GLOBAL ASYMPTOTIC STABILITY OF ROBOT MANIPULATORS WITH LINEAR PID AND PI^{2}D CONTROL

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Abstract In this note we address the problem of set-point control of robot manipulators with uncertain gravity knowledge by combining several previous contributions to PID control. The main contribution is a *linear* PID controller which ensures *global* asymptotic stability of the closed loop. The key feature of the controller is that the integration is started after a short transient. In the case of unmeasurable velocities, a similar "delayed" PI^{2}D controller is shown to *globally* asymptotically stabilize the manipulator.

Keywords: robot control, PID control, global asymptotic stability.

1 Introduction

From [19] it is well known that a PD plus gravity compensation controller can globally asymptotically stabilize a rigid-joints manipulator. This approach has two well known drawbacks: 1) the vector of gravitational forces is assumed to be known accurately and 2) velocity measurements are needed. An *ad hoc* solution to the first problem is to compensate for the gravitational vector with the best estimate available. This method can be used if a (small) bounded steady state error can be tolerated. It is also well known that this error can be eliminated by adding an integrator. While PID control started probably with Nicholas Minorsky in 1922, in marine vessels applications, in robotics, the first *local* asymptotic stability proof of a PID controller is attributed to [3]. For a recent reference on PID (set-point and tracking) control of industrial manipulators see [18]. Concerning the problem of unmeasurable velocities, we know at least the following *linear* dynamic position feedback controllers which appeared independently [9, 1, 5, 6], see also [16] where the concept of *EL controllers* was introduced and which generalizes the results of the previous references. The PI^{2}D controller, which uses the approximate differentiation filter used in [9, 5, 6] together with an integral action was first introduced in [15]. See also [7].

For the case of measurable velocities one can design, with some smart modifications, *nonlinear* PID's which guarantee *global* asymptotic stability. As far as we know, the first nonlinear PID controller is due to [8], where a normalized integral term was used. Indeed, even though the author presented his result as an "adaptive" controller, the latter can be reformulated as a nonlinear PID (see [14]). Arimoto [2] proposed to use a saturated proportional term. This helps in the same way as the normalization to cope with third order terms which appear in the Lyapunov function derivative and impede proving global properties.

As far as we know, there exist no proof of *global* asymptotic stability of a *linear* PID controller in closed loop with a robot manipulator. Based on the discussion at the begining of the Introduction, in this note we prove that a robot manipulator in closed loop with a linear *delayed* PID controller is globally asymptotically stable. Our "delayed PID", in short PI$_d$D, can be understood as a simple PD controller to which an integral action is added after some transient of time. Also, in the case of unmeasurable velocities we show that the integral action of a PI^{2}D controller can be delayed as to guarantee the *global* asymptotic stability of the closed loop. We will call this linear controller, "delayed PI^{2}D", or PI$_d^2$D.

Our approach is inspired by the ideas of *composite control* developed in [13]. The idea of this approach consists of "patching" a global and a local controller. The first drives the solutions to an arbitrarily small domain, while the second, yields local asymptotic stability. See also [20] where the authors propose an algorithm to combine global with local controllers with the aim of improving both robustness and performance.

This note is organized as follows. In Section 2, we first state the problem of interest here and then, we recall some well known but fundamental results for our main contributions. The latter are presented in Section 3. In Section 4 we show some simulations, and Section 5 concludes the note.

Notation. In this note we use $\|\cdot\|$ for the Euclidean norm of vectors and matrices. We denote by k_{p_m} and k_{p_M} the smallest and largest eigenvalues of the matrix K_p.

2 Problem formulation and preliminary results

The rigid-joints robot kinetic energy is given by $T(q,\dot{q}) = \frac{1}{2}\dot{q}^\top D(q)\dot{q}$, where $q \in \mathbb{R}^n$ represents the link positions, $D(q) = D^\top(q) > 0$ is the robot inertia matrix, and the potential energy generating gravity forces is denoted by $U_g(q)$. Applying the Euler-Lagrange equations we obtain the well known model

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \tag{1}$$

where $g(q) := \frac{\partial U_g}{\partial q}(q)$, $C(q,\dot{q})\dot{q}$ represents the Coriolis and centrifugal forces, and $u \in \mathbb{R}^n$ are the applied torques. It is also well known now (see for instance [17]) that the following properties hold

P1 For all $q \in \mathbb{R}^n$ the matrix $D(q)$ is positive definite and, with a suitable factorization (more precisely using the so called Christoffel symbols of the first kind) the matrix $N(q,\dot{q}) = \dot{D}(q) - 2C(q,\dot{q})$ is skew symmetric. Moreover, there exist some positive constants d_m and d_M such that $d_m I < D(q) < d_M I$.

P2 There exists some positive constants k_g and k_v such that for all $q \in \mathbb{R}^n$

$$k_g \geq \sup_{q\in\mathbb{R}^n} \left\| \frac{\partial^2 U_g(q)}{\partial q^2} \right\|, \quad k_v \geq \sup_{q\in\mathbb{R}^n} \left\| \frac{\partial U_g(q)}{\partial q} \right\| . \tag{2}$$

P3 The matrix $C(x,y)$ is bounded in x and linear in y, that is, for all $z \in \mathbb{R}^n$ we have that $C(x,y)z = C(x,z)y$ and $\|C(x,y)\| \leq k_c\|y\|$ with $k_c > 0$.

