Almost global decentralised formation tracking for multiple distinct UAVs

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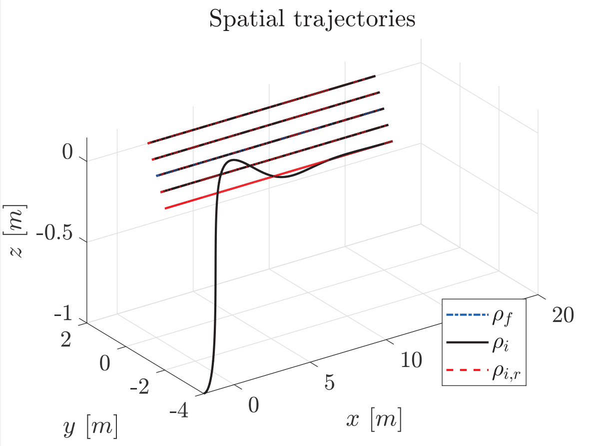
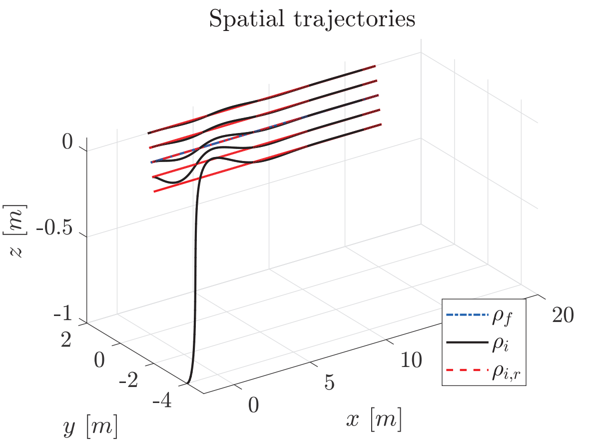
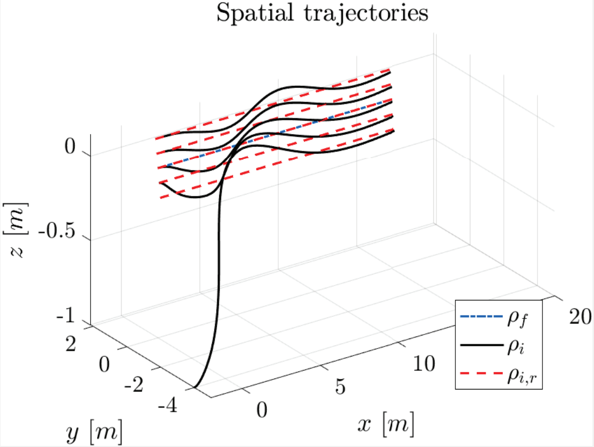
Tracking control: non-social behavior



Formation tracking: More social behavior



Formation control: Social behavior (no tracking)



Distinguishing features

Combination of all of the following ingredients

- Underactuated (not fully actuated)
- Dynamics (not only kinematics)
- Both translational and rotational dynamics
- Consider attitude on SO(3), not Euler angles or quaternions
- Almost global tracking results, not local (positive total thrust)
- Uniform asymptotic stability

Dynamics

Drone dynamics ($m_i > 0, J_i = J_i^T > 0$)

$$\begin{aligned} \dot{\rho}_i &= R_i \nu_i \\ \dot{\nu}_i &= -S(\omega_i)\nu_i + gR_i^T e_3 - \frac{f_i}{m_i} e_3 \\ \dot{R}_i &= R_i S(\omega_i) \\ J_i \dot{\omega}_i &= S(J_i \omega_i)\omega_i + \tau_i \end{aligned}$$

