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Optimization of Container Placement and Production Allocation in a Feed Production System

Master Thesis

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Preface

This thesis covers the results of my final project in the Mechanical Engineering master at the Eindhoven University of Technology. During this project I had the opportunity to study the scheduling optimization of a feed production system at KSE Process Technology. The project has been a great learning experience, for which I would like to thank a few people.

First and foremost I would like to thank Erjen Lefeber, who has been my coach during this project. He has shown to be a great all-round guide throughout this and previous projects. Any student I have spoken that has worked with him, has been able to tell me that he works in a pleasantly clear approach. It is surely notable that he always had time if I had any questions, which proves that he is engaged with his students. His advice has been useful for my academic path and beyond, for which I am genuinely and eternally grateful.

Furthermore, I would like to thank Ivo Adan for giving me the opportunity to do this project and for giving his useful advice and ideas during the meetings we had. Without him, this project would not have been possible.

Naturally, I would like to thank KSE and the people that I have met over there during this year. Most thanks go to Twan van der Borgh, Michiel Willemse and Erik Smulders who have given me advice and feedback during the course of the project. In particular I would like to thank Twan for his constructive criticism and all the useful ideas that go along with it. He always had solutions and advice whenever I encountered a challenge. It goes without doubt to say that this has been of notable worth during this project.

Finally, I would like to thank my family, friends, lab mates and boyfriend, who have been a source of great support and joy during my studies.

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Chapter 1

Introduction

Demand for livestock products has increased steadily in the past fifty years and is estimated to grow even further in the coming decades[44]. This growth has lead to an increase in demand for animal food. To meet this demand producers in the animal feed industry have to increase their production capacity. This can be achieved by expansion of capital goods and man labor and by increase in production efficiency. However, capital goods are a costly investment and man labor can be expensive in some countries. Furthermore, space limitations might restrict possibilities in expanding capital goods in existing factories. Therefore, it is important to consider increasing the production efficiency. Moreover, the reason to apply a certain production planning in factories is often not supported by scientific reasoning, but rather based on experience and those production plannings are only updated out once per year or even carried out once in general. Hence, this research focuses on a systematic production planning optimization in a feed production system.

Optimization of production planning is studied in industries other than the animal feed industry as well. Examples range from the chemical and semiconductor industries to shipbuilding manufacturing[13][26][35].

The graduation project is executed in collaboration with KSE Process Technology. KSE Process Technology is a company in Bladel, Noord-Brabant, specialized in solutions for dosing, weighing and transport of materials in factories. They supply instruments and expertise primarily in the animal feed industry in the form of dosing and weighing installations, automation solutions and services for producers of powders, granulates and liquids[1]. A typical animal feed production system used in the industry is illustrated in Figure 1.1. The figure shows the general steps in an animal feed production plant. First, material is introduced to the system. Most material is supplied in bulk as visualized by the 'Bulk Intake' stage in the figure. The material is stored in silos. A silo is a container intended for storing material in dosing installations. A dosing installation is a machine dedicated to dosing, i.e. weighing, material. One or several steps of dosing take place on these installations as indicated by 'Macro Dosing' and 'Medium and Micro Dosing'. The process continues with mixing and grinding the materials, after which they are removed from the system as illustrated by 'Bulk Outtake'.

This research focuses on optimizing the production planning of the blending line of the animal production feed system, that is, the process of dosing and mixing materials. The intake and outtake of material and grinding is thus not taken into consideration. Furthermore, this research considers a simplified dosing process: it consists of only one dosing stage followed by one mixing stage, meaning that there is simply one step at which materials are added and one step at which materials are mixed in the system. In the dosing stage one or multiple dosing installations are placed parallel to each other. In the mixing stage one mixer is placed behind the dosing stage.

Optimization of total production time on the blending line is achieved by arranging the container placement and production allocation. Assuming a fixed production schedule, rearranging the con-

tainer placement, and therefore the materials, between machines may result in a more efficient production as it may reduce the idle time of the installations. Furthermore, given a set of dosing installations, material may theoretically be weighed on multiple installations. Therefore, rearranging the production allocation over these installations may also result in more efficient production.

The problem definition is explained in more detail in Chapter 2. Furthermore, Chapter 3 introduces some of the background information required to solve the problem. Chapters 4, 5 and 6 discuss solution methods to the problem at stake. Finally, conclusions to these solution methods are drawn and recommendations for future research are given in Chapter 7.

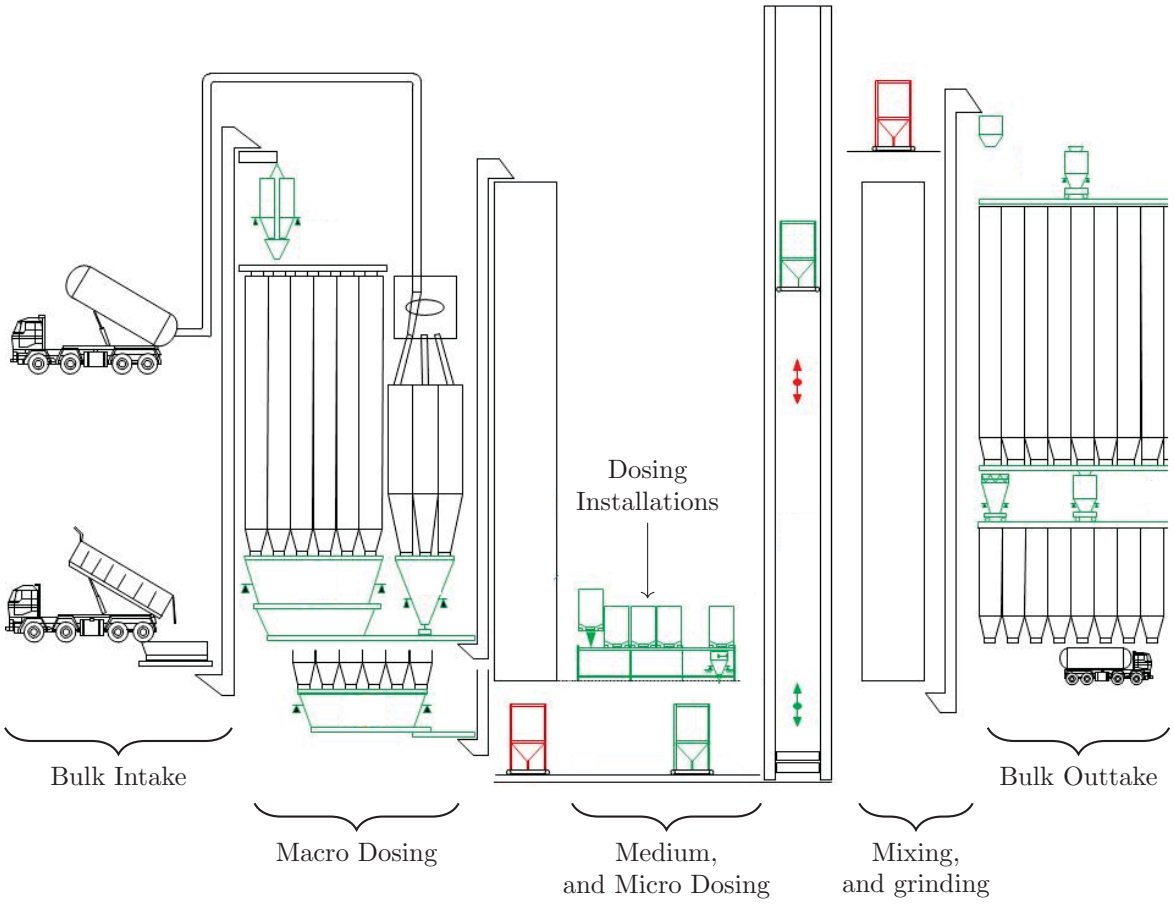


Figure 1.1: General animal feed production process[46]

Chapter 2

Problem definition

In the introduction a typical animal production feed system is presented. This chapter introduces the investigated system at KSE Process Technology. The investigated system is a simplified section of the animal production system as depicted in Chapter 1. First, an overview of the system is given. Then, some components are discussed in more detail. After that, the production that needs to be scheduled on the system is discussed. In the end of the chapter the research question of this project is posed.

2.1 General system

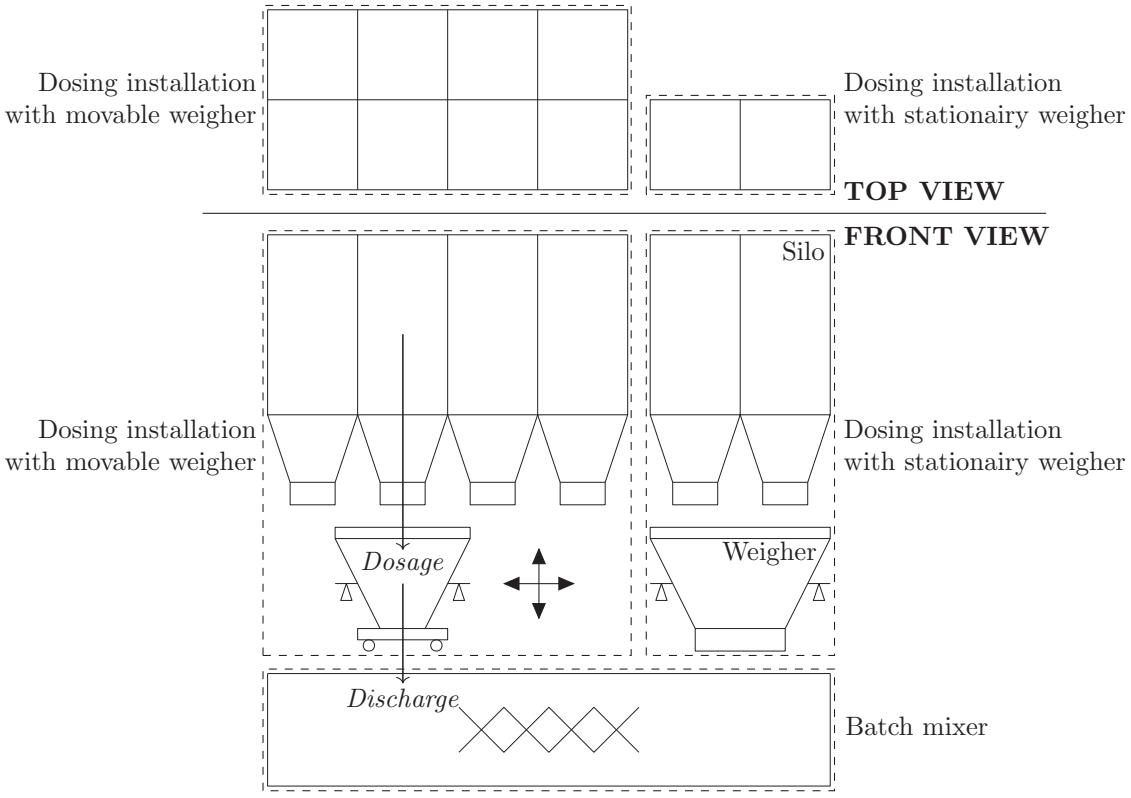


Figure 2.1: System with dosing installations and a batch mixer

The focus of this graduation project is optimization of the production on a blending line. A blending line is the process of dosing and consequently mixing material. In Figure 1.1 in Chapter 1 an

animal feed production process is depicted. The blending line considered in this chapter concerns a simplified production process consisting of solely one dosing stage and one blending stage as seen in Figure 2.1. This is in contrast to the process in Chapter 1 where multiple dosing stages are illustrated. The dosing stage consists of one or multiple dosing installations which weigh raw material. A dosing installation consists of silos and a single weigher. Each silo contains one specific ingredient, referred to as a raw material. It is assumed that a silo has infinite capacity, such that silos do not require replenishing. Also, once raw material is assigned to a silo, it cannot be changed. Furthermore, it is assumed that some raw materials are bound to a certain set of dosing installations or silos while other materials are excluded from a certain set of dosing installations or silos. The weigher weighs raw material that flows from a silo. A single flow of raw material from a silo in the weigher is called a dosage. In addition, it is assumed that a silo has infinite capacity and does not require replenishing. Dosing installations are set parallel to each other and are connected to a single batch mixer which mixes raw material. Material is released from a weigher into the batch mixer. A single flow of material from a weigher in the batch mixer is called a discharge. On one installation only one dosage takes place simultaneously, since only one raw material can be weighed at once. In the total system only one discharge takes place simultaneously.

There are two types of dosing installations: stationary and movable. In a dosing installation with a stationary weigher, the weigher is located at a fixed position. Raw material can flow directly from a silo into the weigher. In a dosing installation with a movable weigher the weigher is not in direct connection with the silos of the installation. The weigher relocates in order to collect or release material. The weigher of one installation cannot collect material from another installation.

In the system production takes place in the form of production orders. A production order is defined as a certain mass of a recipe. Recipes are a certain ratio of a set of raw materials. Hence, production orders are specified as a set of raw materials, in which each raw material in the production order has a specific mass. The production process starts with raw material in silos, which need to be weighed in the weigher and are consequently mixed in the batch mixer. In the system, only one batch may be mixed simultaneously.

Production orders are processed according to the following approach: production orders are split in batches as production orders may not exceed the maximum allowed mass in the batch mixer. Most batches of a production order have equal production quantities. However, the last batch in a production order often may have a different weight and sometimes a smaller sized test batch is performed in advance. Hence, it is assumed that batches in a production order are not equally sized. Each batch contains the exact ratio of raw material as the production order. The production of a batch is carried out by collecting dosages from silos. These dosages are cleared by discharges from the weigher into the mixer.

2.2 Components system

The blending line consists of two types of instruments, namely: a dosing installation and a mixer. Dosing is defined as taking a certain mass of a raw material from the silo by weighing it on the weigher. The dosing time is dependent of a silo dependent dosing constant c^D, the released mass m in the dosage and a silo dependent dosing velocity v^D and the dosing time as a whole is dependent of a dosability factor K_g, which is dependent of raw material g. The dosing time is given as: $K_g(\frac{1}{v_l^D}m + c_l^D)$[s], for a silo l and a raw material g. Due to the weighing accuracy of the weigher a dosage is bound to a required minimum dosing mass. Due to the physical dimensions of a weigher in an installation a dosage is bound to a maximum allowed dosing mass and a maximum allowed dosing volume. Dosing is executed on a dosing installation, which can either contain a stationary or movable weigher. Material is released from the dosing installation in the batch mixer by a discharge. Discharging is defined as emptying the content of the weigher in the batch mixer. The discharge time is dependent of an installation dependent discharge constant c^S, the released mass

m in the discharge, and an installation dependent discharge velocity v^S. The discharge time at a weigher w is hence given as: $\frac{1}{v_w^S} m + c_w^S$[s]. The discharge time is not dependent of a dosability factor as often discharges involve multiple raw materials. The combination of these materials generates a more uniform release time, which is expressed in the discharge constant. Furthermore, the dosability in such cases is dependent of the specific combination of raw materials; it is assumed that this is unknown.

Usually, installations in the system are not all identical in size. An installation is considered to be larger in size if it has a larger maximum allowed dosing mass. If two installations have an equal maximum allowed dosing mass, but a different minimum required dosing mass, then the installation with the largest minimum required dosing mass (i.e., the installation with the most limiting weighing interval) is considered the larger installation of the two.

2.2.1 Batch mixer

Mixing is executed on a batch mixer and the mixing time is considered to be constant with a mixing time constant. It is assumed that the mixing time is the time needed for the batch mixer to mix the raw materials to a homogeneous mixture and the time needed to unload the completed batch from the system. There can be multiple parallel dosing installations but there exists solely one batch mixer in the studied system.

2.2.2 Dosing installation with stationary weigher

In a dosing installation with stationary weigher, the weigher is in direct connection with all silos and the mixer and thus raw material can directly flow from a silo in the weigher.

2.2.3 Dosing installation with movable weigher

In case a movable weigher is involved the mixer is not in direct connection with the silos and the mixer and thus the weigher has to relocate in order to collect raw material from a silo or to release material in the mixer. The relocation time is dependent of the installation dependent acceleration, installation dependent deceleration and maximum velocity of the weigher as well as the displacement of the weigher. It is assumed that the weigher takes the shortest path from one silo to another and that it always accelerates and decelerates with the same acceleration and deceleration. The distance between silos and the distance between silos and the discharge location are assumed to be known. The exact relation between the relocation time and displacement, acceleration, deceleration and maximum velocity of the weigher is given in Appendix A. For the remainder of this report is assumed that the relocation time is known.

2.2.4 Handtip

Not all raw materials are processed by machines. Some materials are added directly into the batch mixer by hand, referred to as a 'handtip'. The reason to do so might be that some materials must be weighed below the minimum required dosing mass of any weigher. Another reason to place a material at the handtip is because their flow properties do not allow these materials to be weighed accurately at a weigher. For this report, materials processed by handtip are considered to be processed on a dosing installation. The parameters for the 'handtip installation' have to be chosen such that in most cases the dosing installations are preferred over the handtip installation to prevent all materials to be placed on the handtip installation. It is assumed that the handtip installation does not have a weight or volume restriction.

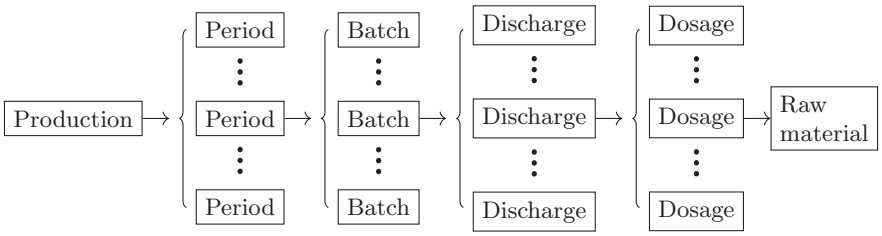


Figure 2.2: Composition production

2.3 Production

The production in the system is given in periods. Periods are assumed consecutive and each period is given by a number of production orders. A production order is the production, i.e. weighing and mixing, of a certain mass of a recipe. A recipe is defined as a certain ratio of a set of raw materials.

Often, a production order exceeds the mass or volume restrictions of the batch mixer, meaning that it cannot be mixed by the batch mixer at once. The production of a production order is therefore split in several batches. A batch is defined as part of a production order that can be processed at once by a batch mixer. Furthermore, in order to generate conveniently sized batches the batch size may additionally be dependent of, for example, the maximum allowed mass or volume in other parts of the system. It is assumed that each batch in a production order consists of the same ratio of raw materials as the production order. The batches of one production order are mixed consecutively in the batch mixer and should be finished before a batch of a next production order can be mixed. As a production order can be expressed in a number of batches of identical recipe, the formulation of production in production orders in batches can be reduced to a formulation of production in batches directly, as shown in Figure 2.2. Thus, from this point on in general production orders are not taken in account anymore. Rather, it is spoken of as batches.

As illustrated earlier in this section, batches are collected in the batch mixer by performing dosages on the weighers of installations. These dosages are released in the mixer by discharges. Hence, each batch is collected by a few discharges from multiple dosing installations, which on its turn are gathered by dosages from silos. Each discharge is performed by a single installation. Multiple discharges are possible from a single installation in one batch. Furthermore, it is assumed that a dosage is executed on one installation only and that the raw material in a dosage is extracted from a single silo.

The order in which the production is carried out is assumed to be fixed according to a given production schedule. The periods and batches are given by the schedule. The recipe of each batch is known.

Furthermore, an installation cannot dose and discharge simultaneously. Dosages are performed consecutively. It is assumed that no material can be added to the batch mixer during mixing. Therefore, all discharges of a batch have to take place before mixing starts. Discharges take place consecutively, meaning that installations may not discharge simultaneously.

The total production spans a number of production periods. The production of each period is carried out consecutively as given in the production schedule. No activity from a next period may be carried out in the current period, meaning that batches in one period, and therefore dosages in that period, may only start if the previous period has ended.

2.3.1 Dosing and discharge order policy

In general, the dosing order of dosages in batches are carried out using the following policy:

- all dosages in a batch at the same installation are executed consecutively. Multiple dosages may be present together in a weigher depending on the maximum allowed mass or volume of the weigher. In case a dosage exceeds the remaining capacity of the weigher, it is forwarded completely to the next discharge. This is illustrated in Example 1;
- dosages at the same installation are carried out in the order they appear in the production schedule;
- the order in which installations may discharge is assumed to be from large to small. All dosages executed at the largest installation are discharged first. Only after the dosages at the largest installation have finished, dosages at the second largest installation may be discharged and so forth. Finally, the dosages at the handtip are discharged. Even though the installations discharge in the given order, dosages at a smaller installation may be dosed before the dosages of a larger installation have been discharged or even dosed, as long as the dosages of the larger installation are discharged before the dosages of the smaller installation.

Summarizing, the periods and batches are carried out in the order as imposed by the production schedule. The dosages of the batches however, are carried out in an alternative manner, such that the number of discharges is limited.

Example 1. ***Forwarding a dosage*** *Assume that several raw materials with a total mass of 1,000[kg] are present in a weigher with a maximum allowed dosage mass of 2,000[kg]. An additional dosage of 3,000[kg] grains should be dosed. Then, the 1,000[kg] of raw materials is discharged first. The grain is consequentially dosed in two dosages. The first dosage is 2,000[kg], which is discharged before the second dosage of 1,000[kg] is performed.*

2.3.2 Production allocation

The decision of how batches are split up over the installation is dependent of the production allocation. Production allocation indicates setting weight bounds for the distribution of dosages on installations. The production allocation is determined for each raw material individually. It is possible that a certain raw material has to be weighed in different quantities in the total production. All dosages between the bounds are performed on the installation, i.e.: each weight class is continuous. For a specific raw material, no overlap in these weight classes is allowed. Note that a consequence of this allocation method is that a raw material from a specific weight class is always assigned to a specific installation even if another installation that is theoretically capable of processing that dosage is available. Hence, the production allocation is defined as the allocation of dosing intervals of raw materials such that:

1. dosing intervals of a raw material are disjoint, that is, each dosing is allowed on one and one installation only;
2. all dosages are allocated;
3. each dosing interval is left-closed, right-open and a convex set.

It is assumed that the lower bound *is* and the upper bound *is not* performed on the installation. An example of a production allocation is given in Example 2.

Example 2. ***Production allocation*** *Assume two dosing installations. Installations A and B have a weigher with a required minimum dosing mass of respectively 10 and 25[kg] and a maximum allowed dosing mass of respectively 1,000 and 2,000[kg]. Assume that grain is a raw material used in production and that it is dosed in the following dosage sizes: 10[kg], 50[kg] and 1,500[kg].*

It is straightforward that dosages of 10[kg] can only be dosed at installation A, because of the required minimum dosing mass restriction. Dosages of 50 and 1,500[kg] can be dosed on either installation. However, dosages of 1,500[kg] have to be split up in two dosages in case they are dosed at installation B. Therefore, three production allocations are shown in Table 2.1 that lead to different allocations of the dosages over the installations. Note that an allocation of 10 and 1500[kg] at installation A and 500[kg] at installation B is not possible, since the allocation should result in weight classes of continuous intervals.

Table 2.1: Possible production allocations

Installation	A	B
Production allocation 1	$[10, -)$	-
Dosages [kg]	10, 50, 1500	-
Production allocation 2	$[10, 1500)$	$[1500, -)$
Dosages [kg]	10, 50	1,500
Production allocation 3	$[10, 50)$	$[50, -)$
Dosages [kg]	10	50, 1500

Moreover, there are special dosing installations for medicines and liquid materials. It is assumed that the allocation for certain raw materials, such as the medicines and liquids, are fixed.

Furthermore, raw material is placed on maximum two machines. In very rare cases a raw material is placed at three installations. Therefore, it is assumed that raw materials can only be placed at two installations, excluding placement at the handtip installation.

2.3.3 Silo assignment

Another decision that should be made in the system besides production allocation is silo assignment. Silo assignment implies the assignment of raw materials to silos. This is only relevant for installations with movable weighers, as in these installations the weigher relocates in order to collect and discharge material, which influences the relocation time as discussed in Section 2.2.

2.4 Research question

The goal of the project is to minimize the makespan for a given production and a given number dosing installations with movable and a given number of dosing installations with stationary weighers. The definition of makespan is given in Section 3.1. Minimization of the makespan may be achieved by optimization of the assignment of the raw materials to the silos on the installations and the optimization of the production allocation over the dosing installations.

With the problem definition clear, several methods to solve this problem are examined, namely MILP, heuristics and a genetic algorithm. An MILP is method applied to optimize problems using a mathematical model. A heuristic is a deterministic algorithm that aims to find a solution according to a set of rules in which these rules often can be seen as a shortcut towards finding (an approximation of) the optimal solution. A genetic algorithm is a meta-heuristic; meta-heuristics are higher-order algorithms that use a combination of rules and often probability to optimize problems. Genetic algorithms do so by a search procedure based on evolution, i.e. natural selection, of a sample set of solutions. The next chapter focuses on the necessary theoretical background to understand these methods.

Chapter 3

Theoretical background

The practical background of the project has been explained in the introduction. This chapter focuses on the theoretical background of the project, or more precisely: scheduling optimization. Scheduling is defined as the arrangement of a set of jobs that are processed in a certain machine environment. Each job consists of a set of tasks or operations. Numerous literature is available on scheduling problems. First, some basic scheduling characteristics are discussed in sections 3.1–3.2. Section 3.3 briefly introduces a framework that may describe and solve optimization problems exactly. Lastly, Section 3.4 describes some deterministic non-exact optimization methods, that is: heuristics, and non-deterministic and non-exact optimization methods, that is: meta-heuristics. In particular, genetic algorithms, a type of meta-heuristic, are described in more detail.

3.1 Optimality criteria

In scheduling optimization several optimality criteria exist. These criteria are set as objective for the scheduling optimization problem and are identified as the measure of performance. Three of them are explained here:

Makespan is the completion time of a certain job or the production in general, that is, the time measured between the start and finish of the job or production. In case makespan is stated without further specification then it mostly implies the completion time of the production in general.

Lateness is defined as the difference between the completion time and due date of a job. The lateness is negative if the completion time falls before the due date and it is positive if the completion time exceeds the due date.

Tardiness is the difference between the completion time and due date of a job in case the completion time exceeds the due date. The tardiness is zero in case the completion time is before or equal to the due date.

Weighted combinations of these criteria are likewise possible as well as other criteria, such as flow time, earliness and waiting time. The latter criteria are not explained here, so for a more extensive discussion consult books such as by Conway, Maxwell and Miller[9] and Baker[4].

Another optimality criterion could be production time. Note that production time might have another definition in other works. However, in this report the production time is considered as the sum of the processing time of each job in the production. The processing time of each job is dependent of the combination of machines used to complete the job and the order in which the components of the jobs are carried out. The difference between makespan and production time is that makespan measures the duration of the actual production and the production time measures the sum of the duration of individual jobs. More specifically, for calculating the makespan

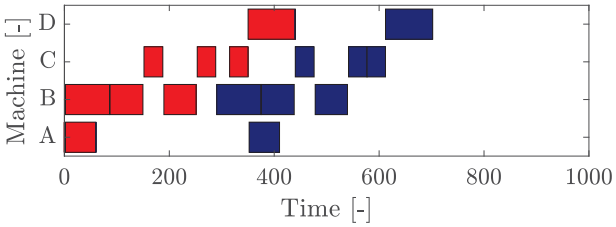


Figure 3.1: Makespan of two jobs (red and blue)

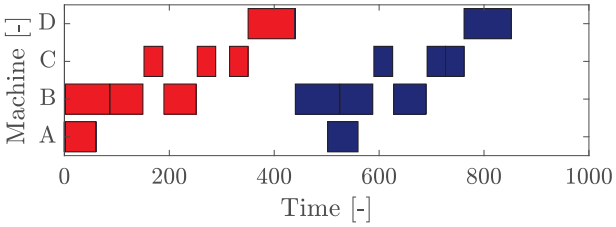


Figure 3.2: Production time of two jobs (red and blue)

of production, the completion time of some jobs may be later than the starting time of later jobs. That is, jobs might be carried out simultaneously during the production. The makespan is generally dependent of the exact order in which jobs are carried out. For calculating the production time, the sum of the processing time of the individual jobs is calculated. Thus, the production time is independent of the job order. The difference between the makespan and production order is visualized in Figures 3.1 and 3.2. In Figure 3.1 the production of two jobs is shown. This results in a makespan of approximately 700 time units. The production time to this production however, is approximately 850 time units, as shown in Figure 3.2.