Set-point control problem with uncertain gravity knowledge: Assume that only an inacurate estimate, $\hat{U}_g(q)$, of the gravitational energy function $U_g(q)$, is available. Assume also that the estimate of the gravitational forces vector, $\hat{g}(q) := \frac{\partial \hat{U}_g}{\partial q}(q)$ satisfies

$$k_v \geq \sup_{q\in\mathbb{R}^n} \|\hat{g}(q)\| , \quad \forall\, q \in \mathbb{R}^n \tag{3}$$

where k_v is defined in property **P2**. Under these conditions design continuous control laws (state feedback) $u = u(q, \dot{q})$ and (dynamic position feedback) $u = u(q, \vartheta)$, $\dot{\vartheta} = f(q, \vartheta)$, such that the closed loop (1) with u is globally asymptotically stable (GAS) at an arbitrary setpoint $(\dot{q}, q, \vartheta) = (0, q_\star, 0)$. In particular, we are interested in *linear* PID-like control laws achieving this goal.

2.1 Preliminary results

For clarity of exposition and to introduce some notation, we reconsider in this section, some well known results on robot control. These are fundamental for the proofs of our main propositions.

First case: measurable velocities. Based on the results of [19] and [21] we present below a simple robustness result *vis-a-vis* the uncertainty of $g(q)$.

Proposition 1 *Consider the robot manipulator model (1) in closed loop with the PD control law*

$$u = -K_p\tilde{q} - K_d\dot{q} + \hat{g}(q_*)\,, \tag{4}$$

where $\tilde{q} := q - q_*$. Let $k_{p_m} > k_g$, then there exists a unique equilibrium point $(\dot{q}, q) = (0, q_s)$ for the closed loop system. The point $(\dot{q}, q) = (0, q_s)$ is globally asymptotically stable for (1), (4) and the steady state error $\tilde{q}_s := q_s - q_*$ satisfies

$$\|\tilde{q}_s\| \le \frac{2k_v}{k_{p_m}}. \tag{5}$$

Sketch of proof. The closed loop equation (1), (4) is given by

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) - \hat{g}(q_*) + K_p\tilde{q} + K_d\dot{q} = 0. \tag{6}$$

The system (6) is Lagrangian with potential energy function

$$U_1(q) := U_g(q) - \hat{U}_g(q_*) - \tilde{q}^\top \hat{g}(q_*) + \frac{1}{2}\tilde{q}^\top K_p\tilde{q}\ .$$

The system (6) has equilibria, $q = q_s$, at the minima of $U_1(q)$, which are solutions of

$$\frac{\partial U_1}{\partial q}(q_s) = 0 \ \Leftrightarrow\ K_p(q_s - q_*) + g(q_s) - \hat{g}(q_*) = 0 \tag{7}$$

moreover, the equilibrium $q = q_s$ is unique if $k_{p_m} > k_g$ where k_g satisfies (2). Global asymptotic stability of the equilibrium $(\dot{q}, q) = (0, q_s)$ immediately follows using Krasovskii-LaSalle's invariance principle taking the time derivative of $V_1(q, \dot{q}) = \frac{1}{2}\dot{q}^\top D(q)\dot{q} + U_1(q)$ which qualifies as a Lyapunov function candidate if $k_{p_m} > k_g$. ■

As it is well known the steady state error $\tilde{q}_s$ can be eliminated by the use of an integrator, this result was firstly proved in [3]. Reformulating (for further analysis) the original contribution of [3] we have

Proposition 2 *Consider the dynamic model (1) in closed loop with the PID control law*

$$\begin{aligned} u &= -K_p\tilde{q} - K_d\dot{q} + \nu & (8)\\ \dot{\nu} &= -K_i\tilde{q}, \qquad \nu(0) = \nu_0 \in \mathbb{R}^n. & (9) \end{aligned}$$

where K_p, K_d, and K_i are diagonal positive definite matrices and $\tilde{q} := q - q_\star$. If K_p is sufficiently large then the closed loop is locally *asymptotically stable at the origin $x :=$ col$[\tilde{q}, \dot{q}, \tilde{\nu}] = 0$, where $\tilde{\nu} := \nu - g(q_*)$.*

Sketch of proof. Choose any positive definite diagonal matrix K_p' and let

$$K_p := K_p' + \frac{1}{\varepsilon} K_i \tag{10}$$

where $\varepsilon > 0$ is a (small) constant to be determined. Then the error dynamics (1), (8), (9) become

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - g(q_*) + K_p'\tilde{q} + K_d\dot{q} = -\frac{1}{\varepsilon}K_i\tilde{q} + \tilde{\nu} \tag{11}$$

$$\dot{\tilde{\nu}} = -K_i\tilde{q}\,. \tag{12}$$

A simple inspection shows that the unique equilibrium of the system (11), (12) is $\tilde{q} = 0$, $\tilde{\nu} = 0$ and $\dot{q} = 0$. To analyze the stability of the closed loop system we use the following Lyapunov function candidate with cross terms (similarly to [12, 22, 10, 2, 15]):

$$V_2(x) := \frac{1}{2}\dot{q}^\top D\dot{q} + U_g - U_{g*} - \tilde{q}^\top g_* + \frac{1}{2}\tilde{q}^\top K_p'\tilde{q} + \frac{\varepsilon}{2}(-\frac{1}{\varepsilon}K_i\tilde{q} + \tilde{\nu})^\top K_i^{-1}(-\frac{1}{\varepsilon}K_i\tilde{q} + \tilde{\nu}) + \varepsilon\tilde{q}^\top D\dot{q}\,,$$

