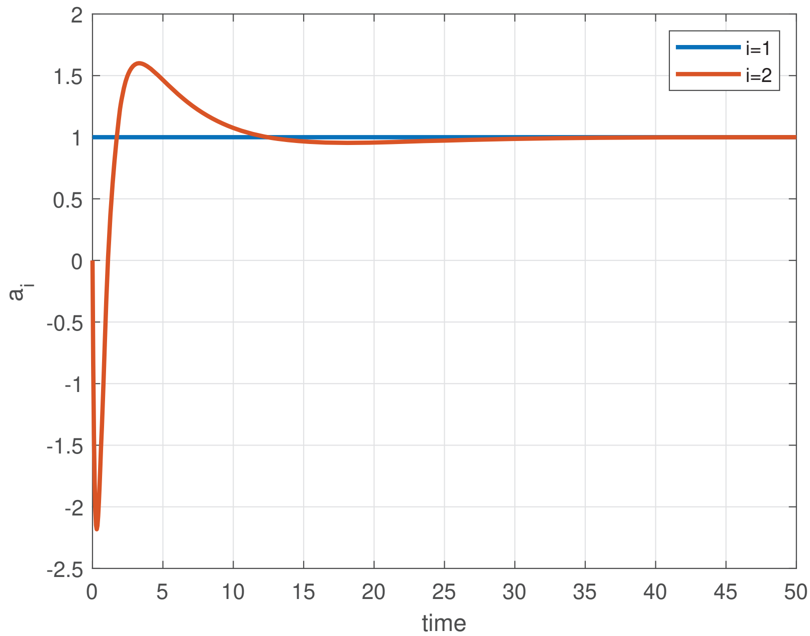

Figure D.2: Estimated a-CACC controller with $k_{im} = 0.02$

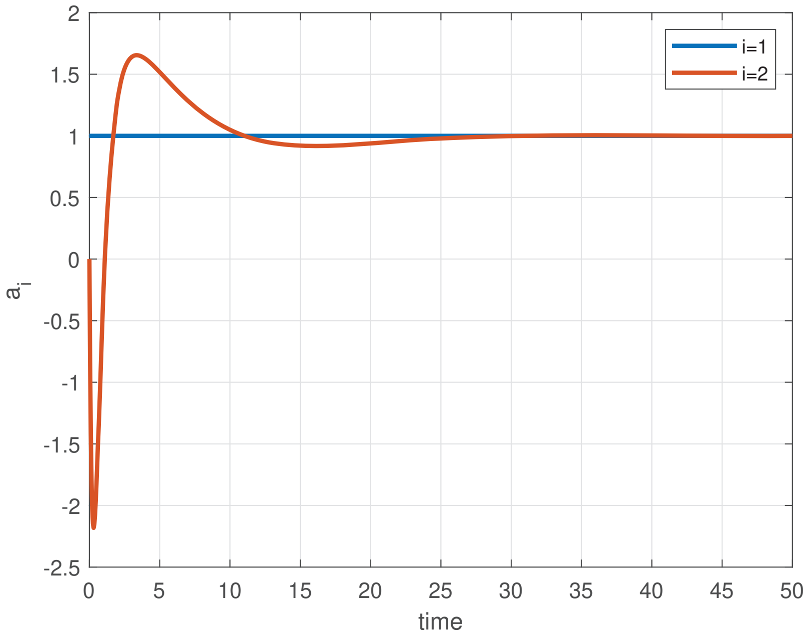

Figure D.3: Estimated a-CACC controller with $k_{im} = 0.03$

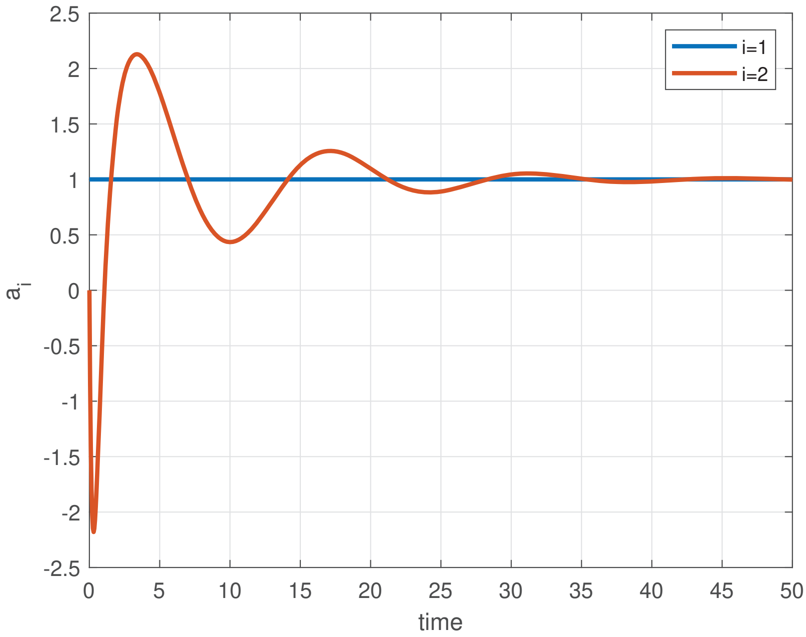

Figure D.4: Estimated a-CACC controller with $k_{im} = 0.13$

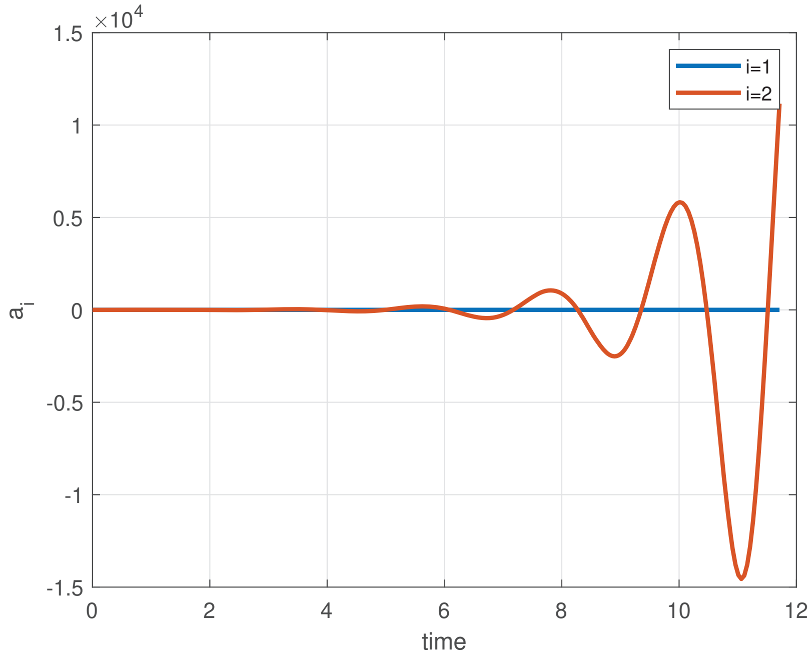

Figure D.5: Estimated a-CACC controller with $k_{im} = 10$

E Tuning of the gain of integral action for the adaptive a-CACC controller

Using the step input defined in Section 5.4 and the values mentioned in Section 5.1 for the dynamics in (4.5), we get the Figures E.1, E.2, E.3, and E.5. The step parameters mentioned in Table 5.2 show that the integral action is not needed here.

