Controller design for switched linear systems with setups

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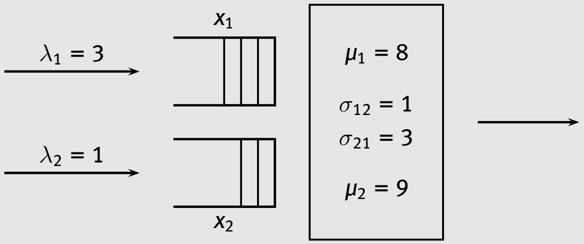
Single machine: switched linear system



- Constant arrival rates λ_1 and λ_2
- Maximal service rates μ_1 and μ_2
- Setup times σ_{12} and σ_{21}
- Buffer contents x_1 and x_2
- Activities: ❶: setup for Type 1 jobs

  ①: serve Type 1 jobs

  ❷: setup for Type 2 jobs

  ②: serve Type 2 jobs

Objective (1)

Determine "optimal periodic orbit"

$$\frac{1}{T}\int_0^T c_1 x_1(t) + c_2 x_2(t)\,\mathrm{d}t$$

Assume $c_1 = c_2 = 1$

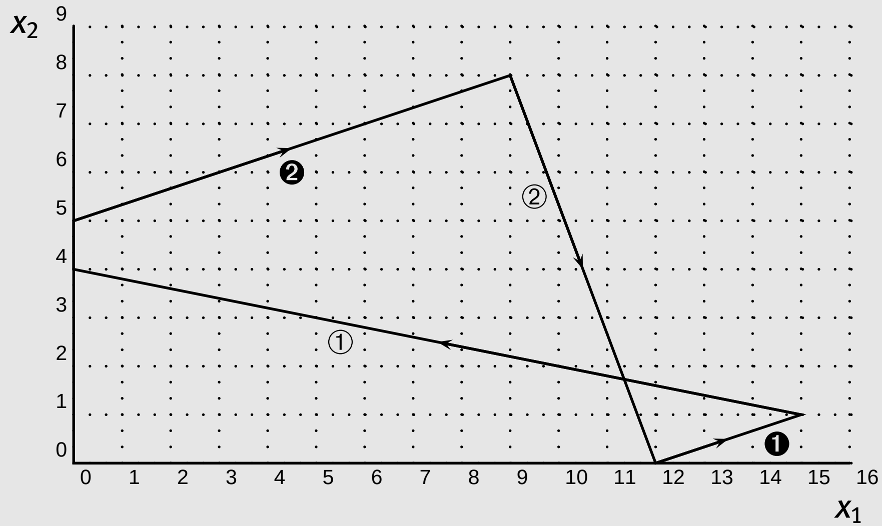
Objective (1)

Determine "optimal periodic orbit"

$$\frac{1}{T}\int_0^T c_1 x_1(t) + c_2 x_2(t)\,\mathrm{d}t$$

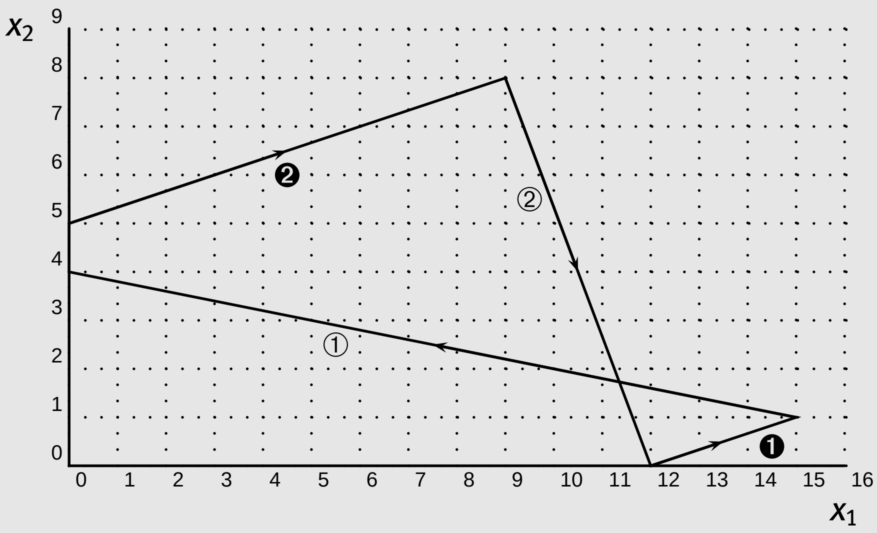
Solution

- Idling policy $\quad c_1\lambda_1(\rho_1+\rho_2) + (c_2\lambda_2 - c_1\lambda_1)(1-\rho_2) < 0$