where we have dropped the arguments and defined $U_{g*} := U_g(q_*)$, $g_* := g(q_*)$. By splitting the kinetic, and part of the potential energy terms as $\tilde{q}^\top K_p'\tilde{q} = (\lambda_1 + \lambda_2 + \lambda_3)\tilde{q}^\top K_p'\tilde{q}$, $\dot{q}^\top D(q)\dot{q} = (\lambda_1 + \lambda_2 + \lambda_3)\dot{q}^\top D(q)\dot{q}$ with $1 > \lambda_i > 0$, $i = 1, 2, 3$, one can show that, if

$$k_{p_m}' \geq \max\left\{\frac{k_g}{\lambda_1}, \frac{\varepsilon^2 d_M}{\lambda_1\lambda_2}\right\}, \tag{13}$$

then the function $V_2(x)$ satisfies the lower bound:

$$V_2(x) \geq \frac{\lambda_3}{2}\tilde{q}^\top K_p'\tilde{q} + \frac{\lambda_2 + \lambda_3}{2}\dot{q}^\top D\dot{q} \tag{14}$$

hence, $V_2(x)$ is positive definite and radially unbounded. Next, using $\|C(z,y)\| \leq k_c\|y\|$ and $\|g(q) - g(q_*)\| \leq k_g\,\|\tilde{q}\|$, the time derivative of $V_2(x)$ along the trajectories of (11), (12) satisfies

$$\dot{V}_2(x) \leq -\left(k_{d_m} - \frac{\varepsilon}{2}k_{d_M} - \varepsilon k_c\,\|\tilde{q}\| - \varepsilon d_M\right)\|\dot{q}\|^2 - \varepsilon\left(k_{p_m}' - k_g - \frac{1}{2}k_{d_M}\right)\|\tilde{q}\|^2 \tag{15}$$

which is negative semidefinite if,

$$k_{d_m} > \varepsilon(k_{d_M} + 2d_M) \tag{16}$$

$$k_{p_m}' > k_g + \frac{1}{2}k_{d_M} \tag{17}$$

$$\|\tilde{q}\| \leq \frac{k_{d_m}}{2\varepsilon k_c}\,. \tag{18}$$

Local asymptotic stability of the origin $x = 0$ follows using Krasovskii-LaSalle's invariance principle (see for instance [11, p. 115]). A domain of attraction can be determined by defining the set

$$B_\delta := \left\{x \in \mathbb{R}^{3n} : V_2(x) \leq \delta\right\} \tag{19}$$

with δ, the largest positive constant such that (18) holds and hence $\dot{V}_2(x) \leq 0$ for all $x \in B_\delta$. Since V_2 is radially unbounded and positive definite, and $\dot{V}_2(x) \leq 0$ for all $x \in B_\delta$, this set is positive invariant (i.e. if $x(0) \in B_\delta$ then $x(t) \in B_\delta$ for all $t \geq 0$) and qualifies as a domain of attraction for $x(t)$.

Second case: *un*measurable velocities. Based on the results of Kelly [10], we briefly present some results similar to those contained in Propositions 1 and 2.

Proposition 3 *Consider the dynamic model (1) in closed loop with the PD control law*

$$u = -K_p\tilde{q} - K_d\vartheta + \hat{g}(q_*) \tag{20}$$

$$\dot{q}_c = -A(q_c + Bq) \tag{21}$$

$$\vartheta = q_c + Bq \tag{22}$$

where A, B, K_d and K_p are diagonal positive definite matrices. Then, if $k_{p_m} > k_g$, the equilibrium point $(\dot{q},\,\vartheta,\,q) = (0,\,0,\,q_s)$ where q_s satisfies (5), of the closed loop system is globally asymptotically stable.

The following proposition can be deduced from the results presented in [15].

Proposition 4 *Consider the robot model (1) in closed loop with the PI²D control law (21), (22), $u = -K_p\tilde{q} - K_d\vartheta + \nu$, and*

$$\dot{\nu} = -K_i(\tilde{q} - \vartheta), \qquad \nu(0) = \nu_0 \in \mathbb{R}^n \ . \tag{23}$$

Let K_p, K_i, K_d, A and B be positive definite diagonal matrices where B is such that $BD(q) = D(q)B > 0$. Under these conditions, we can always find a sufficiently large proportional gain K_p (or sufficiently small K_i) such that the equilibrium $\xi :=$ col$[\tilde{q},\,\dot{q},\,\vartheta,\,\tilde{\nu}] = 0$ is locally *asymptotically stable.*

Sketch of proof. Using (10), the error equations can be written as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - g(q_*) + K_p'\tilde{q} + K_d\vartheta = \tilde{\nu} - \frac{1}{\varepsilon}K_i\tilde{q} \tag{24}$$

$$\dot{\tilde{\nu}} = -K_i(\tilde{q} - \vartheta) \tag{25}$$

$$\dot{\vartheta} = -A\vartheta + B\dot{q} \tag{26}$$

where K_p' is defined by (10). From [15] we know that the Lyapunov function candidate, $V_4(\xi) = V_2(x) + \frac{1}{2}\vartheta^\top K_d B^{-1}\vartheta - \varepsilon\vartheta^\top D(q)\dot{q}$, is positive definite and radially unbounded with a global and unique minimum at the origin, if ε is sufficiently small. We rewrite the conditions derived in [15] with a slight modification convenient for the purpose of this note. Partition the term $\vartheta^\top K_d B^{-1}\vartheta = (\mu_1 + \mu_2)\vartheta^\top K_d B^{-1}\vartheta$ where $0 < \mu_1 + \mu_2 \le 1$, $\mu_i > 0$ with $i = 1, 2$. One can prove that if (13) holds and

