Modeling, Validation and Control of Manufacturing Systems

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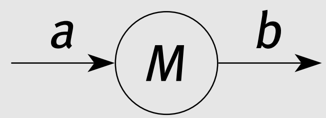
Information and Material Flows in Complex Networks

- Introduction to χ
- From real fab to DEM
- Effective process times
- Control Framework
- Approximation model
- Validation
- Control
- Conclusions

Machine



$\text{proc } M(a : ?\text{lot}, b : !\text{lot}, t_e, c_e^2 : \text{real}) =$

$|[\ u : \rightarrow \text{real}, x : \text{lot}$

$\ |\ u := \Gamma(t_e, c_e^2)$

$\ ; *[\ \text{true} \rightarrow a?x; \Delta \sigma u; b!x]$

$]|$

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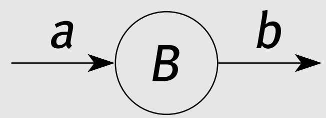
Buffer




```
proc B(a : ?lot, b : !lot) =
|[ x : lot, xs : lot*
 | xs := [ ]
 ; *[ true;          a?x         → xs := xs ++ [x]
    [] len(xs) > 0; b! hd(xs)→ xs := tl(xs)
    ]
]|
```

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Generator and Exit

type lot = real

proc $G(a : !\text{lot}, t_a : \text{real}) = |[\ *[\ \text{true} \longrightarrow a!\tau;\ \Delta t_a]\ ]|$

proc $E(a : ?\text{lot}) =$
$|[\ x : \text{lot}$
$|\ *[\ \text{true} \longrightarrow a?x;\ !\text{“Flow time: ”}, x - \tau, \text{“\textbackslash\ n”}]$
$]|$

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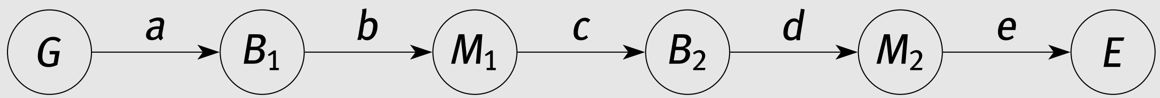
Overall model




```
clus F() =
|[ a, b, c, d, e : −lot
 | G(a, 3.0)
|| B(a, b) || M(b, c, 1.0, 1.0)
|| B(c, d) || M(d, e, 2.0, 1.0)
|| E(e)
]|

xper = |[ F() ]|
```

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Discrete Event Modeling of a real factory

- raw process time t_0 and c_0
- setups t_s and c_s
- TBF t_f and c_f, TTR t_r and c_r
- operator delays
- rework
- ...(!)

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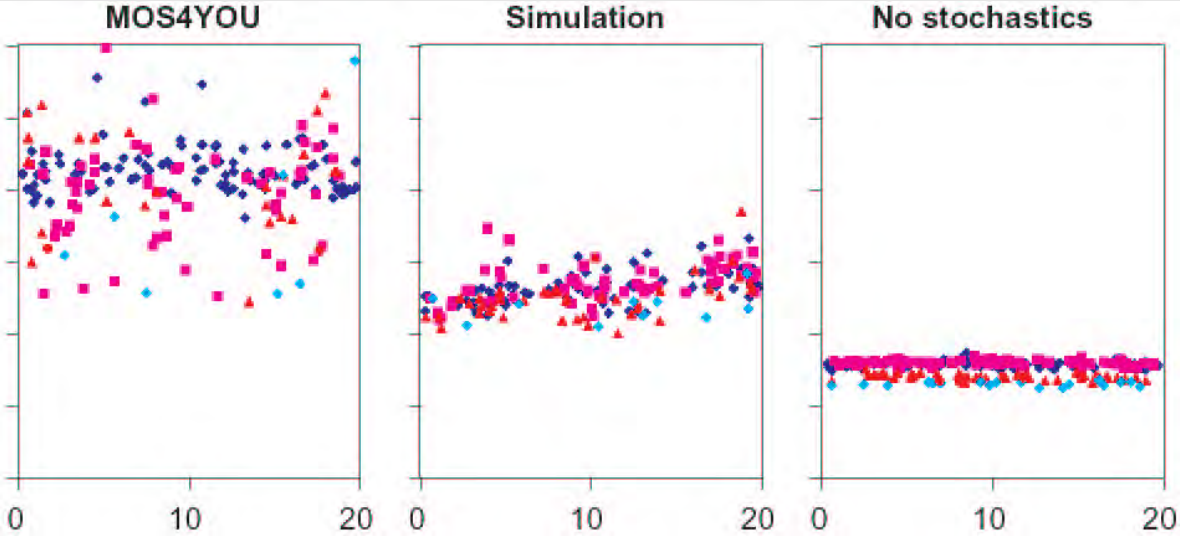
Results



- smaller mean flow time
- smaller variance flow time

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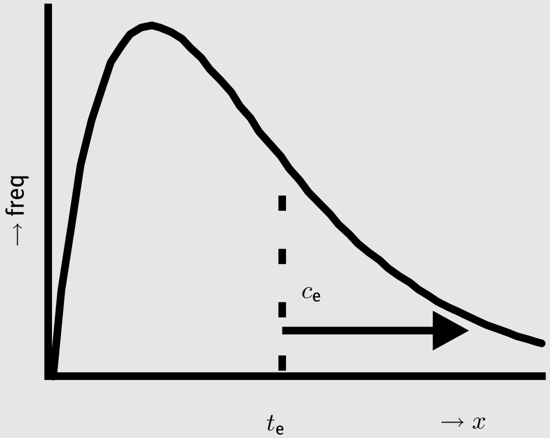
The effective process time method

- raw process time t_0 and c_0
- setups t_s and c_s
- TBF t_f and c_f, TTR t_r and c_r
- operator delays
- rework
- . . . (!)

