EINDHOVEN UNIVERSITY OF TECHNOLOGY

Controller design for flow networks of switched servers with setup times

Erjen Lefeber. UCOCOS Workshop, October 29, 2018, Eindhoven

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Motivation

Problem

How to control these networks?

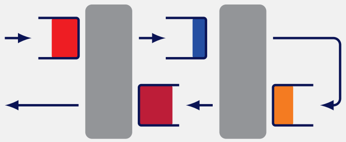
Decisions: When to switch, and to which job-type

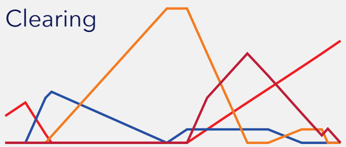
Goals: Minimal number of jobs, minimal flow time

Current approach

Start from policy, analyze resulting dynamics

Kumar, Seidman (1990)





Problem

Current status (after three decades)

Several policies exist that guarantee stability of the network

Remark

Stability is only a prerequisite for a good policy

Open issues

- Do existing policies yield satisfactory network performance?
- How to obtain pre-specified network behavior?

Main subject of study (modest)

Fixed, deterministic flow networks (not evolving, constant inflow)

Approach

Notions from control theory

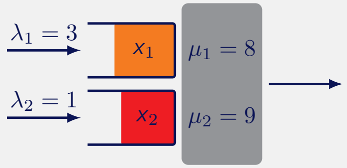
1. Generate feasible reference trajectory
2. Design (static) state feedback controller
3. Design observer
4. Design (dynamic) output feedback controller

Parallels with this problem

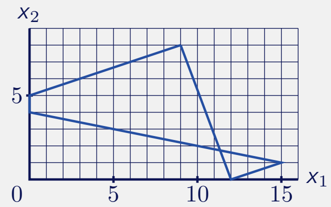
1. Determine desired system behavior
2. Derive non-distributed/centralized controller
3. Can state be reconstructed?
4. Derive distributed/decentralized controller

Problem 1: Generate reference

Single machine



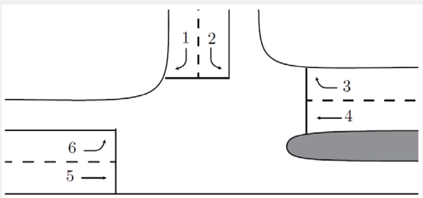
Optimal behavior

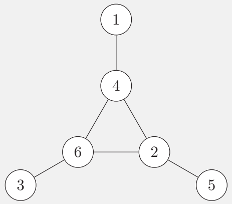


Remarks

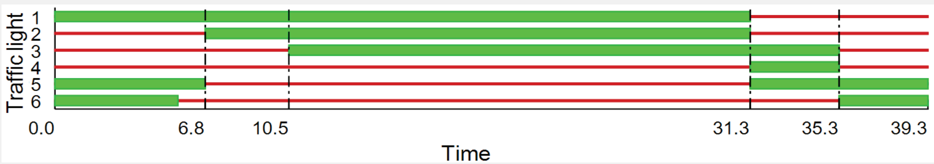
- Many existing policies assume non-idling a-priori
- Slow-mode optimal if $(\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2}) + (\lambda_2 - \lambda_1)(1 - \frac{\lambda_2}{\mu_2}) < 0.$
- Trade-off in wasting capacity: idle ⇔ switch more often

Intersection

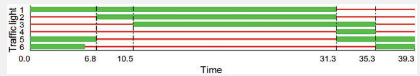




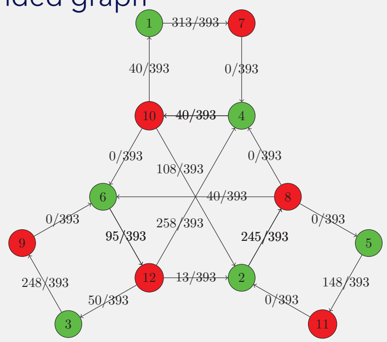
Optimal schedule (data from Grontmij: A2/N279)