$$\varepsilon < \left(\frac{2k_{d_m}\lambda_2\mu_2}{d_M b_M}\right)^{1/2} \tag{27}$$

then $V_4(\xi)$ satisfies the bound

$$V_4(\xi) \ge \frac{\lambda_3}{2}\dot{q}^\top D\dot{q} + \frac{\lambda_3}{2}\tilde{q}^\top K_p'\tilde{q} + \frac{\mu_1}{2}\vartheta^\top K_d B^{-1}\vartheta \ . \tag{28}$$

Furthermore, it has also been shown in [15] that if the position error $\tilde{q}$ and the filter output ϑ satisfy

$$\|\vartheta\| + \|\tilde{q}\| \le \frac{b_m d_m}{2k_c} \tag{29}$$

and if $\varepsilon > 0$ is sufficiently small to satisfy

$$\varepsilon < \min\left\{\frac{(k_{p_m}' - k_g)k_{d_m}a_m}{2b_M\left[k_{p_M}' + k_{d_M} + k_g\right]^2},\ \frac{k_{d_m}a_m d_m}{2[a_m d_M]^2},\ \frac{k_{d_m}a_m}{2b_M k_{d_M}}\right\}, \tag{30}$$

there exist strictly positive constants β_1, β_2, and β_3 such that the time derivative of $V_4(\xi)$ along the closed loop trajectories (24), (25) is bounded by $\dot{V}_4(\xi) \le -\beta_1\|\tilde{q}\|^2 - \beta_2\|\dot{q}\|^2 - \beta_3\|\vartheta\|^2$. *Local* asymptotic stability of $\xi = 0$ can be proven by invoking Krasovskii-LaSalle's invariance principle. A domain of attraction for the system (24), (26) with state $\xi =$ col$[\tilde{q},\,\dot{q},\,\vartheta,\,\tilde{\nu}]$ can be defined as in the proof of Proposition 2, as the set $B_\rho := \left\{\xi \in \mathbb{R}^{4n} : V_4(\xi) \le \rho\right\}$ where ρ is the largest positive constant such that (29) holds and hence $\dot{V}_4(\xi) \le 0$ for all $\xi \in B_\rho$.

3 Main results

We show that one can achieve *global* asymptotic stability with PID and PI^{2}D control by simply *delaying* the integral action.

3.1 First case: measurable velocities

Proposition 5 (PI$_d$D controller) *Consider the robot manipulator model (1) in closed loop with the PI$_d$D controller*

$$u = -K_p\tilde{q} - K_d\dot{q} + \nu \tag{31}$$

$$\dot{\nu} = \begin{cases} 0, & \nu(0)\in\mathbb{R}^n \quad \forall\ 0\le t\le t_s \\ -K_i\tilde{q}, & \nu(t_s)=\nu(0) \quad \forall\ t\ge t_s \end{cases} \tag{32}$$

where K_p, K_d, and K_i are diagonal positive definite matrices. There always exist a finite *time instant $t_s \ge 0$, a sufficiently large proportional gain K_p and/or a sufficiently small integral gain K_i, independent of the initial conditions, such that the closed loop system is* globally *asymptotically stable at the origin $x := col[\dot{q},\tilde{q},\tilde{\nu}] = 0$ where $\tilde{\nu} := \nu - g(q_*)$.*

Remark 1 Notice that, when $\nu(0) = \hat{g}(q_*)$, in its first phase (that is $0 \le t \le t_s$), the delayed PID of Proposition 5 reduces to the robust controller of Proposition 1 which guarantees global *asymptotic* stability of a different equilibrium than desired but it also guarantees that the *steady* position error is confined to the closed ball of radius determined by (5). In its second phase (that is, for all $t \ge t_s$), the delayed PID reduces to the "conventional" PID controller of Proposition 2 with initial conditions $x_0 = x(t_s)$. □

Proof of Proposition 5. From Proposition 1 it follows that during the first phase of the delayed PID, $(\tilde{q},\,\dot{q},\,\tilde{\nu}) \to (\tilde{q}_s,\,0,\,\hat{g}(q_*)-g(q_*))$ as $t\to\infty$. Furthermore, $\tilde{q}_s$ satisfies the upperbound (5). Define the set

$$\Gamma := \left\{ x\in\mathbb{R}^{3n} \,:\, \|\tilde{q}\| \le \frac{2k_v}{k_{p_m}},\ \dot{q}=0,\ \|\tilde{\nu}\|\le 2k_v \right\},$$

then we must find a constant δ so that $\Gamma \subset B_\delta$ where B_δ is defined in (19) and this will yield a suitable t_s to guarantee GAS of the closed loop. Notice that in order to give an explicit value to δ in terms of the control gains, $V_2(x)$ is needed, however the potential energy term $U_g(q)$ is not known explicitly. Therefore, define

$$V_{2M}(x) = \frac{1}{2}\dot{q}^\top D\dot{q} + \frac{1}{2}(k_{p_M}+k_g)\|\tilde{q}\|^2 + \|\tilde{\nu}\|\,\|\tilde{q}\| + \frac{\varepsilon}{2k_{i_m}}\|\tilde{\nu}\|^2 + \varepsilon\tilde{q}^\top D\dot{q} \tag{33}$$

and the set $B_\delta^M := \left\{x\in\mathbb{R}^{3n} : V_{2M}(x)\le\delta\right\}$. Notice that from (10) we have that $V_{2M}(x) \ge V_2(x)$ hence $B_\delta^M \subset B_\delta$. Now we look for a δ such that $\Gamma\subset B_\delta^M \subset B_\delta$, using (33) and (5) it suffices that

$$\delta > \frac{1}{2}(k_{p_M}+k_g)\left(\frac{2k_v}{k_{p_m}}\right)^2 + \frac{4k_v^2}{k_{p_m}} + \frac{2\varepsilon k_v^2}{k_{i_m}}\ . \tag{34}$$