Idea:

Combine all disturbances in one single EPT probability density function



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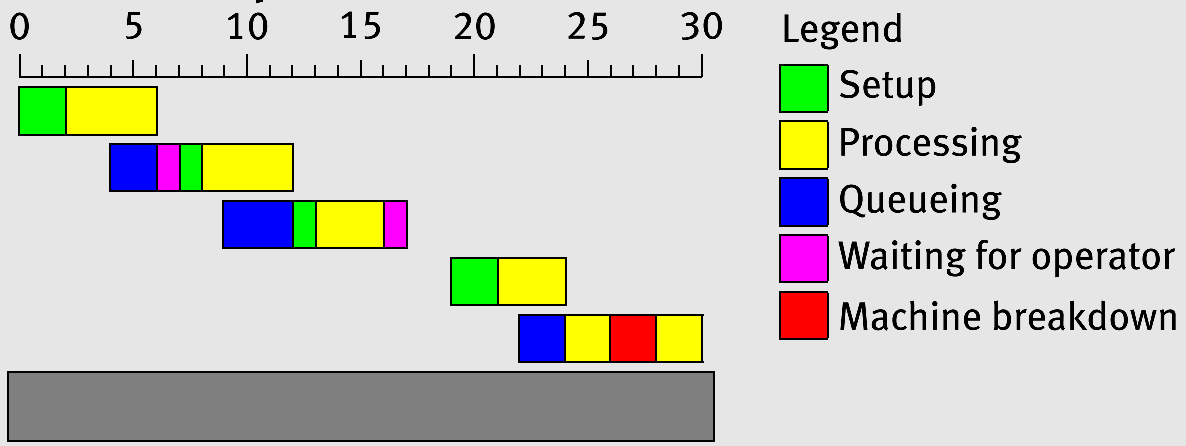
Effective process times



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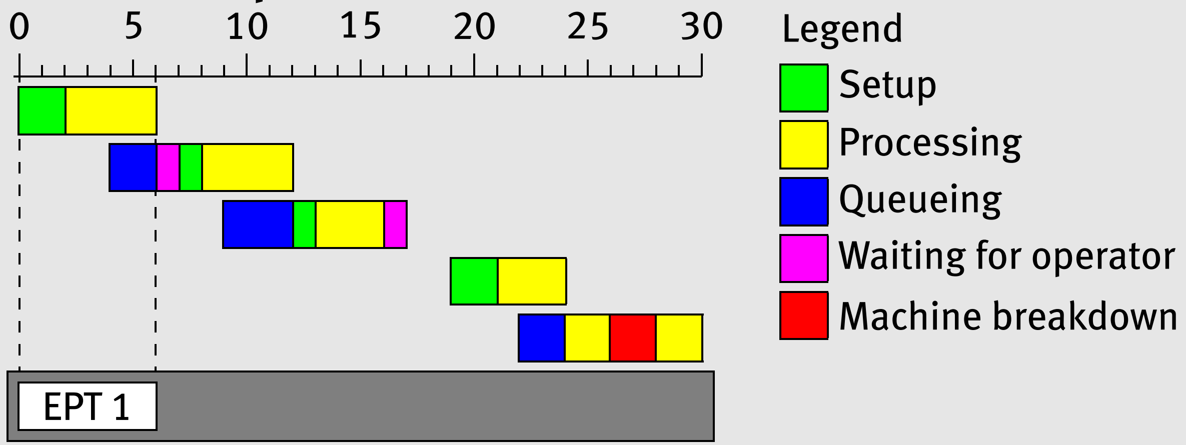

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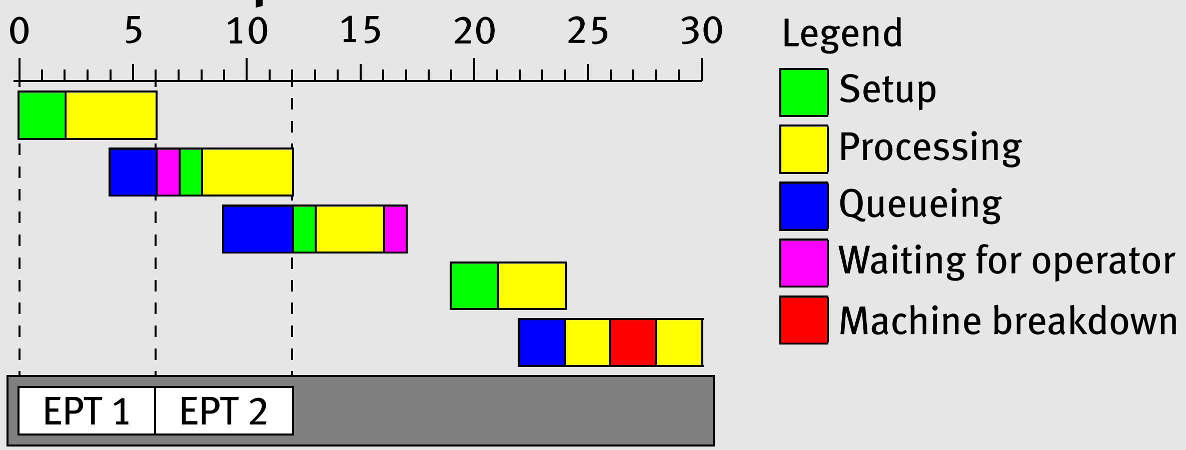