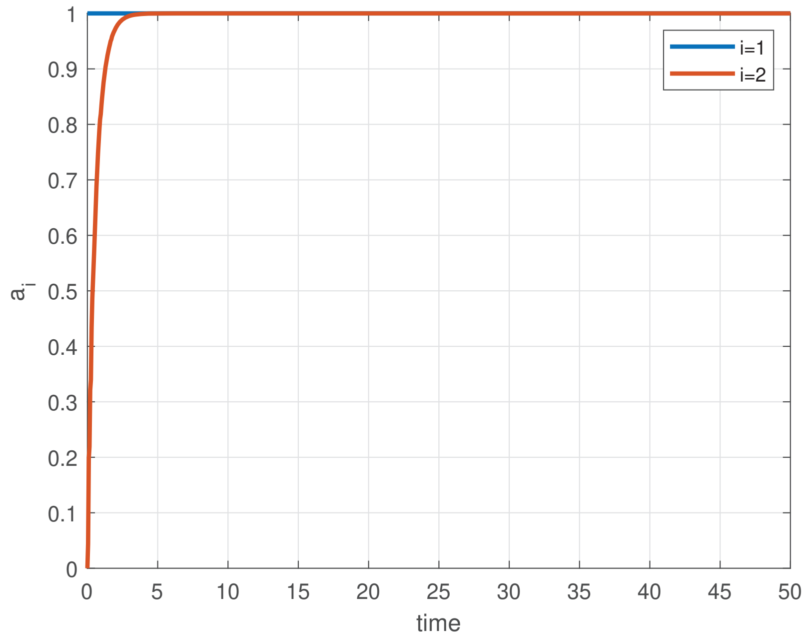

Figure E.1: Adaptive a-CACC controller with $k_{im} = 0$

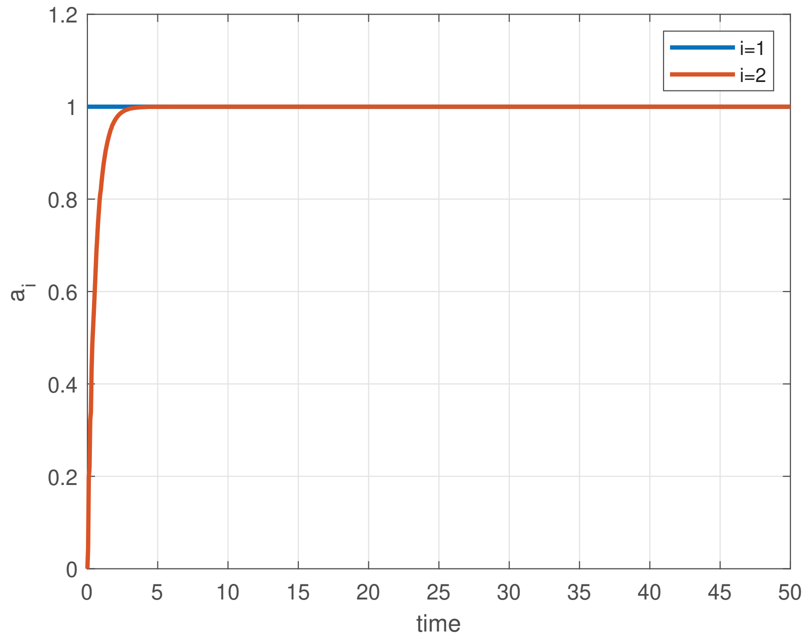

Figure E.2: Adaptive a-CACC controller with $k_{im} = 0.02$

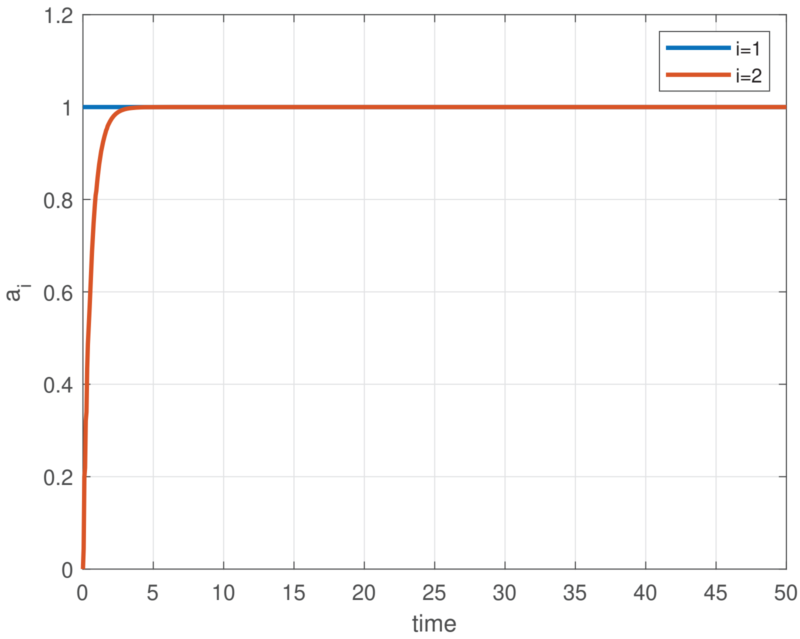

Figure E.3: Adaptive a-CACC controller with $k_{im} = 0.03$

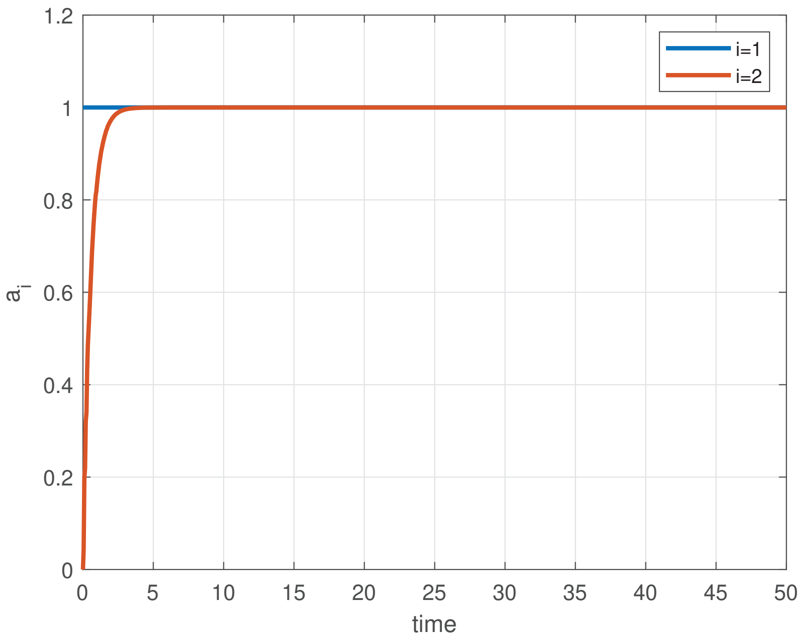

Figure E.4: Adaptive a-CACC controller with $k_{im} = 0.13$

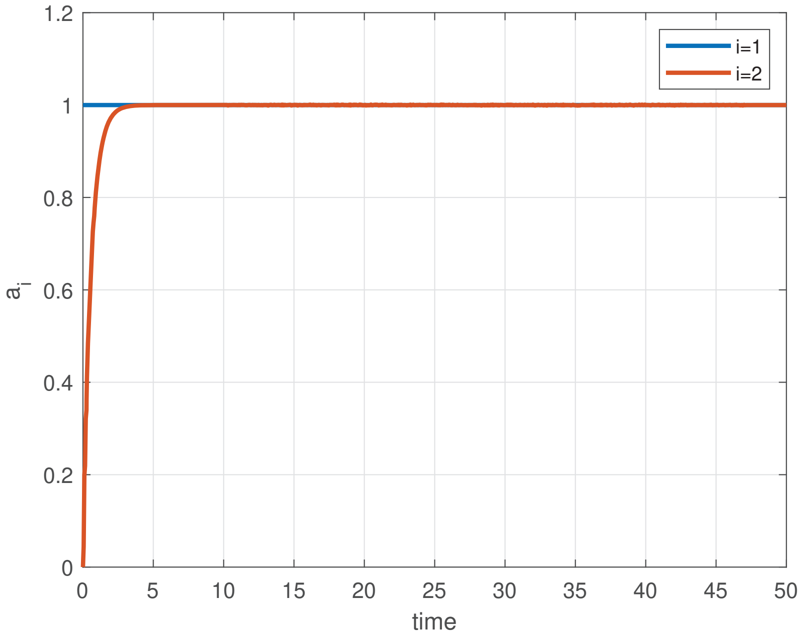

Figure E.5: Adaptive a-CACC controller with $k_{im} = 10$

Declaration concerning the TU/e Code of Scientific Conduct for the Master's thesis

I have read the TU/e Code of Scientific Conduct[i].

I hereby declare that my Master's thesis has been carried out in accordance with the rules of the TU/e Code of Scientific Conduct

Date

08-09-2022

Name

Kashish Singh Pilyal

ID-number

1485830

Signature

Submit the signed declaration to the student administration of your department.

[i] See: https://www.tue.nl/en/our-university/about-the-university/organization/integrity/scientific-integrity/
The Netherlands Code of Conduct for Scientific Integrity, endorsed by 6 umbrella organizations, including the VSNU, can be found here also. More information about scientific integrity is published on the websites of TU/e and VSNU