In words, the lower-bound on δ given above, ensures that the delayed PID controller in its first phase will drive the trajectories into the domain of attraction B_δ in finite time. The second requirement on δ is that $\dot{V}_2(x)$ be negative semi-definite for all $x\in B_\delta$, hence we proceed to calculate an upperbound for δ so that $\dot{V}_2(B_\delta)\le 0$.

From the proof of Proposition 2 (see (14)) we know that (13) implies that $V_2(x)\ge V_{2m}(x)$ where we defined $V_{2m}(x) := 0.5\lambda_3 k'_{p_m}\|\tilde{q}\|^2$. Define the set $B_\delta^m := \left\{x\in\mathbb{R}^{3n} : V_{2m}(x)\le\delta\right\}$. With these definitions we have that $B_\delta \subset B_\delta^m$ hence it suffices to prove that $\dot{V}_2(B_\delta^m)\le 0$. Notice that among the three sufficient conditions (16)-(18) to ensure $\dot{V}_2(x) \le 0$, the only one which affects the definition of the domain of attraction (hence of δ) is (18) thus, it should hold true that

$$\frac{2\delta}{\lambda_3 k'_{p_m}} < \frac{k_{d_m}^2}{4\varepsilon^2 k_c^2} \,. \tag{35}$$

In summary, (34) and (35) suffice to ensure that the trajectories $x(t)$ converge to the domain of attraction B_δ in finite time. Finally, to ensure global asymptotic stability of the origin it suffices to choose the time t_s as the first time moment when the "initial conditions" $x(t_s) \in B_\delta$ that is, $t_s : V_2(x(t_s)) \le \delta$ however, since $V_2(x)$ is not accurately known consider the function

$$\bar{V}_{2M}(x) := \frac{1}{2}\dot{q}^\top D\dot{q} + \frac{1}{2}(k_{p_M} + k_g)\,\|\tilde{q}\|^2 + 2k_v\left(\|\tilde{q}\| + \frac{\varepsilon k_v}{k_{i_m}}\right) + \varepsilon\tilde{q}^\top D\dot{q}$$

which has been defined based on (33) by taking the worst case scenario, that is, when the unknown *constant* error $\tilde{\nu} = \hat{g}(q_*) - g(q_*)$ takes its maximal possible value, $\|\tilde{\nu}\| = 2k_v$. This steady state error is a robustness measure for the the PI$_d$D controller in its first phase, that is, when it works as a PD which drives the system trajectories to a bounded domain. Motivated by this discussion, we define the start-integration time as

$$t_s \;:\; \bar{V}_{2M}(x(t_s)) \le \delta \tag{36}$$

so the proof is completed observing that $\bar{V}_{2M}(x(t)) \ge V_{2M}(x(t))$ for all $t \le t_s$ and (35) holds for sufficiently small ε, hence due to (10) for sufficiently large k_{p_m} and/or sufficiently small $k_{i_M} < \varepsilon$. ∎

The following corollary gives an insight to the practitioner on how to choose the control gains and the switching time t_s to guarantee GAS of the origin.

Corollary 1 *Consider the dynamic model (1) in closed loop with the PI$_d$D control law (31), (32). Let K_p, K_d, and K_i be diagonal positive definite matrices, satisfying $k_{d_m} > \varepsilon(k_{d_M} + 2d_M)$ and $k'_{p_m} > \max\{k_g/\lambda_1\,,\, \varepsilon^2 d_M/\lambda_2\lambda_1\,,\, k_g + 0.5k_{d_M}\}$, (34) and (35). Define the start-integration time t_s as in (36). Under these conditions, the closed loop system is* globally asymptotically stable.

3.2 Second case: *un*measurable velocities

Proposition 6 *[PI$_d^2$D controller] Consider the robot model (1) in closed loop with the PI$_d^2$D control law (21), (22) and*

$$u = -K_p\tilde{q} - K_d\dot{q} + \nu \tag{37}$$

$$\dot{\nu} = \begin{cases} 0, \quad \nu(0) \in \mathbb{R}^n & \forall \;\; 0 \le t \le t_s \\ -K_i(\tilde{q} - \vartheta), \quad \nu(t_s) = \nu(0) & \forall \;\; t \ge t_s \end{cases} \tag{38}$$

Let K_p, K_i, K_d, A and B be positive definite diagonal matrices where B is such that $BD(q) = D(q)B > 0$. Under these conditions, we can always find a finite time instant $t_s \ge 0$, sufficiently large gains K_p, B and/or a sufficiently small integral gain K_i, independent of the initial conditions, such that the closed loop system is globally asymptotically stable *at the origin $\xi := col[\dot{q}, \tilde{q}, \vartheta, \tilde{\nu}] = 0$, where $\tilde{\nu} := \nu - g(q_*)$.*