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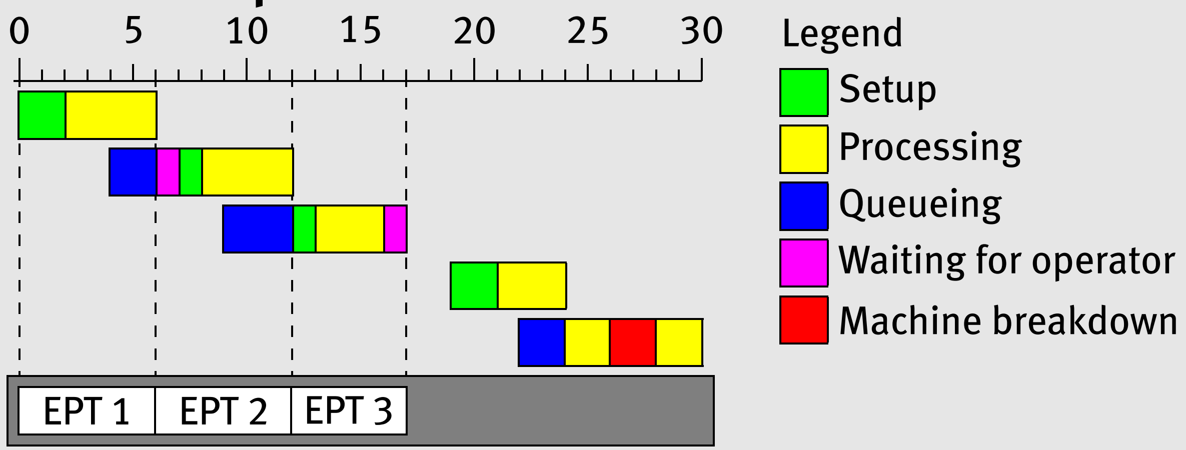

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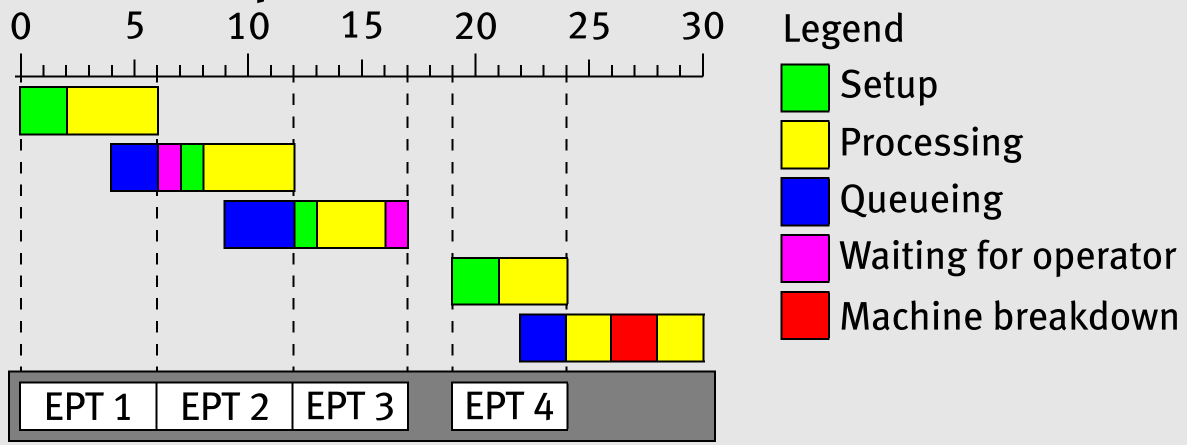

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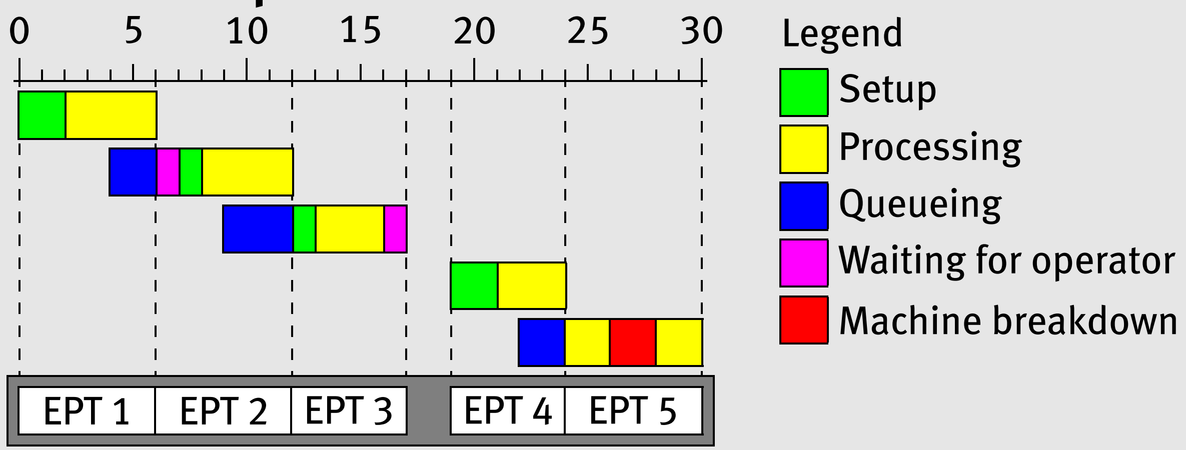

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Control Framework

Manufacturing System

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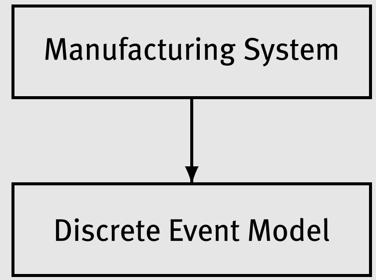