Optimal schedule



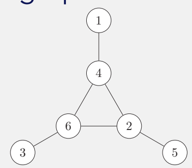
Extended graph



Event times

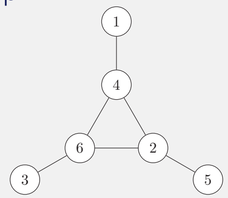
i	$t(i)$	$t(i+6)$	i+6
1	0.0	31.3	7
2	6.8	31.3	8
3	10.5	35.3	9
4	31.3	35.3	10
5	31.3	6.8	11
6	35.3	5.5	12

Conflict graph



Data

- Arrival rates: λ_i
- Service rates: μ_i
- Clearance times: $\sigma_{i,j}$
- Min/max green time: $g_i^{\min}, g_i^{\max}$.
- Min/max period: $T^{\min}, T^{\max}$.
- Conflict graph:



Design variables

- $x(i,j)$ fraction of period from event i to j
- $T' = 1/T$ reciprocal of duration of period

Constraints

- Stable system: $\rho_i = \lambda_i/\mu_i \leq x(i, i+n)$
- Clearance time: $\sigma_{i,j}T' \leq x(i,j)$
- Minimal/maximal green time
- Minimal/maximal period
- Conflict:
  $x(i,i+n)+x(i+n,j)+x(j,j+n)+x(j+n,i) = 1$
- Integer cycle:
  $\sum_{(i,j)\in C^+} x(i,j) - \sum_{(i,j)\in C^-} x(i,j) = z_C$.

Objective

Minimize weighted average delay (Fluid, Webster, Miller, v.d. Broek):

$$\sum_{i=1}^{n} \frac{r_i}{2\lambda_i(1-\rho_i)T}\left(r_i\lambda_i + \frac{s_i^2}{1-\rho_i} + \frac{r_i\rho_i^2 s_i^2 T^2}{(1-\rho_i)(T-r_i)^2((1-\rho_i)T-r_i)}\right)$$

Concluding remarks for Problem 1

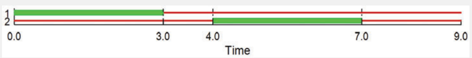
- Mixed integer convex optimization problem.
- Data (Grontmij) of real intersection in the Netherlands with 29 directions:
  - Notebook Intel i5-4300U CPU 1.90GHZ with 16.0GB of RAM, Solver: SCIP 3.2.0
  - Standard implementation: 48 hours.
  - Our approach (plus advanced graph theoretical algorithms): 2 seconds.
- Network of intersections: (conflict) graph with components

Problem 2: Feedback design

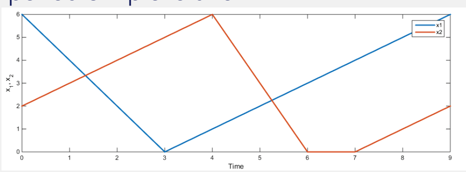
Consider single server with the following system data:

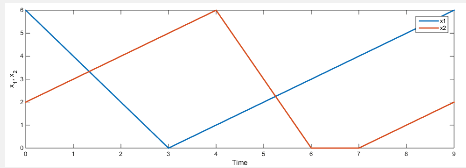
$$\lambda_1 = 1 \qquad \mu_1 = 3 \qquad \sigma_{12} = 1 \qquad \lambda_2 = 1 \qquad \mu_2 = 4 \qquad \sigma_{21} = 2$$

with periodic schedule:



Resulting steady state periodic wip evolution:





From the periodic orbit we determine:

$$\delta_1 = 0 \qquad \delta_2 = 1$$

Policy

Mode 1: After σ_{21}: serve 1 at μ_1 until $x_1(t) = 0$.

Mode 2: After σ_{12}: serve 2 until $x_2(t) = 0$ for at least δ_2.
Serve at μ_2 if $x_2 > 0$, serve at λ_2 if $x_2 = 0$.

Policy

Mode 1: After σ_{21}: serve 1 at μ_1 until $x_1(t) = 0$.