Proof. The proof follows along the lines of the proof of Proposition 5, based on the results obtained in Propositions 3 and 4. Define the set

$$\Gamma' := \left\{\xi \in \mathbb{R}^{4n} \;:\; \|\tilde{q}\| \le \frac{2k_v}{k_{p_m}},\; \dot{q} = \vartheta = 0,\; \|\tilde{\nu}\| \le 2k_v\right\},$$

and denoting the set $B_\rho^M := \{\xi \in \mathbb{R}^{4n} \;:\; V_{4M}(\xi) \le \rho\}$ where $V_{4M}(\xi) := V_{2M}(x) + \frac{1}{2}\vartheta^\top K_d B^{-1}\vartheta - \varepsilon\vartheta^\top D\dot{q}$. Notice from the proof of Proposition 4 that $V_{4M}(\xi) \ge V_4(\xi)$, hence $B_\rho^M \subset B_\rho$. Notice also that

$V_{4M}(\Gamma') = V_{2M}(\Gamma)$ hence $\Gamma' \subset B_\rho^M$ if ρ satisfies a similar bound as (34). We only need to define an upperbound for ρ which ensures that $\dot{V}_4(B_\rho) \le 0$. Let

$$V_{4m}(\xi) := V_{2m}(x) + \frac{\mu_1 k_{d_m}}{2b_M} \|\vartheta\|^2 \ , \tag{39}$$

from (28) we have that $V_4(\xi) \ge V_{4m}(\xi)$ if condition (27) and (13) hold. Consider next the condition established by inequality (29), then analogously to (35) we have that

$$\max\left\{ \left(\frac{2\rho}{\lambda_3 k'_{p_m}}\right), \left(\frac{2\rho b_M}{\mu_1 k_{d_m}}\right) \right\} < \frac{b_m^2 d_m^2}{16 k_c^2}$$

and (30) imply that $\dot{V}_4(\xi) \le 0$ for all ξ such that $V_{4m}(\xi) \le \rho$, hence also for all $\xi \in B_\rho$. In summary, it is sufficient that ρ satisfies

$$\frac{1}{2}(k_{p_M} + k_g)\left(\frac{2k_v}{k_{p_m}}\right)^2 + \frac{4k_v^2}{k_{p_m}} + \frac{2\varepsilon k_v^2}{k_{i_m}} < \rho < \frac{b_m^2 d_m^2}{16k_c^2} \min\left\{ \left(\frac{\lambda_3 k'_{p_m}}{2}\right), \left(\frac{\mu_1 k_{d_m}}{2b_M}\right) \right\}, \tag{40}$$

to ensure that the delayed PI^{2}D controller in its first phase drives the trajectories $\xi(t)$ into the domain of attraction defined for the second phase. Hence, there exists a finite $t_s \ge 0$ ensuring GAS of the origin $\xi = 0$. As in the proof of Proposition 5, considering that for all $t \le t_s$, the gravity compensation error $\|\tilde{\nu}\|$ is a *constant* bounded by $2k_v$, the instant t_s can be chosen as

$$t_s \ : \ \bar{V}_{4M}(\xi(t_s)) \le \rho \tag{41}$$

where

$$\bar{V}_{4M}(\xi) := \frac{1}{2}\dot{q}^\top D\dot{q} + \frac{1}{2}(k_{p_M} + k_g)\|\tilde{q}\|^2 + 2k_v\left(\|\tilde{q}\| + \frac{\varepsilon k_v}{k_{i_m}}\right) + \varepsilon\tilde{q}^\top D\dot{q} + \frac{1}{2}\vartheta^\top K_d B^{-1}\vartheta - \varepsilon\vartheta^\top D\dot{q}.$$

The proof finishes noticing that (40) holds for sufficiently large b_m and sufficiently small ε, hence due to (10) for sufficiently large k_{p_m} and/or sufficiently small $k_{i_M} < \varepsilon$. ■

Remark 2 Notice from (41) that the switching time t_s does indeed depend on the *unmeasurable* velocities $\dot{q}(t_s)$. Hence, the precise *theoretical* result which is contained in Proposition 6 is that "there exists a start-integration time t_s such that the origin $\xi = 0$ is GAS". For practical purposes however, observe that the velocity measurements are *not* used in the controller equations (37) – (38). The start-integration time t_s can then be computed using the best estimate available of the velocity measurement at a *precise* instant. For instance, any t_s such that $\bar{V}_{4M}(\xi(t_s)) < \rho$ where we redefined

$$\bar{V}_{4M}(\xi(t_s)) := \frac{1}{2}\hat{\dot{q}}(t_s)^\top D\hat{\dot{q}} + \frac{1}{2}(k_{p_M} + k_g)\|\tilde{q}(t_s)\|^2 + 2k_v\left(\|\tilde{q}(t_s)\| + \frac{\varepsilon k_v}{k_{i_m}}\right) + \varepsilon\tilde{q}(t_s)^\top D\hat{\dot{q}}(t_s) \ . \tag{42}$$

and $\hat{\dot{q}}(t_s)$ is the best estimate available of $\dot{q}(t_s)$. Such an estimate can be computed for instance from the last two position measurements prior to the moment t_s. □

