Technische Universiteit Eindhoven University of Technology

How to control traffic lights or factories

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Lustrum TW
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Where innovation starts

Technische Universiteit Eindhoven University of Technology

Why Applied Mathematics in Twente?

- Enjoyed mathematics (member Dutch team IMO 1990)

- Interested in application. So no pure mathematics.
- Disliked atmosphere in Delft; Liked campus of UT

Introduction

Abacus

Abacus

- Mathematical Café
- Parents day
- Kaleidoscoopdag (Symposium)
- IKTW 1993 (Twick-In)
- Board 1993–1994
- Ideaal!
- Coco
- Advisory councils
- Cash audit committees
- Former board members day

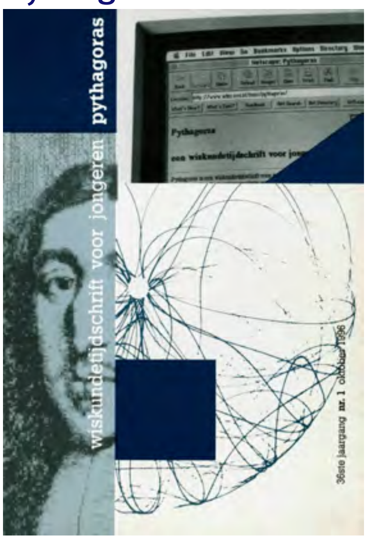
Drienerlo

- Church (elder)

Vierkant voor Wiskunde

vierkant wiskunde

Pythagoras



Graduation

PhD defense

Marriage

Family

Hobby: scuba diving

Problem

How to control these networks?

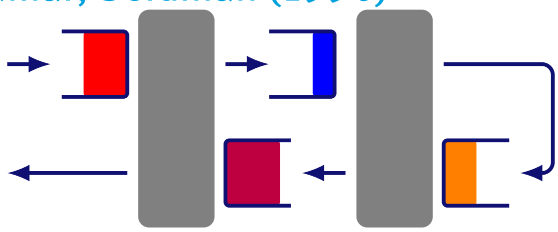
Decisions: When to switch, and to which job-type

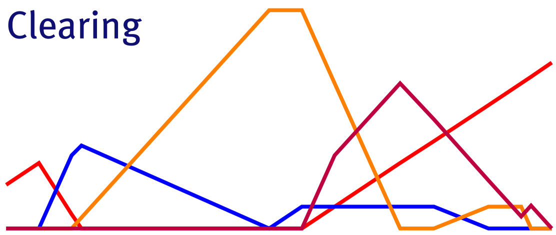
Goals: Minimal number of jobs, minimal flow time

Current approach

Start from policy, analyze resulting dynamics

Kumar, Seidman (1990)

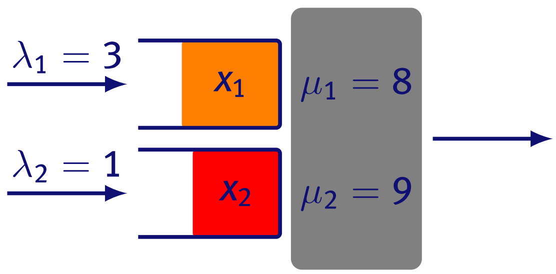




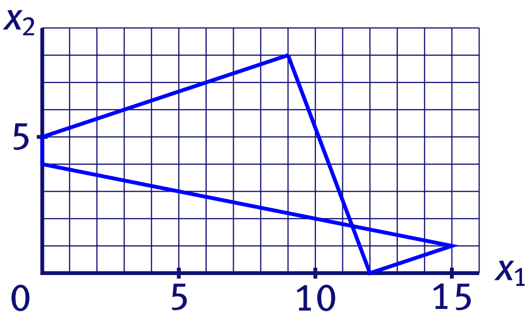
Important questions

- Do existing policies yield satisfactory network performance?
- How to obtain pre-specified network behavior?