Objective (2)

Design feedback which make system converge towards “optimal periodic orbit”



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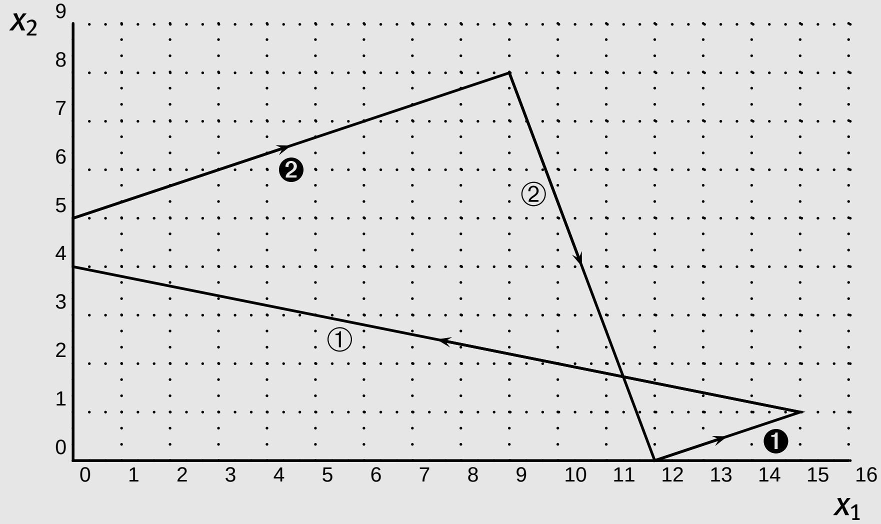

Do not start from policy, then study closed-loop behavior

But start from desired behavior and derive policy

Controller design: main idea

Lyapunov: If energy is decreasing all the time $\Rightarrow$ system should settle down at constant energy level

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Lyapunov: If energy is decreasing all the time $\Rightarrow$ system should settle down at constant energy level

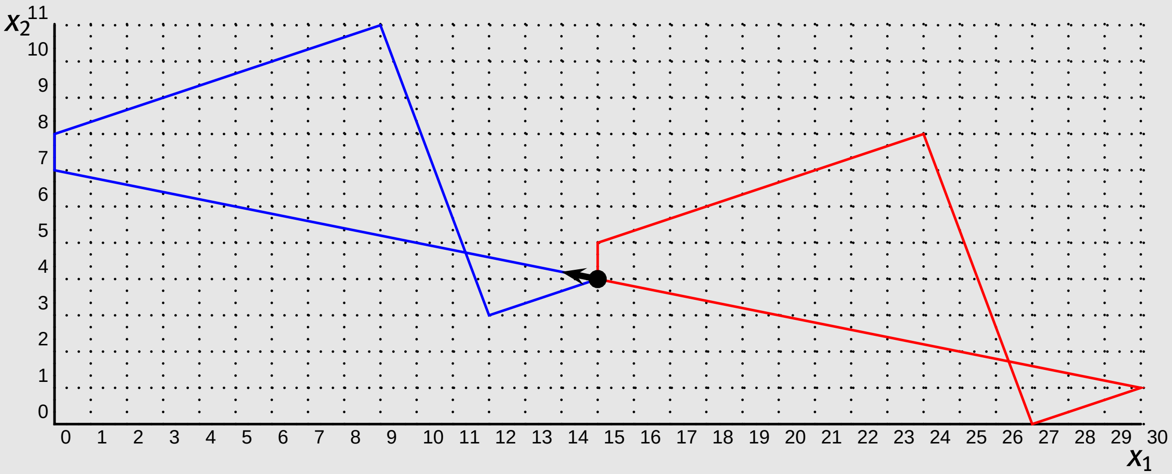
Amount of work in system: $\frac{x_1}{\mu_1} + \frac{x_2}{\mu_2}$

Associate with periodic orbit: mean amount of work.