Corollary 2 *Consider the dynamic model (1) in closed loop with the PI$_d^2$D control law (37)–(38). Let K_p, K_d, and K_i be diagonal positive definite matrices with K_p defined by (10), satisfying (27), (30), and (40). There exists a time instant t_s (for instance given by (41)) such that the closed loop system is* globally *asymptotically stable at the origin $\xi =$ col$[\tilde{q}, \dot{q}, \vartheta, \tilde{\nu}] = 0$.*

4 Simulation results

To illustrate the performance of the PI$_d$D and PI$_d^2$D controllers, we present some MATLAB™ simulations. We compared the delayed PID controller derived in Section 3.1 with the normalized PID of Kelly [8] and the saturated PID controller of Arimoto [2]. We used the model presented in [4], where

$$D(q) = \begin{bmatrix} 8.77 + 1.02\cos q_2 & 0.76 + 0.51\cos q_2 \\ 0.76 + 0.51\cos q_2 & 0.62 \end{bmatrix} \qquad C(q,\dot{q}) = 0.51\sin q_2 \begin{bmatrix} -\dot{q}_2 & -(\dot{q}_1 + \dot{q}_2) \\ \dot{q}_1 & 0 \end{bmatrix}$$

$$g(q) = 9.81 \begin{bmatrix} 7.6\sin q_1 + 0.63\sin(q_1 + q_2) \\ 0.63\sin(q_1 + q_2) \end{bmatrix}$$

For this system we have $d_m = 0.45$, $d_M = 9.96$, $k_c = 1.53$, $k_v = 80.7$, $k_g = 81.2$. For simplicity, we assume to have no better estimate of the gravitational forces vector than $\hat{g}(q) = [0, 0]^\top$. We considered the problem of controlling the manipulator from the position $[2, 0]^\top$ towards $[1, 1]^\top$. For this we used $K_p = 240I$, $K_d = 75I$, $K_i = 150I$, where $K_p' = 120I$. From (10) it follows that $\epsilon = 1.25$. From (13) we see that $V_2(x) \geq 0$ and by choosing $\delta = 290$ condition (34) is met and the solution of the closed loop system is guaranteed to enter the ball $\bar{B}_\delta^M := \{x \in \mathbb{R}^6 \,:\, \bar{V}_{2M}(x) \leq \delta\}$ in finite time. Therefore the existence of t_s as defined in (36) is also guaranteed. Next we selected $\lambda_1 = 0.7$, $\lambda_2 = 0.2$ and $\lambda_3 = 0.1$ so that $V_2(\tilde{q}, \dot{q}, \tilde{\nu}) \geq 6\|\tilde{q}\|^2$ hence from the fact that $V_2(x) \leq \delta$ we can conclude that $\|\tilde{q}\| < 7$, which results into $\dot{V}_2(x) < -2.2\|\dot{q}\|^2 - 1.6\|\tilde{q}\|^2$.

From [14], we know that our selection of gains also guarantees global asymptotic stability of the normalized PID of Kelly [8] and the saturated PID controller of Arimoto [2]. In Figure 1 we show a comparative study in simulations of all three schemes. We see that the partially saturated proportional term leads to a larger overshoot for Arimoto's controller [2], whereas the saturation in Kelly's controller [8] leads to a slower convergence of ν to $g(q_*)$. We can also see the delayed integration (starting at $t_s = 0.2533$) of the PI$_d$D controller.

To make not only a qualitative but also a quantitative comparison among the three controllers, we have evaluated the Integral Square Error (ISE) index

$$ise(t) := \int_0^t \tilde{q}(s)^\top \tilde{q}(s) ds.$$

The result is illustrated at the bottom of Figure 1. The application of Arimoto's controller [2] yields the largest *ise*, seemingly due to the partially saturated proportional term which retards the transient. One can also appreciate that during the first second, the *ise* with Kelly's controller [8] is slightly lower than the *ise* of our delayed PID controller, however, due to the saturation in the integral part the final convergence of Kelly's controller is slower, resulting into a larger *ise*.

In case of unmeasurable velocities we consider the same task. For the PI$_d^2$D we used $K_p = 240I$, $K_d = 75I$, $K_i = 2I$, $A = 15I$, $B = 200I$. The smaller value for K_i in comparison with the state-feedback case is due to the more restrictive inequalities. By choosing $K_p' = 100I$ (which results into $\epsilon = 0.0143$) we have that $V_4 \geq 0$, and using $\lambda_1 = 0.82$, $\lambda_2 = 0.08$, $\lambda_3 = 0.10$, $\mu_1 = 0.95$, $\mu_2 = 0.05$, (39) becomes $V_{4m}(\xi) \geq 5\|\tilde{q}\|^2 + 0.1781\|\vartheta\|^2$. By choosing $\rho = 275$ we meet the left hand side of (40) and are guaranteed to enter the set B_ρ^M and therefore the existence of t_s as defined in (41). Since $V_{4m} \leq \rho$ we have that $\|\tilde{q}\| + \|\vartheta\| \leq 40$ and hence,

$$\dot{V}_4 \leq - \begin{bmatrix} \|\dot{q}\| \\ \|\tilde{q}\| \\ \|\vartheta\| \end{bmatrix}^\top \begin{bmatrix} 0.3974 & 0 & -1.0675 \\ 0 & 0.2679 & -0.6697 \\ -1.0675 & -0.6697 & 4.5535 \end{bmatrix} \begin{bmatrix} \|\dot{q}\| \\ \|\tilde{q}\| \\ \|\vartheta\| \end{bmatrix} < 0 \; .$$

Therefore, if we start integrating as soon as the trajectories are in B_ρ^M we obtain asymptotic stability in the second phase and henceforth, global asymptotic stability of the delayed PI^{2}D controller. As pointed out in Remark 2, an estimate of the velocity value at the instant t_s is needed. Here, $t_s = 25$ since a visual inspection from the plot of $\tilde{q}$ reveals that the velocities are considerably small after 25s. The resulting overall performance is depicted in Figure 2. Notice that the delayed integral action results into a zero position error.