3.2 Job characteristics

Jobs can be described by several characteristics. Some affect the relation between jobs, such as preemption and precedence constraints. Preemption is defined as job splitting, which is the interruption of a job in order to process another job. Precedence constraints describe the precedence relation between jobs, which is the order in which jobs must be processed. Furthermore, jobs can have certain properties such as their release date, processing time and due date.

3.3 Mixed Integer Programming

Mixed Integer Programming (MIP) models are a method to mathematically describe optimization problems such as scheduling problems. Some of the variables in MIP models are constrained to be integers. In optimization problems one can distinguish the objective function, equality and inequality constraints and bound constraints, often written in the following structure:

$$\min f(x) \tag{3.1}$$
$$\text{subject to } g(x) = p \tag{3.2}$$
$$h(x) \leq q \tag{3.3}$$
$$lb \leq x \leq ub \tag{3.4}$$

In MIP, decision variables x consists of a real variables of which some, but not all, are integer. The objective function $f(x)$ describes to what variables the problem should be optimized. Some objectives of scheduling problems have been mentioned in Section 3.1. The constraints describe the environment in which the solution may exist. One distinguishes equality and inequality constraints, which impose an equality or inequality on a combination of variables, respectively given by (3.2) and (3.3) respectively. Bound constraints describe the domain in which variables exist or variables are restricted as shown in (3.4).

A variable x_i may be bounded from below, above or both, meaning that respectively $lb_i > -\infty$, $ub_i < \infty$ or both should hold true. A solution x that satisfies all constraints is called feasible; a feasible solution that yields the minimum value for the objective is called optimal.

Mixed Integer Linear Programming

A class of MIP models are Mixed Integer Linear Programming (MILP) models. In MILP models the variables are solely linearly dependent of each other. Hence, the optimization problem can be written as:

$$\min c^T x \tag{3.5}$$
$$\text{subject to } A_{eq}x = b_{eq} \tag{3.6}$$
$$Ax \leq b \tag{3.7}$$
$$lb \leq x \leq ub \tag{3.8}$$

with x a vector with decision variables. Furthermore, c, b_{eq}, b, lb and ub are vectors and A_{eq} and A are matrices.

A commonly used formulation in MILP is the 'Big M'-notation[30]. The Big M-notation is used to penalize a variable with a sufficiently large positive value M if a constraint is not met. With the Big M notation some conditional constraints can be defined. For example, a constraint that defines "if binary variable $a = 1$, then $y \leq 0$, else $y \geq 0$", is constructed as following:

$$y \leq M(1 - a) \tag{3.9}$$
$$-y \leq Ma \tag{3.10}$$

If $a = 1$, then $y \leq 0$ by (3.9), such that (3.10) becomes redundant. If $a = 0$, then $y \geq 0$ by (3.10), such that (3.9) becomes redundant.

MILPs can be solved using exact algorithms, such as cutting plane and branch and bound methods. The goal of these algorithms is to reduce the number of feasible solutions to analyze in order to find the optimal solution. Cutting plane methods do so by adding constraints in order to refine the feasible solution set and branch and bound methods do so by dividing the solution set into subsets. Extensive material on integer programming is provided by Nemhauser et. al.[32].

3.4 (Meta-)heuristics

Due to the computational complexity of scheduling problems often heuristic search is applied. Heuristic algorithms determine a (sub-)optimal solution to the problem objective in order to find the desired solution quicker than exact methods or to approximate the optimal solution if no exact answer can be found in a reasonable amount of time. Examples of simple heuristics for job scheduling problems are minimum processing time first, maximum processing time first and minimum due date first. Such heuristics are often simple and computationally inexpensive. However, they are mostly only applicable for specific problems or a narrow set of problems and often there exists a trade off between the computation time and accuracy of the solution found.

Other solution methods that have yielded success are meta-heuristics. Meta-heuristics are higher-level general-purpose algorithms which sample a set of solutions and often incorporate randomness in heuristics which can be used as a guiding strategy for developing heuristics for a specific problem. Examples of meta-heuristics are tabu search, simulated annealing and genetic algorithms. Early research on meta-heuristics dates back to the 1960s and 1970s.

In the 1970s Holland[23] introduced the genetic algorithm. Genetic algorithms use operators based on genetic variation and natural selection, that is, these algorithms sample a set of solutions and alter the best of those in order to find better solutions[39]. Genetic algorithms are commonly used for solving scheduling problems. More recent work on genetic algorithms include studies by Reeves[38] in 1995 and Pezzella et al.[36] in 2008.

Tabu search has first been introduced by Glover[14] in the 1980s. Tabu search is a method that samples a set of solutions and repeatedly moves to the best solutions in the neighborhood of these solutions. By declaring recently visited solutions taboo, tabu search may escape local minima in search of the global optimum.

Simulated annealing was proposed by Kirkpatrick et al.[27] in 1983 and by Cerny[8] in 1985. It is an algorithm based on the simulation of the process of physical annealing with solids. Annealing is the process in which a solid in a heat bath is heated up and slowly cooled down in order to achieve a highly regular crystal lattice configuration. Simulated annealing is an iterative algorithm that samples a set of solutions and allows moves to worse solutions with a certain probability in order to escape local minima.

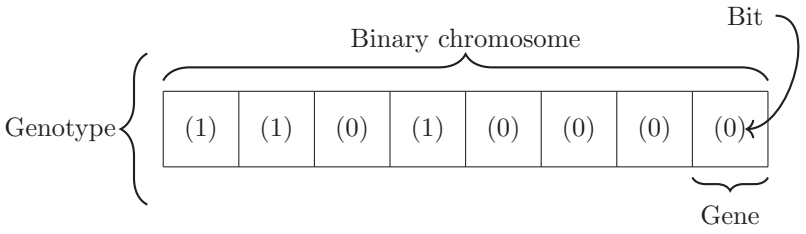
The list of meta-heuristics is plentiful and too extensive to discuss entirely. An introduction on the principles of the genetic algorithm is given in Section 3.4.1. An overview of other methods is given by Osman and Kelly[34].

3.4.1 Genetic algorithm

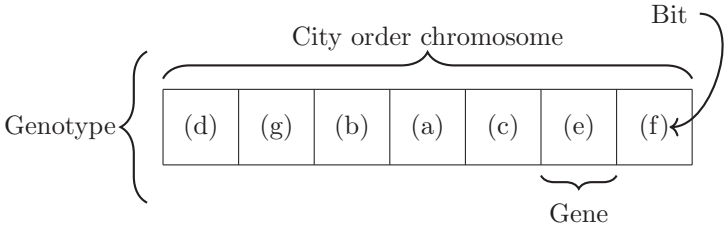
Genetic algorithms are population based global search optimization methods inspired by the evolution theory of Darwin. The heuristic simulates the evolution of a group of individuals. First, it samples a population of solutions, more commonly referred to as individuals. Each individual is evaluated based on its 'fitness' score in which better solutions are correlated with a higher fitness score. Via selection individuals are chosen that may continue to the next operation. Using crossover the selected individuals are recombined with the aim of uniting good traits of individuals. Selected individuals are often referred to as 'parents' and the resulting solutions following from crossover are commonly referred to as 'offspring'. The resulting offspring may or may not replace the parents in the following iteration step, referred to as 'generation'. Furthermore, using mutation the population is diversified by altering individuals. If a certain number of generations is carried out, if some predefined steady state situation is reached or if a certain objective is met, the algorithm stops. Summarizing, a genetic algorithm passes the following steps:

1. An initial population of N individuals is sampled.
2. Each individual n is assigned a fitness score f_n.
3. If k generations are carried out or if objective $f_{\min}$ is met, the algorithm is terminated and the individual with the best fitness score is returned. Otherwise, continue to step 4.
4. Select individuals to perform crossover on. Carry out the crossover and, possibly, remove the parents from the population.
5. Select individuals to perform mutation on. Carry out the mutation.
6. Generate a new generation by collecting the resulting population. The process is repeated from step 2.

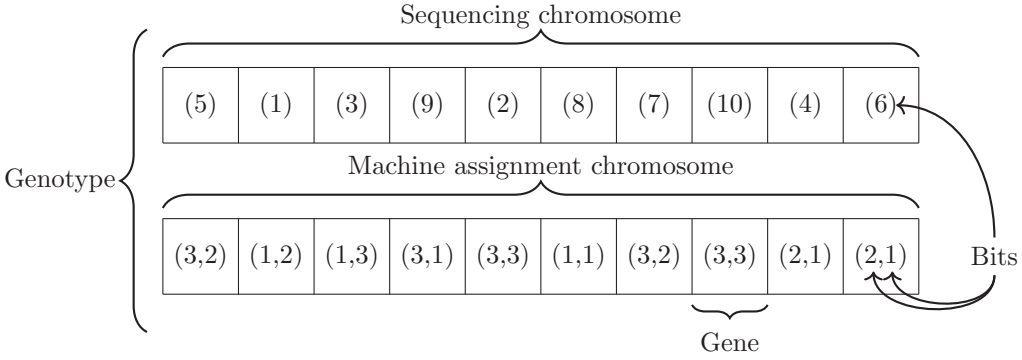
The representation of solutions are key to developing a solid genetic algorithm. Furthermore, numerous fitness assessment, selection, crossover and mutation methods exist. These facets of genetic algorithms are discussed in this section.



(a) Genotype 1, representing a numeric variable written in binary



(b) Genotype 2, representing a Traveling salesman problem solution



(c) Genotype 3, representing a job shop planning solution

Figure 3.3: Examples of genotypes

Representation of solutions

The general set up of a population is given by 'genotypes'. Each individual or solution is expressed by a genotype. Each genotype consists of a number of chromosomes and each chromosome consists of a number of genes. Finally, each gene consists of a number of bits. A few examples of genotypes are shown in Figure 3.3. Figure 3.3a shows the possible architecture of a genotype that represents a value. It consists of one chromosome and 8 genes. For example, if one wishes to optimize a certain function $h(x)$, then a real number x may be encoded by a binary vector. Figure 3.3b shows the possible architecture of a genotype that represents an order. This might represent the order of 6 cities, namely a, b, c, d, e and f, visited in a Traveling salesman problem. Figure 3.3b shows a genotype that might represent the solution of a job shop planning. It consists of 2 chromosomes, each consisting of 10 genes. The first chromosome might represent the sequence of jobs, in which each gene consists of a single bit, and the second chromosome might represent the set of operations required for completing a job, in which each gene consists of two bits. Note that in all following visualizations of genotypes, for example in Figure 3.4, the boxes around genes are omitted for

simplicity.

Fitness assessment

Each individual is assigned a fitness score, which should reflect the objective of the optimization problem. Examples of objectives have been discussed in Section 3.1.

Selection

The fitness assessment is required for the selection operator. In order to carry out crossover, individuals have to be selected based on their fitness. There are various methods to do so, with two of the most used ones being roulette wheel selector and tournament selector.

The roulette wheel selector, also commonly named the fitness proportionate selector is an operator that proportionally scales the probability of being selected with the fitness score. The probability p_i of an individual being selected is given by:

$$p_i = \frac{f_i}{\sum_{n=1}^{N} f_n} \tag{3.11}$$

In the tournament selector a given number or percentage of the population is randomly chosen using a certain 'pressure' parameter. From this sub-population, the individual with the highest fitness score is chosen as one of the parents. This process is referred to as a 'tournament'. The second parent is acquired by repeating the procedure, however, the first selected parent is excluded to avoid repetition. In general, in tournament selection the selection of 1 parent requires 1 'tournament'. In tournament selection, the worst individual is never chosen, and the best individual is chosen in all tournaments it participates in.

Crossover

After parents have been selected crossover is carried out. The crossover operator aims to unify good traits of selected individuals. The applied crossover operator in a specific genetic algorithm is highly dependent of the architecture of the genotype. Numerous crossover operators are available, such as the Partially Mapped Crossover (PMX) operator by Goldberg and Lingle[16] and Cycle Crossover (CX) by Oliver et al.[33] and Position Based Crossover by Syswerda[43]. Two common crossover operators are discussed here, namely: Single Point Crossover (SPX) and Ordered Crossover (OX).

Parent 1	(0)	(0)	(1)	(0)	(0)	(1)	(0)	Parent 2	(1)	(1)	(1)	(1)	(0)	(1)	(1)

(a) Selected parents with $q = 3$

Offspring 1	(0)	(0)	(1)	(1)	(0)	(1)	(1)	Offspring 2	(1)	(1)	(1)	(0)	(0)	(1)	(0)

(b) Resulting offspring

Figure 3.4: Illustration of OX applied to a binary chromosome with crossover point $q = 3$

One of the most used crossover operators is SPX, sometimes referred to as the One Point Crossover (1PX). In SPX, a point q indicates a randomly selected gene from a chromosome of parent 1. Gene 1 until q in that chromosome of parent 1 are copied to the first offspring. Genes $q + 1$ until the end of the chromosome are copied to the second offspring. The location of the copied genes remains equal, meaning that a copied gene q is located at position q in the offspring. An example of SPX is illustrated in Figure 3.4. Instead of crossover at one point, multiple points can be selected, such

Parent 1	(5)	(2)	(4)	(1)	(7)	(6)	(3)	Parent 2	(2)	(6)	(1)	(3)	(7)	(4)	(5)

(a) Selected parents with $q_1 = 1$ and $q_2 = 4$

Parent 2	~~(2)~~	(6)	~~(1)~~	(3)	(7)	~~(4)~~	(5)	Parent 1	(5)	(2)	(4)	~~(1)~~	(7)	~~(6)~~	~~(3)~~

(b) Marked out genes

Offspring 1	(6)	(2)	(4)	(1)	(3)	(7)	(5)	Offspring 2	(5)	(6)	(1)	(3)	(2)	(4)	(7)

(c) Resulting offspring

Figure 3.5: Illustration of SPX applied to a binary chromosome with crossover points $q_1 = 1$ and $q_2 = 4$

as in the Two Point Crossover (2PX).

Another operator is the Order Crossover (OX), first introduced by Davis[12]. In OX two crossover points p_1 and p_2 are randomly selected in a chromosome of parent 1, dividing the chromosome in 3 sections, called 'alleles'. The middle allele from parent 1 is copied to offspring 1 to the exact location in which it occurs in parent 1. The genes of in this allele are marked out in parent 2. The remaining genes in parent 2 are copied to the remaining locations in offspring 1. These genes from parent 2 are likely to occupy different locations in offspring 1 than in parent 2, however, the order of the copied genes is retained. An example of OX is illustrated in Figure 3.5. The OX operator ensures that offspring do not introduce duplicate genes. This is of importance in for example Traveling salesman problems and job scheduling problems.

Mutation

Mutations are applied in genetic algorithms to introduce diversity in the population. A simple type of mutation is carried out by changing each gene of a chromosome with a small probability. Other mutation operators are for example shift mutations and or exchange mutations. Shift mutations shift a randomly chosen gene a random number of places to the left or the right. Exchange mutation exchanges the location of two randomly chosen genes of a chromosome. Note that the significant difference between crossover and mutation is that a crossover operator exchanges information between individuals and a mutation operation alters information within a single individual.

Theory from this chapter is applied in the following chapters, especially Chapters 4–6, which discuss various solution methods to the problem introduced in Chapter 2. Three solution methods are discussed, namely MILP, deterministic heuristics and genetic algorithms. The next chapter focuses on the first of these solution: MILP.

Chapter 4

MILP

The problem posed in Chapter 2 can be translated in a Mixed Integer Linear Programming (MILP) formulation. In short, MILP is a type of mathematical model using linear equations with partly integer variables to solve optimization problems. Some background on MILP is provided in Chapter 3. First, the relevant data and assumptions needed to solve the problem, as introduced in the previous chapter, is summarized and structured. Second, the required design variables are explained. Then, the constraints and objective function are presented. Finally, the chapter concludes on the practical functionality of the proposed MILP.

4.1 Required input data

The relevant input data needed to solve the MILP as introduced in the previous chapter is given below together with the structure in which it is required in the MILP formulation as presented in Section 4.3.

- W weighers with required minimum dosing mass $m_w^{\min}$, maximum allowed dosage mass $m_w^{\max}$ en maximum allowed dosing volume $V_w^{\max}$ for weigher $w \in \{1, ..., W\}$.
- G raw materials.
- L silos, in which $L \geq \max(G, W)$.
- P periods.
- B batches.
- S_b discharges in each batch b with $\sum_{b=1}^{B} S_b$ batches in total, in which $\sum_{b=1}^{B} S_b \geq W$.
- D_b dosages in each batch b with $\sum_{b=1}^{B} D_b$ dosages in total. Each dosage d has weight m_d, volume V_d, in which $D_b \geq \max(L, S_b), \quad \forall b$.
- Δ is a $D \times G$ allocation matrix of D dosages to G raw materials, i.e.:
  $\Delta_{d,g} = \begin{cases} 1, & \text{if dosage } d \text{ consists of raw material } g \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each dosage consists of exactly 1 raw material $\sum_{g=1}^{G} \Delta_{d,g} = 1, \quad \forall d$.
- Λ is an $L \times W$ allocation matrix of L silos W weighers, i.e.:
  $\Lambda_{l,w} = \begin{cases} 1, & \text{if silo } l \text{ belongs to weigher } w \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each silo belongs to exactly 1 weigher $\sum_{w=1}^{W} \Lambda_{l,w} = 1, \quad \forall l$.

- Θ is a $\sum_{b=1}^{B} S_b \times B$ allocation matrix of $\sum_{b=1}^{B} S_b$ discharges to B batches, i.e.:
  $\Theta_{s,b} = \begin{cases} 1, & \text{if discharge } s \text{ in batch } b \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each discharge belongs to exactly 1 batch $\sum_{b=1}^{B} \Theta_{s,b} = 1, \quad \forall s.$
- Ψ is a $\sum_{b=1}^{B} D_b \times B$ allocation matrix of $\sum_{b=1}^{B} D_b$ dosages to B batches, i.e.:
  $\Psi_{d,b} = \begin{cases} 1, & \text{if dosage } b \text{ in batch } b \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each dosage belongs to exactly 1 batch $\sum_{b=1}^{B} \Theta_{d,b} = 1, \quad \forall d.$
- Ω is a $B \times P$ allocation matrix of B batches to P periods, i.e.:
  $\Omega_{b,p} = \begin{cases} 1, & \text{if batch } b \text{ in period } p \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each batch belongs to exactly 1 period $\sum_{p=1}^{P} \Omega_{b,p} = 1, \quad \forall b.$
- Set of silos at a weigher: $\mathcal{L}_w = \{0\} \cup \{l | \Lambda_{l,w} = 1\}$, in which silo 0 represents the discharge location.
- γ^0 set of {silo l and corresponding raw material g}-combinations that exclude raw material g at silo l.
- γ^1 set of {silo l and corresponding raw material g}-combinations that forces raw material g at silo l.
- η_g the maximum number of silos at which a raw material g occurs.
- Setup time from location l_1 to location l_2: Σ_{l_1,l_2} for $l_1, l_2 \in \mathcal{L}_w$, $\forall w$. In this case, the setup time solely consists of the relocation time due to driving from location l_1 to location l_2.
- Dosing time for x[kg] of raw material g at silo l, dependent of the dosing constant $c^D_{l,g}$ and dosing speed $v^D_{l,g}$: $\frac{1}{v^D_{l,g}} x + c^D_{l,g}$[s]. Using the dosability factor K_g, the silo and raw material dependent dosing constant and dosing speed are expressed by $c^D_{l,g} = K_g c^D_l$ and $v^D_{l,g} = \frac{v^D_l}{K_g}$.
- Discharge time for x[kg] at weigher w: $\frac{1}{v^S_w} x + c^S_w$[s].
- Completion time of the last discharge (from a previous planning) at weigher w: $C^S_{0,w}$.
- Completion time of the last discharge (from a previous planning): $C^S_0 = \max_w C^S_{0,w}$.
- M a sufficient large value, at least larger than the mass of the largest batch.
- δ a sufficient small value, at least smaller than the smallest weighing accuracy among all weighers.

4.2 Assumptions

Besides the assumptions presented in the previous section the following time based assumptions are taken in consideration:

- If period $p_1 < p_2$, then p_1 is completed before p_2.
- If batch $b_1 < b_2$, then b_1 is completed before b_2.
- If dosages $d_1 < d_2$ both in discharge s, then d_1 is completed before d_2

- If discharge $s_1 < s_2$, then s_1 is completed before s_2 (without loss of generality).
- Dosages are discharged in order, that is, d_2 is never discharged before d_1 for $d_1 < d_2$.

Note that relaxation of the last assumption would not simplify the problem in terms of number of variables and constraints. In Appendix B an MILP is presented in which dosages of a batch may be discharged in any order and in which the order of production order is also unconstrained. This leads to three extra design variables and five extra constraints.

Furthermore, the MILP with only one period is a simplified MILP formulation from the one given in this chapter. Alternatively, the MILP for multiple periods can be seen as an MILP concerning one period in which the batches of all periods are carried out consecutively in one period. The MILP concerning only one period is given in Appendix C. Note that the number of design variables remain equal to the case with multiple variables.

Moreover, the dosing order policy, as introduced in Section 2.3.1, is altered to simplify the MILP. In the MILP it is assumed that the periods, batches are carried out in the exact order as imposed by the production schedule. The dosages of the batches should arrive in the batch mixer in the order as imposed by the production schedule. Consecutive dosages may arrive at the mixer simultaneously.

More precisely, this implies the following: if dosage d_1 is listed before dosage d_2 in the production schedule and both dosages take place in batch b, then d_1 arrives before or at the same time at the batch mixer as d_2. Thus, the following two situations are possible:

1. If d_1 and d_2 take place at the same dosing installation, then they may be discharged together and they may arrive at the mixer simultaneously if the capacity of the weigher allows so, as is the case with the general dosing and discharge order policy.
2. If d_1 and d_2 do not take place at the same installations, then d_1 arrives at the mixer before d_2. This does not imply that dosages are carried out in order. The installation carrying out d_2 may start dosing d_2 before d_1 is carried out on the other installation, as long as d_1 is discharged before d_2.

For the solution methods other than MILP, the general dosing order policy, as introduced in Section 2.3.1, is applied.

4.3 Design variables

The binary variables to this MILP formulation are:

- $Y^{DS}_{d,s} = \begin{cases} 1, & \text{if dosage } d \text{ takes place in discharge } s \\ 0, & \text{otherwise.} \end{cases}$
- $Y^{SW}_{s,w} = \begin{cases} 1, & \text{if discharge } s \text{ is executed at weigher } w \\ 0, & \text{otherwise.} \end{cases}$
- $Y^{DL}_{d,l} = \begin{cases} 1, & \text{if dosage } d \text{ is extracted from silo } l \\ 0, & \text{otherwise.} \end{cases}$
- $Y^{LG}_{l,g} = \begin{cases} 1, & \text{if silo } l \text{ contains raw material } g \\ 0, & \text{otherwise.} \end{cases}$
- $Y^{S}_{s} = \begin{cases} 1, & \text{if discharge } s \text{ contains one or more dosages} \\ 0, & \text{otherwise.} \end{cases}$

- $F_{w_1,w_2,g} = \begin{cases} 1, & \text{if } f_{w_1,g}^{WG_2} \leq f_{w_2,g}^{WG_1} \\ 0, & \text{otherwise.} \end{cases}$

The continuous variables to this MILP formulation are:

- $f_{w,g}^{WG_1}$= lower limit of an allowed dosing interval of raw material g at weigher w
- $f_{w,g}^{WG_2}$= upper limit of an allowed dosing interval of raw material g at weigher w
- p_d^D = duration of dosage d.
- p_s^S = duration of discharge s.
- C_d^D = completion time of dosage d.
- C_s^S = completion time of discharge s.
- S_{d_1,d_2} = setup time due to driving from dosage d_1 to dosage $d_2 > d_1$, in which dosage 0 represents the discharge.
- C_b^B = completion time of batch b.
- C = completion time of the schedule, i.e.: the total makespan.

4.4 Constraints

The MILP is built around:

- binary variables $Y_{d,s}^{DS}, Y_{s,w}^{SW}, Y_{d,l}^{DL}, Y_{l,g}^{LG}$ and Y_s^S, which fix the location of raw materials and execution order of dosages;
- binary variable $F_{w_1,w_2,g}$ and continuous variables $f_{w,g}^{WG_1}$ and $f_{w,g}^{WG_2}$, which fix the allowed dosing interval of raw materials at installations;
- time variables $p_d^D, p_s^S, C_d^D, C_s^S, S_{d_1,d_2}, C_b^B$ and C, which are fixed by variables mentioned above.

The constraints describing the posed problem of Chapter 2 are introduced below.

Equality constraints
Below, the equality constraints of the proposed MILP are presented. Equations 4.1–4.5 represent constraints that describe how the production on the system is tied to the physical design of the system. Equations 4.6 and 4.7 represent constraints that describe the architecture of the production itself. The physical limitations of the system are described by (4.8).

Due to several reasons some raw materials may be excluded from or forced on certain silos. Therefore, silo l may not contain raw material g if this is specified in γ^0 and silo l must contain raw material g if this is specified in γ^1, which results in:

$$Y_{l,g}^{LG} = 0 \qquad \forall (l,g) \in \gamma^0, \tag{4.1}$$

$$Y_{l,g}^{LG} = 1 \qquad \forall (l,g) \in \gamma^1. \tag{4.2}$$

It is required that each discharge s is executed at exactly one weigher w, each dosage d originates from exactly one silo l, each silo l contains exactly one raw material g and each dosage d is executed in exactly one discharge s. Hence, the following four constraints are introduced:

$$\sum_{w=1}^{W} Y_{s,w}^{SW} = 1 \qquad \forall s \in \{1, ..., S\}, \tag{4.3}$$

$$\sum_{l=1}^{L} Y_{d,l}^{DL} = 1 \qquad \forall d \in \{1, ..., D\}, \tag{4.4}$$

$$\sum_{g=1}^{G} Y_{l,g}^{LG} = 1 \qquad \forall l \in \{1, ..., L\}, \tag{4.5}$$

$$\sum_{s=1}^{S} Y_{d,s}^{DS} = 1 \qquad \forall d \in \{1, ..., D\}. \tag{4.6}$$

Note that these constraints do not rule out that a discharge may consist of multiple dosages, a weigher may perform multiple discharges, multiple dosages may originate from a silo and a raw material may be placed on multiple silos.

Furthermore, if batch b contains dosage d, but does not contain discharge s, then dosage d cannot take place in discharge s. Similarly, if batch b does not contain dosage d, but does contain discharge s, then dosage d cannot take place in discharge s:

$$\begin{aligned} Y_{d,s}^{DS} = 0 \qquad & \text{if } \Theta_{s,b} + \Psi_{d,b} = 1, \\ & \forall s \in \{1, ..., S\}, \\ & \forall d \in \{1, ..., D\}, \\ & \forall b \in \{1, ..., B\}. \end{aligned} \tag{4.7}$$

Note that (4.7) does not apply in case dosage d and discharge s both take place in batch b or in case dosage d and discharge s both do not take place in b.

If the mass of dosage d is less than the minimum required mass on weigher w due to the weighing accuracy, then dosage d may not take place at weigher w:

$$\begin{aligned} \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} = 0 \qquad & \text{if } m_w^{\min} > m_d, \\ & \forall w \in \{1, ..., W\}, \\ & \forall d \in \{1, ..., D\}. \end{aligned} \tag{4.8}$$

Note that in order to check whether dosage d takes place at weigher w no extra binary variabele $Y_{d,w}^{DW}$ is required, since $Y_{d,w}^{DW} = \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL}$. In words: dosages that take place at weigher w can be expressed by the product of the silos at weigher w and the dosages that are performed at those silos. Thus, if a dosage may not take place at weigher w then it must hold true that the silo at which dosage d is performed is not located at weigher w.