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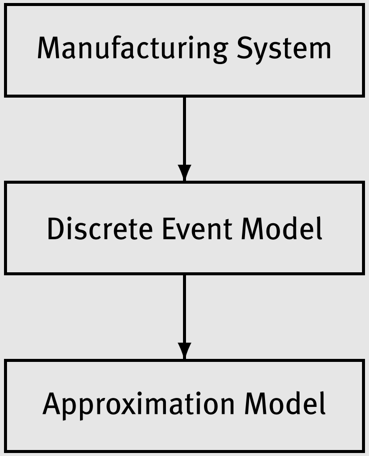

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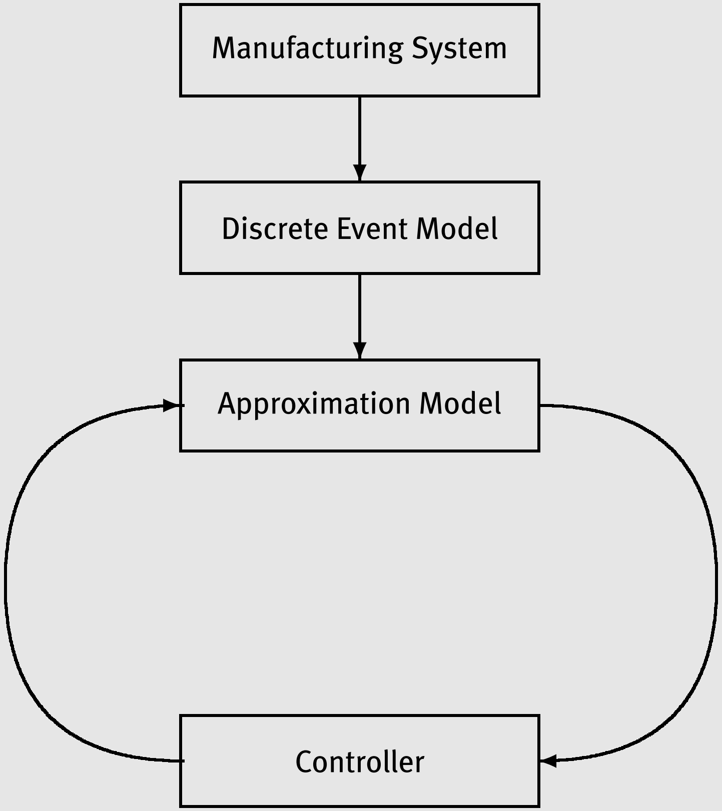

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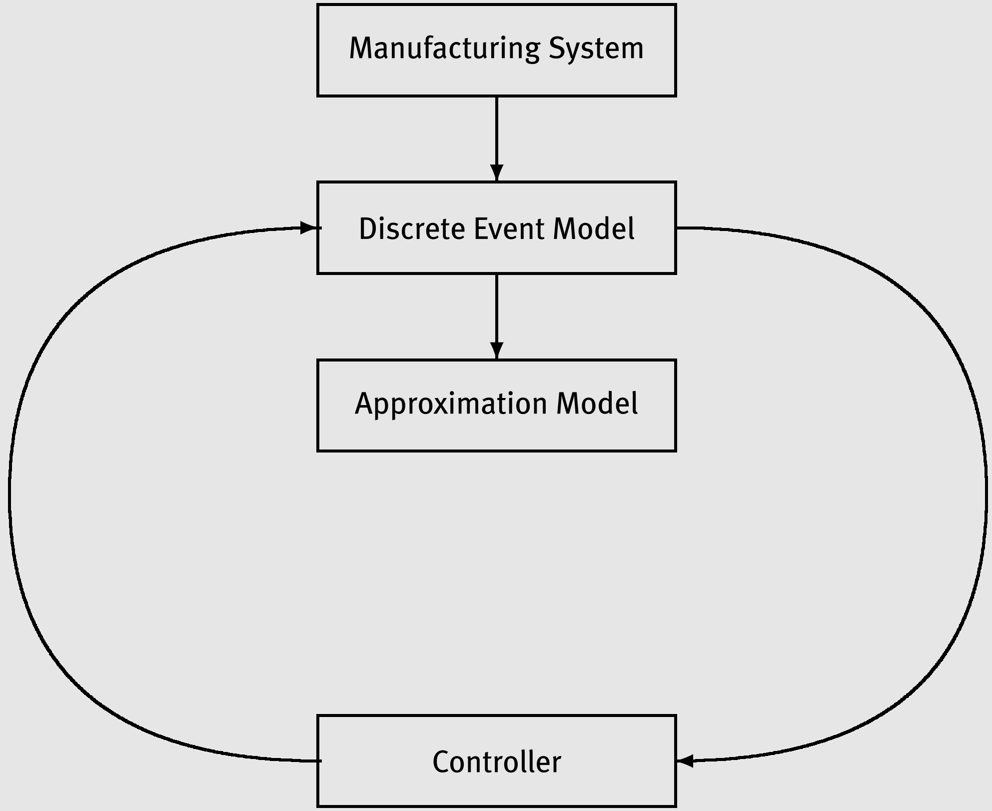