Mode 2: After σ_{12}: serve 2 until $x_2(t) = 0$ for at least δ_2.
Serve at μ_2 if $x_2 > 0$, serve at λ_2 if $x_2 = 0$.

State at start mode 1: $X = \begin{bmatrix} x_1(t_i) & x_2(t_i) \end{bmatrix}^T$.

State at end mode 1: $\mathcal{T}_1 X = \begin{bmatrix} x_2(t_{i+1}) & x_1(t_{i+1}) \end{bmatrix}^T$.

Duration of mode: $\sigma_{21} + \frac{x_1(t_i)+\lambda_1\sigma_{21}}{\mu_1-\lambda_1} = \frac{x_1(t_i)+\mu_1\sigma_{21}}{\mu_1-\lambda_1}$.

$$\mathcal{T}_1 X = \begin{bmatrix} \frac{\lambda_2}{\mu_1-\lambda_1} & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} \frac{\lambda_2\mu_1\sigma_{21}}{\mu_1-\lambda_1} \\ 0 \end{bmatrix}.$$

Policy

Mode 1: After σ_{21}: serve 1 at μ_1 until $x_1(t) = 0$.

Mode 2: After σ_{12}: serve 2 until $x_2(t) = 0$ for at least δ_2.
Serve at μ_2 if $x_2 > 0$, serve at λ_2 if $x_2 = 0$.

State at start mode 2: $X = \begin{bmatrix} x_2(t_i) & x_1(t_i) \end{bmatrix}^T$.

State at end mode 2: $\mathcal{T}_2 X = \begin{bmatrix} x_1(t_{i+1}) & x_2(t_{i+1}) \end{bmatrix}^T$.

Duration of mode: $\sigma_{12} + \frac{x_2(t_i)+\lambda_2\sigma_{12}}{\mu_2-\lambda_2} + \delta_2 = \frac{x_2(t_i)+\mu_2\sigma_{12}}{\mu_2-\lambda_2} + \delta_2$.

$$\mathcal{T}_2 X = \begin{bmatrix} \frac{\lambda_1}{\mu_2-\lambda_2} & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} \frac{\lambda_1\mu_2\sigma_{12}}{\mu_2-\lambda_2} + \lambda_1\delta_2 \\ 0 \end{bmatrix}.$$

Mappings:

$$\mathcal{T}_1 X = \begin{bmatrix} \frac{\lambda_2}{\mu_1-\lambda_1} & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} \frac{\lambda_2\mu_1\sigma_{21}}{\mu_1-\lambda_1} \\ 0 \end{bmatrix}$$

$$\mathcal{T}_2 X = \begin{bmatrix} \frac{\lambda_1}{\mu_2-\lambda_2} & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} \frac{\lambda_1\mu_2\sigma_{12}}{\mu_2-\lambda_2} + \lambda_1\delta_2 \\ 0 \end{bmatrix}$$

Monodromy operator $M = \mathcal{T}_2 \circ \mathcal{T}_1$:

$$\begin{aligned} MX &= \begin{bmatrix} \frac{\lambda_1}{\mu_2-\lambda_2} & 1 \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} \frac{\lambda_2}{\mu_1-\lambda_1} & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} \frac{\lambda_2\mu_1\sigma_{21}}{\mu_1-\lambda_1} \\ 0 \end{bmatrix} \right) + \begin{bmatrix} \frac{\lambda_1\mu_2\sigma_{12}}{\mu_2-\lambda_2} + \lambda_1\delta_2 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{\lambda_1\lambda_2}{(\mu_1-\lambda_1)(\mu_2-\lambda_2)} & \frac{\lambda_1}{\mu_2-\lambda_2} \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} \frac{\lambda_1\lambda_2\mu_1\sigma_{21}}{(\mu_1-\lambda_1)(\mu_2-\lambda_2)} + \frac{\lambda_1\mu_2\sigma_{12}}{\mu_2-\lambda_2} + \lambda_1\delta_2 \\ 0 \end{bmatrix} \end{aligned}$$