Inequality constraints
It is undesired to place a raw material on a large number of silos as this indicates that the given set of dosing installations with its corresponding specifications is not fit for the given production. Restricting the number of silos that may contain raw material g could influence the outcome of the MILP optimization, the makespan, negatively. Even though the set of dosing installations is considered a fixed input parameter, it is penalized in the makespan and thus returning the indication that the given input parameters should be changed. To make this come to expression in the MILP formulation the number of silos that contain raw material g is restricted to maximum η_g:

$$\sum_{l=1}^{L} Y_{l,g}^{LG} \leq \eta_g \qquad \forall g \in \{1, ..., G\}. \tag{4.9}$$

It must hold true that if (1) dosage d takes place at silo l and (2) raw material g is located at silo l, then dosage d must contain raw material g. Moreover, if dosage d does not contain raw material g, statement (1) and/or statement (2) must be false, resulting in the following inequality:

$$1 + \Delta_{d,g} \geq Y_{d,l}^{DL} + Y_{l,g}^{LG} \qquad \begin{array}{r} \forall d \in \{1, ..., D\}, \\ \forall g \in \{1, ..., G\}, \\ \forall l \in \{1, ..., L\}. \end{array} \tag{4.10}$$

Furthermore, it must hold true that if (1) dosage d takes place in discharge s and (2) discharge s takes place at weigher w, then dosage d must be executed by weigher w. Moreover, if dosage d does not take place at weigher w, statement (1) and/or statement (2) must be false, resulting in the following inequality:

$$1 + \sum_{l=1}^{L} Y_{d,l}^{DL} \Lambda_{l,w} \geq Y_{d,s}^{DS} + Y_{s,w}^{SW} \qquad \begin{array}{r} \forall d \in \{1, ..., D\}, \\ \forall s \in \{1, ..., S\}, \\ \forall w \in \{1, ..., W\}. \end{array} \tag{4.11}$$

If dosage d takes place in discharge s and discharge s takes place at weigher w, then dosage d completes at least $S_{0,d} + p_d^D$ later than starting condition $C_{0,w}^S$:

$$M(Y_{s,w}^{SW} + Y_{d,s}^{DS} - 2) + C_{0,w}^S + S_{0,d} + p_d^D \leq C_d^D \qquad \begin{array}{r} \forall d \in \{1, ..., D\}, \\ \forall s \in \{1, ..., S\}, \\ \forall w \in \{1, ..., W\}. \end{array} \tag{4.12}$$

If dosages $d_1 < d_2$ both take place in discharge s, then dosage d_2 completes at least $S_{d_1,d_2} + p_{d_2}^D$ later than dosage d_1:

$$M(Y_{d_1,s}^{DS} + Y_{d_2,s}^{DS} - 2) + C_{d_1}^D + S_{d_1,d_2} + p_{d_2}^D \leq C_{d_2}^D \qquad \begin{array}{r} \forall d_1 < d_2 \in \{1, ..., D\}, \\ \forall s \in \{1, ..., S\}. \end{array} \tag{4.13}$$

Similarly, if discharges $s_1 < s_2$ both take place at weigher w and dosage d takes place in discharge s_2, then dosage d completes at least $S_{0,d} + p_d^D$ later than discharge s_1:

$$M(Y_{s_1,w}^{SW} + Y_{s_2,w}^{SW} + Y_{d,s_2}^{DS} - 3) + C_{s_1}^S + S_{0,d} + p_d^D \leq C_d^D \qquad \begin{array}{r} \forall d \in \{1, ..., D\}, \\ \forall s_1 < s_2 \in \{1, ..., S\}, \\ \forall w \in \{1, ..., W\}. \end{array} \tag{4.14}$$

If batch b_1 takes place in $p_1 < p_2$ and dosage d takes place in period p_2, then dosage d completes at least $S_{0,d} + p_d^D$ later than batch b_1:

$$M(\Omega_{b_1,p_1} + \Psi_{d,b_2} + \Omega_{b_2,p_2} - 3) + C_{b_1}^B + p_d^D + S_{0,d} \leq C_d^D \qquad \begin{array}{r} \forall d \in \{1, ..., D\}, \\ \forall b_1 < b_2 \in \{1, ..., B\}, \\ \forall p_1 < p_2 \in \{1, ..., P\}. \end{array} \tag{4.15}$$

Note that in order to check whether dosage d takes place in period p_2, data parameters Ψ_{d,b_2} and Ω_{b_2,p_2} are used.

Discharge s completes at least p_s^S later than discharge $s-1$:

$$C_{s-1}^S + p_s^S \leq C_s^S \qquad \forall s \in \{1, ..., S\}. \tag{4.16}$$

Note that $s = 1$ requires data parameter C_0^S.

If discharge s takes place in batch $b_2 > b_1$, then discharge s completes at least p_s^S later than $C_{b_1}^B$:

$$C_{b_1}^B + p_s^S + M(\Theta_{s,b_2} - 1) \leq C_s^S \qquad \begin{array}{r} \forall s \in \{1, ..., S\}, \\ \forall b_1 < b_2 \in \{1, ..., B\}. \end{array} \tag{4.17}$$

If dosage d takes place in discharge s, then discharge s completes at least $S_{d,0} + p_s^S$ later than dosage d:

$$M(Y_{d,s}^{DS} - 1) + C_d^D + S_{d,0} + p_s^S \leq C_s^S \qquad \begin{array}{r} \forall d \in \{1, ..., D\}, \\ \forall s \in \{1, ..., S\}. \end{array} \tag{4.18}$$

If dosage d containing raw material g takes place at silo l, then the dosing time is determined by the dosing constant and the mass of the dosage m_d:

$$M(Y_{d,l}^{DL} + Y_{l,g}^{LG} - 2) + c_{l,g}^D + \frac{m_d}{v_{l,g}^D} \leq p_d^D \qquad \begin{array}{r} \forall d \in \{1, ..., D\}, \\ \forall g \in \{1, ..., G\}, \\ \forall l \in \{1, ..., L\}. \end{array} \tag{4.19}$$

Furthermore, if discharge s takes place at weigher w, then the discharge time is determined by the discharge constant and the mass $\sum_{d=1}^{D} m_d Y_{d,s}^{DS}$ of all dosages in that discharge:

$$M(Y_{s,w}^{SW} - 1) + c_w^S Y_s^S + \frac{1}{v_w^S} \sum_{d=1}^{D} m_d Y_{d,s}^{DS} \leq p_s^S \qquad \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\}. \tag{4.20}$$

If no dosages take place in a discharge, then the discharge is empty, i.e.: $Y_s^S = 0$. This results in:

$$\sum_{d=1}^{D} Y_{d,s}^{DS} \geq Y_s^S \qquad \forall s \in \{1, ..., S\}. \tag{4.21}$$

On the other hand, if one or more dosages take place in a discharge, then that discharge is not empty, i.e.: $Y_s^S = 1$. This results in:

$$\frac{1}{D} \sum_{d=1}^{D} Y_{d,s}^{DS} \geq Y_s^S \qquad \forall s \in \{1, ..., S\}. \tag{4.22}$$

Only if dosage d contains more mass than the required minimum dosage mass $m_w^{\min}$ of weigher w, dosage d may take place on that weigher:

$$\sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \leq 1 \qquad \text{if } m_w^{\min} \leq m_d, \; \forall w \in \{1, ..., W\}, \; \forall d \in \{1, ..., D\}. \tag{4.23}$$

If discharge s takes place at weigher w, then the mass and volume of all dosages in discharge s cannot not exceed the maximum allowed dosage mass and volume of weigher w:

$$M(Y_{s,w}^{SW} - 1) + \sum_{d=1}^{D} m_d Y_{d,s}^{DS} \leq m_w^{\max} \qquad \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\}, \tag{4.24}$$

$$M(Y_{s,w}^{SW} - 1) + \sum_{d=1}^{D} V_d Y_{d,s}^{DS} \leq V_w^{\max} \qquad \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\}. \tag{4.25}$$

The lower and upper limit of the allowed dosing interval of raw material g at weigher w may not be larger than the maximum allowed dosage mass of weigher w:

$$f_{w,g}^{WG_1} \leq m_w^{\max} \qquad \forall w \in \{1,...,W\}, \; \forall g \in \{1,...,G\}, \tag{4.26}$$

$$f_{w,g}^{WG_2} \leq m_w^{\max} \qquad \forall w \in \{1,...,W\}, \; \forall g \in \{1,...,G\}. \tag{4.27}$$

The dosing interval of raw material g at weigher w_1 may not overlap with the interval at w_2. Thus, if the allowed dosing interval of raw material g at weigher w_1 is smaller or equal the interval at weigher w_2, then the upper limit at weigher w_1 should be smaller or equal to the lower limit at weigher w_2:

$$f_{w_2,g}^{WG_1} + M(1 - F_{w_1,w_2,g}) \geq f_{w_1,g}^{WG_2} \qquad \forall w_1 \neq w_2 \in \{1,...,W\}, \; \forall g \in \{1,...,G\}. \tag{4.28}$$

If the dosing interval of raw material g at weigher w_1 is smaller than the interval at w_2, then $F_{w_1,w_2,g} = 1$ and $F_{w_2,w_1,g} = 0$. If the dosing interval of raw material g at weigher w_2 is smaller than the interval at w_1, then $F_{w_1,w_2,g} = 0$ and $F_{w_2,w_1,g} = 1$. If no dosages of raw material g take place at either weighers w_1 or w_2, then both intervals are 'empty', i.e.: $f_{w_1,g}^{WG_1} = f_{w_1,g}^{WG_2} = f_{w_2,g}^{WG_1} = f_{w_2,g}^{WG_2} = 0$, resulting in, $F_{w_1,w_2,g} = F_{w_2,w_1,g} = 1$. This means in any situation either one or both variables $F_{w_1,w_2,g}$ and $F_{w_2,w_1,g}$ should equal 1:

$$F_{w_1,w_2,g} + F_{w_2,w_1,g} \geq 1 \qquad \forall w_1 < w_2 \in \{1,...,W\}, \; \forall g \in \{1,...,G\}. \tag{4.29}$$

If a raw material g is not placed on weigher w, then the interval is 'empty', i.e.: $f_{w,g}^{WG_1} = f_{w,g}^{WG_2} = 0$. To check whether raw material g is placed on weigher w, binary variable $Y_{g,w}^{GW}$ is needed, which comes to expression using $\sum_{d=1}^{D} \Delta_{d,g} Y_{d,w}^{DW}$ in which $Y_{d,w}^{DW} = \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL}$.

$$M \sum_{d=1}^{D} \Delta_{d,g} \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \geq f_{w,g}^{WG_1} \qquad \forall w \in \{1,...,W\}, \; \forall g \in \{1,...,G\}, \tag{4.30}$$

$$M \sum_{d=1}^{D} \Delta_{d,g} \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \geq f_{w,g}^{WG_2} \qquad \forall w \in \{1,...,W\}, \; \forall g \in \{1,...,G\}. \tag{4.31}$$

If dosage d takes place at weigher w and if dosage d contains raw material g, then the mass of dosage d must be contained in the allowed dosing interval of raw material g at weigher w, i.e.: the mass of dosage d should be larger than the lower limit and smaller than the upper limit of the dosing interval:

$$m_d - \delta + M(2 - Y^{DW}_{d,w} - \Delta_{d,g}) \geq f^{WG_1}_{w,g} \qquad \forall w \in \{1, ..., W\}, \forall g \in \{1, ..., G\}, \forall d \in \{1, ..., D\}, \tag{4.32}$$

$$m_d + M(Y^{DW}_{d,w} + \Delta_{d,g} - 2) \leq f^{WG_2}_{w,g} \qquad \forall w \in \{1, ..., W\}, \forall g \in \{1, ..., G\}, \forall d \in \{1, ..., D\}. \tag{4.33}$$

Consider that the mass m_d of dosage d consists of raw material g. Then m_d may not correspond to both the lower limit if the dosing interval of raw material g at weigher w_1 and the upper limit of raw material g at weigher w_2, since dosage d is only allowed to be executed on one installation. To prevent this, at least one of the limits of the weighers should be excluded from executing dosage d. Therefore, in (4.32), the lower limit of the allowed dosing interval of raw material g at weigher w should be smaller than m_d with a margin δ.

If dosages $d_1 < d_2$ take place at silos l_1 and l_2 respectively, the setup time should be equal to Σ_{l_1,l_2}:

$$M(Y^{DL}_{d_1,l_1} + Y^{DL}_{d_2,l_2} - 2) + \Sigma_{l_1,l_2} \leq S_{d_1,d_2} \qquad \forall d_1 < d_2 \in \{1, ..., D\}, \forall l_1 \in \{1, ..., L\}, \forall l_2 \in \{1, ..., L\}. \tag{4.34}$$

Similarly, for silo 0 this results in:

$$M(Y^{DL}_{d,l} - 1) + \Sigma_{0,l} \leq S_{0,d} \qquad \forall d \in \{1, ..., D\}, \forall l \in \{1, ..., L\}, \tag{4.35}$$

$$M(Y^{DL}_{d,l} - 1) + \Sigma_{l,0} \leq S_{d,0} \qquad \forall d \in \{1, ..., D\}, \forall l \in \{1, ..., L\}. \tag{4.36}$$

Dosages must be executed in order, i.e.: if dosages $d_1 < d_2$ take place in discharges s_1 and s_2 respectively, then discharge s_2 can take place at earliest together with discharge s_1:

$$M(Y^{DS}_{d_1,s_1} + Y^{DS}_{d_2,s_2} - 2) + C^S_{s_1} \leq C^S_{s_2} \qquad \forall d_1 < d_2 \in \{1, ..., D\}, \forall s_1 \in \{1, ..., S\}, \forall s_2 \in \{1, ..., S\}. \tag{4.37}$$

If discharge s takes place in batch b, batch b finishes at least later than $C^S_s + c^B$:

$$C^S_s + c^B + M(\Theta_{s,b} - 1) \leq C^B_b \qquad \forall s \in \{1, ..., S\}, \forall b \in \{1, ..., B\}. \tag{4.38}$$

The final batch in the final period determines the total makespan:

$$C_b^B \leq C \qquad \forall b \in \{1, ..., B\}. \tag{4.39}$$

Some of the continuous variables have to be lower-bounded:

$$f_{w,g}^{WG_1} \geq 0 \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}, \tag{4.40}$$

$$f_{w,g}^{WG_2} \geq 0 \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{4.41}$$

Note that all other continuous variables have already been (implicitly) lower-bounded by their constraints.

Optional constraints
The following constraints are not necessary to solve the optimization problem. However, they reduce the number of possible solutions or make an implicit constraint explicit.

It is assumed that placing a raw material more than once on an installation probably does not significantly improve the performance as the weigher can only be located at one silo simultaneously. The benefit of placing a raw material more than once is a potential decrease in driving time in an installation with a movable weigher. Furthermore, placing a raw material more than once on an installation implies that another raw material cannot be placed on that installation. Placing another raw material on the installation decreases the potential idle time of that installation. It is assumed that the reduction in makespan from a decrease in idle time exceeds the reduction from a decrease in driving time. Hence, it is assumed that a raw material g can only occur once on a dosing installation with weigher w:

$$\sum_{l=1}^{L} \Lambda_{l,w} Y_{l,g}^{LG} \leq 1 \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{4.42}$$

The lower limit of a dosing interval of raw material g at weigher w must be smaller or equal to the upper limit of raw material g at weigher w:

$$f_{w,g}^{WG_1} \leq f_{w,g}^{WG_2} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{4.43}$$

Note that this implicitly follows from (4.32) and (4.33).

Furthermore, all time-based continuous variables cannot be negative. Thus:

$$p_d^D \geq 0 \qquad \forall d \in \{1, ..., D\}, \tag{4.44}$$

$$p_s^S \geq 0 \qquad \forall s \in \{1, ..., S\}, \tag{4.45}$$

$$S_{d_1,d_2} \geq 0 \qquad \forall d_1 \in \{1, ..., D\}, \; \forall d_2 \in \{1, ..., D\}, \tag{4.46}$$

$$C_d^D \geq 0 \qquad \forall d \in \{1, ..., D\}, \tag{4.47}$$

$$C_s^S \geq 0 \qquad \forall s \in \{1, ..., S\}, \tag{4.48}$$

$$C_b^B \geq 0 \qquad \forall b \in \{1, ..., B\}, \tag{4.49}$$

$$C \geq 0. \tag{4.50}$$

These variables are already implicitly lower-bound. For example: p_d^D is constrained by (4.19), in which all terms are explicitly positive except for $M(Y_{d,l}^{DL} + Y_{l,g}^{LG} - 2)$. However, from (4.4) and (4.5) it follows that there is always some silo l and some raw material g for which $Y_{d,l} = Y_{l,g} = 1$. Therefore, p_d^D is always lower-bounded by at least 0.

4.5 Objective function

The objective function follows straightforward from the constraints as the objective is to minimize the makespan:

$$\min \; C \tag{4.51}$$

4.6 Complete MILP

The MILP constraints and objective function are described in sections 4.4 and 4.5. Hence, the optimization problem can be formulated as proposed in Section 3.3:

$$\begin{aligned}
&\min C \\
&\text{subject to } Y_{l,g}^{LG} = 0 && \forall (l,g) \in \gamma^0, \\
&\qquad Y_{l,g}^{LG} = 1 && \forall (l,g) \in \gamma^1, \\
&\qquad \sum_{w=1}^{W} Y_{s,w}^{SW} = 1 && \forall s \in \{1, ..., S\}, \\
&\qquad \sum_{l=1}^{L} Y_{d,l}^{DL} = 1 && \forall d \in \{1, ..., D\}, \\
&\qquad \sum_{g=1}^{G} Y_{l,g}^{LG} = 1 && \forall l \in \{1, ..., L\},
\end{aligned}$$

$$\sum_{s=1}^{S} Y_{d,s}^{DS} = 1 \qquad \forall d \in \{1, ..., D\},$$

$$Y_{d,s}^{DS} = 0 \qquad \text{if } \Theta_{s,b} + \Psi_{d,b} = 1, \; \forall s \in \{1, ..., S\}, \; \forall b \in \{1, ..., B\}, \; \forall d \in \{1, ..., D\},$$

$$\sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} = 0 \qquad \text{if } m_w^{\min} > m_d, \; \forall w \in \{1, ..., W\}, \; \forall d \in \{1, ..., D\},$$

$$\sum_{l=1}^{L} Y_{l,g}^{LG} \leq \eta_g \qquad \forall g \in \{1, ..., G\},$$

$$1 + \sum_{l=1}^{L} Y_{d,l}^{DL} \Lambda_{l,w} \geq Y_{d,s}^{DS} + Y_{s,w}^{SW} \qquad \forall d \in \{1, ..., D\}, \; \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\},$$

$$1 + \sum_{l=1}^{L} Y_{d,l}^{DL} \Lambda_{l,w} \geq Y_{d,s}^{DS} + Y_{s,w}^{SW} \qquad \forall d \in \{1, ..., D\}, \; \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\},$$

$$M(Y_{s,w}^{SW} + Y_{d,s}^{DS} - 2) + C_{0,w}^{S} + S_{0,d} + p_d^D \leq C_d^D \qquad \forall d \in \{1, ..., D\}, \; \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\},$$

$$M(Y_{d_1,s}^{DS} + Y_{d_2,s}^{DS} - 2) + C_{d_1}^{D} + S_{d_1,d_2} + p_{d_2}^D \leq C_{d_2}^D \qquad \forall d_1 < d_2 \in \{1, ..., D\}, \; \forall s \in \{1, ..., S\},$$

$$M(Y_{s_1,w}^{SW} + Y_{s_2,w}^{SW} + Y_{d,s_2}^{DS} - 3) + C_{s_1}^{S} + S_{0,d} + p_d^D \leq C_d^D \qquad \forall d \in \{1, ..., D\}, \; \forall s_1 < s_2 \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\},$$

$$M(\Omega_{b_1,p_1} + \Psi_{d,b_2} + \Omega_{b_2,p_2} - 3) + C_{b_1}^{B} + p_d^D + S_{0,d} \leq C_d^D \qquad \forall d \in \{1, ..., D\}, \; \forall b_1 < b_2 \in \{1, ..., B\}, \; \forall p_1 < p_2 \in \{1, ..., P\},$$

$$C_{s-1}^{S} + p_s^S \leq C_s^S \qquad \forall s \in \{1, ..., S\},$$

$$C_{b_1}^{B} + p_s^S + M(\Theta_{s,b_2} - 1) \leq C_s^S \qquad \forall s \in \{1, ..., S\}, \; \forall b_1 < b_2 \in \{1, ..., B\},$$

$$M(Y_{d,s}^{DS} - 1) + C_d^D + S_{d,0} + p_s^S \leq C_s^S \qquad \forall d \in \{1, ..., D\}, \; \forall s \in \{1, ..., S\},$$

$$M(Y_{d,l}^{DL} + Y_{l,g}^{LG} - 2) + c_{l,g}^{D} + \frac{m_d}{v_{l,g}^{D}} \leq p_d^D \qquad \forall d \in \{1, ..., D\}, \; \forall g \in \{1, ..., G\}, \; \forall l \in \{1, ..., L\},$$

$$M(Y_{s,w}^{SW}-1)+c_w^S Y_s^S+\frac{1}{v_w^S}\sum_{d=1}^{D} m_d Y_{d,s}^{DS} \le p_s^S \qquad \forall s\in\{1,...,S\},$$
$$\forall w\in\{1,...,W\},$$

$$\sum_{d=1}^{D} Y_{d,s}^{DS} \ge Y_s^S \qquad \forall s\in\{1,...,S\},$$

$$\frac{1}{D}\sum_{d=1}^{D} Y_{d,s}^{DS} \ge Y_s^S \qquad \forall s\in\{1,...,S\},$$

$$\sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \le 1 \qquad \text{if } m_w^{\min}\le m_d,$$
$$\forall w\in\{1,...,W\},$$
$$\forall d\in\{1,...,D\},$$

$$M(Y_{s,w}^{SW}-1)+\sum_{d=1}^{D} m_d Y_{d,s}^{DS} \le m_w^{\max} \qquad \forall s\in\{1,...,S\},$$
$$\forall w\in\{1,...,W\},$$

$$M(Y_{s,w}^{SW}-1)+\sum_{d=1}^{D} V_d Y_{d,s}^{DS} \le V_w^{\max} \qquad \forall s\in\{1,...,S\},$$
$$\forall w\in\{1,...,W\},$$

$$f_{w,g}^{WG_1} \le m_w^{\max} \qquad \forall w\in\{1,...,W\},$$
$$\forall g\in\{1,...,G\},$$

$$f_{w,g}^{WG_2} \le m_w^{\max} \qquad \forall w\in\{1,...,W\},$$
$$\forall g\in\{1,...,G\},$$

$$f_{w_2,g}^{WG_1}+M(1-F_{w_1,w_2,g}) \ge f_{w_1,g}^{WG_2} \qquad \forall w_1\ne w_2\in\{1,...,W\},$$
$$\forall g\in\{1,...,G\},$$

$$F_{w_1,w_2,g}+F_{w_2,w_1,g} \ge 1 \qquad \forall w_1<w_2\in\{1,...,W\},$$
$$\forall g\in\{1,...,G\},$$

$$M\sum_{d=1}^{D}\Delta_{d,g}\sum_{l=1}^{L}\Lambda_{l,w}Y_{d,l}^{DL} \ge f_{w,g}^{WG_1} \qquad \forall w\in\{1,...,W\},$$
$$\forall g\in\{1,...,G\},$$

$$M\sum_{d=1}^{D}\Delta_{d,g}\sum_{l=1}^{L}\Lambda_{l,w}Y_{d,l}^{DL} \ge f_{w,g}^{WG_2} \qquad \forall w\in\{1,...,W\},$$
$$\forall g\in\{1,...,G\},$$

$$m_d-\delta+M(2-Y_{d,w}^{DW}-\Delta_{d,g}) \ge f_{w,g}^{WG_1} \qquad \forall w\in\{1,...,W\},$$
$$\forall g\in\{1,...,G\},$$
$$\forall d\in\{1,...,D\},$$

$$m_d+M(Y_{d,w}^{DW}+\Delta_{d,g}-2) \le f_{w,g}^{WG_2} \qquad \forall w\in\{1,...,W\},$$
$$\forall g\in\{1,...,G\},$$
$$\forall d\in\{1,...,D\},$$

$$M(Y_{d_1,l_1}^{DL}+Y_{d_2,l_2}^{DL}-2)+\Sigma_{l_1,l_2} \le S_{d_1,d_2} \qquad \forall d_1<d_2\in\{1,...,D\},$$
$$\forall l_1\in\{1,...,L\},$$
$$\forall l_2\in\{1,...,L\},$$

$$M(Y_{d,l}^{DL} - 1) + \Sigma_{0,l} \leq S_{0,d} \qquad \forall d \in \{1, ..., D\},$$
$$\forall l \in \{1, ..., L\},$$
$$M(Y_{d,l}^{DL} - 1) + \Sigma_{l,0} \leq S_{d,0} \qquad \forall d \in \{1, ..., D\},$$
$$\forall l \in \{1, ..., L\},$$
$$M(Y_{d_1,s_1}^{DS} + Y_{d_2,s_2}^{DS} - 2) + C_{s_1}^{S} \leq C_{s_2}^{S} \qquad \forall d_1 < d_2 \in \{1, ..., D\},$$
$$\forall s_1 \in \{1, ..., S\},$$
$$\forall s_2 \in \{1, ..., S\},$$
$$C_s^S + c^B + M(\Theta_{s,b} - 1) \leq C_b^B \qquad \forall s \in \{1, ..., S\},$$
$$\forall b \in \{1, ..., B\},$$
$$C_b^B \leq C \qquad \forall b \in \{1, ..., B\},$$
$$f_{w,g}^{WG_1} \geq 0 \qquad \forall w \in \{1, ..., W\},$$
$$\forall g \in \{1, ..., G\},$$
$$f_{w,g}^{WG_2} \geq 0 \qquad \forall w \in \{1, ..., W\},$$
$$\forall g \in \{1, ..., G\},$$
and optionally $$\sum_{l=1}^{L} \Lambda_{l,w} Y_{l,g}^{LG} \leq 1 \qquad \forall w \in \{1, ..., W\},$$
$$\forall g \in \{1, ..., G\},$$
$$f_{w,g}^{WG_1} \leq f_{w,g}^{WG_2} \qquad \forall w \in \{1, ..., W\},$$
$$\forall g \in \{1, ..., G\},$$
$$p_d^D \geq 0 \qquad \forall d \in \{1, ..., D\},$$
$$p_s^S \geq 0 \qquad \forall s \in \{1, ..., S\},$$
$$S_{d_1,d_2} \geq 0 \qquad \forall d_1 \in \{1, ..., D\},$$
$$\forall d_2 \in \{1, ..., D\},$$
$$C_d^D \geq 0 \qquad \forall d \in \{1, ..., D\},$$
$$C_s^S \geq 0 \qquad \forall s \in \{1, ..., S\},$$
$$C_b^B \geq 0 \qquad \forall b \in \{1, ..., B\},$$
$$C \geq 0$$
where $Y_{d,s}^{D,S}, Y_{s,w}^{SW}, Y_{d,l}^{DL}, Y_{l,g}^{LG}, Y_s^S, F_{w_1,w_2,g}$ binary variables

4.7 Practical functionality

The MILP formulation as presented in this chapter has a relatively large number of variables. A typical system developed by KSE might processes $50,000$ batches which consist of 1000 different recipes with a total of 500 raw materials within a year. To make a conservative estimation of the number of variables a smaller model of $5,000$ batches consisting of 100 recipes and 50 raw materials is regarded. Assume that each recipe consists of 5 raw materials, a total number of 3 dosing installations with a collective 50 silos, each batch is able to be processed in 1 discharge. Again, these are all conservative estimations. This model renders a production with 3 dosing installations, 50 silos, 50 raw materials, $5,000$ batches, $5,000$ discharges and $25,000$ dosages, leading to a total of $7.5 \cdot 10^8$ design variables.