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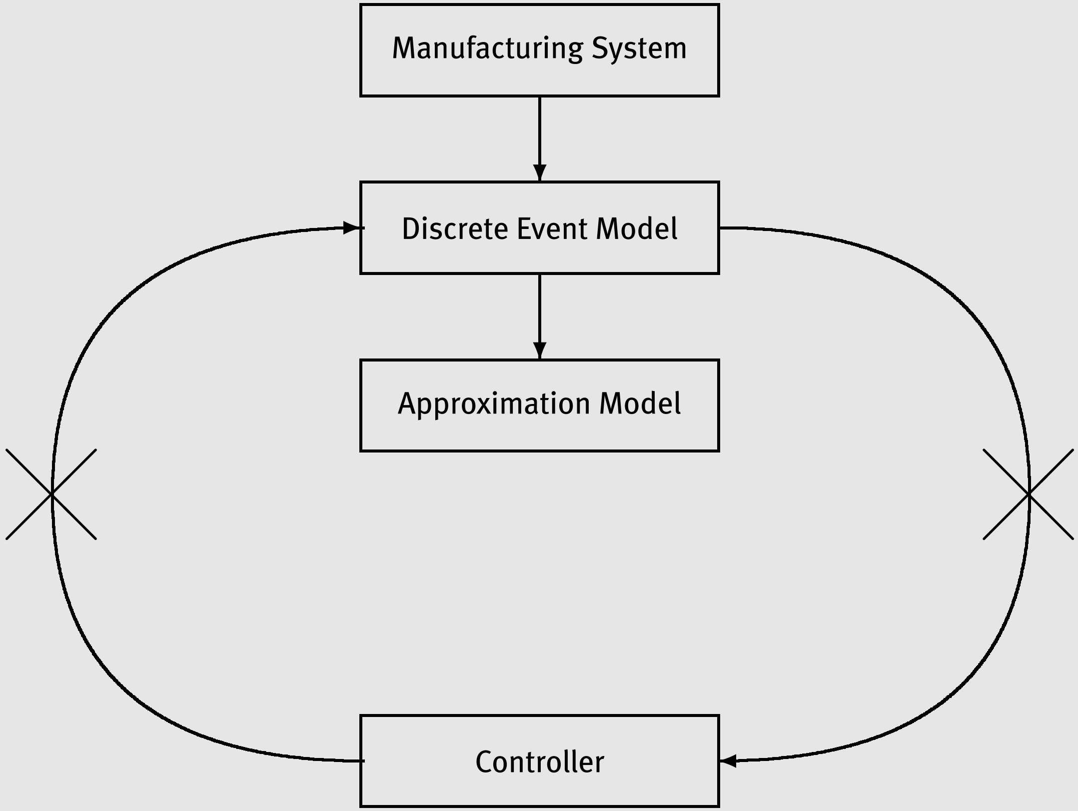

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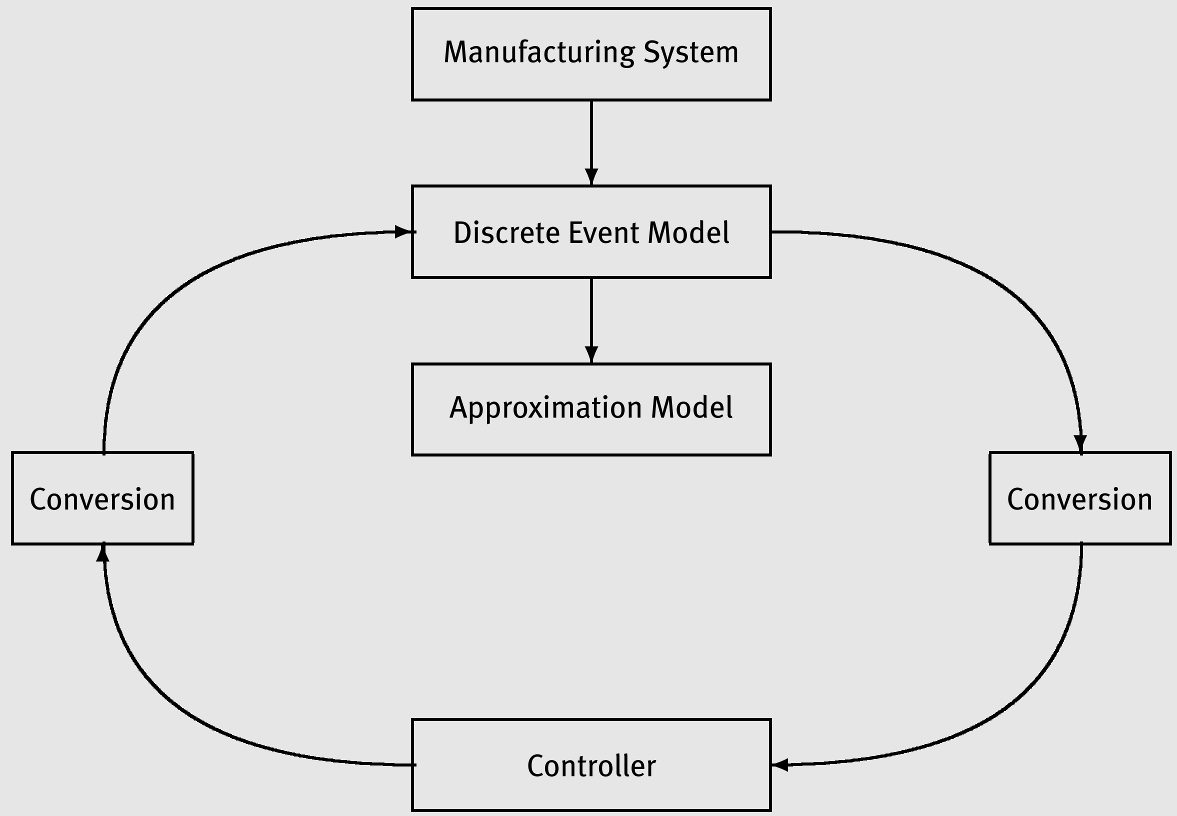