Monodromy operator $M = \mathcal{T}_2 \circ \mathcal{T}_1$:

$$MX = \begin{bmatrix} \frac{\lambda_1\lambda_2}{(\mu_1-\lambda_1)(\mu_2-\lambda_2)} & \frac{\lambda_1}{\mu_2-\lambda_2} \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} \frac{\lambda_1\lambda_2\mu_1\sigma_{21}}{(\mu_1-\lambda_1)(\mu_2-\lambda_2)} + \frac{\lambda_1\mu_2\sigma_{12}}{\mu_2-\lambda_2} + \lambda_1\delta_2 \\ 0 \end{bmatrix}$$

Since $x_2 = 0$, we restrict M to $x_2 = 0$ (using $\rho_i = \lambda_i/\mu_i$) resulting in $\tilde{M}$:

$$x_1(t_{i+2}) = \tilde{M} x_1(t_i) = \underbrace{\frac{\rho_1\rho_2}{(1-\rho_1)(1-\rho_2)}}_{A} x_1(t_i) + \underbrace{\frac{\rho_1\rho_2\mu_1\sigma_{21}}{(1-\rho_1)(1-\rho_2)} + \frac{\lambda_1\sigma_{12}}{1-\rho_2} + \lambda_1\delta_2}_{b}$$

For $\rho_1 + \rho_2 < 1$ we have $A < 1$, and therefore $\lim_{i\to\infty} x_1(t_i) = x_1^* = \frac{b}{1-A}$.

Stability region

Fixed time controller: stabilizing for $\lambda_1 < 1$, $\lambda_2 < \frac{4}{3}$.

Dynamic controller: stabilizing for $\frac{1}{3}\lambda_1 + \frac{1}{4}\lambda_2 < 1$

Operators for server with 3 buffers

$$\mathcal{T}_1 X = \begin{cases} \begin{bmatrix} \frac{(1-\theta_1)\lambda_3}{\mu_1-\lambda_1} & 0 & 1 \\ \theta_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} \frac{\lambda_3(\mu_1-\theta_1\lambda_1)\sigma_{31}}{\mu_1-\lambda_1} \\ \theta_1\lambda_1\sigma_{31} \\ 0 \end{bmatrix} & \text{if } \frac{(1-\theta_1)x_1(t_i)+(\mu_1-\theta_1\lambda_1)\sigma_{31}}{\mu_1-\lambda_1} \geq \frac{x_2(t_i)}{\mu_2-\lambda_2} \\ \begin{bmatrix} 0 & \frac{\lambda_3}{\mu_2-\lambda_2} & 1 \\ \theta_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ \theta_1\lambda_1\sigma_{31} \\ 0 \end{bmatrix} & \text{if } \frac{(1-\theta_1)x_1(t_i)+(\mu_1-\theta_1\lambda_1)\sigma_{31}}{\mu_1-\lambda_1} \leq \frac{x_2(t_i)}{\mu_2-\lambda_2} \end{cases}$$

$$\mathcal{T}_2 X = \begin{cases} \begin{bmatrix} \frac{(1-\theta_3)\lambda_2}{\mu_3-\lambda_3} & 0 & 1 \\ \theta_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} \frac{\lambda_2(\mu_3-\theta_3\lambda_3)\sigma_{23}}{\mu_3-\lambda_3} \\ \theta_3\lambda_3\sigma_{23} \\ 0 \end{bmatrix} & \text{if } \frac{(1-\theta_3)x_3(t_i)+(\mu_3-\theta_3\lambda_3)\sigma_{23}}{\mu_3-\lambda_3} \geq \frac{x_1(t_i)}{\mu_1-\lambda_1} + \delta_1 \\ \begin{bmatrix} 0 & \frac{\lambda_2}{\mu_1-\lambda_1} & 1 \\ \theta_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} \lambda_2\delta_1 \\ \theta_3\lambda_3\sigma_{23} \\ 0 \end{bmatrix} & \text{if } \frac{(1-\theta_3)x_3(t_i)+(\mu_3-\theta_3\lambda_3)\sigma_{23}}{\mu_3-\lambda_3} \leq \frac{x_1(t_i)}{\mu_1-\lambda_1} + \delta_1 \end{cases}$$