The number of possible combinations and thus outcomes increases exponentially with each addi-

tional variable. Therefore, it can be assumed that for the size of the proposed problem no solution can be found within acceptable time using the MILP as stated above.

Analyzing the number of variables resulting from each variable group as presented in 4.3, it can be seen that a relative large number of variables emanate from the setup time S_{d_1,d_2}. As has been suggested in Section 4.4, the driving time plays a minor role in optimizing the given problem. Hence, by removing the setup time S_{d_1,d_2} may decrease the problem size. However, even if all time-based variables were removed, the problem would still yield $1.3 \cdot 10^8$ variables for the conservative sized problem as presented in this section.

It is concluded that the MILP cannot be of practical use due to the large number of input variables. Hence, heuristics have to be found to speed up the optimization. Heuristics are algorithms that approximate the optimal solution with rules that usually shortcut the quantity of analyzed data. A heuristics-based approach is introduced in the next chapter.

Chapter 5

Heuristics

Heuristics are methods to solve problems which are too large to be solved by classic approaches or which are relatively difficult to optimize exactly. Therefore, heuristics often tend to approximate the optimal solution by approaching the solution using short-cuts. In the previous chapter, it is shown that MILP optimization cannot find a solution within acceptable time. Hence, heuristics are considered in this chapter as a solution method.

5.1 Reformulation problem

As stated in Chapter 4 optimizing the setup times due to driving time are a minor component of optimizing the makespan. Therefore, the results from this chapter reflect the system as if all machines were dosing installations with stationary weighers, meaning that the specific silo assignment within an installation is not considered in this chapter. Thus, the remaining factor contributing to the makespan is the production allocation as defined in Section 2.3.

Furthermore, to limit the computational burden, the production time rather than the makespan is regarded as optimization objective. The difference between the two optimality criteria is described in Section 3.1. It is assumed that the production time provides a reasonable approximation of the makespan as the makespan of each batch is relatively large in comparison to the additional time introduced by the production time.

5.2 General heuristic

The heuristics proposed in this chapter are deterministic, meaning that for a given input always the same output is achieved. Thus, no randomness is involved in these algorithms. The heuristics determine the production allocation, that is, the allocation of dosing intervals of raw materials to installations and they are partially based on the current manual production allocation process at KSE. The proposed heuristics are the descending mass heuristic, the descending frequency heuristic and the descending recipe occupancy heuristic, which are based on the mass of raw materials in the production schedule, the dosing frequency of raw material in the production schedule and the number of recipes a raw material occurs in, respectively.

The three heuristics are similar in approach. Hence, a general heuristic is defined that applies to all three heuristics. The difference between the descending mass heuristic, the descending frequency heuristic and the descending recipe occupancy heuristic is discussed in Section 5.3.

For these heuristics it is important to notice that installations are sorted on descending size. The definition of the size of installations is discussed in Section 2.2. If two installations are exactly equal in size, it is arbitrary which installation is sorted before the other. In this chapter, the handtip, liquid and medicinal dosing installations are differentiated from the other installations.

The handtip, liquid and medicinal installations are referred to distinctively. Other installation are referred to collectively as 'regular installations'.

The general heuristic is defined as following:

1. Raw materials that are pre-allocated to a specific installation, e.g.: medicinal and liquid materials, are placed on their respective installations. Hence, these materials are not regarded anymore in the allocation of the heuristic.

2. All raw materials dosages below the smallest possible required minimum dosing mass, i.e., $\min_w m_w^{\min}$, are placed at the hand tip installation, since they cannot be dosed by any regular installation. If a raw material is also dosed in dosages larger than the minimum dosing mass, then, at this point, only the interval below the required minimum mass is placed at the handtip installation. An illustration of such a situation is shown in Example 3.

3. Sort the remaining raw materials in $\mathcal{V}$ according to the desired sorting method.

4. Allocate single installation raw material: allocate the raw material according to $\mathcal{V}$ in the largest possible regular installation that completely satisfies the total dosing interval of that raw material if possible. Only if the raw material can be allocated, it is removed from $\mathcal{V}$. Note that the resulting materials in $\mathcal{V}$ remain in descending order of the sorting method from step 3.

5. Allocate multi installation raw material: raw materials that cannot be allocated to a single regular installation have to be divided over multiple installations. Allocate raw materials in the order in which they are enlisted in $\mathcal{V}$. It is chosen that raw materials can be split over at most two regular installations and the handtip installation. Use the weighted sum method as introduced in Section 5.7 to determine the dosing intervals at the dosing installations where each raw material should be placed.

6. Raw materials that have not been allocated, i.e., the remaining raw materials in $\mathcal{V}$, are placed in the handtip installation. This situation may occur, for example, if in the single installation allocation in step 4 all silos are allocated and no vacant silos remain.

7. Reallocate raw materials to remaining vacant silos: the silos that are not appointed by a raw material at this point are used to split the allocation of a raw material that has been appointed to a single regular installation over two regular installations. Only raw materials that have not been assigned to multiple installations yet may be split to avoid raw materials to be assigned to more than two regular installations. The vacant silos are evaluated in order of descending installation size. Only raw materials at installations smaller than the installation of the vacant silo are considered for potential split up. Using a priority score, as introduced in Section 5.8, it is determined which raw material should be chosen for split up. Next, use the weighted sum method, as has been applied in step 5 to determine the corresponding intervals in which the allocation of the preferred raw material should be split.

Note that for the heuristics it is not necessary to define the number of silos in the handtip installation. The following sections describe the steps of the heuristic in further detail. The sorting method of step 3 is discussed in Section 5.3. Three sorting methods are proposed, each yielding a different heuristic. The allocation of raw materials that can be placed on a single installation is discussed in Section 5.4. The allocation of raw material that must be placed at multiple installations is discussed in Section 5.5. Reallocation of raw materials to vacant silos is discussed in Section 5.6.

Additionally, the heuristics can be expanded with a local optimization:

8. Swap raw material between regular installations as described in Section 5.9. By swapping materials between installations it is aimed to place ingredients that occur in the same recipe

on different installations. Placing ingredients in the same recipe over different installations may reduce idle time, as this spreads dosages, and thus dosage times, parallel over installations instead of appointing dosages in series on a single installation.

The local optimization is described in further detail in Section 5.9.

Example 3. ***Pre-allocation of raw materials in handtip*** *Assume two dosing installations. Installations A and B have a weigher with a required minimum dosing mass of respectively 10 and 25[kg]. Assume that grain is a raw material used in production and that it is dosed in the following dosage sizes: 5[kg], 15[kg] and 1,500[kg]. This implies that at least all dosages of 5[kg] should be placed at the handtip installation according to step 2.*

5.3 Sorting method

After raw materials that are pre-allocated to an installation are placed and raw materials that are dosed below $\min_w m_w^{\min}$ are placed, i.e., step 1 and 2 of the general heuristic as described in Section 5.2, the remaining raw materials are sorted in $\mathcal{V}$. The difference between the descending mass heuristic, the descending frequency heuristic and the descending recipe occupancy heuristic comes to expression in the sorting method. The descending mass heuristic is based on the sorting of raw material in descending order of total mass dosed in the production schedule. Similarly, the descending frequency heuristic and descending recipe occupancy heuristic are based on the sorting of raw material in descending order of total number of dosages in production and descending order of total number of recipes a raw material occurs in, respectively. The three sorting methods are described in further detail in this section.

5.3.1 Descending mass heuristic

For the descending mass heuristic step 3 of the general heuristic as described in Section 5.2 is defined as following:

3. Sort the raw material in $\mathcal{V}$ in descending order of total mass dosed in production. A raw material ranked high in $\mathcal{V}$ is considered a large raw material and a material ranked low in $\mathcal{V}$ is considered a small raw material.

5.3.2 Descending frequency heuristic

For the descending frequency heuristic step 3 of the general heuristic as described in Section 5.2 is defined as following:

3. Sort the raw material in $\mathcal{V}$ in descending order of total number of dosages in production. A raw material ranked high in $\mathcal{V}$ is considered a large raw material and a material ranked low in $\mathcal{V}$ is considered a small raw material.

5.3.3 Descending recipe occupancy heuristic

Instead of sorting the raw materials in $\mathcal{V}$ in descending order of mass or frequency, another sorting method is considered: raw materials are sorted in descending order of recipes in which they occur. The number of dosages or the mass of the dosages are not relevant, mere the total number recipes in which they occur. Hence, step 3 of the general heuristic as described in Section 5.2 is changed to:

3. Sort the raw material in $\mathcal{V}$ in descending order of total number of recipes in which they occur. A raw material ranked high in $\mathcal{V}$ is considered a large raw material and a material ranked low in $\mathcal{V}$ is considered a small raw material.

5.4 Single installation allocation

As the number of silos are limited, the raw materials that can be allocated to a single regular installation, are placed on a single regular installation. After the raw materials are sorted according to the desired sorting method in step 3 of the general heuristic as described in Section 5.2, the allocation of raw materials to a single installation is carried out. The procedure for the allocation of single installation materials is shown in pseudo-code in Algorithm 1 on page 47.

The procedure is carried out as follows: raw materials are analyzed according to the sorting in $\mathcal{V}$. For each raw material g the largest regular dosing installation with vacant silos that fits all dosages of g is searched, which is described in lines 4–42 of Algorithm 1. A few outcomes are possible:

- If there is such a dosing installation, the raw material is placed and removed from $\mathcal{V}$.
- If there is no regular dosing installation that fits all dosages of g, then the raw material remains in $\mathcal{V}$.
- If there are one or more regular dosing installations that fit all dosages of g, but those do not have any vacant silos, then g remains in $\mathcal{V}$. However, this does not necessarily imply that g cannot be placed at a single installation, as another raw material may be reallocated to another installation in favor of g. Hence, some additional steps are carried out in order to explore this possibility.

In the latter case, additional possibilities are explored in order to place g, as described in lines 15–42 of Algorithm 1. It is assumed that an already placed raw material h may only be reallocated to a smaller installation in favor of g, as h would have been placed in a larger installation in the first placed if this were possible. The following possibilities are analyzed consecutively:

a. If no vacant silos remain in any regular dosing installation, then no raw material can be reallocated to another installation in favor of g. Hence, g is placed in the handtip installation.

b. If g fits the smallest regular dosing installation, then there are no smaller regular dosing installations. Hence, g is placed in the handtip installation.

c. If there are vacant silos in installations smaller than installation w, which is the smallest regular installation that fits all dosages of g, then the smallest raw material h is searched that can be reallocated to a smaller regular installation. If h exists, then h is reallocated, g is placed in w and g is removed from $\mathcal{V}$. It is attempted to reallocate h to the largest regular installation smaller than w. An illustration of this situation is shown in Example 4.

After the single installation allocation procedure has been carried out as described in this section, raw materials may remain in $\mathcal{V}$. It is attempted to allocate the remaining materials using the multi installation allocation procedure as described in the following section.

Example 4. ***Re-allocation of raw materials in single installation allocation*** *Assume three dosing installations A, B and C with each 3 silos. A and C are respectively the largest and smallest installation of the three. Barley, corn and soybean are placed at A. Whey, wheat and sugar are placed at B. Salt and vitamin B are placed at C. Relevant material data and machine properties are shown in Tables 5.1 and 5.2. Assume that in this case the descending frequency heuristic is applied. Thus, raw materials are sorted based on the number of dosages. A raw material, limestone, with a number of 500 dosages, must be allocated. Assume that the minimum and maximum dosage sizes of limestone are 200 and 700 kg respectively. Hence, limestone may fit in installation A or B. However, there are no silos available in either installations. According to the descending frequency heuristic, during the single installation allocation, it is then attempted to place limestone at installation B by re-allocating a raw material from B to C. The smallest raw material in this case is sugar, as it has the smallest number of dosages from all raw materials*

placed at B. Assume that the minimum and maximum dosage sizes of sugar are 75 and 200 kg respectively, which is in accordance with the allowed dosing limits of installation C. Hence, sugar is reallocated from B to C and limestone is placed at B.

Table 5.1: Raw material data

Raw material	Installation	Number of dosages
Barley	A	10.000
Corn	A	5.000
Soybean	A	5.000
Whey	B	2.000
Wheat	B	1.000
Sugar	B	750
Salt	C	700
Vitamin B	C	650

Table 5.2: Dosing installation properties

Installation	$m^{\min}$	$m^{\max}$
A	100	2000
B	50	1200
C	2	500

5.5 Multi installation allocation

Raw materials that cannot be placed on a single installation, because not all dosages fit in a single regular installation or because no silo at a desired installation is available, remain in $\mathcal{V}$. These raw materials are analyzed in the multi installation allocation procedure. This procedure is globally shown in pseudo-code in Algorithm 2 on page 48.

Similar to the single installation allocation, raw materials are analyzed according to the sorting in $\mathcal{V}$, as described by lines 1–2 in Algorithm 2. For each raw material g, the multi installation allocation is only carried out if not all silos of regular dosing installations have been filled, which is expressed by line 3. All possible configurations c of 2 regular installations that g can be divided over are analyzed. A configuration k of g is defined as a set of two regular dosing installations over which the production allocation of g has to be divided. In case allocation of g to 2 regular installations does not suffice to cover the full production allocation of g, additional allocation of dosing interval $range_0$ to the handtip installation exists. It is aimed to avoid allocation of dosages as much as possible to the handtip installation. Hence, in case allocation to the handtip is unavoidable, the upper limit of $range_0$ for each configuration k always corresponds to the minimum required weighing mass $m_w^{\min}$ of the smallest installation in k, such that as little as possible is allocated to the handtip installation. The configurations from c that allocated as little as possible to the handtip installation are given by c_{sub}, as shown in line 5. To each configuration k in c_{sub} the optimal production allocation is defined using the weighed sum method, see lines 6–8. This production allocation is assigned a certain score. The exact procedure of finding the optimal production allocation and assigning this score to a configuration k of g is illustrated in Section 5.7. The production allocation of the configuration with the highest score is described by regular installations w_1 and $w_2 < w_1$ and their respective dosing intervals $range_1$ and $range_2$, which is shown in lines 9–12. For the placement of g several possibilities exist, as illustrated in lines 14–33, which are analyzed consecutively:

a. If there are silos vacant in w_1, g is placed at w_1 with $range_1$.

b. If there are no silos available, then it is checked whether $range_1$ of g is larger than or equal to the smallest raw material h at w_1 according to the applied sorting method. If this is true, then g is placed at w_1 with $range_1$. It is attempted to reallocate h to the next smaller dosing installation $w_1 - 1$. If this is not possible, because $w_1 - 1$ does not have vacant silos or because the dosing interval of h at w_1 does not correspond to the weigher limits at $w_1 - 1$, then h is placed at the handtip installation.

c. If there are no silos available and if $g < h$, then it is attempted to place g at $w_1 - 1$. If this is not possible, then g is placed at the handtip installation.

These steps are repeated for the allocation of g in interval $range_2$ at w_2.

After the multi installation allocation procedure has been carried out as described in this section, steps 6 and 7 of the heuristic are executed. Step 6 is straight forward: all remaining raw materials in $\mathcal{V}$ are placed in the handtip installation. Step 7, the reallocation of raw materials to remaining empty silos, is explained in the next section.

5.6 Reallocation to vacant silos

Remaining raw materials in $\mathcal{V}$ after steps 1–5, are placed in the handtip allocation. This situation may occur if after the singe installation allocation procedure no vacant silos remain. However, if after step 5 vacant silos do exist, then a raw material should be allocated to those silos. This procedure is shown in pseudo-code in Algorithm 3 on page 49.

For each regular dosing installation i in descending order of installation size, except the smallest regular installation, it is checked whether there are vacant silos available. If this is true for i, then each raw material g allocated at a regular installation smaller than i is assigned a priority score $prior_g$, as described by lines 2–5. All these raw materials and scores are collected in g_{list} and $prior_{list}$ respectively. This priority score resembles the urgency of a raw material to be reallocated to installation i. The procedure of calculating the priority score is illustrated in Section 5.8. For each vacant silo s at i, the raw material g in $prior_{list}$ that has the highest urgency $prior_{\max} = \max_g prior_g$ is searched. Note that g should be allocated to exclusively one regular installation j. Otherwise, reallocation is not possible as a raw material may only be placed at most at 2 regular dosing installations. The optimal production allocation for g at i and j is found using the weighted sum method, which is explained in Section 5.7. According to that production allocation, part of the dosing interval of g at j is reallocated to i. If $prior_{\max} = 0$, then no dosages can be reallocated to installation i. In that case, the search for reallocating a raw material to i is aborted and the next (smaller) regular dosing installation is analyzed for potential reallocation, as shown in lines 8–10.

After the reallocation of raw materials to empty silos has taken place, additionally, local optimization may be performed using the swap procedure. The swap procedure is explained in the Section 5.9.

5.7 Weighted sum method

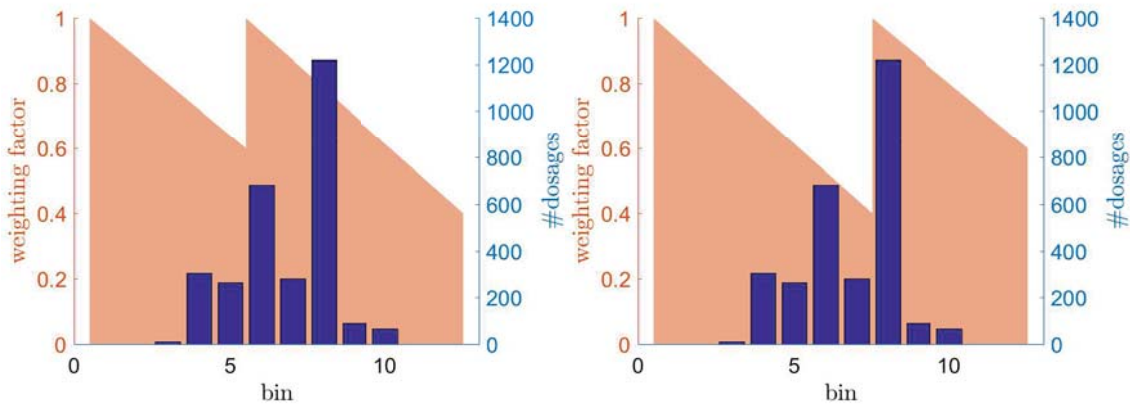
The multi installation allocation procedure as presented in Section 5.5 and the reallocation procedure as presented in Section 5.6 use the weighted sum method to find the best production allocation. In the multi installation allocation procedure, a raw material g must be allocated to at most two regular installations. In the reallocation procedure a section of the dosing interval of

a raw material g is reallocated to a vacant silo in another installation. If the two chosen installations for g operate in an overlapping weight range, then more than one distribution of the dosing intervals of g exists.

To find the optimal production allocation of g the weighted sum method is applied. In this case, the optimal distribution is defined such that the number of dosages of g executed at the installations is highest towards the lower weight limit of the installation, the required minimum dosing mass $m_w^{\min}$. It is aimed to avoid allocation of relatively large dosages to an installation as this leaves little space for additional dosages in the weigher, which leads to a high number of discharges and thus, a longer makespan.

For a given configuration k of a raw material g the weighted sum method searches the highest score and correlation production allocation. The configuration of a raw material implicates the set of dosing installations, w_1 and w_2, at which the raw material is allocated. Placement at two regular dosing installations at most are allowed for each raw material g. Furthermore, additional placement with dosing interval $range_0$ at the handtip installation is allowed.

The weighted sum method is based on the concept that dosages of a raw material g are distributed in a number of weight bins. Depending on the proposed production allocation, a weighting factor is applied to each bin. The resulting weighted sum s represents the score for that specific production allocation.



(a) Weighting factor distribution for cutoff between bin 5 and 6

(b) Weighting factor distribution for cutoff between bin 7 and 8

Figure 5.1: Two examples of weighting factor distributions in two dosing intervals

Figure 5.1 shows the distribution of the dosages of an arbitrary raw material g in 12 weight bins. A low bin number corresponds to a dosage with low mass and a high bin number corresponds to a dosage with high mass. Figure 5.1a shows that dosages in bins 1–5 are assigned to one dosing installation and dosages in bins 6–12 are assigned to another dosing installation. Figure 5.1b shows another production distribution with bins 1–7 and 8–12 to the respective dosing installations. The mass at which the dosing interval of the first installation ends and the second installation starts is referred to as the cutoff point.

The weighting factor decreases for increasing bin numbers in the specific dosing interval. The resulting weighted sum represents the score for that specific production allocation. Furthermore, to ensure that all dosages are assigned within the dosing limits of the installations, the weight bins must be distributed such that the dosages of a bin are always fully included or excluded at a

dosing installation.

The procedure for applying the weighted sum method is globally shown in pseudo-code in Algorithm 4 on page 49. For a given configuration k of a raw material g the weighted sum method returns the highest score s and correlation production allocation, i.e. dosing intervals $range_1$ and $range_2$ at dosing installations w_1 and w_2 respectively, as described by line 1 in Algorithm 4. Lines 3–4 involve finding the cutoff points for configuration k. Cutoff points may be placed before and after common bins between w_1 and w_2. Hence, if there are a common bins, then $a + 1$ possible cutoff points exist. For each of the $a + 1$ cutoff points the score is calculated, yielding an optimal production allocation for g. It is taken into account that if part of the production allocation of g has been already allocated to the handtip installation, then $range_1$ and $range_2$ should be chosen such that no overlap exists with $range_0$. This is described in lines 5–13.

5.8 Priority score

In order to carry out step 7, the reallocation of raw materials to empty silos, a priority score is calculated for each raw material allocated at an installation smaller than the installation with the empty silo. Based on that priority score it is chosen which raw material should be partially moved to the installation with the empty silo.

The priority of which raw material g should be moved from dosing installation $j < i$ to an empty silo in dosing installation i is based on the concept that it is attempted to avoid allocation of relatively large dosages at an installation, that is, dosages that are weighted near the maximum weighing mass of the installation, as this leaves little space for additional dosages in the weigher. Hence, a raw material g that is allocated at j with relatively large dosages that can be moved to the larger installation i is assigned a relatively high priority score.

Similar to the weighted sum method as described in Section 5.7, dosages of a raw material g are distributed over weight bins and a weighting factor is applied, which results in a weighted sum that, in this case, represents the priority score $prior_{list}$. As for the priority score large dosages should be prioritized, the weighing factor increases for increasing bin numbers, opposed to the situation of the weighted sum method with descending weighting factors. Only common bins between i and j are taken into consideration for the priority score. In Figure 5.2 an example is shown of weighting factors for a random raw material distribution with common bins 7–10. As can be seen, a weighting factor is applied to the common bins only.

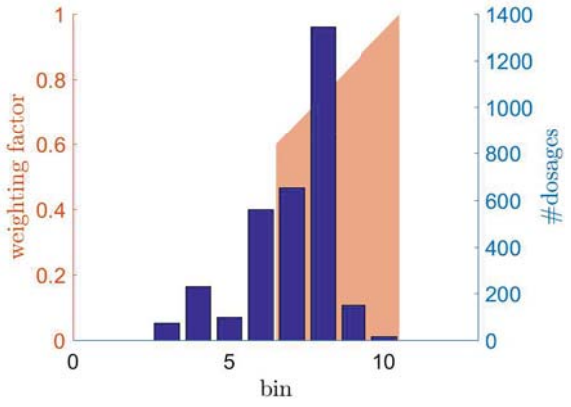


Figure 5.2: Example of weighting factor for a raw material distribution with common bins 7–10

5.9 Swap procedure

Steps 1–7 in the descending mass heuristic and descending frequency heuristic do not consider the recipe coherence of raw materials at installations. In this context coherence is defined as the extent in which the raw materials on an installation occur in the same recipe. To take into account that raw materials in the same recipe should be placed over different installations, i.e., a low recipe coherence is desired, step 8, the swap procedure, is introduced. By doing so, it is aimed to divide the dosage times of a batch parallel over a number of installations, rather than having a serial dosage times on a single dosing installation and thus reducing the batch time and consequently the makespan. The batch time is defined as the time required to complete a batch. An indication of an undesirable high recipe coherence at an installation is thus a large batch time. To this end, the swap procedure is developed.

As stated in Section 5.1, the production time is regarded as optimization objective. The swap procedure aims to swap two raw materials g and h between two dosing installations w_1 and w_2 in order to reduce the production time. To reduce the production time efficiently, i.e., finding the two raw materials g and h for which a swap reduces the production time the most, it is assumed that raw material g should preferably be searched at the busiest dosing installation. To approximate the business, the occupation time of each installation is calculated. The occupation time $occupation_i$ of an installation i is that part of the production time that installation i is involved in. Hence, $occupation_i$ is the sum of the production times of the batches that involve i.

The swap procedure is schematically visualized in pseudo-code in Algorithm 5 on page 50. The maximum number of iterations $iter$ that may be carried out for the swap procedure has to be defined first. Also the production time resulting from steps 1–7 is calculated. This is denoted in line 1. For each iteration, first, the occupation time $occupation_i$ for each installation i is calculated, see lines 3–5. As shown in line 6, each installation i is ranked in $\mathcal{W}$ to descending occupation time and they are analyzed in this order. For installation $w_1 = i$, the largest raw material g, according to the applied sorting method, is searched and for installation $w_2 = w_1 - 1$, the smallest raw material h, according to the applied sorting method, is searched. This is described in lines 8–11. If the dosages of g at w_1 can be reallocated to w_2 and the dosages of h in w_2 can be reallocated to w_1, then the production time for this potential swap is calculated. If this swap leads to a smaller production time, then the swap is executed and iteration k is ended, as shown in lines 14–18. If during an iteration no single swap is made, then the local optimization procedure is aborted, as presented in lines 21–23.

Note that a low recipe coherence is not always desired. In case of the installations of KSE the discharge times are relatively small compared to the collective dosing times in a batch. In such situations the time reduction by performing dosages in parallel outweighs the added time of potential extra discharges. Thus, in this report it is assumed that a low recipe coherence is desired.

The following section discusses the results of applying the heuristic including the swap procedure to four case studies.

5.10 Results

The heuristic as described in the previous sections, including local optimization using the swap procedure, has been applied to four case studies provided by KSE. See Table 5.3 on page 50 for some properties of the case studies. The results are discussed in this section.

The heuristic is divided in two distinct processes. The first process is the initial optimization, which is described by steps 1–7 of the heuristics as shown in Section 5.2. In the initial optimization raw material are appointed an initial allocation such that each raw material is allocated and

no empty silos remain. The second process is the local optimization, which is described by step 8 of the heuristics as shown in Section 5.2. In the local optimization step raw materials are reallocated such that a better production time is achieved. For each of the two processes one of three sorting methods, descending mass, descending frequency and descending recipe occupancy, may be applied. This results in at most nine different solutions. For the local optimization a maximum of 20 iterations is allowed. The applied weighting factors for the weighted sum method and the priority score are given in Appendix D.