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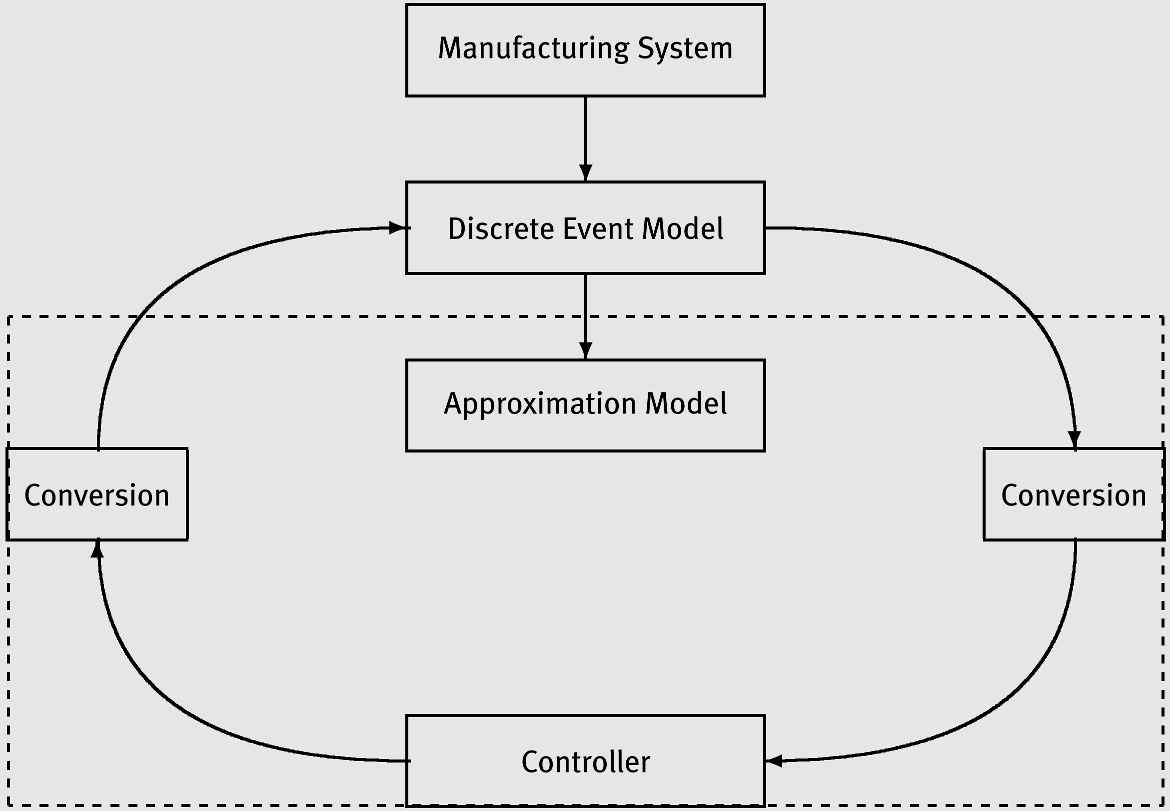

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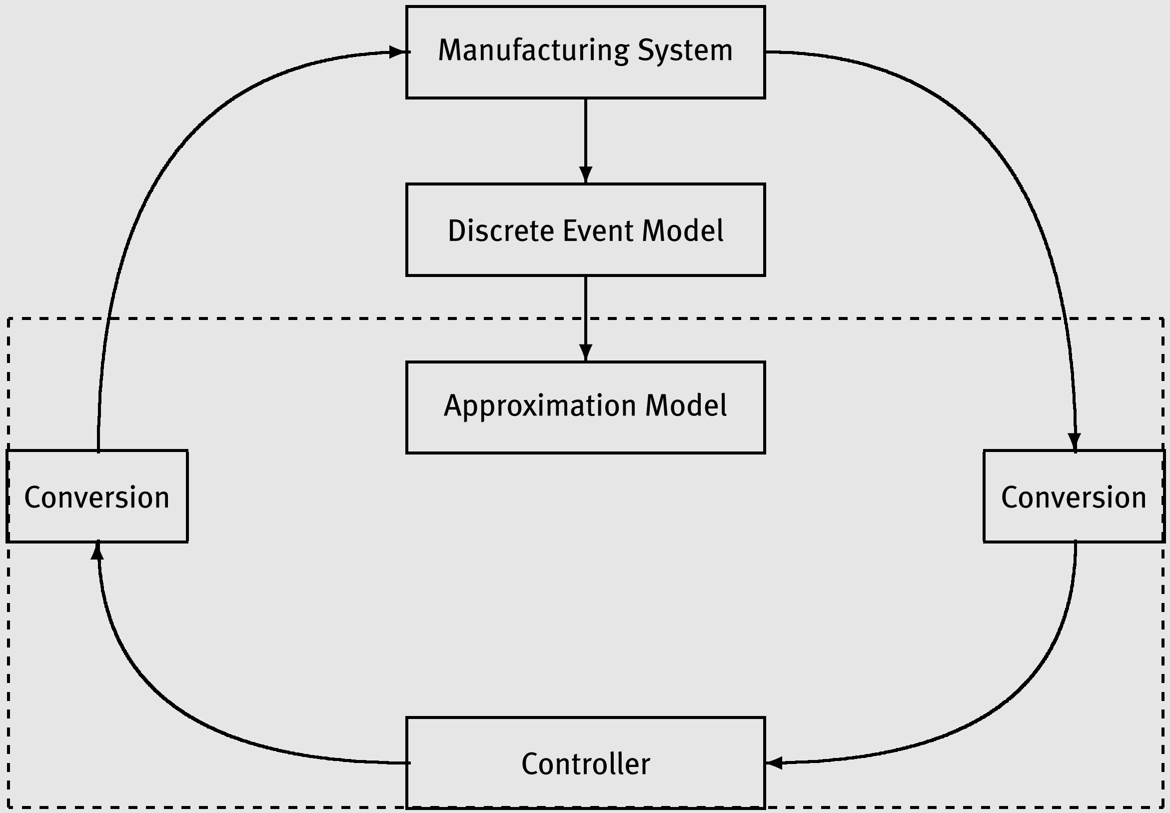