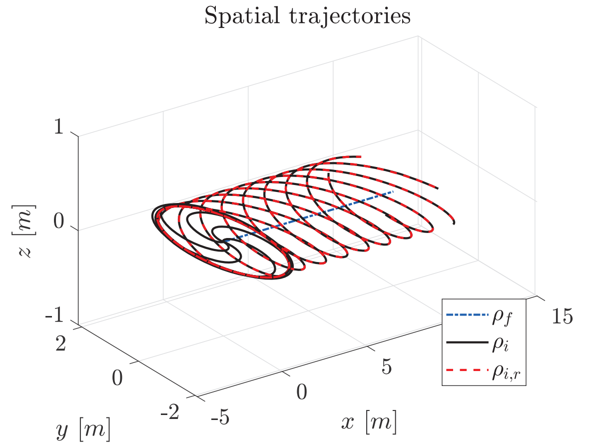
Reference dynamics ($0 < f_{r,i}^{\min} \leq f_{r,i}(t)$)

$$\begin{aligned} \dot{\rho}_{r,i} &= R_{r,i} \nu_{r,i} \\ \dot{\nu}_{r,i} &= -S(\omega_{r,i})\nu_{r,i} + gR_{r,i}^T e_3 - \frac{f_{r,i}}{m_i} e_3 \\ \dot{R}_{r,i} &= R_{r,i} S(\omega_{r,i}) \\ J_i \dot{\omega}_{r,i} &= S(J_i \omega_{r,i})\omega_{r,i} + \tau_{r,i} \end{aligned}$$

Formation frame: ρ_f, R_f.

Position expressed in formation frame: $p_{r,i} = R_f^T(\rho_{r,i} - \rho_f) \Leftrightarrow \rho_{r,i} = R_f(p_{r,i} + \rho_f)$.

Formation frame



Problem formulation (1 of 3)

Define (translational) errors expressed in formation frame:

$$p_{e,i} = p_{r,i} - p_i = R_f^T(\rho_{r,i} - \rho_i) \qquad v_{e,i} = R_f^T(R_{r,i}\nu_{r,i} - R_i\nu_i)$$

Define (bi-directional) communication topology:

$$a_{ij} = a_{ji} = \begin{cases} 1 & \text{if UAV } i \text{ and } j \text{ exchange positional information} \\ 0 & \text{otherwise} \end{cases}$$

$$b_{ij} = b_{ji} = \begin{cases} 1 & \text{if UAV } i \text{ and } j \text{ exchange velocity information} \\ 0 & \text{otherwise} \end{cases}$$

Assumption: Graphs with incidence matrices $A = [a_{ij}]$, $B = [b_{ij}]$ are connected

Problem formulation (2 of 3)

Formation forming:

$$\lim_{t\to\infty} \|p_{e,i}(t) - p_{e,j}(t)\| = 0 \qquad \lim_{t\to\infty} \|v_{e,i}(t) - v_{e,j}(t)\| = 0$$

In case of formation tracking, i.e., $\lim_{t\to\infty} \|p_{e,i}\| = \lim_{t\to\infty} \|v_{e,i}\| = 0$ also attitude tracking, i.e.

$$\lim_{t\to\infty} R_{e,i}(t) = I_3 \qquad \lim_{t\to\infty} \omega_{e,i} = 0$$

where $R_{e,i} = R_i^T R_{r,i}$ and $\omega_{e,i} = \omega_{r,i} - R_{e,i}^T \omega_i$

Problem formulation (3 of 3)

Find controllers

$$f_i = f_i^0(p_{e,i}, v_{e,i}, R_{e,i}, \omega_{e,i}, t) + \sum_{j=1}^{n} a_{ij} f_{ij}^a(p_{e,i}, R_{e,i}, \omega_{e,i}, p_{e,j}, t) + b_{ij} f_{ij}^b(v_{e,i}, R_{e,i}, \omega_{e,i}, v_{e,j}, t)$$

$$\tau_i = \tau_i^0(p_{e,i}, v_{e,i}, R_{e,i}, \omega_{e,i}, t) + \sum_{j=1}^{n} a_{ij} \tau_{ij}^a(p_{e,i}, R_{e,i}, \omega_{e,i}, p_{e,j}, t) + b_{ij} \tau_{ij}^b(v_{e,i}, R_{e,i}, \omega_{e,i}, v_{e,j}, t)$$

which guarantee formation forming and in case of formation tracking also attitude tracking

NB: incorporate trade-off between formation forming and reference tracking.