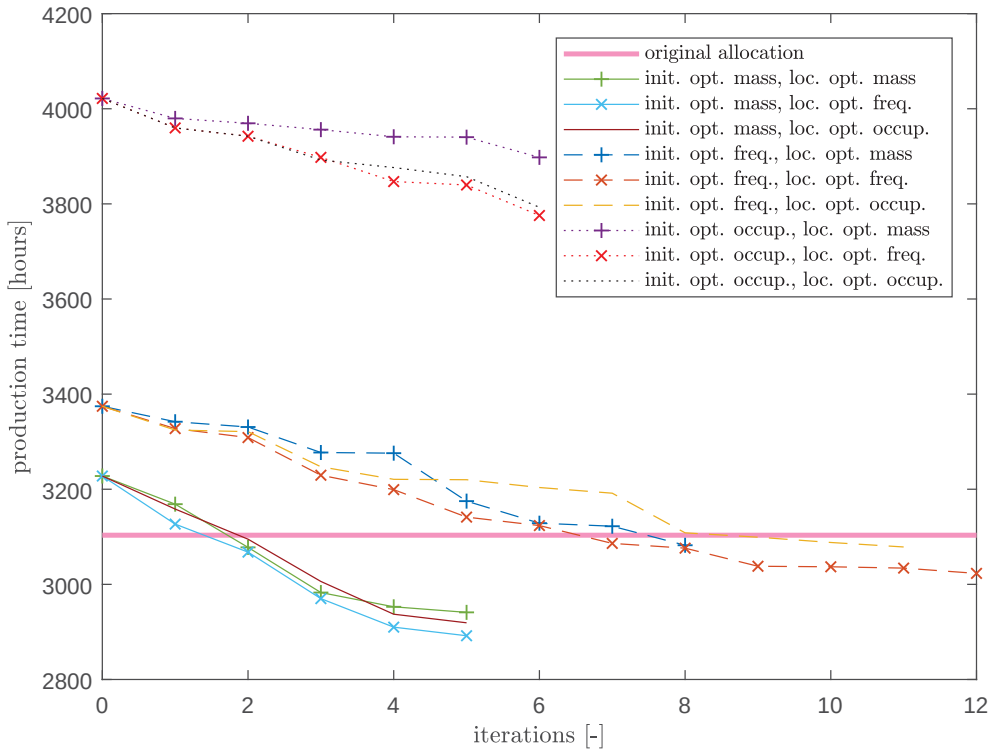


Figure 5.3: Results case study 1

Figures 5.3, 5.4 and 5.5 show the results for the heuristics applied to case studies 1, 2 and 4. The production time given by the initial optimization is given by iteration 0. As three different sorting methods have been applied to the initial optimization, three different solutions are possible. To each of these initial optimization solutions local optimization has been applied, resulting in at most nine different solutions. No local optimization is possible to the solutions of case study 3, hence, no figure is given for this case study. Numerical results for the best performing heuristic for case studies 1–4 are given in Table 5.4 on page 50. Additional numeric results are provided in Appendix D. Furthermore, a production allocation has been assigned by hand to each of the case studies. These allocations are appointed as 'original allocations' in the results and are given by a bold red line in the figures. The maximum number of iterations is 20. However, it should be concluded that for neither of the case studies this number of iterations is carried out, because according to the heuristics no more swaps are possible. At most 12 iterations have been carried out, as shown in case study 1 by the result of the heuristic and local optimization based on sorting by descending frequency.

It can be seen that for case studies 1 and 4 similar or better results are achieved by the heuristics

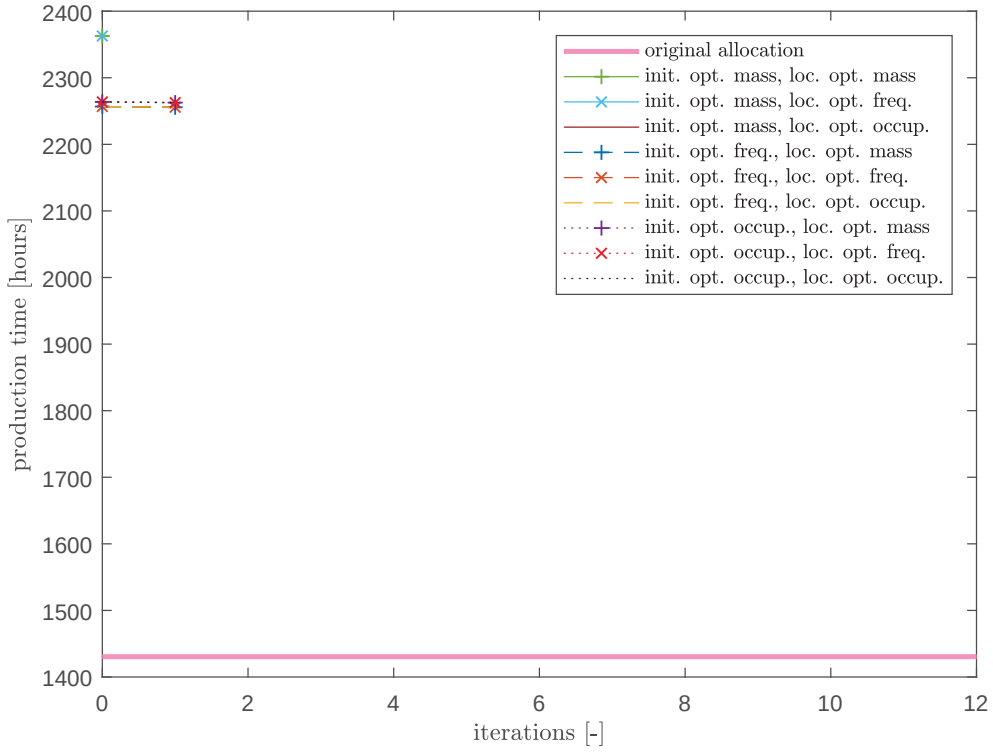


Figure 5.4: Results case study 2

compared to the original allocation. For case study 1 the maximum reduction in production time compared to the original allocation is realized by an initial optimization using descending mass sorting and a local optimization using descending frequency sorting. The reduction is 211.4 hours, which is a decrease of 7.1% in production time. For case study 2 the maximum reduction in production time compared to the original allocation is realized by using descending mass sorting during both the initial optimization and local optimization. The reduction is 0.5% in production time. For case studies 2 and 3 the heuristics lead to a significantly worse result than the original allocation by 57.7% and 38.5% respectively. In case study 3 the local optimization sorting method is irrelevant, all three sorting methods lead to the exact same solution. This is related to the fact that only one iteration has been able to be carried out, which resulted in the same swap for each of the sorting methods.

The local optimization is limited or not carried out if the largest raw material cannot be swapped with the smallest raw material, according to the sorting method, from one installation to another. This may be due to several factors. If the number of installations is limited, as is the case in case study 3, then the number of potential installations at which swaps may take place is limited. Furthermore, if the installations have a minimum dosing mass and maximum allowed dosing mass limit such that this leaves a relatively small common dosing interval, then the probability that a raw material can be moved from one installation to another decreases significantly. Also, note that first all possible raw materials that may be allocated to a single installation during the single installation allocation procedure are placed. If there are relatively many possible single installation raw materials, then this procedure leaves no unoccupied silos and the multi installation allocation is practically not carried out. Hence, resulting raw materials in $\mathcal{V}$ after the single installation allocation procedure are placed at the handtip installation. In general, there is a correlation be-

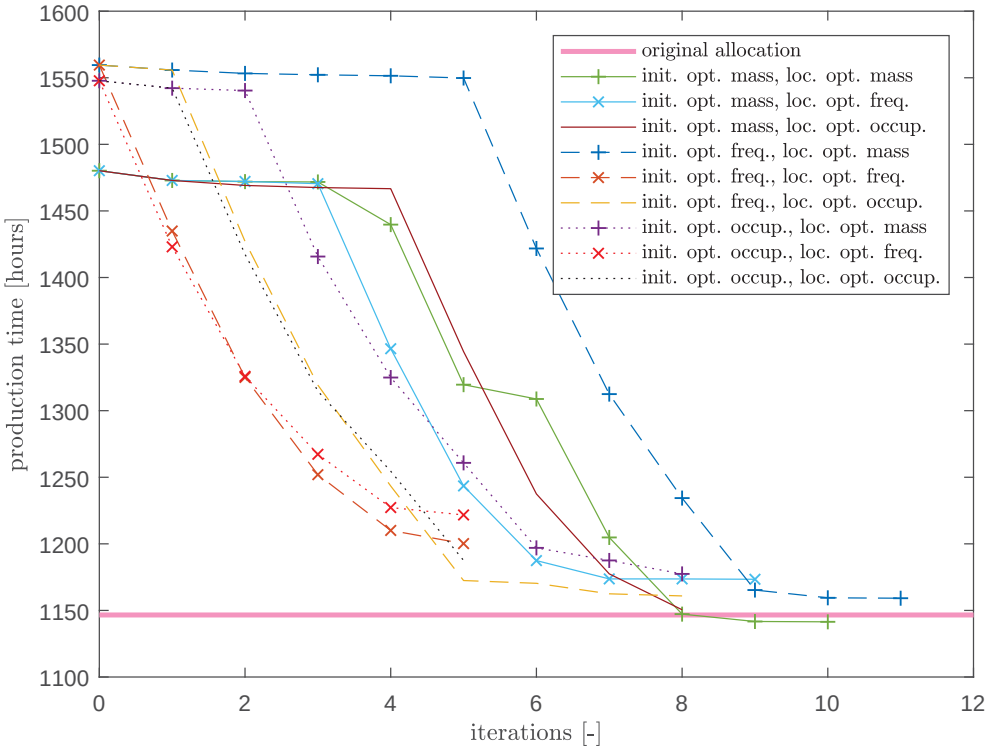


Figure 5.5: Results case study 4

tween the size of the raw material according to the applied sorting method and the dosing interval of these raw materials. Large raw materials have higher dosing intervals than small raw materials. Hence, in general, large raw materials are placed at bigger installations with higher allowed dosing mass limits and small raw materials are placed at smaller installations. In case no multi installation raw materials are allocated however, and a relatively large number of small single installation raw materials exists in the production, then relatively small raw materials are allocated to the installations. Hence, the probability is relatively large that the dosing interval of the smallest raw material at installation w_2 cannot be placed at the larger installation w_1 as intended by the swap procedure as the minimum required dosing mass at w_2 is too large for the raw material to be reallocated. Due to this reason, few or no swaps are performed for case studies 2 and 3.

Moreover, in that case, the resulting raw materials after the single installation allocation procedure may consist of raw materials that are required to be allocated to multiple regular installations and may consist of raw materials that may be placed at a single installation, but are not able to as all silos have already been occupied. In the first case, the raw material may be large or small, according to the applied sorting method. In the second case, the raw materials are relatively small as the largest of the single installation raw materials have been placed. Hence, relatively large raw materials, the multi installation raw materials, are placed in the handtip installation, resulting in a relatively high production time as seen in the results of case studies 2 and 3. In case study 4 this misplacement of relatively large raw materials to the handtip is true to a lesser extent compared to case studies 2 and 3. Therefore, case study 4 results in a similar, but not significantly better, result compared to the original allocation.

Furthermore, it should be noted from Table 5.4 that no sorting method is superior, indicating that

the desired sorting method is highly dependent of the nature of the raw materials, batches and dosing installations involved in the production schedule.

It should be concluded that the heuristics yields unsatisfactory results as in 2 out of 4 case studies significantly worse solutions are given by the heuristics compared to the original hand made allocation. This is due to the fact that the nature of the production data in each case study is different, meaning that the heuristics do not cover, or do not cover well enough, all aspects of the production in the case studies that contribute to the makespan. The following section provides some recommendations to improve the heuristics.

5.11 Recommendations heuristic

To improve the heuristics several recommendations are proposed:

- As concluded from the results in Section 5.10 the multi installations allocation procedure practically does not take place if all silos are occupied after the single installation allocation procedure. Due to this, it is probable that raw materials that are sorted high in $\mathcal{V}$, according to the concerning sorting method, are placed in the handtip installation and the swap procedure is not or scarcely carried out, which is undesired. Hence, line 4 in Algorithm 2 on page 48 should be removed in order to carry out the multi installation allocation procedure, such that small single installation raw materials can be reallocated to the handtip and larger raw materials can be placed in the regular installations.
- After the initial optimization has taken place the production allocation for multi installation raw materials could be redistributed. The initial allocation that is proposed by the weighted sum method is merely an approximation of a good solution, i.e. a low production time. After the initial optimization process, the production allocation for each raw material should be redistributed if this leads to a production time reduction. The raw materials could be analyzed in order of the applied sorting method.
- In the multi installation allocation procedure only one configuration is analyzed for the eventual allocation. If this allocation is not possible, the dosing intervals of the raw materials are placed in the handtip installation. Analyzing more configurations if placement in regular installations is not (fully) possible could improve results.
- As defined in the problem definition in Chapter 2, installations do have volume restrictions next to mass restrictions. The volume restrictions have not been implemented yet, however, this could be done straight forwardly: at all stages in the heuristics that involve some type of mass analysis, the volume restriction should be analyzed as well.
- The redistribution of raw materials to vacant silos as introduced in Section 5.6 is based on the priority score and the weighted sum method. However, the production time can be involved directly instead of the weighted sum method to find the optimal redistribution of dosages. Also, only one raw material, namely the raw material with the highest priority score, is chosen to be analyzed for redistribution. This could be extended to a number of raw materials.
- The heuristic is split up in two processes, the initial optimization and local optimization. However, in the initial optimization the sorting method is involved in both the single and the multi installation allocation. Therefore, the heuristics may be split up even further, in three processes in total. In that case, the remaining raw materials in $\mathcal{V}$ should be resorted to the according sorting method after the single installation allocation procedure.
- In these heuristics three sorting methods are applied based on descending mass, descending frequency and descending recipe occupancy. A hybridization by combining the sorting methods may improve results. In that case all three aspects are taken into account. This

hybridization could be defined as: $am_g + bf_g + co_g$, with m_g the total mass used, f_g the frequency and o_g the recipe occupancy of a raw material g and a, b and c weighting factors.

- In the proposed heuristics, the handtip installation has not been taken into account in the local optimization. Doing this might improve results.
- The allocation procedure of raw materials is divided into steps such that single installation allocation is performed first and multi installation allocation is performed hereafter. However, instead the allocation procedure may be analyzed strictly to the order of raw materials in $\mathcal{V}$. Each raw material according to sorting order could be analyzed consecutively. If a raw material cannot be placed by a single installation allocation, then directly a check may be performed if the raw material can be placed by a multi installation allocation.
- The swap procedure is a fairly rigid process as it only evaluates the largest and smallest raw materials in two dosing installations. The swap procedure could be extended by evaluating more raw materials if swaps are not possible. This might be done by evaluating swaps with the second (or more) largest and smallest raw materials in addition. Alternatively, random swaps may be considered as well. Also, swaps between other dosing installations may also be carried out in order to increase performance, as in the current heuristics only swaps between consecutive installations $w_1 = i$ and $w_2 = i - 1$ are considered.
- Lastly, the provided case studies all have an original allocation with which the heuristic results are compared. To this original allocation the local optimization, as introduced in Section 5.9, might be applied. This may result in better solutions. This method however, can only be used if an original allocation is available, which is not always the case.

As has been indicated, the results of the heuristics vary highly dependent on the nature of the raw materials, batches and dosing installations involved in the production schedule. Hence, a deterministic heuristic might not provide a robust solution: there may always be a situation in which the deterministic heuristic as proposed in this chapter leads to unsatisfactory results, even if the recommendations stated above are implemented. Hence, the following chapter introduces a meta-heuristic, namely the genetic algorithm.

Algorithm 1: Pseudo-code for the allocation procedure of raw material that can be appointed to a single dosing installation. Note that 'break' immediately terminates the execution of the closest for or while loop in which it occurs

```
for materialindex = 1:length(V) do
    initialization: w = 0, placed = 0;
    g = V(materialindex);
    for each regular dosing installation i in descending order of installation size do
        if dosing interval of g fits at i then
            w = i;
            if there are vacant silos at w then
                place g at w;
                placed = 1;
                remove g from V;
                break;
            end
        end
    end
    if w > 0, placed == 0 then
        if there are no vacant silos in any regular installation then
            place g at the handtip installation;
            remove g from V;
            break;
        else if w == 1 then
            place g at the handtip installation;
            remove g from V;
        else if there are vacant silos in regular installations < w then
            for each raw material h in w in descending size depending on sorting method
             do
                for each regular installation j < w in descending order of installation size
                 do
                    if h fits in j then
                        remove h from w;
                        place h at j;
                        place g at w;
                        placed = 1;
                        remove g from V;
                        break;
                    end
                end
                if placed==1 then
                    break;
                end
            end
        else
            do nothing;
        end
    end
end
```

Algorithm 2: Pseudo-code for the allocation procedure of raw materials that should be appointed to multiple dosing installations. Note that installation w_1 is larger than installation w_2 and that 'break' immediately terminates the execution of the closest for or while loop in which it occurs

```
for materialindex = 1:length(V) do
    g = V(materialindex);
    if any vacant silos in regular installations available then
        c = all configurations raw material g can be divided over concerning at most 2
          regular installations and the handtip installation;
        c_sub = all configurations from c with the smallest possible dosing interval range_0
          placed at the handtip installation;
        for each configuration k in c_sub do
            find production allocation with highest score for each configuration k of g using
              the weighted sum method;
        end
        w_1 = largest installation of configuration with highest score;
        w_2 = smallest installation of configuration with highest score;
        range_1 = dosing interval at w_1 of configuration with highest score;
        range_2 = dosing interval at w_2 of configuration with highest score;
        for installationindex=1:2 do
            if silos available in w_installationindex then
                place g in w_installationindex;
            else
                h = smallest raw material at w_installationindex;
                if g ≥ h according to sorting method then
                    place g at w_installationindex with range_installationindex;
                    if h can be reallocated to the (smaller) next regular installation
                      w_installationindex − 1 then
                        remove h from w_installationindex;
                        place h at w_installationindex − 1;
                    else
                        place h at the handtip installation;
                    end
                else
                    if g can be placed at the (smaller) next regular installation
                      w_installationindex − 1 then
                        place g at w_installationindex − 1 with dosing interval
                          range_installationindex;
                    else
                        place g at the handtip installation with dosing interval
                          range_installationindex;
                    end
                end
            end
        end
        remove g from V;
    end
end
```

Algorithm 3: Pseudo-code for the reallocation procedure of raw materials to vacant silos. Note that 'break' immediately terminates the execution of the closest for or while loop in which it occurs

```
for each regular dosing installation i, except the smallest regular installation, in
  descending order of installation size do
    if there are vacant silos at i then
        g_list = all raw materials allocated to silos at regular installations < i;
    end
    prior_list = priority score based on sorting method;
    for each vacant silo s at i do
        find raw material g in prior_list with prior_max = max score_list that is allocated at
          regular dosing installations j;
        if prior_max == 0 then
            break;
        end
        find production allocation with highest score for configuration of g at i and j using
          the weighted sum method;
        place g at i;
    end
end
```

Algorithm 4: Pseudo-code of the weighted sum method. Note that installation w_1 is larger than installation w_2

```
Weighed sum method (range_0, k);
Input  : handtip dosing interval range_0,
         configuration k of raw material g at regular installations w_1 and w_2 < w_1;
Output: score s,
         dosing interval range_1 at w_2,
         dosing interval range_2 at w_2;
initialization: s = 0;
find common bins i between w_1 and w_2;
define cutoff points j_list based on i;
for each cutoff point j in j_list do
    find dosing intervals range_{1,j} to w_1 and range_{2,j} to w_2 for cutoff point j, taking into
      account that range_0 is already allocated to the handtip interval;
    calculate the score s_j for the production distribution with range_{1,j} and range_{2,j};
    if s_j > s then
        s = s_j;
        range_1 = range_{1,j};
        range_2 = range_{2,j};
    end
end
```

Algorithm 5: Pseudo-code for the swap procedure of the local optimization procedure. Note that installation w_1 is larger than installation w_2 and that 'break' immediately terminates the execution of the closest for or while loop in which it occurs

```
initialization: iter, pt_old = production time of allocation;
for k = 1:iter do
    for each regular dosing installation i, except the smallest regular dosing installation do
        occupation_i = occupation time at each installation;
    end
    W = order of installations based on descending occupation time;
    for each installation i in the order of W do
        w_1 = i;
        w_2 = i − 1;
        g = largest raw material in w_1 according to sorting method;
        h = smallest raw material in w_2 according to sorting method;
        if g can be reallocated to w_2 and h can be reallocated to w_1 then
            pt_new = production time if g is reallocated to w_2 and h is reallocated to w_1;
            if pt_new < pt_old then
                swap raw materials g and h;
                pt_old = pt_new;
                break;
            end
        end
    end
    if no swap is made during iteration k then
        break;
    end
end
```

Table 5.3: Case study properties

Case study	Number of regular installations	Total number of silos	Number of recipes	Total number of batches	Number of raw materials
1	6	90	43	28.642	99
2	5	66	523	12.218	178
3	3	65	50	6.858	116
4	4	87	471	10.648	191

Table 5.4: Minimum achieved production time [hours] for best performing heuristics

Case study	Initial allocation	Local optimization	Production time [hours]
1	Descending mass	Descending frequency	2891.9
2	Descending frequency	-	2255.7
3	Descending recipe occupancy	All	1550.0
4	Descending mass	Descending mass	1141.4

Chapter 6

Genetic algorithm

The solution method proposed in the previous chapter has not shown to be a robust algorithm to solve the problem posed in Chapter 2. The deterministic heuristic in the previous chapter falls short as the nature of each production schedule is different and, therefore, requires a solution of a different nature. Hence, a meta-heuristic is proposed in this chapter. Meta-heuristics are higher-level general-purpose algorithms that often incorporate randomness in order to find good solutions. Some background information on meta-heuristics, in particular genetic algorithms, can be found in Chapter 3. Genetic algorithms are widely used in scheduling problems and are inspired by the process of natural selection. The possibility of applying these algorithms to the problem of Chapter 2 are investigated in this chapter.

6.1 General genetic algorithm

The design of the genetic algorithm resembles the architecture posed in Section 3.4.1: an initial population is created, each individual is assigned a fitness score, based on some criteria the simulation is terminated or not. If the simulation is not terminated, crossover and mutation take place by a selection procedure and a new generation is created from this. This process is repeated until the simulation is terminated. The genetic algorithm proposed in this chapter is made suitable for the required production allocation of the posed problem in Chapter 2. However, likewise to the heuristic in Chapter 5 the specific silo assignment within a dosing installation is not considered. Hence, it is assumed that all dosing installations have stationary weighers exclusively, which omits relocation times.

First, a set of initial solutions should be created. This could be achieved by taking the solutions of the deterministic heuristic as described in Chapter 5. Alternatively, in some situations extra solutions can be created by excluding local optimization in the heuristic.

These solutions should be represented in a suitable format for genetic algorithms, that is, in the form of genotypes. The proposed representation of solutions for the genetic algorithm is addressed in Section 6.2.

The fitness function may correspond to the makespan, as applied in the MILP in Chapter 4 and as required in the problem definition. However, as applied in the deterministic heuristic of the previous chapter, in order to limit computational burden, production time is another possible objective.

For the recombination of solutions a custom crossover is introduced, which is discussed in Section 6.3. Since the initial population introduced in the genetic algorithm is not guaranteed to be diverse, it is chosen to introduce two types of mutation operators. These operators are explained in Section 6.4. Tournament selection is chosen as selection operator for finding individuals for recombination.

After recombination has taken place a new generation is generated by collecting the resulting population. It is chosen to limit the population size to $N_{\max}$ to prevent population explosion. If the population size exceeds $N_{\max}$ after recombination, tournament selection takes place to reduce the population size to $N_{\max}$. Furthermore, it is chosen to retain the parents after recombination has taken place.

6.2 Representation of solutions

Due to the uncommon requirement of the production allocation as presented in Chapter 2, a custom representation of the solutions is required. In genetic algorithms solutions are encoded in the form of genotypes. Several options are discussed in this section. The visualization of the genotypes follow the same convention as proposed in Chapter 3, that is, chromosomes in a genotype are listed vertically and genes in a chromosome are listed horizontally. Note that, contrary to the genotypes shown in Figure 3.3, in this chapter, the boxes around genes are omitted for simplicity.

Option 1

The genotype consists of a number of chromosomes, each representing a dosing installation. The position of each chromosome represents a specific installation. The number of genes in each chromosome represents the number of silos at that dosing installation. Each gene consists of two bits. The first bit represents the raw material and the second bit represents the lower limit of the dosing interval of that raw material on that dosing installation. The production allocation can be deducted from the lower limits only, as it is assumed that dosing intervals of a raw material are disjoint and continuous. If a raw material is only placed at a single installation w, then the raw material is only present once in the genotype with a lower limit of 0. The accompanying upper limit in that case is the maximum allowed dosing mass $m_w^{\max}$. Thus, the dosing interval in that case is $[0, m_w^{\max})$, or simply $[0, -)$. If a raw material is placed on multiple installations, then the raw material is present multiple times in the genotype. The raw material on a specific installation is lower bounded by the presented lower limit. The upper limit of the raw material on that installation is then deducted from the next smallest lower limit of that raw material.

Table 6.1 shows an example of such a genotype. It represents a solution with 3 dosing installations: installation 1 has 4 silos, installation 2 has 2 silos and installation 3 has 1 silo. Raw material 1 is placed at installations 1, 2 and 3, with dosing intervals $[67, -), [0, 33)$ and $[33, 67)$ respectively. Materials 2, 3 and 4 are placed at installation 1. Material 5 is placed at installation 2.

Table 6.1: Example of a genotype of option 1

(4,0)	(3,0)	(1,67)	(2,0)
(5,0)	(1,0)		
(1,33)			

Table 6.2: Example of a genotype of option 2

(67)	(0)	(0)	(0)	(-1)
(0)	(-1)	(-1)	(-1)	(0)
(33)	(-1)	(-1)	(-1)	(-1)

Option 2

The genotype consists of a number of chromosomes, each representing a dosing installation. The position of each chromosome represents a specific installation. The number of genes in each chromosome resembles the total number raw materials. The position of each gene in a chromosome represents a specific raw material. Each gene consists of a single bit. The bit represents the lower limit of the dosing interval of the raw material expressed by that specific gene. If a raw material is not placed on an installation, the gene is marked -1. Table 6.2 shows an example of such a genotype. It represents the same solution as used for Option 1 as shown in Table 6.1.

Table 6.3: Example of a genotype of option 3

(2,0)	(3,33)	(1,67)
(1,0)		
(1,0)		
(1,0)		
(1,0)		

Option 3

The genotype consists of a number of chromosomes, each representing a raw material. The position of each chromosome represents a specific material. The number of genes in a chromosome expresses the total number of dosing intervals of a raw material, that is, it represents the number of installations a raw material is placed on. Each gene consists of two bits representing the installation and lower limit of the dosing interval respectively. The total number of genes in the genotype represents the total number of silos in the set of dosing installations. Table 6.3 shows an example of such a genotype. It represents the same solution as used for Option 1 as shown in Table 6.1.

Option 4

The genotype consists of a number of chromosomes, each representing a raw material. The position of each chromosome represents a specific material. The number of genes in each chromosome resembles the number of dosing installations. The position of each gene in a chromosome represents a specific installation. Each gene consists of a single bit, representing the lower limit of the dosing interval of the raw material expressed by that specific gene. This produces the transposed genotype of Option 2.

Option 5

A variant to Option 4 can be made by differentiating between raw materials that cannot be placed on an installation because:

1. the installation properties do not allow it, or;
2. it is simply not placed at the installation.

In Option 4, materials that are not placed at an installation are marked -1 in the gene. Instead, in Option 5, genes can be marked -2 if a raw material is never allowed at the installation and marked -1 if it is allowed at the installation, but it is simply not placed at it.

Table 6.4: Examples of genotypes of option 5; each row or column correspond to a raw material or dosing installation respectively

(a) Genotype 1

(7)	(0)	(3)
(0)	(-1)	(-2)
(0)	(-1)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

(b) Genotype 2

(8)	(-1)	(0)
(0)	(-1)	(-2)
(0)	(-1)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

(c) Genotype 3

(8)	(-1)	(0)
(0)	(-1)	(-2)
(-1)	(0)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

Three examples of genotypes are given in Table 6.4. These examples are used in Section 6.3 to illustrate crossover operations. In genotype 1 it can be seen that raw material 1 is placed at installations 1, 2 and 3. The lower limit at installation 2 is 0 and the lower limit at installation 3

is 3. Hence, the dosing interval of raw material 1 at installation 2 is $[0, 3)$. As the lower limit at installation 1 is 7, the dosing interval of raw material 1 at installation 3 is $[3, 7)$ and the dosing interval at installation 1 is $[7, -)$. Furthermore, it can be seen that raw material 2 is placed at installation 1 only. It is not placed at installation 2 and it may not be placed at installation 3. Hence, raw material 2 is placed at installation 1 only with dosing interval $[0, -)$. Raw materials 3 and 4 are placed at installation 1 only, but they are allowed to be placed at installations 2 and 3. Raw material 5 is placed at installation 2 and it is allowed at installation 3, but it is not allowed at installation 1.