Single machine



Optimal periodic behavior



Remarks

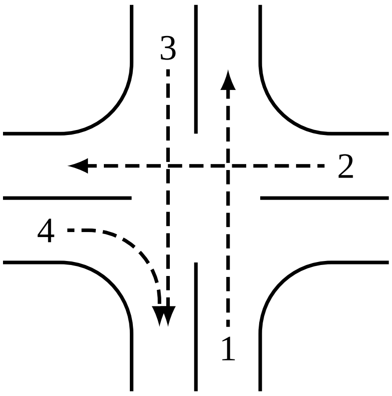
- Many existing policies assume non-idling a-priory
- Slow-mode optimal if $(\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2}) + (\lambda_2 - \lambda_1)(1 - \frac{\lambda_2}{\mu_2}) < 0$.
- Trade-off in wasting capacity: idle $\Leftrightarrow$ switch more often

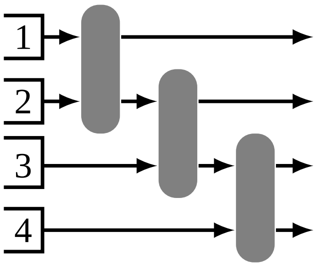
Resulting Controller

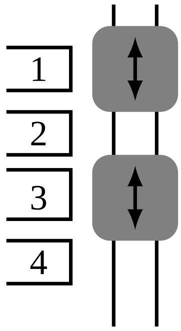
Two modes:

- When serving type 1:
  1. empty buffer
  2. serve until x ≥ 5
- When serving type 2:
  1. empty buffer
  2. serve until x ≥ 12

Examples

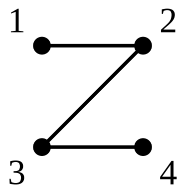




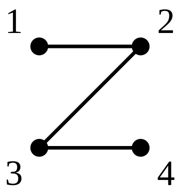


- Intersection
- Hot ingots
- Constrained polling

Conflict graph:



Conflict graph



Clearance times

$\sigma_{1,2} = 1 \quad \sigma_{2,3} = 2 \quad \sigma_{3,4} = 1$

$\sigma_{2,1} = 0 \quad \sigma_{3,2} = 0 \quad \sigma_{4,3} = 0$

Minimal green times

$g_1^{\min} = 4 \quad g_3^{\min} = 4$

$g_2^{\min} = 4 \quad g_4^{\min} = 4$

Arrival/Service rates

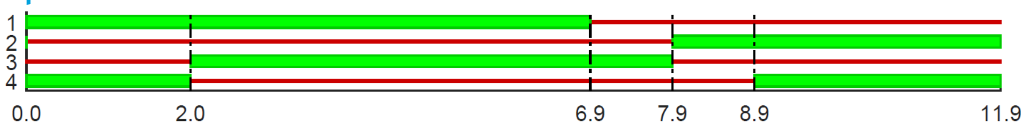
$\lambda_1 = 1200 \quad \mu_1 = 3800$

$\lambda_2 = 400 \quad \mu_2 = 1900$

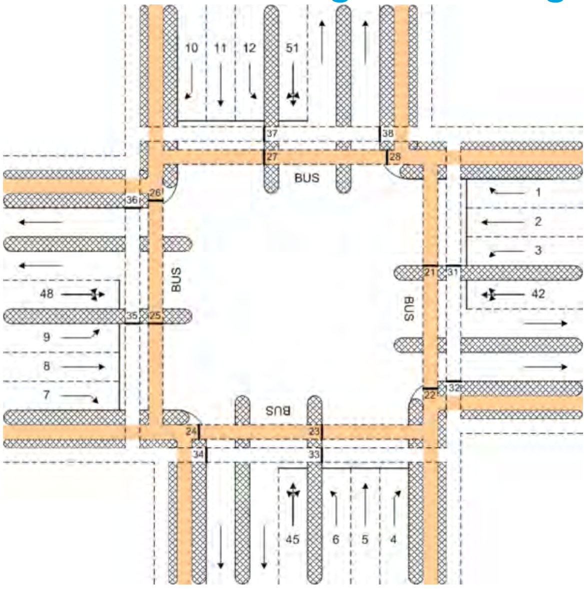
$\lambda_3 = 1200 \quad \mu_3 = 5360$

$\lambda_4 = 400 \quad \mu_4 = 3400$

Optimal periodic schedule



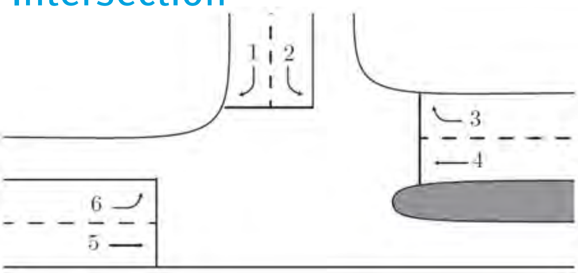
Standard numbering of traffic lights in then Netherlands



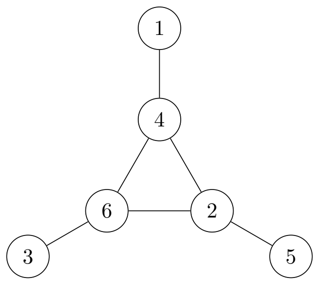
1–12 Cars (clockwise)
21-28 Bicycles
31-38 Pedestrians
41-52 Public Transport

Higher numbers when more than one intersection per controller, e.g., 61–72 Cars, etc.

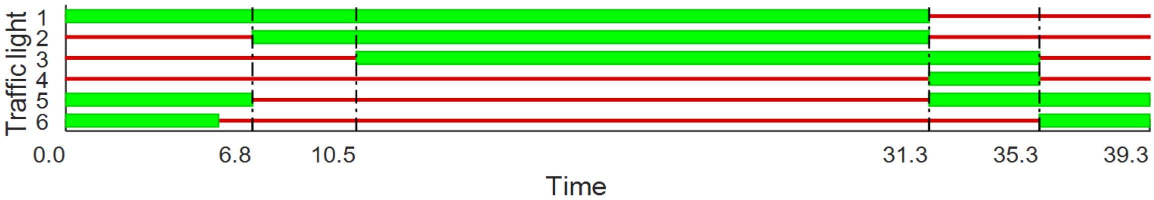
Intersection



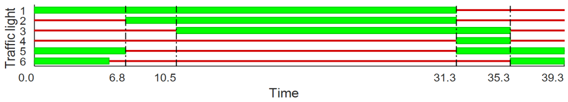
Conflict graph:



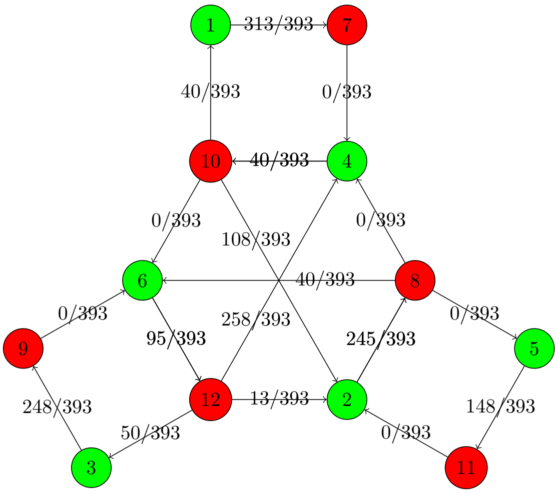
Optimal schedule (data from Grontmij: A2/N279)



Optimal schedule



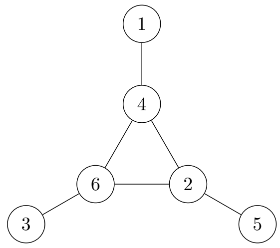
Extended graph



Event times

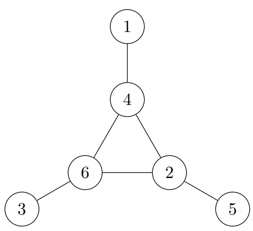
i	$t(i)$	$t(i+6)$	i+6
1	0.0	31.3	7
2	6.8	31.3	8
3	10.5	35.3	9
4	31.3	35.3	10
5	31.3	6.8	11
6	35.3	5.5	12