Main idea

Two steps:

1. Use virtual input to achieve formation forming (tracking) of translational dynamics
2. Achieve realisation of virtual input, and (if possible) attitude tracking

Difficulties step 1

- Time-varying dynamics (dependent on ω_f).
- Non-zero virtual input (solved using saturation)

Issue with combining step 1 and step 2

- Stability of cascade

Position tracking control

Position tracking error dynamics

$$\dot{p}_{e,i} = -S(\omega_f)p_{e,i} + v_{e,i}$$
$$\dot{v}_{e,i} = -S(\omega_f)v_{e,i} + u_{e,i},$$

where

$$u_{e,i} = -R_f^T \left(\frac{f_{r,i}}{m_i} R_{r,i} - \frac{f_i}{m_i} R_i \right) e_3.$$

Some more ingredients

- $s : \mathbb{R} \to \mathbb{R}$ in C^2 satisfies $s'(0) > 0$ and $\int s(x)dx$ radially unbounded.
- $\sigma : \mathbb{R}^n \to \mathbb{R}^n$: $\sigma(e) = s(e^T e)e$
- If $\|\sigma(e)\| \leq M$ for all $e \in \mathbb{R}^n$, σ is called saturation function.

Some observations:

- Possible candidates: $\sigma(e) = k_0 e$ and $\sigma(e) = \frac{k_0 k_\infty e}{\sqrt{k_\infty^2 + k_0^2 e^T e}}$ for $k_0 > 0$, $k_\infty > 0$.
- $\sigma(-e) = -\sigma(e)$
- If $a_{ij} = a_{ji}$ and $\sigma_{ij}(x) = \sigma_{ji}(x)$ then: $\sum_{i=1}^n \sum_{j=1}^n a_{ij}\sigma_{ij}(x_i - x_j) = 0$.

Lemma (cf. Ren, 2008, Lemma 3.1): If $a_{ij} = a_{ji}$ and $\sigma_{ij}(x) = \sigma_{ji}(x)$ then:
$\sum_{i=1}^n \sum_{j=1}^n a_{ij}(x_i - x_j)^T \sigma_{ij}(y_i - y_j) = 2 \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i^T \sigma_{ij}(y_i - y_j)$.

Position tracking result

Consider the position tracking error dynamics

$$\dot{p}_{e,i} = -S(\omega_f)p_{e,i} + v_{e,i}$$
$$\dot{v}_{e,i} = -S(\omega_f)v_{e,i} + u_{e,i},$$

with virtual input

$$u_{e,i} = -\alpha_i \sigma_{p_i}(p_{e,i}) - \beta_i \sigma_{v_i}(v_{e,i}) - \sum_{j=1}^{n} \left[a_{ij}\sigma_{p_{ij}}(p_{e,i} - p_{e,j}) + b_{ij}\sigma_{v_{ij}}(v_{e,i} - v_{e,j}) \right]$$

where $\sigma_{p_{ij}}(x) = \sigma_{p_{ji}}(x)$. Then we have for ω_f and $\dot{\omega}_f$ bounded:

- If $\alpha = \beta = 0$: Formation forming
- If $\alpha_i \geq 0$, $\beta_i \geq 0$, $\sum_{i=1}^{n} \alpha_i > 0$, $\sum_{i=1}^{n} \beta_i > 0$: formation tracking

Matrosov like theorem (Loría et.al., 2005)

Consider the dynamical system

$$\dot{x} = f(t, x) \qquad x(t_0) = x_0 \qquad f(t, 0) = 0 \qquad (*)$$

$f : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ loc. bdd., cont. a.e., loc. unif. cont. in t. If there exist