Proposed genotype format

Key to solving the problem stated in Chapter 2 is to find the optimal production allocation of raw materials at installations. In this genetic algorithm the genotype format of Option 5 is considered as representation of the solutions. The reason to do so is because Options 2, 4 and 5 have chromosomes and genes, in which their position directly represents a raw material and dosing installation, hence only need genes consisting of a single bit. Finally, Option 5 allows the genotype to be encoded with more information about the reason why a raw material is not placed on a dosing installation.

6.3 Crossover operator

A custom crossover operator is considered as the representation of solutions requires a unique genotype format due to the uncommon requirement of the production allocation as presented in Chapter 2. The example genotypes as shown in Table 6.4 are used to illustrate the operations of the proposed crossover operator. The crossover is carried out as following:

1. Copy identical negative values from the parents. For example, take genotypes 1 and 2 from Table 6.4. The corresponding result is shown in Table 6.5.

Table 6.5: Crossover (step 1). Identical negative values of parents 1 and 2 as shown in Tables 6.4a and 6.4b

-	-	-
-	(-1)	(-2)
-	(-1)	(-1)
-	(-1)	(-1)
(-2)	-	(-1)

Table 6.6: Crossover (step 2). Filled in raw materials based on parents 1 and 2 as shown in Tables 6.4a and 6.4b

-	-	-
(0)	(-1)	(-2)
(0)	(-1)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

2. Raw materials that can only be placed on one dosing installation, must be placed. Practically, this implies that chromosomes that have only one unfilled gene are filled with a 0, see Table 6.6. For clarification reasons, the representation of the filled in genes show the complete dosing interval if needed.

Table 6.7: Crossover (step 3). Identical intervals of parents 1 and 2 as shown in Tables 6.4a and 6.4b

$[8, \infty)$	$\emptyset$	$[3, 7)$
(0)	(-1)	(-2)
(0)	(-1)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

3. In the resulting (partially) unfilled chromosomes, choose x chromosomes to alter. All chromosomes that are not chosen to be altered are filled in completely corresponding to one of both parents. Roulette selector is used to choose the parents. If the number of (partially) unfilled chromosomes is $\leq x$, then all unfilled chromosomes are chosen. Keep the identical intervals of the chosen chromosomes, see Table 6.7. This means that the coinciding intervals between both parents are maintained.

4. Divide all residual intervals. From this all possible offspring follow, see Table 6.8.

Table 6.8: Crossover (step 4). All possible offspring from parents 1 and 2 as shown in Tables 6.4a and 6.4b

(a) Offspring 1.

$[7,\infty)$	$[0,3)$	$[3,7)$
(0)	(-1)	(-2)
(0)	(-1)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

(b) Offspring 2

$[7,\infty)$	$\emptyset$	$[0,7)$
(0)	(-1)	(-2)
(0)	(-1)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

(c) Offspring 3

$[8,\infty)$	$[7,8)$	$[0,7)$
(0)	(-1)	(-2)
(0)	(-1)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

(d) Offspring 4

$[8,\infty)$	$[0,3)$	$[3,8)$
(0)	(-1)	(-2)
(0)	(-1)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

(e) Offspring 5

$[8,\infty)$	$\emptyset$	$[0,8)$
(0)	(-1)	(-2)
(0)	(-1)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

5. Omit offspring that exceed the number of available silos of any dosing installation.

6. In this genetic algorithm it is chosen to retain the parents after recombination has taken place. As can be seen, the resulting offspring in Table 6.8 does include two offspring that are identical to the parents, namely: offspring 1 and 5. Therefore, the these offspring should be omitted to prevent duplicate individuals appearing in the population after crossover has taken place.

In the case that two individuals with no identical intervals are combined a new random distribution is chosen. An example of crossover between parents 2 and 3, see genotypes 2 and 3 in Tables 6.4b and 6.4c, with such outcome is shown in Tables 6.9 and 6.10. In Table 6.9 it is shown that there are no coinciding intervals for raw material 3 at installations 1 and 2. In Table 6.10 the offspring are shown that offspring 1 and 2 result from a randomly chosen distribution.

Table 6.9: Crossover (step 3). Identical intervals based on parents 2 and 3 from Tables 6.4b and 6.4c

(80)	(-1)	(0)
(0)	(-1)	(-2)
$\emptyset$	$\emptyset$	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

Table 6.10: Crossover (step 4). Possible offspring based on parents 2 and 3 from Tables 6.4b and 6.4c

(a) Offspring 1

(80)	(-1)	(0)
(0)	(-1)	(-2)
(0)	(4)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

(b) Offspring 2

(80)	(-1)	(0)
(0)	(-1)	(-2)
(4)	(0)	(-1)
(0)	(-1)	(-1)
(-2)	(0)	(-1)

6.4 Mutation operator

In Section 6.1 it is mentioned that mutation is of great importance as a diverse initial population is not guaranteed. As stated in the problem definition of Chapter 2, optimization can be achieved by two measures: assignment of a raw material to certain installations and addressing the production allocation on these installations. Hence, two types of mutation operators are considered: limit expansion mutation and exchange mutation. Limit expansion mutation alters the dosing intervals and exchange mutation alter the assignment of a raw raw material to a certain installation.

The mutation operator chooses a gene with probability $P(m)$, such that:

1. A genotype is chosen from the population with a probability $P_G(m)$;
2. An eligible chromosome is chosen from the selected genotype with probability $P_C(m)$. Note that only chromosomes with two or more values ≥ 1 are eligible;
3. A positive and non-zero gene is chosen from the selected chromosome with probability $P_g(m)$;

in which $\sqrt[3]{P(m)} = P_G(m) = P_C(m) = P_g(m)$.

Limit expansion mutation alters the value of a gene with a predefined percentage up or down. It is randomly chosen whether the value is altered up or down, unless the value of the gene is equal to the minimum required dosing mass or maximum allowed dosing mass limit of the installation it is placed on. In that case the value can only be altered up or down respectively. In case the value of the gene is not equal to the minimum required or maximum allowed limit of the installation and the alteration leads to a value greater than the maximum allowed limit or smaller than the minimum required limit, then the new value is chosen to be equal to the maximum allowed or minimum required limit of the installation respectively.

Exchange mutation moves a gene, the primary mutation gene that is chosen with probability $P(m)$, from one installation to another eligible installation and simultaneously moves another eligible randomly chosen gene from that installation back to the first installation. This implies that a gene exchanges location with another gene within a chromosome. If the mutation results in an invalid genotype, another exchange with the primary mutation gene is searched. A number of z attempts is carried out before the mutation operation is aborted.

6.5 Implementation

Implementation of the genetic algorithm has not been (fully) carried out and remains open for future research. A library for genetic algorithms exists in Java, namely Jenetics[2], developed by Franz Wilhelmstötter. The library[1] provides a structure for genetic algorithms with straightfor-

[1]Note that in release 4.1.0 of the Jenetics library a subtle bug exists in the class 'CustomCrossover', in which it is assumed that the number of genes is constant over all genotype instances. This has been addressed to the developer of Jenetics and is fixed in release 4.2.0.

ward and common genotypes and operators. Jenetics also leaves possibilities for adding custom genotypes and operators. However, the nature of the library allows implementation of the proposed genetic algorithm for more experienced Java programmers.

The input parameters required for the genetic algorithm, such as the ratio of individuals selected for crossover and mutation, highly depend on the specific set of dosing installations and production schedule. The process of finding these parameters is often experimental, however, it is advised to apply a relatively high crossover and mutation ratio due to the possible low diversity in the initial population.

Furthermore, tournament selection has been chosen as the appropriate selector as it excludes the worst individual from being chosen. Other selectors, such as roulette wheel selection, should be studied as well.

Genetic algorithms might perform better and certainly more robustly than deterministic algorithms, if the right parameters are chosen. As no results have been produced, no conclusions can be drawn from the performance of the proposed genetic algorithm.

Finally, if it is desirable to include dosing installations with movable weighers in the genetic algorithm, the specific silo placement within an installation should be taken into account. This can be achieved by using a genotype format resembling that of Option 1 in Section 6.2. In that case, the location of the gene within the chromosome may resemble the specific silo at an installation in which the raw material is placed. Note that altering the genotype format does mostly require changing the crossover and/or mutation operator.

Chapter 7

Conclusions and recommendations

This research focuses on optimizing the blending line of the animal production feed system, that is, the process of dosing and mixing materials. The intake and outtake of material and grinding is thus not taken into consideration. Furthermore, this research considers a simplified dosing process: it consists of only one dosing stage followed by one mixing stage, meaning that there is simply one step at which materials are added and one step at which materials are mixed in the system. In the dosing stage one or multiple dosing installations are placed parallel to each other. In the mixing stage one mixer is placed behind the dosing stage.

Optimization of total production time on the blending line is achieved by arranging the container placement and production allocation. Assuming a fixed production schedule, rearranging the container placement, and therefore the materials, between machines may result in a more efficient production as it may reduce the idle time of the installations. Furthermore, for a given a set of dosing installations, a material may theoretically be weighed on multiple installations. Therefore, rearranging the production allocation over these installations may also result in a more efficient production. Next to dosing installations, materials may also be processed by hand, i.e. they may be 'handtipped'. This process is executed in parallel with the dosing installations and may, hence, be regarded as a custom type of installation in the system. It is assumed that handtipping is a relatively slow process.

Three different approaches are investigated in order to solve the optimization problem. First, an MILP has been developed. However, due to the size of the optimization problem a sheer number of design variables are required in the MILP. Hence, the MILP is of no practical use. Second, a deterministic heuristic is proposed. The heuristic is based on the allocation of raw materials in roughly 3 steps: first, materials that may be allocated at a single dosing installation are placed; second, materials that must be allocated to multiple installations are placed; finally, a local optimization takes place by swapping materials between installations in order to decrease the production time. Four cases studies have been made with diverse results. The deterministic heuristic achieves a satisfactory result in specific situations. To improve the performance of the heuristics a number of recommendations have been introduced in Section 5.11. However, the main issue with the case studies that fail to generate a satisfactory solution is that after materials are placed on a single installation all installations are completely occupied. Thus, the materials that must be allocated to multiple installations are not evaluated to be placed at an installation and must, thus, be handtipped, which results in a high production time. The solution to correct this is given as the first recommendation in Section 5.11. Applying this correction is fairly straightforward and may yield better results. The implementation of this correction remains open for future research. Furthermore, the results for the heuristics applied to the case studies are compared to existing results that are obtained by hand at KSE. The heuristic results are satisfactory in case they are better than the existing results. Part of the heuristic optimization involves local optimization. This local optimization may be applied to the existing results in order to find better results. This application of the local optimization has not been studied and remains open for future research.

Nonetheless, the performance of the deterministic heuristics vary highly dependent on the nature of the production schedule and involved dosing installations. Hence, a deterministic heuristic might not provide a robust solution. Therefore, another type of heuristic, the genetic algorithm, is introduced. Genetic algorithms are meta-heuristics, which are higher-level general-purpose algorithms that often incorporate randomness in order to find good solutions. Genetic algorithms are widely used in scheduling problems and are inspired by the process of natural selection. A proposal is introduced for the format of the genetic algorithm suitable for this research project. The implementation of the algorithm remains open for future research.

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Appendix A

Derivation relocation time equations

This appendix focuses on the derivation of the equations for the displacement time for dosing installations with movable weighers as given in Section 2.2.

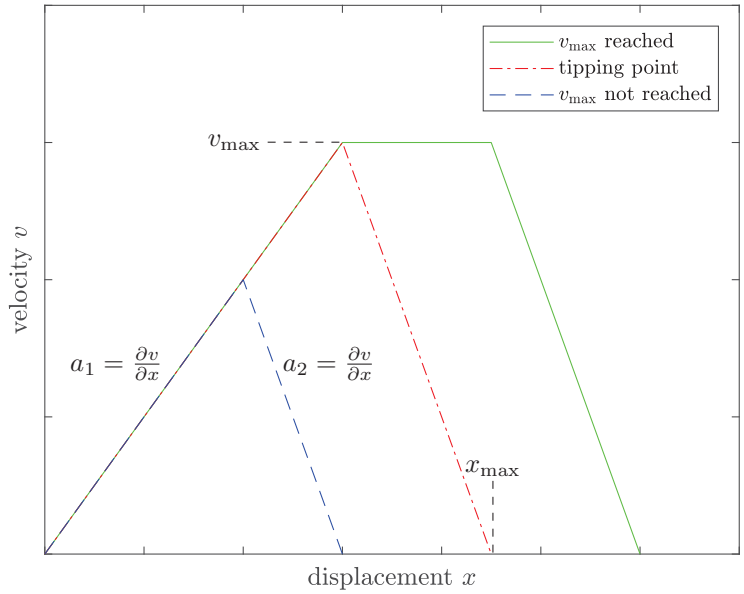


Figure A.1: Different velocity paths depending on the magnitude of displacement x

The displacement time J_{l_1,l_2} is the duration a movable weigher needs to relocate from location l_1 to l_2. It is assumed that the weigher takes the shortest path from l_1 to l_2. The weigher starts up from standstill at l_1 with a constant acceleration a_1. If it reaches the maximum velocity $v_{\max}$ it continues its path with $v_{\max}$ until it starts to decelerate. The weigher decelerates with a constant deceleration a_2 until it reaches destination l_2. This velocity path is indicated by the green line in Figure A.1. If the weigher travels a distance x_{l_1,l_2} lower than $x_{\max}$, then it does not it reach $v_{\max}$ so it decelerates immediately after acceleration. This velocity path is shown by the blue line in Figure A.1. The red line indicates the tipping point between the two types of velocity paths. Hence, the equation for the displacement time J_{l_1,l_2} depends on the traveled distance x_{l_1,l_2} which consecutively determines the type of velocity path taken.

The tipping point between the two types of paths occurs at $x_{l_1,l_2} = x_{\max}$. In order to calculate

$x_{\max}$ the following equations of motion are required:

$$x = \frac{1}{2}at^2 + v_0 t + x_0, \tag{A.1a}$$

$$v = at + v_0. \tag{A.1b}$$

In order to travel a distance $x = x_{\max}$ the weigher accelerates with $a = a_1$ to $v = v_{\max}$ and it decelerates with $a = a_2$ to $v = 0$. Define t_1 and t_2 as the time interval during which acceleration and deceleration take place. Assuming that the weigher has no initial displacement or velocity, $x_{\max}$ can be calculated using (A.1a):

$$x_{\max} = \frac{1}{2}(a_1 t_1^2 + a_2 t_2^2). \tag{A.2}$$

Time intervals t_1 and t_2 are described by (A.1b) via

$$t_1 = \frac{v_{\max}}{a_1}, \tag{A.3a}$$

$$t_2 = \frac{v_{\max}}{a_2}. \tag{A.3b}$$

Hence, substituting this result in (A.4) gives

$$x_{\max} = \frac{v_{\max}^2}{2}(\frac{1}{a_1} + \frac{1}{a_2}). \tag{A.4}$$

If $x \leq x_{\max}$, then the weigher accelerates with a_1 to v during time interval t_1 after which it decelerates from v with a_2 during time interval t_2. Using (A.1a) and (A.1b) the following relations are found:

$$x = \frac{1}{2}a_1 t_1^2 + \frac{1}{2}a_2 t_2^2, \tag{A.5a}$$

$$\left.\begin{array}{l} v = a_1 t_1 \\ v = a_2 t_2 \end{array}\right\} a_1 t_1 = a_2 t_2 \tag{A.5b}$$

Substituting these equations in each other gives the displacement time J_{l_1,l_2} for $x \leq x_{\max}$:

$$J_{l_1,l_2} = t_1 + t_2 = \sqrt{2x(\frac{1}{a_1} + \frac{1}{a_2})}. \tag{A.6}$$

If $x > x_{\max}$, then the weigher accelerates with a_1 to $v_{\max}$ during time interval t_1, travels with a constant velocity $v_{\max}$ during time interval t_{12} and decelerates with a_2 from $v_{\max}$ during time interval t_2. From Figure A.1 it can be seen that the collective distance covered during acceleration and deceleration equals $x_{\max}$. Therefore, the distance covered during t_{12} is equal to $x - x_{\max}$. Using the equation of motion

$$x = vt - \frac{1}{2}at^2 + x_0 \tag{A.7}$$

and substituting (A.4) the following expression for t_{12} is found:

$$t_{12} = \frac{x}{v_{\max}} - \frac{v_{\max}}{2}(\frac{1}{a_1} + \frac{1}{a_2}) \tag{A.8}$$

The expressions for t_1 and t_2 are equal to those for the tipping point. Hence, combining (A.3a), (A.3b) and (A.8) yields the following relation for J_{l_1,l_2}:

$$J_{l_1,l_2} = \frac{x}{v_{\max}} + \frac{v_{\max}}{2a_1} + \frac{v_{\max}}{2a_2}. \tag{A.9}$$

Appendix B

Relaxation assumption order of dosages and production orders

This appendix focuses on a variation of the MILP presented in Chapter 4 in which the execution order of dosages within a batch are unconstrained and the execution order of production orders are unconstrained. First, the relevant data and assumptions are summarized and structured that are needed to solve the problem as introduced in Chapter 2 with relaxation of the assumptions that dosages have to arrive at the batch mixer in order. Second, the required design variables are explained. Finally, the constraints and objective function are presented. Constraints B.3 and B.13–B.16 are added to make the relaxation of the execution order valid.

B.1 Required input data

The relevant input data required in the MILP formulation as presented in Section B.3 is given below.

- W weighers with required minimum dosing mass $m_w^{\min}$, maximum allowed dosage mass $m_w^{\max}$ en maximum allowed dosing volume $V_w^{\max}$ for weigher $w \in \{1, ..., W\}$.
- G raw materials.
- L silos, in which $L \geq \max(G, W)$.
- P periods.
- B batches.
- S_b discharges in each batch b with $\sum_{b=1}^{B} S_b$ batches in total, in which $\sum_{b=1}^{B} S_b \geq W$
- D_b dosages in each batch b with $\sum_{b=1}^{B} D_b$ dosages in total. Each dosage d has weight m_d, volume V_d, in which $D_b \geq \max(L, S_b), \quad \forall b$.
- Δ is a $D \times G$ allocation matrix of D dosages to G raw materials, i.e.:
  $\Delta_{d,g} = \begin{cases} 1, & \text{if dosage } d \text{ consists of raw material } g \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each dosage consists of exactly 1 raw material $\sum_{g=1}^{G} \Delta_{d,g} = 1, \quad \forall d$.
- Λ is an $L \times W$ allocation matrix of L silos W weighers, i.e.:
  $\Lambda_{l,w} = \begin{cases} 1, & \text{if silo } l \text{ belongs to weigher } w \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each location belongs to exactly 1 weigher $\sum_{w=1}^{W} \Lambda_{l,w} = 1, \quad \forall l$.

- Ψ is a $\sum_{b=1}^{B} D_b \times B$ allocation matrix of $\sum_{b=1}^{B} D_b$ dosages to B batches, i.e.:
  $\Psi_{d,b} = \begin{cases} 1, & \text{if dosage } b \text{ in batch } b \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each dosage belongs to exactly 1 batch $\sum_{b=1}^{B} \Theta_{d,b} = 1, \quad \forall d.$
- Ω is a $B \times P$ allocation matrix of B batches to P periods, i.e.:
  $\Omega_{b,p} = \begin{cases} 1, & \text{if batch } b \text{ in period } p \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each batch belongs to exactly 1 period $\sum_{p=1}^{P} \Omega_{b,p} = 1, \quad \forall b.$
- Set of silos at a weigher: $\mathcal{L}_w = \{0\} \cup \{l | \Lambda_{l,w} = 1\}$, in which silo 0 represents the discharge location.
- γ^0 set of {silo l and corresponding raw material g}-combinations that exclude raw material g at silo l.
- γ^1 set of {silo l and corresponding raw material g}-combinations that forces raw material g at silo l.
- η_g the maximum number of silos at which a raw material g occurs.
- Setup time from location l_1 to location l_2: Σ_{l_1,l_2} for $l_1, l_2 \in \mathcal{L}_w$, $\forall w$. In this case, the setup time solely consists of the relocation time due to driving from location l_1 to location l_2.
- Dosing time for x[kg] of raw material g at silo l, dependent of the dosing constant $c^D_{l,g}$ and dosing speed $c^D_{l,g}$: $\frac{1}{v^D_{l,g}} x + c^D_{l,g}$[s]. Using the dosability factor K_g, the silo and raw material dependent dosing constant and dosing speed are expressed by $c^D_{l,g} = K_g c^D_l$ and $v^D_{l,g} = \frac{v^D_l}{K_g}$.
- Discharge time for x[kg] at weigher w: $\frac{1}{v^S_w} x + c^S_w$[s].
- Completion time of the last discharge (from a previous planning) at weigher w: $C^S_{0,w}$.
- Completion time of the last discharge (from a previous planning): $C^S_0 = \max_w C^S_{0,w}$.
- M a sufficient large value, at least larger than the mass of the largest batch.
- δ a sufficient small value, at least smaller than the smallest weighing accuracy among all weighers.

B.2 Assumptions

Besides the assumptions presented in the previous section the following time based assumptions are taken in consideration:

- If period $p_1 < p_2$, then p_1 is completed before p_2.
- If batch $b_1 < b_2$ and both take place in the same production order, then b_1 is completed before b_2.
- If discharge $s_1 < s_2$, then s_1 is completed before s_2 (without loss of generality).

Furthermore, the dosing order policy, as introduced in Section 2.3.1, is altered to simplify the MILP. In the MILP it is assumed that the periods, batches are carried out in the exact order as imposed by the production schedule. The dosages of the batches should arrive in the batch mixer in the order as imposed by the production schedule. Consecutive dosages may arrive at the mixer

simultaneously.

More precisely, this implies the following: if dosage d_1 is listed before dosage d_2 in the production schedule and both dosages take place in batch b, then d_1 arrives before or at the same time at the batch mixer as d_2. Thus, the following two situations are possible:

1. If d_1 and d_2 take place at the same dosing installation, then they may be discharged together and they may arrive at the mixer simultaneously if the capacity of the weigher allows so, as is the case with the general dosing and discharge order policy.
2. If d_1 and d_2 do not take place at the same installations, then d_1 arrives at the mixer before d_2. This does not imply that dosages are carried out in order. The installation carrying out d_2 may start dosing d_2 before d_1 is carried out on the other installation, as long as d_1 is discharged before d_2.

For the solution methods other than MILP, the general dosing order policy, as introduced in Section 2.3.1, is applied.

B.3 Design variables

The binary variables to this MILP formulation are:

- $X^J_{j_1,j_2} = \begin{cases} 1, & \text{if production order } j_1 \text{ takes place before } j_2 > j_1 \text{ in the same period} \\ 0, & \text{otherwise.} \end{cases}$
- $X^D_{d_1,d_2} = \begin{cases} 1, & \text{if dosage } d_1 \text{ takes place before } d_2 > d_1 \text{ in the same discharge} \\ 0, & \text{otherwise.} \end{cases}$
- $Y^{SB}_{s,b} = \begin{cases} 1, & \text{if discharge } s \text{ takes place in batch } b \\ 0, & \text{otherwise.} \end{cases}$
- $Y^{DS}_{d,s} = \begin{cases} 1, & \text{if dosage } d \text{ takes place in discharge } s \\ 0, & \text{otherwise.} \end{cases}$
- $Y^{SW}_{s,w} = \begin{cases} 1, & \text{if discharge } s \text{ is executed at weigher } w \\ 0, & \text{otherwise.} \end{cases}$
- $Y^{DL}_{d,l} = \begin{cases} 1, & \text{if dosage } d \text{ is extracted from silo } l \\ 0, & \text{otherwise.} \end{cases}$
- $Y^{LG}_{l,g} = \begin{cases} 1, & \text{if silo } l \text{ contains raw material } g \\ 0, & \text{otherwise.} \end{cases}$
- $Y^S_s = \begin{cases} 1, & \text{if discharge } s \text{ contains one or more dosages} \\ 0, & \text{otherwise.} \end{cases}$
- $F_{w_1,w_2,g} = \begin{cases} 1, & \text{if } f^{WG_2}_{w_1,g} \leq f^{WG_1}_{w_2,g} \\ 0, & \text{otherwise.} \end{cases}$

The continuous variables to this MILP formulation are:

- $f^{WG_1}_{w,g}$= lower limit of an allowed dosing interval of raw material g at weigher w
- $f^{WG_2}_{w,g}$= upper limit of an allowed dosing interval of raw material g at weigher w

- p_d^D = duration of dosage d.
- p_s^S = duration of discharge s.
- C_d^D = completion time of dosage d.
- C_s^S = completion time of discharge s.
- S_{d_1,d_2} = setup time due to driving from dosage d_1 to dosage $d_2 > d_1$, in which dosage 0 represents the discharge.
- C_b^B = completion time of batch b.
- C = completion time of the schedule, i.e.: the total makespan.

B.4 Constraints

The MILP is built around:

- binary variables X_{j_1,j_2}^J, X_{d_1,d_2}^D, $Y_{s,b}^{SB}$, $Y_{d,s}^{DS}$, $Y_{s,w}^{SW}$, $Y_{d,l}^{DL}$, $Y_{l,g}^{LG}$ and Y_s^S, which fix the location of raw materials and execution order of dosages and production orders;
- binary variable $F_{w_1,w_2,g}$ and continuous variables $f_{w,g}^{WG_1}$ and $f_{w,g}^{WG_2}$, which fix the allowed dosing interval of raw materials at installations;
- time variables $p_d^D, p_s^S, C_d^D, C_s^S, S_{d_1,d_2}, C_b^B$ and C, which are fixed by variables mentioned above.

The constraints describing the posed problem of Chapter 2 with relaxation of the execution order are introduced below.

Equality constraints
Below, the equality constraints of the proposed MILP are presented. Equations B.1–B.6 represent constraints that describe how the production on the system is tied to the physical design of the system. Equations B.7 and B.8 represent constraints that describe the architecture of the production itself. The physical limitations of the system are described by (B.9).

Due to several reasons some raw materials may be excluded from or forced on certain silos. Therefore, silo l may not contain raw material g if this is specified in γ^0 and silo l must contain raw material g if this is specified in γ^1, which results in:

$$Y_{l,g}^{LG} = 0 \qquad \forall (l,g) \in \gamma^0, \tag{B.1}$$

$$Y_{l,g}^{LG} = 1 \qquad \forall (l,g) \in \gamma^1. \tag{B.2}$$

It is required that each discharge s is executed in exactly one batch b, each discharge s is executed at exactly one weigher w, each dosage d originates from exactly one silo l, each silo l contains exactly one raw material g and each dosage d is executed in exactly one discharge s. Hence, the following five constraints are introduced:

$$\sum_{b=1}^{B} Y_{s,b}^{SB} = 1 \qquad \forall s \in \{1, ..., S\}, \tag{B.3}$$

$$\sum_{w=1}^{W} Y_{s,w}^{SW} = 1 \qquad \forall s \in \{1, ..., S\}, \tag{B.4}$$

$$\sum_{l=1}^{L} Y_{d,l}^{DL} = 1 \qquad \forall d \in \{1, ..., D\}, \tag{B.5}$$

$$\sum_{g=1}^{G} Y_{l,g}^{LG} = 1 \qquad \forall l \in \{1, ..., L\}, \tag{B.6}$$

$$\sum_{s=1}^{S} Y_{d,s}^{DS} = 1 \qquad \forall d \in \{1, ..., D\}, \tag{B.7}$$

Note that these constraints do not rule out that a discharge may consist of multiple dosages, a weigher may perform multiple discharges, multiple dosages may originate from a silo and raw material may be placed on multiple silos.