Lyapunov function candidate

For given point X

- Of all curves going through X, take the one with minimal mean amount of work



Lyapunov function candidate

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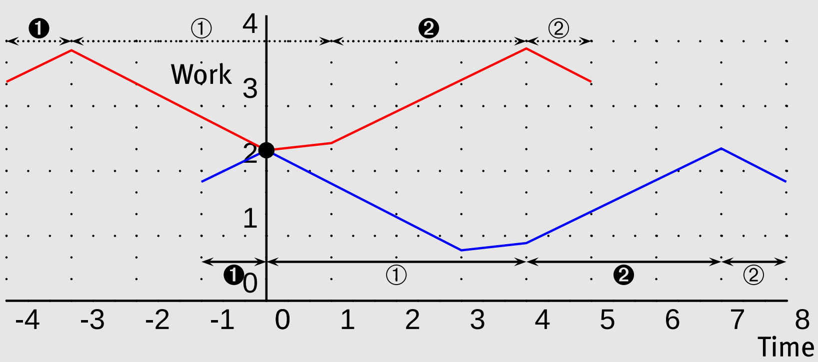


Lyapunov function candidate

For given point X

- Of all curves going through X, take the one with minimal mean amount of work
- Subtract from this amount of work, that of the desired periodic orbit
- This number is the value $V(X)$ of the Lyapunov function candidate in X

Controller design

Of all possible control-actions, take the one which makes $V(X)$ decrease the most.

Resulting controller

- If in Mode 1: Stay in this mode until both $x_1 = 0$ and $x_2 \geq 5$. Then switch to Mode 2.
- If in Mode 2: Stay in this mode until both $x_2 = 0$ and $x_1 \geq 12$. Then switch to Mode 1.

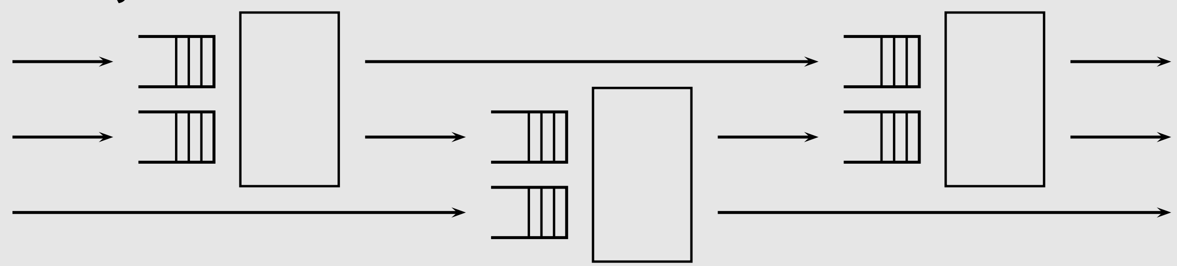
Resulting controller (finite buffers)

If $x_1 \leq x_1^{max}$ and $x_2 \leq x_2^{max}$ with $x_1^{max} \geq 15$ and $x_2^{max} \geq 8$:

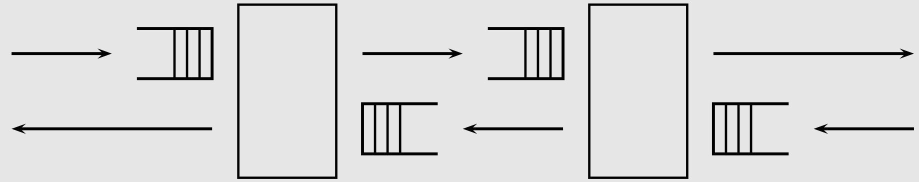
- If in Mode 1: Stay in this mode until either both $x_1 = 0$ and $x_2 \geq 5$ or $x_2 = x_2^{max} - 3$. Then switch to Mode 2.
- If in Mode 2: Stay in this mode until either both $x_2 = 0$ and $x_1 \geq 12$ or $x_1 = x_1^{max} - 3$. Then switch to Mode 1.