- j differentiable functions $V_i : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$, bounded in t, and
- continuous functions $Y_i : \mathbb{R}^n \to \mathbb{R}$ for $i \in \{1, 2, \ldots j\}$ such that
  - V_1 is positive definite and radially unbounded,
  - $\dot{V}_i(t, x) \leq Y_i(x)$, for all $i \in \{1, 2, \ldots, j\}$,
  - $Y_i(x) = 0$ for $i \in \{1, 2, \ldots, k-1\}$ implies $Y_k(x) \leq 0$, for all $k \in \{1, 2, \ldots, j\}$,
  - $Y_i(x) = 0$ for all $i \in \{1, 2, \ldots, j\}$ implies $x = 0$,

then the origin $x = 0$ of (*) is uniformly globally asymptotically stable.

Proof (first claim: formation forming)

$$V_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} V_{\sigma_{p_{ij}}}(p_{e,i} - p_{e,j}) + \frac{1}{2n}(v_{e,i} - v_{e,j})^T (v_{e,i} - v_{e,j})$$

$$\dot{V}_1 = -\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(v_{e,i} - v_{e,j})^T \sigma_{v_{ij}}(v_{e,i} - v_{e,j}) = Y_1(v_{e,i} - v_{e,j})$$

$$V_2 = -\sum_{i=1}^{n}\sum_{j=1}^{n}(v_{e,i} - v_{e,j})^T(\dot{v}_{e,i} - \dot{v}_{e,j})$$

$$\dot{V}_2 \leq \underbrace{-\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\sum_{k=1}^{n} a_{ik}\sigma_{p_{ik}}(p_{e,i} - p_{e,k}) - \sum_{k=1}^{n} a_{jk}\sigma_{p_{jk}}(p_{e,j} - p_{e,k})\right]^2 + M\sum_{i=1}^{n}\sum_{j=1}^{n}\|v_{e,i} - v_{e,j}\|}_{Y_2(p_{e,i}-p_{e,j},v_{e,i}-v_{e,j})}$$

Proof (second claim: formation tracking)

$$V_1 = \sum_{i=1}^{n} \alpha_i V_{\sigma_{p_i}}(p_{e,i}) + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} V_{\sigma_{p_{ij}}}(p_{e,i} - p_{e,j}) + \frac{1}{2}\sum_{i=1}^{n} v_{e,i}^T v_{e,i}$$

$$\dot{V}_1 = -\sum_{i=1}^{n} \beta_i v_{e,i}^T \sigma_{v_i}(v_{e,i}) - \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(v_{e,i} - v_{e,j})^T \sigma_{v_{ij}}(v_{e,i} - v_{e,j}) = {\color{red} Y_1(v_{e,i})}$$

$$V_2 = -\sum_{i=1}^{n} v_{e,i}^T \dot{v}_{e,i}$$

$$\dot{V}_2 \leq -\sum_{i=1}^{n}\left[\alpha_i \sigma_{p_i}(p_{e,i}) + \sum_{j=1}^{n} a_{ij}\sigma_{p_{ij}}(p_{e,i} - p_{e,j})\right]^2 + \sum_{i=1}^{n} M\|v_{e,i}\| = {\color{red} Y_2(p_{e,i}, v_{e,i})}$$

Attitude control (1)

Since $u_{e,i} = -R_f^T \left(\frac{f_{r,i}}{m_i} R_{r,i} - \frac{f_i}{m_i} R_i \right) e_3$ we ideally need f_i and R_i to satisfy:

$$f_i R_i e_3 = m_i R_f u_{e,i} + f_{r,i} R_{r,i} e_3 \qquad \text{so take } f_i = \| m_i R_f u_{e,i} + f_{r,i} R_{r,i} e_3 \|$$

Define desired thrust direction $f_{d,i} = \frac{m_i R_f u_{e,i} + f_{r,i} R_{r,i} e_3}{\| m_i R_f u_{e,i} + f_{r,i} R_{r,i} e_3 \|}$.

By using saturation functions in u_e, we can guarantee that $f_{d,i_3} > 0$ since $0 < f_{r,i}^{\min} \leq f_{r,i}(t)$.