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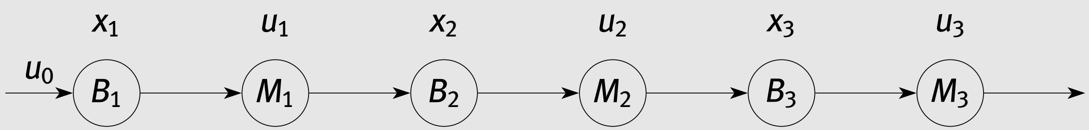
Approximation model



$$x_1(k+1) = x_1(k) + u_0(k) - u_1(k)$$

$$x_2(k+1) = x_2(k) + u_1(k) - u_2(k)$$

$$x_3(k+1) = x_3(k) + u_2(k) - u_3(k)$$

or

$$\dot{x}_1(t) = u_0(t) - u_1(t) \qquad\qquad \dot{x}_1(t) = u_0(t) - u_1(t)$$

$$\dot{x}_2(t) = u_1(t) - u_2(t) \qquad \text{or} \qquad \dot{x}_2(t) = u_1(t-\tau_1) - u_2(t)$$

$$\dot{x}_3(t) = u_2(t) - u_3(t) \qquad\qquad \dot{x}_3(t) = u_2(t-\tau_2) - u_3(t)$$

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Modeling manufacturing flow

- density $\rho(x, t)$,
- speed $v(x, t)$,
- flow $u(x, t) = \rho(x, t)v(x, t)$,
- Conservation of mass: $\frac{\partial \rho}{\partial t}(x, t) + \frac{\partial \rho v}{\partial x}(x, t) = 0$.
- Boundary condition: $u(0, t) = \lambda(t)$

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Modeling manufacturing flow

Armbruster, Marthaler, Ringhofer (2002):

- Single queue: $\frac{1}{v(x,t)} = \frac{1}{\mu}(1 + \int_0^1 \rho(s,t)\,\mathrm{d}s)$
- Single queue: $\frac{\partial \rho v}{\partial t}(x,t) + \frac{\partial \rho v^2}{\partial x}(x,t) = 0$

$$\rho v^2(0,t) = \frac{\mu \cdot \rho v(0,t)}{1 + \int_0^1 \rho(s,t)\,\mathrm{d}s}$$

- Re-entrant: $v(x,t) = v_0 \left(1 - \frac{\int_0^1 \rho(s,t)\,\mathrm{d}s}{W_{\max}}\right)$
- Re-entrant: $\frac{\partial \rho v}{\partial t}(x,t) + \frac{\partial \rho v^2}{\partial x}(x,t) = 0$

$$\rho v^2(0,t) = \rho v(0,t) \cdot v_0 \left(1 - \frac{\int_0^1 \rho(s,t)\,\mathrm{d}s}{W_{\max}}\right)$$

Lefeber (2003):

- Line of m identical queues: $v(x,t) = \frac{\mu}{m + \rho(x,t)}$

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Validation

- Line of 15 identical machines
- Infinite queues
- FIFO-policy
- Exponential Effective Processing Times
- Step-response (initially empty, start rate λ)
- Model 1, 2, 5 versus averaged discrete event

Rampup to 50% utilization (averaged discrete event)

Rampup to 50% utilization (validation studies)

Rampup to 25% utilization (validation studies)

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Concluding remarks

Rampup to 50% with non-exponential arrivals ($c_a^2 = 9$)

- Correct steady state behavior
- Better description transient needed
- Second moment and correlation needs to be included

While keeping the following:

- No backward-flow allowed (cf. Daganzo '95)
- No negative density
- Stable steady states
  - i.e., constant feed rate $\longrightarrow$ equilibrium
  - also, equilibrium meets relations queueing theory

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- Number of machines $n = 10$
- Mean processing time: 0.5h
- Desired $u = 0.75$ (1.5 lot per h)
- Initial WIP $x_i(0) = 0$

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Nonlinear model

$$
\begin{aligned}
x_1(k+1) &= x_1(k) - \frac{\mu x_1(k)}{1+x_1(k)} + u(k) \\
x_2(k+1) &= x_2(k) - \frac{\mu x_2(k)}{1+x_2(k)} + \frac{\mu x_1(k)}{1+x_1(k)} \\
&\vdots \\
x_n(k+1) &= x_n(k) - \frac{\mu x_n(k)}{1+x_n(k)} + \frac{\mu x_{n-1}(k)}{1+x_{n-1}(k)}
\end{aligned}
$$

$$
y(k) = \frac{\mu x_n(k)}{1+x_n(k)}
$$

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MPC controller design

- Prediction horizon $p = 100$h
- Control horizon $p = 5$h
- Control constant over periods of 1h
- Time sampling: 40 steps per 1h

Cost function:

$$\min_{u} \sum_{i=0}^{p} ||y(k+i|k) - y_{\text{des}}||_Q^2$$

Constraints:

$$0 \leq u(k) \leq 2 \qquad 0 \leq \frac{\mu x_i(k)}{1 + x_i(k)} \leq 2$$

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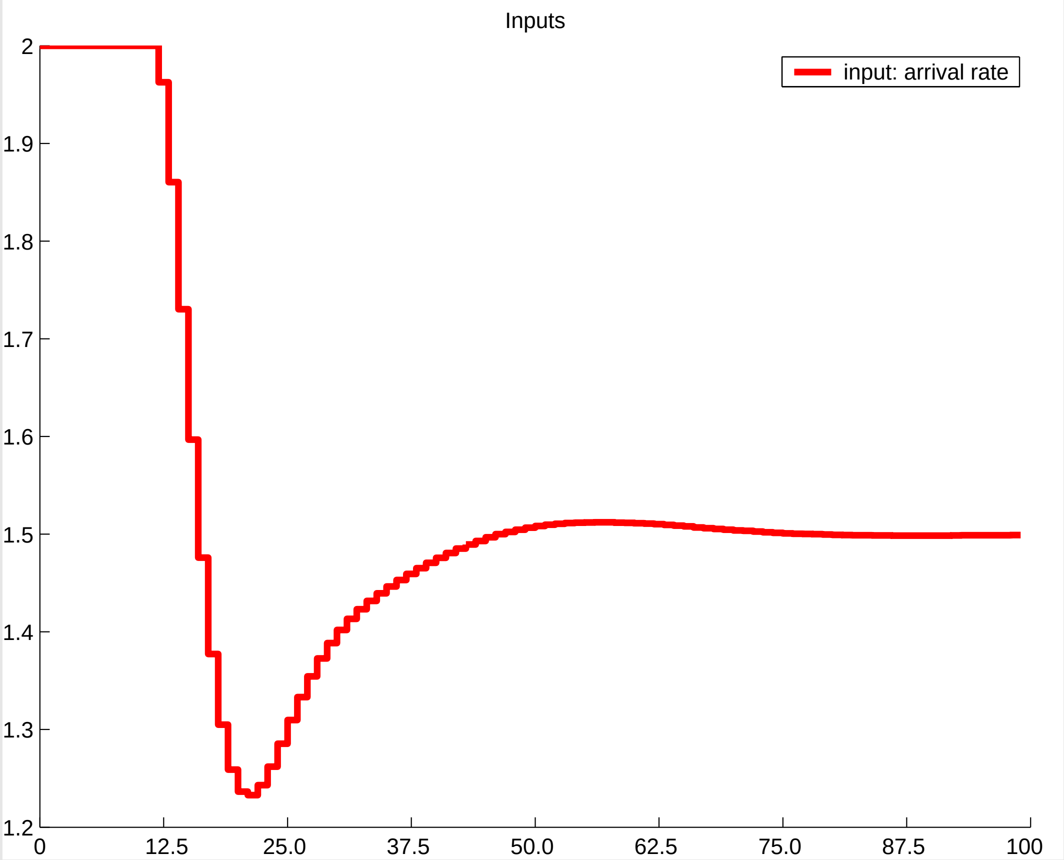
MPC based controller design



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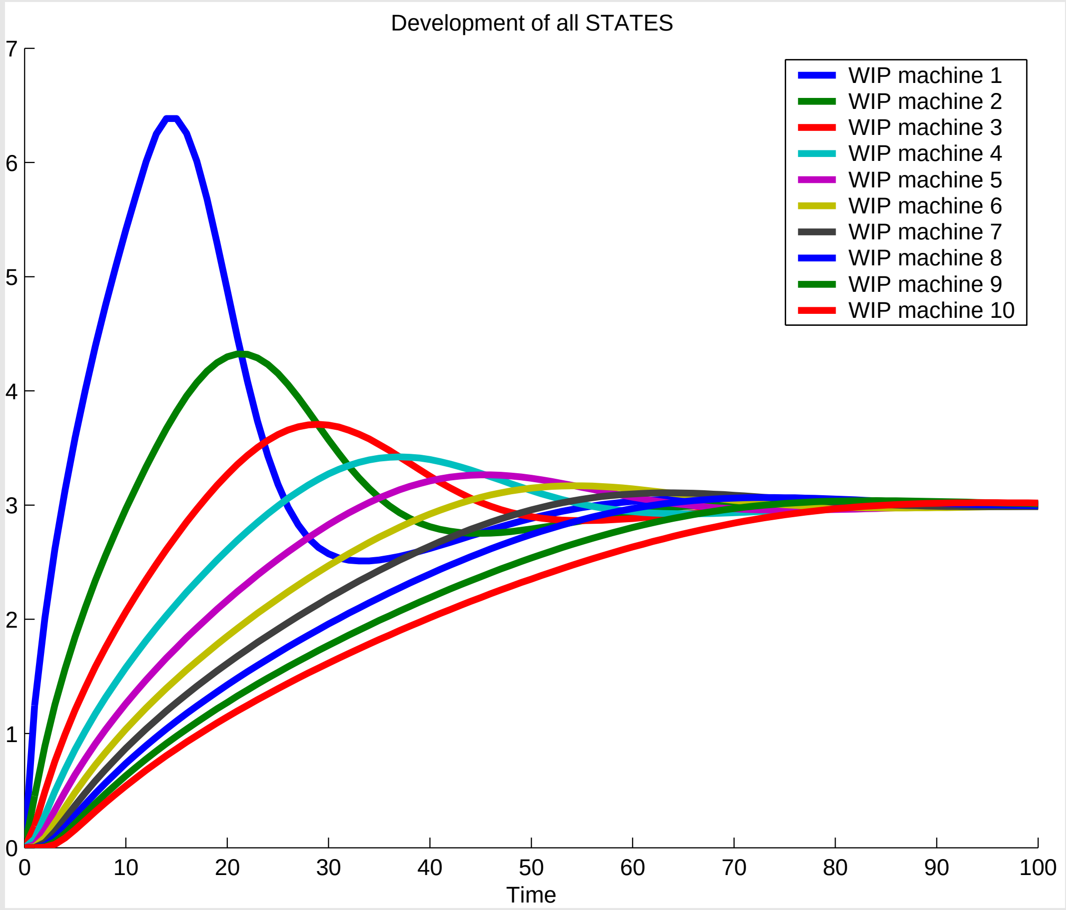

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Conclusions

- Shown how to use χ for building DE model.
- EPT can be used to get from real data to simple queueing network model
- Control framework
- Validation of PDE models
  - Transient needs improvement
  - Steady-state for changing variance
- MPC control seems promising