Networks

An acyclic network

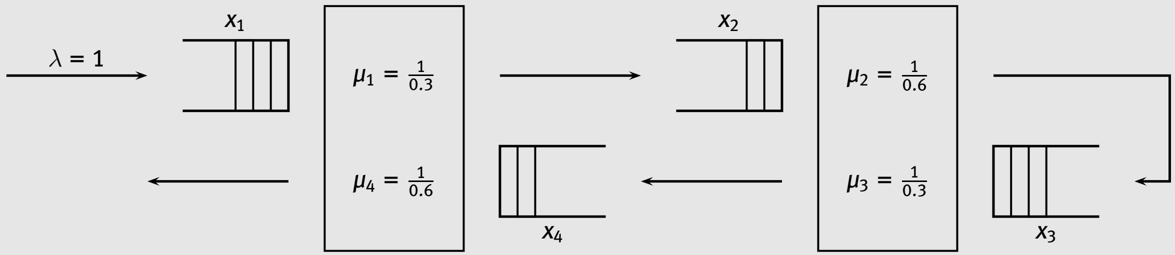


A non-acyclic network

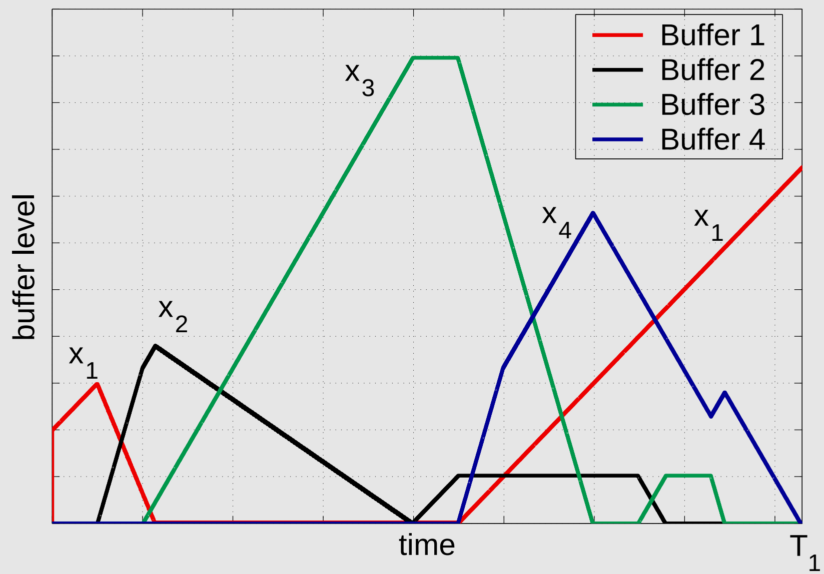


Kumar-Seidman

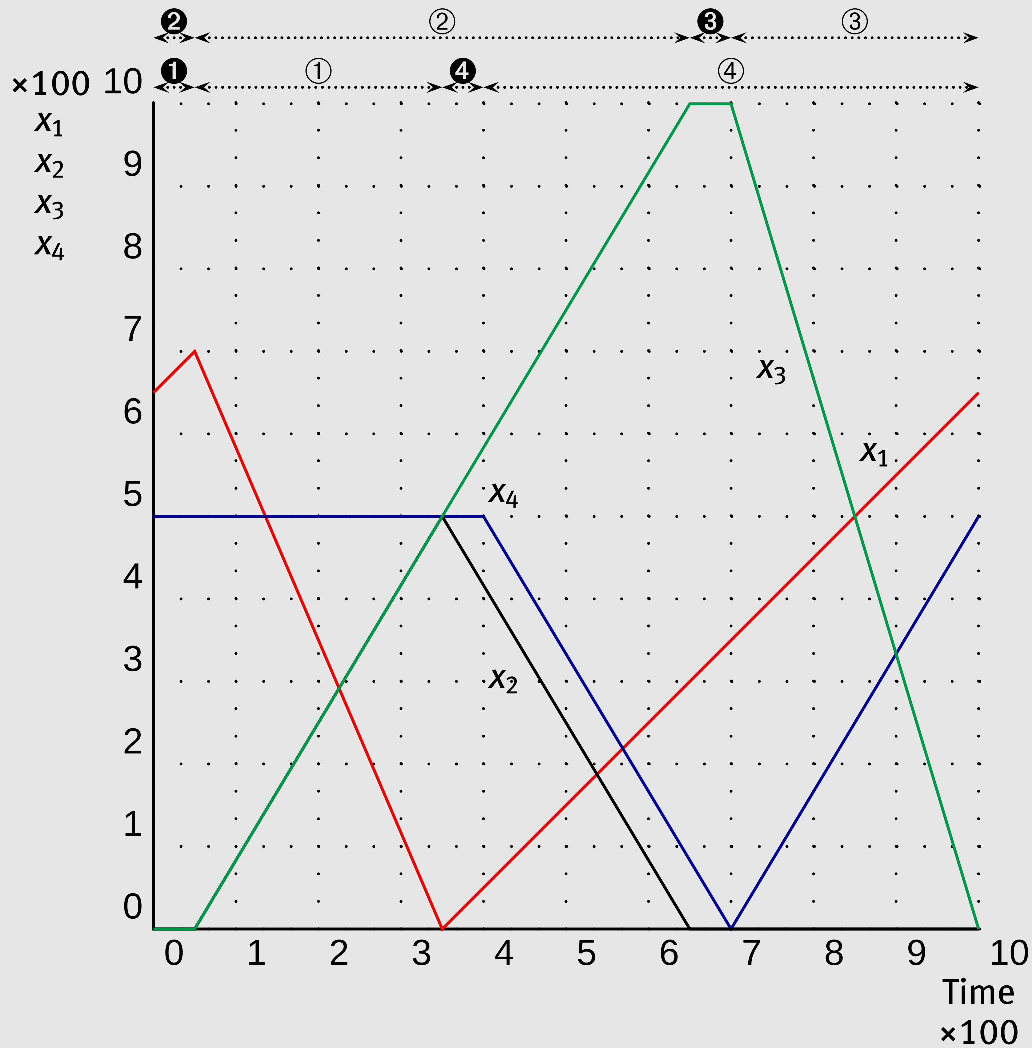
System



Clearing results in



Desired periodic orbit



Resulting controller

- If initially in Mode (1,3): Switch to Mode (1,2).
- If in Mode (1,2): Stay in this mode until $x_1 = 0$. Then switch to Mode (4,2).
- If in Mode (4,2): Stay in this mode until either $x_2 = 0$ or $x_4 \leq 83\frac{1}{3}$. Then switch to Mode (4,3).
- If in Mode (4,3): Stay in this mode until $x_3 = 0$. Then switch to Mode (1,2).

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Even though $\dot{V} \leq 0$ we do not have $\lim_{t\to\infty} V(t) = 0$.

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Even though $\dot{V} \leq 0$ we do not have $\lim_{t\to\infty} V(t) = 0$.

Modified controller

- If initially in Mode (1,3): Switch to Mode (1,2).
- If in Mode (1,2): Stay in this mode until $x_1 = 0$. Then switch to Mode (4,2).
- If in Mode (4,2): Stay in this mode until both $x_2 = 0$ and $x_4 \leq 83\frac{1}{3}$. Then switch to Mode (4,3).
- If in Mode (4,3): Stay in this mode until $x_3 = 0$. Then switch to Mode (1,2).

Now have $\lim_{t\to\infty} V(t) = 0$ (but not always $\dot{V} \leq 0$).

Conclusions

- Given a desired periodic orbit, a policy has been *derived*.
- Buffer constraints can be taken into account easily
- Transportation delays might also be incorporated
- Policy is not necessarily clearing
- Policy is neither gated nor k-limited
- Policy is global (non-autonomous)