Conflict graph



Data

- Arrival rates: λ_i
- Service rates: μ_i
- Clearance times: $\sigma_{i,j}$
- Minimal/maximal green time: $g_i^{\min}$, $g_i^{\max}$.
- Minimal/maximal period: $T^{\min}$, $T^{\max}$.
- Conflict graph:



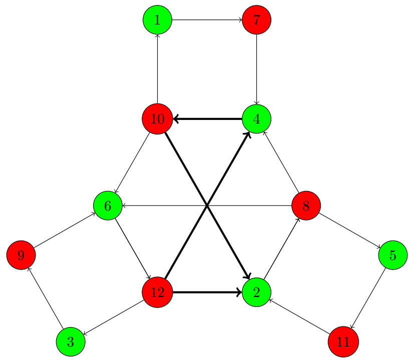
Design variables

- $x(i, j)$ fraction of period from event i to event j.
- T' reciprocal of duration of period, i.e. $T' = 1/T$.

Constraints

- Stable system: $\rho_i = \lambda_i/\mu_i \leq x(i, i+n)$
- Minimal/maximal green time: $g_i^{\min} T' \leq x(i, i+n) \leq g_i^{\min} T'$
- Clearance time: $\sigma_{i,j} T' \leq x(i, j)$
- Minimal/maximal period: $1/T^{\max} \leq T' \leq 1/T^{\min}$
- Conflict: $x(i, i+n) + x(i+n, j) + x(j, j+n) + x(j+n, i) = 1$
- Integer cycle: $\sum_{(i,j)\in C^+} x(i, j) - \sum_{(i,j)\in C^+} x(i, j) = z_C$.

Extended graph



Cycle

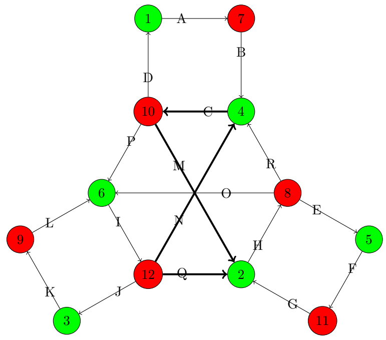
Cycle: $\{(4, 10), (10, 2), (12, 2), (12, 4)\}$

$C^+ = \{(4, 10), (10, 2), (12, 4)\}$.
$C^- = \{(12, 2)\}$.

Integer cycle constraint:
$\sum_{(i,j)\in C^+} x(i, j) - \sum_{(i,j)\in C^+} x(i, j) = z_C$.

Only for cycles from integer cycle base.

Extended graph



Cycle

$$
\begin{array}{cccccccccccccccccc}
A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R \\
\end{array}
$$

$$
[0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0]
$$

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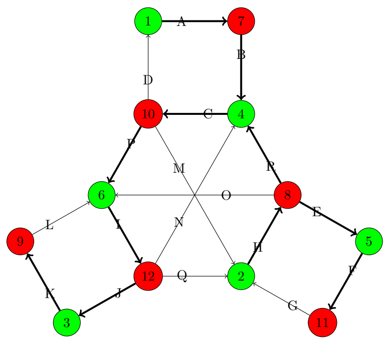

Cycle basis

$$
\begin{array}{cccccccccccccccccc}
A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R \\
{[1} & 1 & 1 & \underline{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0] \\
{[0} & 0 & 0 & 0 & 1 & 1 & \underline{1} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0] \\
{[0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & \underline{1} & 0 & 0 & 0 & 0 & 0 & 0] \\
{[0} & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \underline{1} & 0 & 0 & 0 & 0 & 1] \\
{[0} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \underline{1} & 0 & 1 & 0 & 0] \\
{[0} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{1} & 1 & 0 & 1] \\
{[0} & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \underline{1} & 1]
\end{array}
$$

Observation 1

Let graph consist of 1 component, n edges and m vertices. Then

- Spanning tree consists of $n - 1$ vertices.
- Dimension of cycle basis: $m - n + 1$.