Furthermore, if batch b contains dosage d, but does not contain discharge s, then dosage d cannot take place in discharge s. Similarly, if batch b does not contain dosage d, but does contain discharge s, then dosage d cannot take place in discharge s:

$$Y_{d,s}^{DS} = 0 \qquad \text{if } \Theta_{s,b} + \Psi_{d,b} = 1, \quad \forall s \in \{1, ..., S\}, \quad \forall d \in \{1, ..., D\}, \quad \forall b \in \{1, ..., B\}. \tag{B.8}$$

Note that (B.8) does not apply in case dosage d and discharge s both take place in batch b or in case dosage d and discharge s both do not take place in b.

If the mass of dosage d is less than the minimum required mass on weigher w due to the weighing accuracy, then dosage d may not take place at weigher w:

$$\sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} = 0 \qquad \text{if } m_w^{\min} > m_d, \quad \forall w \in \{1, ..., W\}, \quad \forall d \in \{1, ..., D\}. \tag{B.9}$$

Note that in order to check whether dosage d takes place at weigher w no extra binary variabele $Y^{DW}_{d,w}$ is required, since $Y^{DW}_{d,w} = \sum_{l=1}^{L} \Lambda_{l,w} Y^{DL}_{d,l}$. In words: dosages that take place at weigher w can be expressed by the product of the silos at weigher w and the dosages that are performed at those silos. Thus, if a dosage may not take place at weigher w then it must hold true that the silo at which dosage d is performed is not located at weigher w.

Inequality constraints

It is undesired to place a raw material on a large number of silos as this indicates that the given set of dosing installations with its corresponding specifications is not fit for the given production. Restricting the number of silos that may contain raw material g could influence the outcome of the MILP optimization, the makespan, negatively. Even though the set of dosing installations is considered fixed input parameters, it is penalized in the makespan and thus returning the indication that the given input parameters should be changed. To make this come to expression in the MILP formulation the number of silos that contain raw material g is restricted to maximum η_g:

$$\sum_{l=1}^{L} Y^{LG}_{l,g} \leq \eta_g \qquad \forall g \in \{1, ..., G\}. \tag{B.10}$$

It must hold true that if (1) dosage d takes place at silo l and (2) raw material g is located at silo l, then dosage d must contain raw material g. Moreover, if dosage d does not contain raw material g, statement (1) and/or statement (2) must be false, resulting in the following inequality:

$$1 + \Delta_{d,g} \geq Y^{DL}_{d,l} + Y^{LG}_{l,g} \qquad \begin{array}{r} \forall d \in \{1, ..., D\}, \\ \forall g \in \{1, ..., G\}, \\ \forall l \in \{1, ..., L\}. \end{array} \tag{B.11}$$

Furthermore, it must hold true that if (1) dosage d takes place in discharge s and (2) discharge s takes place at weigher w, then dosage d must be executed by weigher w. Moreover, if dosage d does not take place at weigher w, statement (1) and/or statement (2) must be false, resulting in the following inequality:

$$1 + \sum_{l=1}^{L} Y^{DL}_{d,l} \Lambda_{l,w} \geq Y^{DS}_{d,s} + Y^{SW}_{s,w} \qquad \begin{array}{r} \forall d \in \{1, ..., D\}, \\ \forall s \in \{1, ..., S\}, \\ \forall w \in \{1, ..., W\}. \end{array} \tag{B.12}$$

If dosage d_1 takes place in discharge s and dosage d_2 does not take place in discharge s, then $X_{d_1,d_2} = 0$. If dosage d_1 does not take place in discharge s and dosage d_2 does take place in discharge s, then $x_{d_1,d_2} = 0$. If d_1 and d_2 both do not take place in s, then $X_{d_1,d_2} = 0$. Only if d_1 and d_2 both take place in s, then $X_{d_1,d_2} = 1$.

$$Y_{d_1,p} - Y_{d_2,p} \leq 1 - X_{d_1,d_2} \qquad \begin{array}{r} \forall d_1 < d_2 \in \{1, ..., D\}, \\ \forall p \in \{1, ..., P\}. \end{array} \tag{B.13}$$

$$-Y_{d_1,p} + Y_{d_2,p} \leq 1 - X_{d_1,d_2} \qquad \begin{array}{r} \forall d_1 < d_2 \in \{1, ..., D\}, \\ \forall p \in \{1, ..., P\}. \end{array} \tag{B.14}$$

If production order j_1 takes place in period p and production order j_2 does not take place in period p, then $X_{j_1,j_2} = 0$. If production order j_1 does not take place in period p and production order j_2 does take place in period p, then $x_{j_1,j_2} = 0$. If j_1 and j_2 both do not take place in p, then $X_{j_1,j_2} = 0$. Only if j_1 and j_2 both take place in p, then $X_{j_1,j_2} = 1$.

$$Y_{j_1,p} - Y_{j_2,p} \leq 1 - X_{j_1,j_2} \qquad \forall d_1 < d_2 \in \{1,...,D\}, \forall p \in \{1,...,P\}. \tag{B.15}$$

$$-Y_{j_1,p} + Y_{j_2,p} \leq 1 - X_{j_1,j_2} \qquad \forall d_1 < d_2 \in \{1,...,D\}, \forall p \in \{1,...,P\}. \tag{B.16}$$

If dosage d takes place in discharge s and discharge s takes place at weigher w, then dosage d completes at least $S_{0,d} + p_d^D$ later than startingcondition $C_{0,w}^S$:

$$M(Y_{s,w}^{SW} + Y_{d,s}^{DS} - 2) + C_{0,w}^S + S_{0,d} + p_d^D \leq C_d^D \qquad \forall d \in \{1,...,D\}, \forall s_1 < s_2 \in \{1,...,S\}, \forall w \in \{1,...,W\}. \tag{B.17}$$

If dosages $d_1 < d_2$ both take place in discharge s, then dosage d_2 completes at least $S_{d_1,d_2} + p_{d_2}^D$ later than dosage d_1:

$$M(Y_{d_1,s}^{DS} + Y_{d_2,s}^{DS} - 2) + C_{d_1}^D + S_{d_1,d_2} + p_{d_2}^D \leq C_{d_2}^D \qquad \forall d_1 < d_2 \in \{1,...,D\}, \forall s \in \{1,...,S\}. \tag{B.18}$$

Similarly, if discharges $s_1 < s_2$ both take place at weigher w and dosage d takes place in discharge s_2, then dosage d completes at least $S_{0,d} + p_d^D$ later than discharge s_1:

$$M(Y_{s_1,w}^{SW} + Y_{s_2,w}^{SW} + Y_{d,s_2}^{DS} - 3) + C_{s_1}^S + S_{0,d} + p_d^D \leq C_d^D \qquad \forall d \in \{1,...,D\}, \forall s_1 < s_2 \in \{1,...,S\}, \forall w \in \{1,...,W\}. \tag{B.19}$$

If batch b_1 takes place in $p_1 < p_2$ and dosage d takes place in period p_2, then dosage d completes at least $S_{0,d} + p_d^D$ later than batch b_1:

$$M(\Omega_{b_1,p_1} + \Psi_{d,b_2} + \Omega_{b_2,p_2} - 3) + C_{b_1}^B + p_d^D + S_{0,d} \leq C_d^D \qquad \forall d \in \{1,...,D\}, \forall b_1 < b_2 \in \{1,...,B\}, \forall p_1 < p_2 \in \{1,...,P\}. \tag{B.20}$$

Note that in order to check whether dosage d takes place in period p_2, data parameters Ψ_{d,b_2} and Ω_{b_2,p_2} are used.

Discharge s completes at least p_s^S later than discharge $s-1$:

$$C_{s-1}^S + p_s^S \leq C_s^S \qquad \forall s \in \{1,...,S\}. \tag{B.21}$$

Note that $s = 1$ requires data parameter C_0^S.

If discharge s takes place in batch $b_2 > b_1$, then discharge s completes at least p_s^S later than $C_{b_1}^B$:

$$C_{b_1}^B + p_s^S + M(\Theta_{s,b_2} - 1) \leq C_s^S \qquad \forall s \in \{1, ..., S\}, \; \forall b_1 < b_2 \in \{1, ..., B\}. \tag{B.22}$$

If dosage d takes place in discharge s, then discharge s completes at least $S_{d,0} + p_s^S$ later than dosage d:

$$M(Y_{d,s}^{DS} - 1) + C_d^D + S_{d,0} + p_s^S \leq C_s^S \qquad \forall d \in \{1, ..., D\}, \; \forall s \in \{1, ..., S\}. \tag{B.23}$$

If dosage d containing raw material g takes place at silo l, then the dosing time is determined by the dosing constant and the mass of the dosage m_d:

$$M(Y_{d,l}^{DL} + Y_{l,g}^{LG} - 2) + c_{l,g}^D + \frac{m_d}{v_{l,g}^D} \leq p_d^D \qquad \forall d \in \{1, ..., D\}, \; \forall g \in \{1, ..., G\}, \; \forall l \in \{1, ..., L\}. \tag{B.24}$$

Furthermore, if discharge s takes place at weigher w, then the discharge time is determined by the discharge constant and the mass $\sum_{d=1}^{D} m_d Y_{d,s}^{DS}$ of all dosages in that discharge:

$$M(Y_{s,w}^{SW} - 1) + c_w^S Y_s^S + \frac{1}{v_w^S} \sum_{d=1}^{D} m_d Y_{d,s}^{DS} \leq p_s^S \qquad \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\}. \tag{B.25}$$

If no dosages take place in a discharge, then the discharge is empty, i.e.: $Y_s^S = 0$. This results in:

$$\sum_{d=1}^{D} Y_{d,s}^{DS} \geq Y_s^S \qquad \forall s \in \{1, ..., S\}. \tag{B.26}$$

On the other hand, if one or more dosages take place in a discharge, then that discharge is not empty, i.e.: $Y_s^S = 1$. This results in:

$$\frac{1}{D} \sum_{d=1}^{D} Y_{d,s}^{DS} \geq Y_s^S \qquad \forall s \in \{1, ..., S\}. \tag{B.27}$$

Only if dosage d contains more mass than the required minimum dosage mass $m_w^{\min}$ of weigher w, dosage d may take place on that weigher:

$$\sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \leq 1 \qquad \text{if } m_w^{\min} \leq m_d, \; \forall w \in \{1, ..., W\}, \; \forall d \in \{1, ..., D\}. \tag{B.28}$$

If discharge s takes place at weigher w, then the mass and volume of all dosages in discharge s cannot not exceed the maximum allowed dosage mass and volume of weigher w:

$$M(Y_{s,w}^{SW} - 1) + \sum_{d=1}^{D} m_d Y_{d,s}^{DS} \leq m_w^{\max} \qquad \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\}, \tag{B.29}$$

$$M(Y_{s,w}^{SW} - 1) + \sum_{d=1}^{D} V_d Y_{d,s}^{DS} \leq V_w^{\max} \qquad \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\}. \tag{B.30}$$

The lower and upper limit of the allowed dosing interval of raw material g at weigher w may not be larger than the maximum allowed dosage mass of weigher w:

$$f_{w,g}^{WG_1} \leq m_w^{\max} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}, \tag{B.31}$$

$$f_{w,g}^{WG_2} \leq m_w^{\max} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{B.32}$$

The dosing interval of raw material g at weigher w_1 may not overlap with the interval at w_2. Thus, if the allowed dosing interval of raw material g at weigher w_1 is smaller or equal the interval at weigher w_2, then the upper limit at weigher w_1 should be smaller or equal to the lower limit at weigher w_2:

$$f_{w_2,g}^{WG_1} + M(1 - F_{w_1,w_2,g}) \geq f_{w_1,g}^{WG_2} \qquad \forall w_1 \neq w_2 \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{B.33}$$

If the dosing interval of raw material g at weigher w_1 is smaller than the interval at w_2, then $F_{w_1,w_2,g} = 1$ and $F_{w_2,w_1,g} = 0$. If the dosing interval of raw material g at weigher w_2 is smaller than the interval at w_1, then $F_{w_1,w_2,g} = 0$ and $F_{w_2,w_1,g} = 1$. If no dosages of raw material g take place at either weighers w_1 or w_2, then both intervals are 'empty', i.e.: $f_{w_1,g}^{WG_1} = f_{w_1,g}^{WG_2} = f_{w_2,g}^{WG_1} = f_{w_2,g}^{WG_2} = 0$, resulting in, $F_{w_1,w_2,g} = F_{w_2,w_1,g} = 1$. This means in any situation either one or both variables $F_{w_1,w_2,g}$ and $F_{w_2,w_1,g}$ should equal 1:

$$F_{w_1,w_2,g} + F_{w_2,w_1,g} \geq 1 \qquad \forall w_1 < w_2 \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{B.34}$$

If a raw material g is not placed on weigher w, then the interval is 'empty', i.e.: $f_{w,g}^{WG_1} = f_{w,g}^{WG_2} = 0$. To check whether raw material g is placed on weigher w, binary variable $Y_{g,w}^{GW}$ is needed, which comes to expression using $\sum_{d=1}^{D} \Delta_{d,g} Y_{d,w}^{DW}$ in which $Y_{d,w}^{DW} = \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL}$.

$$M \sum_{d=1}^{D} \Delta_{d,g} \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \geq f_{w,g}^{WG_1} \qquad \forall w \in \{1, ..., W\}, \forall g \in \{1, ..., G\}, \tag{B.35}$$

$$M \sum_{d=1}^{D} \Delta_{d,g} \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \geq f_{w,g}^{WG_2} \qquad \forall w \in \{1, ..., W\}, \forall g \in \{1, ..., G\}. \tag{B.36}$$

If dosage d takes place at weigher w and if dosage d contains raw material g, then the mass of dosage d must be contained in the allowed dosing interval of raw material g at weigher w, i.e.: the mass of dosage d should be larger than the lower limit and smaller than the upper limit of the dosing interval:

$$m_d - \delta + M(2 - Y_{d,w}^{DW} - \Delta_{d,g}) \geq f_{w,g}^{WG_1} \qquad \forall w \in \{1, ..., W\}, \forall g \in \{1, ..., G\}, \forall d \in \{1, ..., D\}, \tag{B.37}$$

$$m_d + M(Y_{d,w}^{DW} + \Delta_{d,g} - 2) \leq f_{w,g}^{WG_2} \qquad \forall w \in \{1, ..., W\}, \forall g \in \{1, ..., G\}, \forall d \in \{1, ..., D\}. \tag{B.38}$$

Consider that the mass m_d of dosage d consists of raw material g. Then m_d may not correspond to both the lower limit if the dosing interval of raw material g at weigher w_1 and the upper limit of raw material g at weigher w_2, since dosage d is only allowed to be executed on one installation. To prevent this, at least one of the limits of the weighers should be excluded from executing dosage d. Therefore, in (B.37), the lower limit of the allowed dosing interval of raw material g at weigher w should be smaller than m_d with a margin δ.

If dosages $d_1 < d_2$ take place at silos l_1 and l_2 respectively, the setup time should be equal to Σ_{l_1,l_2}:

$$M(Y_{d_1,l_1}^{DL} + Y_{d_2,l_2}^{DL} - 2) + \Sigma_{l_1,l_2} \leq S_{d_1,d_2} \qquad \forall d_1 < d_2 \in \{1, ..., D\}, \forall l_1 \in \{1, ..., L\}, \forall l_2 \in \{1, ..., L\}. \tag{B.39}$$

Similarly, for silo 0 this results in:

$$M(Y_{d,l}^{DL} - 1) + \Sigma_{0,l} \leq S_{0,d} \qquad \forall d \in \{1, ..., D\}, \forall l \in \{1, ..., L\}, \tag{B.40}$$

$$M(Y_{d,l}^{DL} - 1) + \Sigma_{l,0} \leq S_{d,0} \qquad \forall d \in \{1, ..., D\}, \forall l \in \{1, ..., L\}. \tag{B.41}$$

Dosages must be executed in order, i.e.: if dosages $d_1 < d_2$ take place in discharges s_1 and s_2 respectively, then discharge s_2 can take place at earliest together with discharge s_1:

$$M(Y^{DS}_{d_1,s_1} + Y^{DS}_{d_2,s_2} - 2) + C^S_{s_1} \leq C^S_{s_2} \qquad \forall d_1 < d_2 \in \{1,...,D\}, \forall s_1 \in \{1,...,S\}, \forall s_2 \in \{1,...,S\}. \tag{B.42}$$

If discharge s takes place in batch b, batch b finishes at least later than $C^S_s + c^B$:

$$C^S_s + c^B + M(\Theta_{s,b} - 1) \leq C^B_b \qquad \forall s \in \{1,...,S\}, \forall b \in \{1,...,B\}. \tag{B.43}$$

The final batch in the final period determines the total makespan:

$$C^B_b \leq C \qquad \forall b \in \{1,...,B\}. \tag{B.44}$$

Some of the continuous variables have to be lower-bounded:

$$f^{WG_1}_{w,g} \geq 0 \qquad \forall w \in \{1,...,W\}, \forall g \in \{1,...,G\}, \tag{B.45}$$

$$f^{WG_2}_{w,g} \geq 0 \qquad \forall w \in \{1,...,W\}, \forall g \in \{1,...,G\}. \tag{B.46}$$

Note that all other continuous variables have already been (implicitly) lower-bounded by their constraints.

Optional constraints
The following constraints are not necessary to solve the optimization problem. However, they reduce the number of possible solutions or make a implicit constraint explicit.

It is assumed that placing a raw material more than once on an installation probably does not significantly improve the performance as the weigher can only be located at one silo simultaneously. The benefit of placing a raw material more than once is a potential decrease in driving time in an installation with a movable weigher. Furthermore, placing a raw material more than once on an installation implies that another raw material cannot be placed on that installation. Placing another raw material on the installation decreases the potential idle time of that installation. It is assumed that the reduction in makespan from a decrease in idle time exceeds the reduction from a decrease in driving time. Hence, it is assumed that a raw material g can only occur once on a dosing installation with weigher w:

$$\sum_{l=1}^{L} \Lambda_{l,w} Y^{LG}_{l,g} \leq 1 \qquad \forall w \in \{1,...,W\}, \forall g \in \{1,...,G\}. \tag{B.47}$$

The lower limit of a dosing interval of raw material g at weigher w must be smaller or equal to the upper limit of raw material g at weigher w:

$$f_{w,g}^{WG_1} \leq f_{w,g}^{WG_2} \qquad \forall w \in \{1, ..., W\}, \forall g \in \{1, ..., G\}. \tag{B.48}$$

Note that this implicitly follows from (B.37) and (B.38).

Furthermore, all time-based continuous variables cannot be negative. Thus:

$$p_d^D \geq 0 \qquad \forall d \in \{1, ..., D\}, \tag{B.49}$$

$$p_s^S \geq 0 \qquad \forall s \in \{1, ..., S\}, \tag{B.50}$$

$$S_{d_1,d_2} \geq 0 \qquad \forall d_1 \in \{1, ..., D\}, \forall d_2 \in \{1, ..., D\}, \tag{B.51}$$

$$C_d^D \geq 0 \qquad \forall d \in \{1, ..., D\}, \tag{B.52}$$

$$C_b^B \geq 0 \qquad \forall b \in \{1, ..., B\}, \tag{B.53}$$

$$C \geq 0. \tag{B.54}$$

These variables are already implicitly lower-bound. For example: p_d^D is constrained by (B.24), in which all terms are explicitly positive except for $M(Y_{d,l}^{DL} + Y_{l,g}^{LG} - 2)$. However, from (B.5) and (B.6) it follows that there is always some silo l and some raw material g for which $Y_{d,l} = Y_{l,g} = 1$. Therefore, p_d^D is always lower-bounded by at least 0.

B.5 Objective function

The objective function follows straightforward from the constraints as the objective is to minimize the makespan:

$$\min C \tag{B.55}$$

Appendix C

MILP concerning 1 period

In the following appendix a simplification of the MILP of Chapter 4 is presented in case the number of periods is 1. This leads to a decrease in the number of variables.

C.1 Required input data

The relevant input data required in the MILP formulation as presented in Section C.3 is given below.

- W weighers with required minimum dosing mass $m_w^{\min}$, maximum allowed dosage mass $m_w^{\max}$ en maximum allowed dosing volume $V_w^{\max}$ for weigher $w \in \{1, ..., W\}$.
- G raw materials.
- L silos, in which $L \geq \max(G, W)$.
- B batches.
- S_b discharges in each batch b with $\sum_{b=1}^{B} S_b$ batches in total, in which $\sum_{b=1}^{B} S_b \geq W$
- D_b dosages in each batch b with $\sum_{b=1}^{B} D_b$ dosages in total. Each dosage d has weight m_d, volume V_d, in which $D_b \geq \max(L, S_b), \quad \forall b$.
- Δ is a $D \times G$ allocation matrix of D dosages to G raw materials, i.e.:
  $\Delta_{d,g} = \begin{cases} 1, & \text{if dosage } d \text{ consists of raw material } g \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each dosage consists of exactly 1 raw material $\sum_{g=1}^{G} \Delta_{d,g} = 1, \quad \forall d$.
- Λ is an $L \times W$ allocation matrix of L silos W weighers, i.e.:
  $\Lambda_{l,w} = \begin{cases} 1, & \text{if silo } l \text{ belongs to weigher } w \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each location belongs to exactly 1 weigher $\sum_{w=1}^{W} \Lambda_{l,w} = 1, \quad \forall l$.
- Θ is a $\sum_{b=1}^{B} S_b \times B$ allocation matrix of $\sum_{b=1}^{B} S_b$ discharges to B batches, i.e.:
  $\Theta_{s,b} = \begin{cases} 1, & \text{if discharge } s \text{ in batch } b \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each discharge belongs to exactly 1 batch $\sum_{b=1}^{B} \Theta_{s,b} = 1, \quad \forall s$.

- Ψ is a $\sum_{b=1}^{B} D_b \times B$ allocation matrix of $\sum_{b=1}^{B} D_b$ dosages to B batches, i.e.:
  $\Psi_{d,b} = \begin{cases} 1, & \text{if dosage } b \text{ in batch } b \\ 0, & \text{otherwise.} \end{cases}$
  Furthermore, it should hold that each dosage belongs to exactly 1 batch $\sum_{b=1}^{B} \Theta_{d,b} = 1, \quad \forall d.$
- Set of silos at a weigher: $\mathcal{L}_w = \{0\} \cup \{l | \Lambda_{l,w} = 1\}$, in which silo 0 represents the discharge location.
- γ^0 set of {silo l and corresponding raw material g}-combinations that exclude raw material g at silo l.
- γ^1 set of {silo l and corresponding raw material g}-combinations that forces raw material g at silo l.
- η_g the maximum number of silos at which a raw material g occurs.
- Setup time from location l_1 to location l_2: Σ_{l_1,l_2} for $l_1, l_2 \in \mathcal{L}_w$, $\forall w$. In this case, the setup time solely consists of the relocation time due to driving from location l_1 to location l_2.
- Dosing time for x[kg] of raw material g at silo l, dependent of the dosing constant $c^D_{l,g}$ and dosing speed $c^D_{l,g}$: $\frac{1}{v^D_{l,g}} x + c^D_{l,g}$[s]. Using the dosability factor K_g, the silo and raw material dependent dosing constant and dosing speed are expressed by $c^D_{l,g} = K_g c^D_l$ and $v^D_{l,g} = \frac{v^D_l}{K_g}$.
- Discharge time for x[kg] at weigher w: $\frac{1}{v^S_w} x + c^S_w$[s].
- Completion time of the last discharge (from a previous planning) at weigher w: $C^S_{0,w}$.
- Completion time of the last discharge (from a previous planning): $C^S_0 = \max_w C^S_{0,w}$.
- M a sufficient large value, at least larger than the mass of the largest batch.
- δ a sufficient small value, at least smaller than the smallest weighing accuracy among all weighers.

C.2 Assumptions

Besides the assumptions presented in the previous section the following time based assumptions are taken in consideration:

- If batch $b_1 < b_2$, then b_1 is completed before b_2.
- If dosages $d_1 < d_2$ both in discharge s, then d_1 is completed before d_2
- If discharge $s_1 < s_2$, then s_1 is completed before s_2 (without loss of generality).
- Dosages are discharged in order, that is, d_2 is never discharged before d_1 for $d_1 < d_2$.

Furthermore, the dosing order policy, as introduced in Section 2.3.1, is altered to simplify the MILP. In the MILP it is assumed that the periods, batches are carried out in the exact order as imposed by the production schedule. The dosages of the batches should arrive in the batch mixer in the order as imposed by the production schedule. Consecutive dosages may arrive at the mixer simultaneously.

More precisely, this implies the following: if dosage d_1 is listed before dosage d_2 in the production schedule and both dosages take place in batch b, then d_1 arrives before or at the same time at the batch mixer as d_2. Thus, the following two situations are possible:

1. If d_1 and d_2 take place at the same dosing installation, then they may be discharged together and they may arrive at the mixer simultaneously if the capacity of the weigher allows so, as is the case with the general dosing and discharge order policy.

2. If d_1 and d_2 do not take place at the same installations, then d_1 arrives at the mixer before d_2. This does not imply that dosages are carried out in order. The installation carrying out d_2 may start dosing d_2 before d_1 is carried out on the other installation, as long as d_1 is discharged before d_2.

For the solution methods other than MILP, the general dosing order policy, as introduced in Section 2.3.1, is applied.

C.3 Design variables

The binary variables to this MILP formulation are:

- $Y_{d,s}^{DS} = \begin{cases} 1, & \text{if dosage } d \text{ takes place in discharge } s \\ 0, & \text{otherwise.} \end{cases}$
- $Y_{s,w}^{SW} = \begin{cases} 1, & \text{if discharge } s \text{ is executed at weigher } w \\ 0, & \text{otherwise.} \end{cases}$
- $Y_{d,l}^{DL} = \begin{cases} 1, & \text{if dosage } d \text{ is extracted from silo } l \\ 0, & \text{otherwise.} \end{cases}$
- $Y_{l,g}^{LG} = \begin{cases} 1, & \text{if silo } l \text{ contains raw material } g \\ 0, & \text{otherwise.} \end{cases}$
- $Y_{s}^{S} = \begin{cases} 1, & \text{if discharge } s \text{ contains one or more dosages} \\ 0, & \text{otherwise.} \end{cases}$
- $F_{w_1,w_2,g} = \begin{cases} 1, & \text{if } f_{w_1,g}^{WG_2} \leq f_{w_2,g}^{WG_1} \\ 0, & \text{otherwise.} \end{cases}$

The continuous variables to this MILP formulation are:

- $f_{w,g}^{WG_1}$= lower limit of an allowed dosing interval of raw material g at weigher w
- $f_{w,g}^{WG_2}$= upper limit of an allowed dosing interval of raw material g at weigher w
- p_d^D = duration of dosage d.
- p_s^S = duration of discharge s.
- C_d^D = completion time of dosage d.
- C_s^S = completion time of discharge s.
- S_{d_1,d_2} = setup time due to driving from dosage d_1 to dosage $d_2 > d_1$, in which dosage 0 represents the discharge.
- C_b^B = completion time of batch b.
- C = completion time of the schedule, i.e.: the total makespan.

C.4 Constraints

The MILP is built around:

- binary variables $Y_{d,s}^{DS}, Y_{s,w}^{SW}, Y_{d,l}^{DL}, Y_{l,g}^{LG}$ and Y_s^S, which fix the location of raw materials and execution order of dosages;
- binary variable $F_{w_1,w_2,g}$ and continuous variables $f_{w,g}^{WG_1}$ and $f_{w,g}^{WG_2}$, which fix the allowed dosing interval of raw materials at installations;
- time variables $p_d^D, p_s^S, C_d^D, C_s^S, S_{d_1,d_2}, C_b^B$ and C, which are fixed by variables mentioned above.

The constraints describing the posed problem of Chapter 2 concerning 1 period are introduced below.