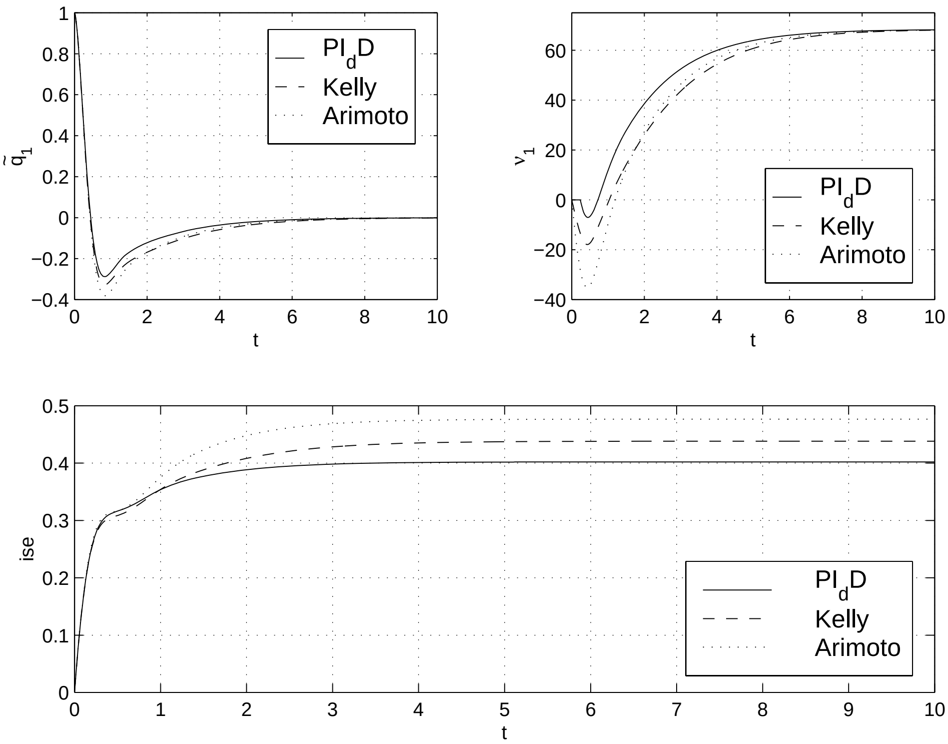

Figure 1: A comparative performance study. The PI_dD versus nonlinear PIDs.

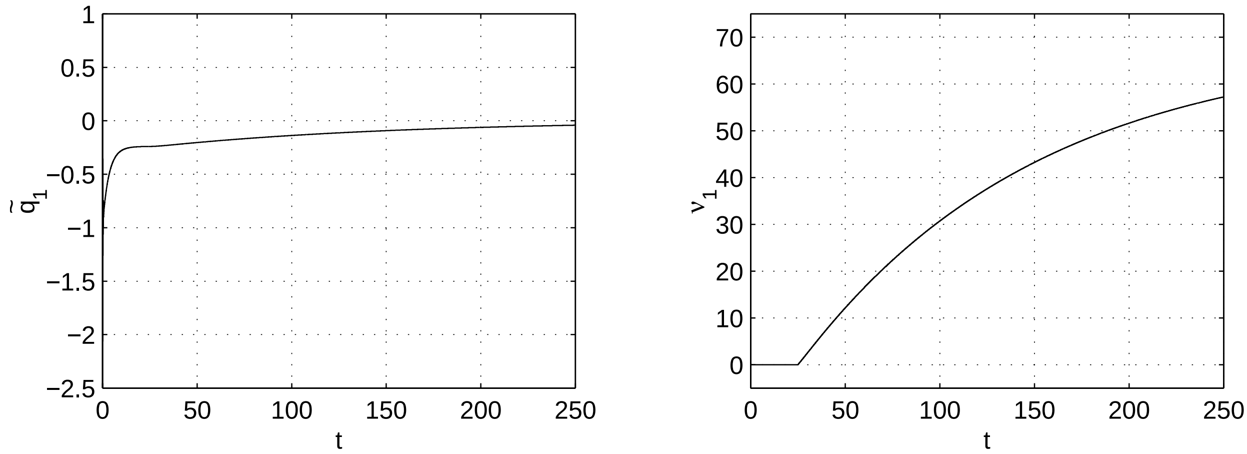

Figure 2: Performance with the delayed PI^2D controller.

5 Concluding remarks

We have addressed the practically important problem of *global* asymptotic stabilization of robot manipulators with uncertain gravity knowledge. Our main contribution is the proof that GAS is possible with *linear* PID (for the state feedback case) and PI^{2}D (if velocities are unavailable) controllers by simply delaying the integral action. From a theoretical point of view we have shown for both the state and position feedback case, that there exists a "start-integration time" t_s such that GAS is guaranteed. From a practical point of view, we have given criteria on how to choose the instant t_s and the control gains. Finally, we have shown in simulations the potential advantages of our schemes vis-a-vis existing *non*linear PID controllers.

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