Define desired attitude: $R_{d,i} = \begin{bmatrix} 1 - \frac{f_{d,i_1}^2}{1+f_{d,i_3}} & -\frac{f_{d,i_1} f_{d,i_2}}{1+f_{d,i_3}} & f_{d,i_1} \\ -\frac{f_{d,i_1} f_{d,i_2}}{1+f_{d,i_3}} & 1 - \frac{f_{d,i_2}^2}{1+f_{d,i_3}} & f_{d,i_2} \\ -f_{d,i_1} & -f_{d,i_2} & f_{d,i_3} \end{bmatrix} \in \mathrm{SO}(3)$

Attitude control (2)

Correspondingly, let $\omega_{d,i} = \left[-\dot{f}_{d,i_2} + \frac{f_{d,i_2}\dot{f}_{d,i_3}}{1+f_{d,i_3}} \quad \dot{f}_{d,i_1} - \frac{f_{d,i_1}\dot{f}_{d,i_3}}{1+f_{d,i_3}} \quad \frac{f_{d,i_2}\dot{f}_{d,i_1} - f_{d,i_1}\dot{f}_{d,i_2}}{1+f_{d,i_3}} \right]^T$.

Define the following attitude error and angular velocity errors:

$$\tilde{R}_i = R_{d,i}^T(R_{r,i}^T R_i) \qquad \tilde{\omega}_i = \omega_i - R_i^T R_{r,i}\omega_{r,i} - \tilde{R}_i^T \omega_{d,i},$$

Then the controller

$$\tau_i = -S(J_i\omega_i)\omega_i + J_i R_i^T R_{r,i}\dot{\omega}_{r,i} - J_i S(\tilde{\omega}_i)[\omega_i - \tilde{\omega}_i] - K_{\omega i}\tilde{\omega}_i J_i \tilde{R}_i^T [S(\omega_{d,i})R_{d,i}^T \omega_{r,i} - \dot{\omega}_{d,i}] + \sum_{j=1}^{3} k_{ji}(e_j \times \tilde{R}_i^T e_j),$$

with $K_{\omega i} = K_{\omega i}^T > 0$, and $k_{ji} > 0$ distinct (i.e., $k_{1i} \neq k_{2i} \neq k_{3i} \neq k_{1i}$), renders the resulting equilibrium point $(\tilde{R}_i, \tilde{\omega}_i) = (I, 0)$ both UaGAS and ULES.

Cascade analysis

Overall closed-loop dynamics:

$$\begin{aligned}
\dot{p}_{e,i} &= -S(\omega_f)p_{e,i} + v_{e,i} \\
\dot{v}_{e,i} &= -S(\omega_f)v_{e,i} + u_{e,i} - \frac{f_i}{m_i}(I - \tilde{R}_i^T)e_3 \\
\dot{\tilde{R}}_i &= \tilde{R}_i S(\tilde{\omega}_i) \\
J_i\dot{\tilde{\omega}}_i &= -K_{\omega i}\tilde{\omega}_i + \sum_{j=1}^{3} k_{ji}(e_j \times \tilde{R}_i^T e_j)
\end{aligned}$$

Boundedness of solutions follows from $\dot{V} \leq \sum_{i=1}^{n} v_{e,i}^T \frac{f_i}{m_i}(I - \tilde{R}_i)e_3 \leq$ $\leq M\sqrt{V}\sum_{i=1}^{n} \|I_3 - \tilde{R}_i\|$, so $\sqrt{V(t)} - \sqrt{V(t_0)} \leq \bar{M}(\tilde{R}_i(t_0), \tilde{\omega}_i(t_0))$ due to ULES

Conclusions/Concluding remarks

- decentralized controller
- *uniform* almost global asymptotic stability
- considering attitude on SO(3).
- illustrated trade-off between individual trajectory tracking (non-social) and formation forming (social)
- currently students work on experiments in lab (using Parrot Mambo drones)
- there is no need to exchange velocity information between UAVs since we can use an observer for reconstructing those