$$\mathcal{T}_3 X = \begin{cases} \begin{bmatrix} \frac{(1-\theta_2)\lambda_1}{\mu_2-\lambda_2} & 0 & 1 \\ \theta_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \text{if } \frac{(1-\theta_2)x_2(t_i)}{\mu_2-\lambda_2} \geq \frac{x_3(t_i)}{\mu_3-\lambda_3} + \delta_3 \\ \begin{bmatrix} 0 & \frac{\lambda_1}{\mu_3-\lambda_3} & 1 \\ \theta_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} \lambda_1\delta_3 \\ 0 \\ 0 \end{bmatrix} & \text{if } \frac{(1-\theta_2)x_2(t_i)}{\mu_2-\lambda_2} \leq \frac{x_3(t_i)}{\mu_3-\lambda_3} + \delta_3 \end{cases}$$

Problem 2: General case

Stability analysis of monodromy operator $M = \mathcal{T}_n \circ \cdots \circ \mathcal{T}_2 \circ \mathcal{T}_1$ is cumbersome.

Useful result by Feoktistova, Matveev, Lefeber, Rooda (2012)

Let $\mathcal{T}$ be an operator which:

- is piecewise affine, i.e. $\mathcal{T}x = A_i x + b_i$ for $x \in \{P_i x \leq q_i\}$,
- is continuous,
- is monotone, i.e. $A_i \geq 0$,
- is strictly dominated, i.e. $b_i > 0$,
- has a fixed point, i.e. there exists x^* such that $x^* = \mathcal{T}x^*$,

then

- the fixed point is unique, and
- attracts all solutions of $x_{k+1} = \mathcal{T}x_k$; $x_0 \in \mathbb{R}^n_+$, i.e. $\lim_{k\to\infty} x_k = x^*$.

Problem 2: General case

Useful Lemma's

Composition: $\mathcal{T}_2 \circ \mathcal{T}_1 : A_2(A_1 x + b_1) + b_2 = \underbrace{A_2 A_1}_{A} x + \underbrace{A_2 b_1 + b_2}_{b}$.

- Composition of piecewise affine operators is piecewise affine.
- Composition of continuous operators is continuous.
- Composition of monotone dominated ($b_i \geq 0$) operators is monotone dominated.

Consequence

If $\mathcal{T}_1, \ldots, \mathcal{T}_n$, are piecewise affine continuous monotone dominated, then $M = \mathcal{T}_n \circ \cdots \circ \mathcal{T}_1$ is piecewise affine continuous monotone dominated.

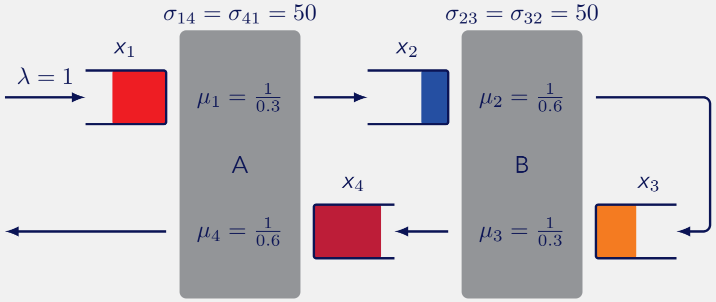
In that case we only need to show that M is strictly dominated and has a fixed point.

Problem 2: General case

Concluding remarks

- Switching rules determine (mode)operators. Can relatively easily be chosen to be piecewise affine continuous monotone dominated
- Only need to show that M is strictly dominated (chose initial mode cleverly) and has a fixed point.
- Robustness against parameters (only requirement on parameters: existence of fixed point).