Equality constraints
Below, the equality constraints of the proposed MILP are presented. Equations C.1–C.5 represent constraints that describe how the production on the system is tied to the physical design of the system. Equations C.6 and C.7 represent constraints that describe the architecture of the production itself. The physical limitations of the system are described by (C.8).

Due to several reasons some raw materials may be excluded from or forced on certain silos. Therefore, silo l may not contain raw material g if this is specified in γ^0 and silo l must contain raw material g if this is specified in γ^1, which results in:

$$Y_{l,g}^{LG} = 0 \qquad \forall (l,g) \in \gamma^0, \tag{C.1}$$

$$Y_{l,g}^{LG} = 1 \qquad \forall (l,g) \in \gamma^1. \tag{C.2}$$

It is required that each discharge s is executed at exactly one weigher w, each dosage d originates from exactly one silo l, each silo l contains exactly one raw material g and each dosage d is executed in exactly one discharge s. Hence, the following four constraints are introduced:

$$\sum_{w=1}^{W} Y_{s,w}^{SW} = 1 \qquad \forall s \in \{1,...,S\}, \tag{C.3}$$

$$\sum_{l=1}^{L} Y_{d,l}^{DL} = 1 \qquad \forall d \in \{1,...,D\}, \tag{C.4}$$

$$\sum_{g=1}^{G} Y_{l,g}^{LG} = 1 \qquad \forall l \in \{1,...,L\}, \tag{C.5}$$

$$\sum_{s=1}^{S} Y_{d,s}^{DS} = 1 \qquad \forall d \in \{1,...,D\}. \tag{C.6}$$

Note that these constraints do not rule out that a discharge may consist of multiple dosages, a weigher may perform multiple discharges, multiple dosages may originate from a silo and a raw material may be placed on multiple silos.

Furthermore, if batch b contains dosage d, but does not contain discharge s, then dosage d cannot take place in discharge s. Similarly, if batch b does not contain dosage d, but does contain discharge s, then dosage d cannot take place in discharge s:

$$Y_{d,s}^{DS} = 0 \qquad \text{if } \Theta_{s,b} + \Psi_{d,b} = 1, \quad \forall s \in \{1, ..., S\}, \quad \forall d \in \{1, ..., D\}, \quad \forall b \in \{1, ..., B\}. \tag{C.7}$$

Note that (C.7) does not apply in case dosage d and discharge s both take place in batch b or in case dosage d and discharge s both do not take place in b.

If the mass of dosage d is less than the minimum required mass on weigher w due to the weighing accuracy, then dosage d may not take place at weigher w:

$$\sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} = 0 \qquad \text{if } m_w^{\min} > m_d, \quad \forall w \in \{1, ..., W\}, \quad \forall d \in \{1, ..., D\}. \tag{C.8}$$

Note that in order to check whether dosage d takes place at weigher w no extra binary variabele $Y_{d,w}^{DW}$ is required, since $Y_{d,w}^{DW} = \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL}$. In words: dosages that take place at weigher w can be expressed by the product of the silos at weigher w and the dosages that are performed at those silos. Thus, if a dosage may not take place at weigher w then it must hold true that the silo at which dosage d is performed is not located at weigher w.

Inequality constraints

It is undesired to place a raw material on a large number of silos as this indicates that the given set of dosing installations with its corresponding specifications is not fit for the given production. Restricting the number of silos that may contain raw material g could influence the outcome of the MILP optimization, the makespan, negatively. Even though the set of dosing installations is considered fixed input parameters, it is penalized in the makespan and thus returning the indication that the given input parameters should be changed. To make this come to expression in the MILP formulation the number of silos that contain raw material g is restricted to maximum η_g:

$$\sum_{l=1}^{L} Y_{l,g}^{LG} \leq \eta_g \qquad \forall g \in \{1, ..., G\}. \tag{C.9}$$

It must hold true that if (1) dosage d takes place at silo l and (2) raw material g is located at silo l, then dosage d must contain raw material g. Moreover, if dosage d does not contain raw material g, statement (1) and/or statement (2) must be false, resulting in the following inequality:

$$1 + \Delta_{d,g} \geq Y_{d,l}^{DL} + Y_{l,g}^{LG} \qquad \forall d \in \{1, ..., D\}, \quad \forall g \in \{1, ..., G\}, \quad \forall l \in \{1, ..., L\}. \tag{C.10}$$

Furthermore, it must hold true that if (1) dosage d takes place in discharge s and (2) discharge s takes place at weigher w, then dosage d must be executed by weigher w. Moreover, if dosage d does not take place at weigher w, statement (1) and/or statement (2) must be false, resulting in the following inequality:

$$1 + \sum_{l=1}^{L} Y_{d,l}^{DL} \Lambda_{l,w} \geq Y_{d,s}^{DS} + Y_{s,w}^{SW} \qquad \forall d \in \{1, ..., D\}, \forall s \in \{1, ..., S\}, \forall w \in \{1, ..., W\}. \tag{C.11}$$

If dosage d takes place in discharge s and discharge s takes place at weigher w, then dosage d completes at least $S_{0,d} + p_d^D$ later than startingcondition $C_{0,w}^S$:

$$M(Y_{s,w}^{SW} + Y_{d,s}^{DS} - 2) + C_{0,w}^S + S_{0,d} + p_d^D \leq C_d^D \qquad \forall d \in \{1, ..., D\}, \forall s \in \{1, ..., S\}, \forall w \in \{1, ..., W\}. \tag{C.12}$$

If dosages $d_1 < d_2$ both take place in discharge s, then dosage d_2 completes at least $S_{d_1,d_2} + p_{d_2}^D$ later than dosage d_1:

$$M(Y_{d_1,s}^{DS} + Y_{d_2,s}^{DS} - 2) + C_{d_1}^D + S_{d_1,d_2} + p_{d_2}^D \leq C_{d_2}^D \qquad \forall d_1 < d_2 \in \{1, ..., D\}, \forall s \in \{1, ..., S\}. \tag{C.13}$$

Similarly, if discharges $s_1 < s_2$ both take place at weigher w and dosage d takes place in discharge s_2, then dosage d completes at least $S_{0,d} + p_d^D$ later than discharge s_1:

$$M(Y_{s_1,w}^{SW} + Y_{s_2,w}^{SW} + Y_{d,s_2}^{DS} - 3) + C_{s_1}^S + S_{0,d} + p_d^D \leq C_d^D \qquad \forall d \in \{1, ..., D\}, \forall s_1 < s_2 \in \{1, ..., S\}, \forall w \in \{1, ..., W\}. \tag{C.14}$$

Note that in order to check whether dosage d takes place in period p_2, data parameters Ψ_{d,b_2} and Ω_{b_2,p_2} are used.

Discharge s completes at least p_s^S later than discharge $s-1$:

$$C_{s-1}^S + p_s^S \leq C_s^S \qquad \forall s \in \{1, ..., S\}. \tag{C.15}$$

Note that $s = 1$ requires data parameter C_0^S.

If discharge s takes place in batch $b_2 > b_1$, then discharge s completes at least p_s^S later than $C_{b_1}^B$:

$$C_{b_1}^B + p_s^S + M(\Theta_{s,b_2} - 1) \leq C_s^S \qquad \forall s \in \{1, ..., S\}, \forall b_1 < b_2 \in \{1, ..., B\}. \tag{C.16}$$

If dosage d takes place in discharge s, then discharge s completes at least $S_{d,0} + p_s^S$ later than dosage d:

$$M(Y_{d,s}^{DS} - 1) + C_d^D + S_{d,0} + p_s^S \leq C_s^S \qquad \forall d \in \{1, ..., D\}, \; \forall s \in \{1, ..., S\}. \tag{C.17}$$

If dosage d containing raw material g takes place at silo l, then the dosing time is determined by the dosing constant and the mass of the dosage m_d:

$$M(Y_{d,l}^{DL} + Y_{l,g}^{LG} - 2) + c_{l,g}^D + \frac{m_d}{v_{l,g}^D} \leq p_d^D \qquad \forall d \in \{1, ..., D\}, \; \forall g \in \{1, ..., G\}, \; \forall l \in \{1, ..., L\}. \tag{C.18}$$

Furthermore, if discharge s takes place at weigher w, then the discharge time is determined by the discharge constant and the mass $\sum_{d=1}^{D} m_d Y_{d,s}^{DS}$ of all dosages in that discharge:

$$M(Y_{s,w}^{SW} - 1) + c_w^S Y_s^S + \frac{1}{v_w^S} \sum_{d=1}^{D} m_d Y_{d,s}^{DS} \leq p_s^S \qquad \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\}. \tag{C.19}$$

If no dosages take place in a discharge, then the discharge is empty, i.e.: $Y_s^S = 0$. This results in:

$$\sum_{d=1}^{D} Y_{d,s}^{DS} \geq Y_s^S \qquad \forall s \in \{1, ..., S\}. \tag{C.20}$$

On the other hand, if one or more dosages take place in a discharge, then that discharge is not empty, i.e.: $Y_s^S = 1$. This results in:

$$\frac{1}{D} \sum_{d=1}^{D} Y_{d,s}^{DS} \geq Y_s^S \qquad \forall s \in \{1, ..., S\}. \tag{C.21}$$

Only if dosage d contains more mass than the required minimum dosage mass $m_w^{\min}$ of weigher w, dosage d may take place on that weigher:

$$\sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \leq 1 \qquad \text{if } m_w^{\min} \leq m_d, \; \forall w \in \{1, ..., W\}, \; \forall d \in \{1, ..., D\}. \tag{C.22}$$

If discharge s takes place at weigher w, then the mass and volume of all dosages in discharge s cannot not exceed the maximum allowed dosage mass and volume of weigher w:

$$M(Y_{s,w}^{SW} - 1) + \sum_{d=1}^{D} m_d Y_{d,s}^{DS} \leq m_w^{\max} \qquad \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\}, \tag{C.23}$$

$$M(Y_{s,w}^{SW} - 1) + \sum_{d=1}^{D} V_d Y_{d,s}^{DS} \leq V_w^{\max} \qquad \forall s \in \{1, ..., S\}, \; \forall w \in \{1, ..., W\}. \tag{C.24}$$

The lower and upper limit of the allowed dosing interval of raw material g at weigher w may not be larger than the maximum allowed dosage mass of weigher w:

$$f_{w,g}^{WG_1} \leq m_w^{\max} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}, \tag{C.25}$$

$$f_{w,g}^{WG_2} \leq m_w^{\max} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{C.26}$$

The dosing interval of raw material g at weigher w_1 may not overlap with the interval at w_2. Thus, if the allowed dosing interval of raw material g at weigher w_1 is smaller or equal the interval at weigher w_2, then the upper limit at weigher w_1 should be smaller or equal to the lower limit at weigher w_2:

$$f_{w_2,g}^{WG_1} + M(1 - F_{w_1,w_2,g}) \geq f_{w_1,g}^{WG_2} \qquad \forall w_1 \neq w_2 \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{C.27}$$

If the dosing interval of raw material g at weigher w_1 is smaller than the interval at w_2, then $F_{w_1,w_2,g} = 1$ and $F_{w_2,w_1,g} = 0$. If the dosing interval of raw material g at weigher w_2 is smaller than the interval at w_1, then $F_{w_1,w_2,g} = 0$ and $F_{w_2,w_1,g} = 1$. If no dosages of raw material g take place at either weighers w_1 or w_2, then both intervals are 'empty', i.e.: $f_{w_1,g}^{WG_1} = f_{w_1,g}^{WG_2} = f_{w_2,g}^{WG_1} = f_{w_2,g}^{WG_2} = 0$, resulting in, $F_{w_1,w_2,g} = F_{w_2,w_1,g} = 1$. This means in any situation either one or both variables $F_{w_1,w_2,g}$ and $F_{w_2,w_1,g}$ should equal 1:

$$F_{w_1,w_2,g} + F_{w_2,w_1,g} \geq 1 \qquad \forall w_1 < w_2 \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{C.28}$$

If a raw material g is not placed on weigher w, then the interval is 'empty', i.e.: $f_{w,g}^{WG_1} = f_{w,g}^{WG_2} = 0$. To check whether raw material g is placed on weigher w, binary variable $Y_{g,w}^{GW}$ is needed, which comes to expression using $\sum_{d=1}^{D} \Delta_{d,g} Y_{d,w}^{DW}$ in which $Y_{d,w}^{DW} = \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL}$.

$$M \sum_{d=1}^{D} \Delta_{d,g} \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \geq f_{w,g}^{WG_1} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}, \tag{C.29}$$

$$M \sum_{d=1}^{D} \Delta_{d,g} \sum_{l=1}^{L} \Lambda_{l,w} Y_{d,l}^{DL} \geq f_{w,g}^{WG_2} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{C.30}$$

If dosage d takes place at weigher w and if dosage d contains raw material g, then the mass of dosage d must be contained in the allowed dosing interval of raw material g at weigher w, i.e.: the mass of dosage d should be larger than the lower limit and smaller than the upper limit of the dosing interval:

$$m_d - \delta + M(2 - Y_{d,w}^{DW} - \Delta_{d,g}) \geq f_{w,g}^{WG_1} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}, \; \forall d \in \{1, ..., D\}, \tag{C.31}$$

$$m_d + M(Y_{d,w}^{DW} + \Delta_{d,g} - 2) \leq f_{w,g}^{WG_2} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}, \; \forall d \in \{1, ..., D\}. \tag{C.32}$$

Consider that the mass m_d of dosage d consists of raw material g. Then m_d may not correspond to both the lower limit if the dosing interval of raw material g at weigher w_1 and the upper limit of raw material g at weigher w_2, since dosage d is only allowed to be executed on one installation. To prevent this, at least one of the limits of the weighers should be excluded from executing dosage d. Therefore, in (C.31), the lower limit of the allowed dosing interval of raw material g at weigher w should be smaller than m_d with a margin δ.

If dosages $d_1 < d_2$ take place at silos l_1 and l_2 respectively, the setup time should be equal to Σ_{l_1,l_2}:

$$M(Y_{d_1,l_1}^{DL} + Y_{d_2,l_2}^{DL} - 2) + \Sigma_{l_1,l_2} \leq S_{d_1,d_2} \qquad \forall d_1 < d_2 \in \{1, ..., D\}, \; \forall l_1 \in \{1, ..., L\}, \; \forall l_2 \in \{1, ..., L\}. \tag{C.33}$$

Similarly, for silo 0 this results in:

$$M(Y_{d,l}^{DL} - 1) + \Sigma_{0,l} \leq S_{0,d} \qquad \forall d \in \{1, ..., D\}, \; \forall l \in \{1, ..., L\}, \tag{C.34}$$

$$M(Y_{d,l}^{DL} - 1) + \Sigma_{l,0} \leq S_{d,0} \qquad \forall d \in \{1, ..., D\}, \; \forall l \in \{1, ..., L\}. \tag{C.35}$$

Dosages must be executed in order, i.e.: if dosages $d_1 < d_2$ take place in discharges s_1 and s_2 respectively, then discharge s_2 can take place at earliest together with discharge s_1:

$$M(Y^{DS}_{d_1,s_1} + Y^{DS}_{d_2,s_2} - 2) + C^S_{s_1} \leq C^S_{s_2} \qquad \forall d_1 < d_2 \in \{1,...,D\}, \forall s_1 \in \{1,...,S\}, \forall s_2 \in \{1,...,S\}. \tag{C.36}$$

If discharge s takes place in batch b, batch b finishes at least later than $C^S_s + c^B$:

$$C^S_s + c^B + M(\Theta_{s,b} - 1) \leq C^B_b \qquad \forall s \in \{1,...,S\}, \forall b \in \{1,...,B\}. \tag{C.37}$$

The final batch in the final period determines the total makespan:

$$C^B_b \leq C \qquad \forall b \in \{1,...,B\}. \tag{C.38}$$

Some of the continuous variables have to be lower-bounded:

$$f^{WG_1}_{w,g} \geq 0 \qquad \forall w \in \{1,...,W\}, \forall g \in \{1,...,G\}, \tag{C.39}$$

$$f^{WG_2}_{w,g} \geq 0 \qquad \forall w \in \{1,...,W\}, \forall g \in \{1,...,G\}. \tag{C.40}$$

Note that all other continuous variables have already been (implicitly) lower-bounded by their constraints.

Optional constraints
The following constraints are not necessary to solve the optimization problem. However, they reduce the number of possible solutions or make a implicit constraint explicit.

It is assumed that placing a raw material more than once on an installation probably does not significantly improve the performance as the weigher can only be located at one silo simultaneously. The benefit of placing a raw material more than once is a potential decrease in driving time in an installation with a movable weigher. Furthermore, placing a raw material more than once on an installation implies that another raw material cannot be placed on that installation. Placing another raw material on the installation decreases the potential idle time of that installation. It is assumed that the reduction in makespan from a decrease in idle time exceeds the reduction from a decrease in driving time. Hence, it is assumed that a raw material g can only occur once on a dosing installation with weigher w:

$$\sum_{l=1}^{L} \Lambda_{l,w} Y^{LG}_{l,g} \leq 1 \qquad \forall w \in \{1,...,W\}, \forall g \in \{1,...,G\}. \tag{C.41}$$

The lower limit of a dosing interval of raw material g at weigher w must be smaller or equal to the upper limit of raw material g at weigher w:

$$f_{w,g}^{WG_1} \leq f_{w,g}^{WG_2} \qquad \forall w \in \{1, ..., W\}, \; \forall g \in \{1, ..., G\}. \tag{C.42}$$

Note that this implicitly follows from (C.31) and (C.32).

Furthermore, all time-based continuous variables cannot be negative. Thus:

$$p_d^D \geq 0 \qquad \forall d \in \{1, ..., D\}, \tag{C.43}$$

$$p_s^S \geq 0 \qquad \forall s \in \{1, ..., S\}, \tag{C.44}$$

$$S_{d_1,d_2} \geq 0 \qquad \forall d_1 \in \{1, ..., D\}, \; \forall d_2 \in \{1, ..., D\}, \tag{C.45}$$

$$C_d^D \geq 0 \qquad \forall d \in \{1, ..., D\}, \tag{C.46}$$

$$C_b^B \geq 0 \qquad \forall b \in \{1, ..., B\}, \tag{C.47}$$

$$C \geq 0. \tag{C.48}$$

These variables are already implicitly lower-bound. For example: p_d^D is constrained by (C.18), in which all terms are explicitly positive except for $M(Y_{d,l}^{DL} + Y_{l,g}^{LG} - 2)$. However, from (C.4) and (C.5) it follows that there is always some silo l and some raw material g for which $Y_{d,l} = Y_{l,g} = 1$. Therefore, p_d^D is always lower-bounded by at least 0.

C.5 Objective function

The objective function follows straightforward from the constraints as the objective is to minimize the makespan:

$$\min C \tag{C.49}$$

Appendix D

Additional results heuristics

In this appendix additional data is provided to the heuristics as presented in Chapter 5. Furthermore, additional figures and numerical results of the heuristic solutions to the four case studies as given in Section 5.10 are provided.

The applied weight factors for the weighted sum method are given in Table D.1. If a number of bins n that are involved in a certain dosing interval, then the weighting factors for these bins correspond to the n first numbers in Table D.1. The first two bins both have a weighting factor of 1, because the greatest bulk of dosages is usually spread over two or three bins. In that way, it is ensured that these bins are incorporated in the larger dosing installation. The weighting factor for calculating the priority score is simply the absolute bin number. In this case, if two bins have the same number of dosages, it is ensured that the larger bin is always prioritized.

Table D.1: Weighting factors for the weighted sum method

Bin	1	2	3	4	5	6	7 and above
Weighting factor	1	1	0.5	0.33	0.25	0.2	0.1

Figures D.1, D.2 and D.3 show the results of the applied heuristics for case studies 1, 2 and 4. In each of these figures, the results are separated by the type of sorting method applied in the initial optimization. In case study 2 no local optimization has taken place, hence, no figure of case study 3 is provided. Numerical results for each of the different heuristics for case studies 1–4 are given in Tables D.2–D.5.

Table D.2: Results case study 1, minimum achieved production time [hours] for each of the applied heuristics. The production time for the original allocation is 3103.3 hours

Init. opt. \ Loc. opt.	Descending mass	Descending frequency	Descending rec. occup.
Descending mass	2941.1	2891.9	2919.2
Descending frequency	3082.2	3023.1	3078.6
Descending rec. occup.	3897.8	3775.6	3792.8

Finally, Table D.6 shows the mean batch time of the heuristic with the minimum production time for each case study.

Table D.3: Results case study 2, minimum achieved production time [hours] for each of the applied heuristics. The production time for the original allocation is 1430.5 hours

Init. opt. \ Loc. opt.	Descending mass	Descending frequency	Descending rec. occup.
Descending mass	2362.7	2362.7	2362.7
Descending frequency	2255.7	2255.7	2255.7
Descending rec. occup.	2262.7	2262.7	2262.7

Table D.4: Results case study 3, only initial optimization is possible for this case study

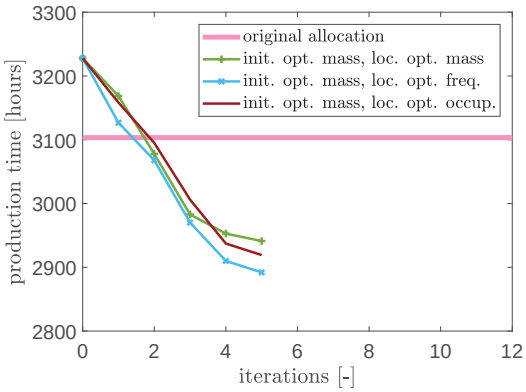
Sorting method	Descending mass	Descending frequency	Descending rec. occup.	Original allocation
Production time [hours]	1772.6	1769.2	1550.0	1119.2

Table D.5: Results case study 4, minimum achieved production time [hours] for each of the applied heuristics. The production time for the original allocation is 1146.6 hours

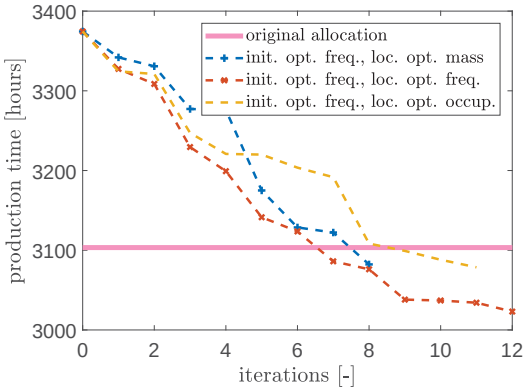
Init. opt. \ Loc. opt.	Descending mass	Descending frequency	Descending rec. occup.
Descending mass	1141.4	1173.4	1150.5
Descending frequency	1159.2	1200.2	1160.9
Descending rec. occup.	1177.4	1221.7	1187.6

Table D.6: Mean batch time of the best performing heuristic for each case study

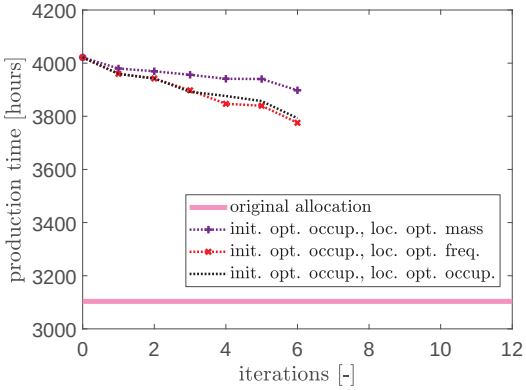
Case study	1	2	3	4
Mean batch time [s]	363.5	664.6	813.6	385.9



(a) Sorting method applied to the initial optimization is descending mass

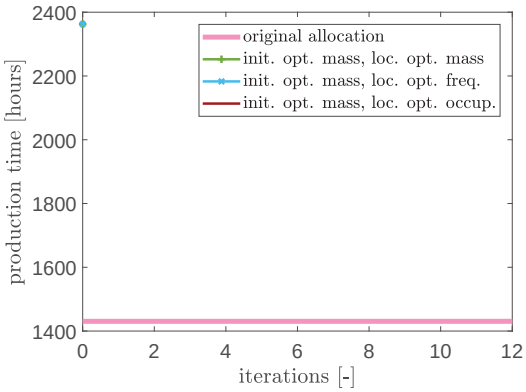


(b) Sorting method applied to the initial optimization is descending frequency

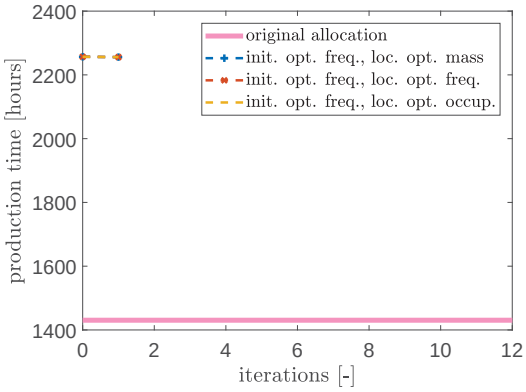


(c) Sorting method applied to the initial optimization is descending recipe occupancy

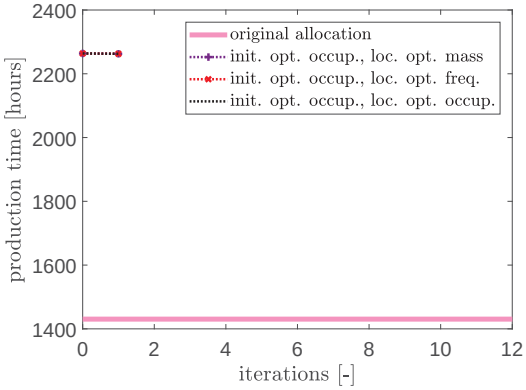
Figure D.1: Results case study 1



(a) Sorting method applied to the initial optimization is descending mass

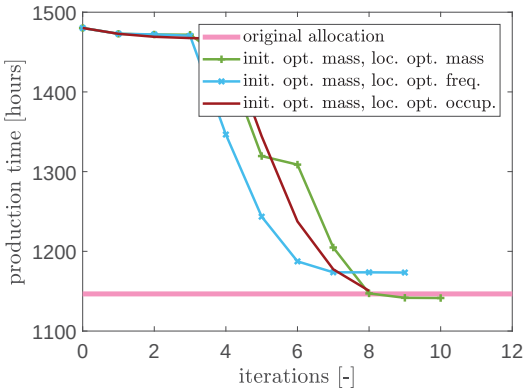


(b) Sorting method applied to the initial optimization is descending frequency

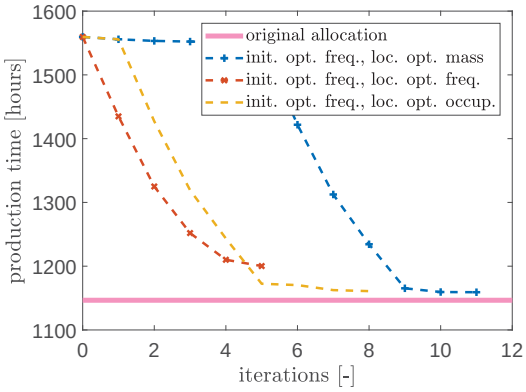


(c) Sorting method applied to the initial optimization is descending recipe occupancy

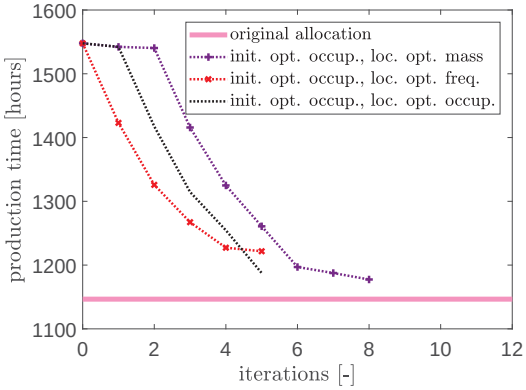
Figure D.2: Results case study 2



(a) Sorting method applied to the initial optimization is descending mass



(b) Sorting method applied to the initial optimization is descending frequency



(c) Sorting method applied to the initial optimization is descending recipe occupancy

Figure D.3: Results case study 4