Observation 2

Only integer combination of cycles is allowed.

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Constraints

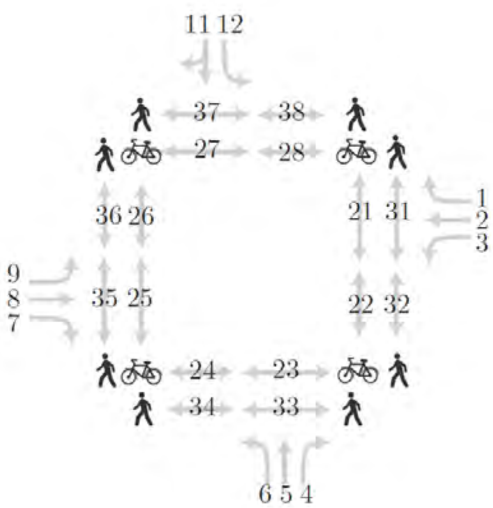
- Stable system: $\rho_i = \lambda_i / \mu_i \leq x(i, i+n)$
- Minimal/maximal green time: $g_i^{\min} T' \leq x(i, i+n) \leq g_i^{\min} T'$
- Clearance time: $\sigma_{i,j} T' \leq x(i, j)$
- Minimal/maximal period: $1/T^{\max} \leq T' \leq 1/T^{\min}$
- Conflict: $x(i, i+n) + x(i+n, j) + x(j, j+n) + x(j+n, i) = 1$
- Integer cycle: $\sum_{(i,j)\in C^+} x(i,j) - \sum_{(i,j)\in C^+} x(i,j) = z_C$.

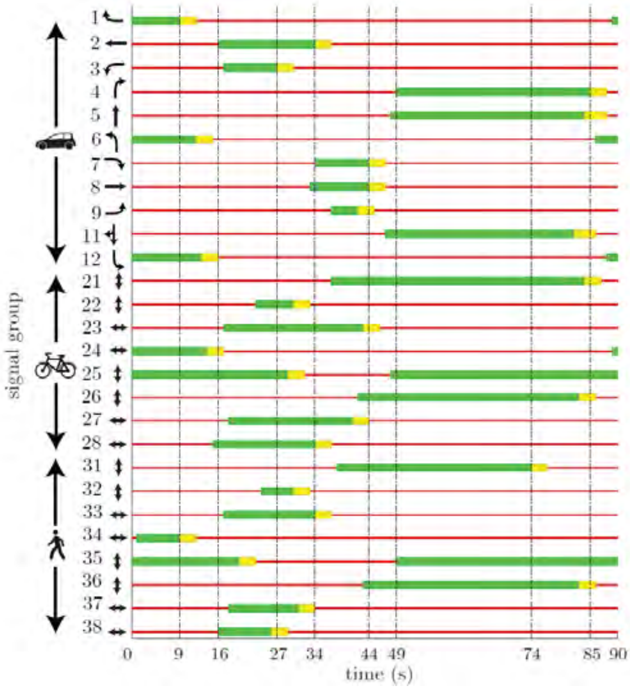
Objective

Minimize weighted average delay (Fluid, Webster, Miller, v.d. Broek):

$$\sum_{i=1}^{n} \frac{r_i}{2\lambda_i(1-\rho_i)T} \left(r_i \lambda_i + \frac{s_i^2}{1-\rho_i} + \frac{r_i \rho_i^2 s_i^2 T^2}{(1-\rho_i)(T-r_i)^2((1-\rho_i)T - r_i)} \right)$$

Real life example





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Subsequent steps (mathematically more involved)

- Given optimal periodic behavior
- How to control system towards optimal behavior?
- Solution
  - Identify modes
  - Determine duration of modes (function of state)
  - Prove that resulting controller works
- We have developed general method.
  - Convergence to optimal behavior
  - Robustness for deviations in arrival rates, service rates, clearance/setup times

Future

- Cars will drive autonomously
- No traffic lights anymore
- Cars resolve the problems themselves (wireless communication)
- Underlying mathematics: same as for traffic lights
- Result...