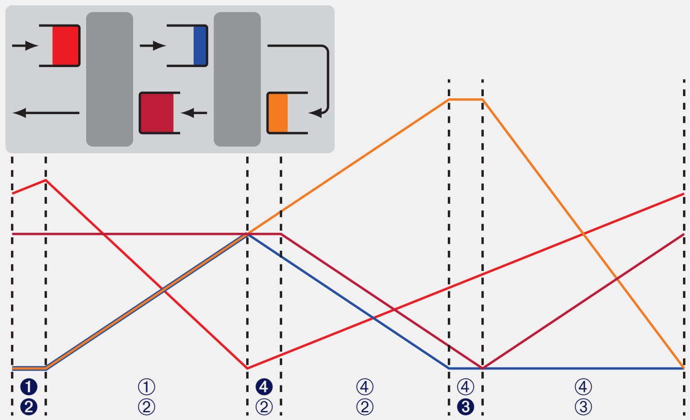
Illustration problems 3 and 4: Kumar-Seidman case



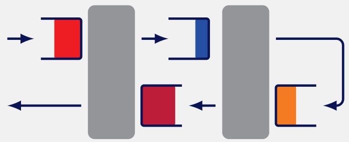
Observation

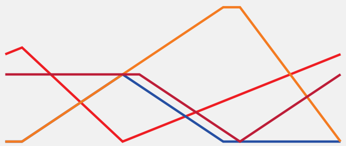
Sufficient capacity (consider period of at least 1000).

Desired behavior



Resulting controller (solving Problem 2)





Mode (1,2): to (4,2) when both $x_1 = 0$ and $x_2 + x_3 \geq 1000$

Mode (4,2): to (4,3) when both $x_2 = 0$ and $x_4 \leq 83\frac{1}{3}$

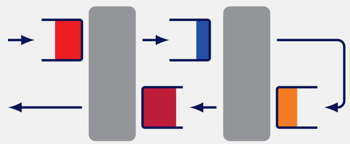
Mode (4,3): to (1,2) when $x_3 = 0$

Remarks

- Non-distributed/centralized controller
- Can be implemented using only synchronisation signals between servers

Observability

Network



Assumptions

- Clearing policy used for machine B
- At $t = t_1$: ③ starts
- At $t = t_2 > t_1$: ③ stops

System state can be reconstructed at machine A

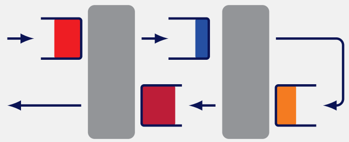
- $x_3(t_2) = 0$, and $x_3(t_1 - 50) = x_3(t_1) = (t_2 - t_1)/0.6$
- $x_2(t_1 - 50) = 0$, and $x_2(t_2) = \int_{t_1-50}^{t_2} u_1(\tau)\,\mathrm{d}\tau$

Observation

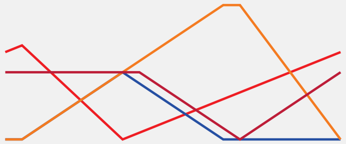
Observablity determined by network topology

Distributed controller

Network



Desired behavior



Distributed controller

Serving 1: Serve at least 1000 jobs until $x_1 = 0$, then switch. Let $\bar{x}_1$ be nr of jobs served.

Serving 2: Serve at least 1000 jobs until $x_2 = 0$, then switch.

Serving 4: Let $\bar{x}_4$ be nr of jobs in Buffer 4 after setup. Serve $\bar{x}_4 + \frac{1}{2}\bar{x}_1$ jobs, then switch.

Serving 3: Empty buffer, then switch.

Conclusions

Control theory inspired approach

1. Determine desired system behavior (trajectory generation; Fleuren)
2. Derive non-distributed/centralized controller (state feedback; Feoktistova)
3. Determine observability/observer
4. Derive distributed/decentralized controller (output feedback)

Advantage

Problems can be considered separately

Centralized control

Approach can deal with
- Arbitrary networks
- Finite buffers
- Transportation delays

Decentralized control

- Observer based approach results in new, tailor-made controllers that perform better

Future work

Research

- Centralized control
  - Derive class of controllers (instead of only one)
- Decentralized control
  - Observability (including tests)
  - Observer design
  - Stability analysis of distributed policies
- Stochastic extensions
  - Analyze performance of derived (de)centralized controllers for stochastic queueing networks