Modeling, Validation and Control of Manufacturing Systems

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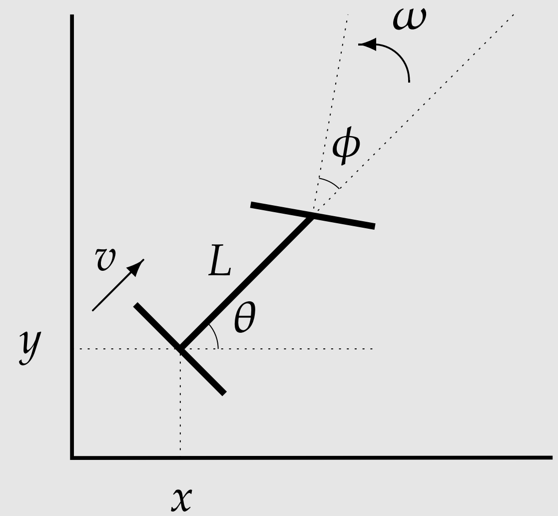
A control example

$$\dot{x} = v\cos\theta$$

$$\dot{y} = v\sin\theta$$

$$\dot{\theta} = \frac{v}{L}\tan\phi$$

$$\dot{\phi} = \omega$$



Identification

Apply some inputs, measure outputs $\Rightarrow$ determine L.

A control example

Trajectory planning

$$\dot{x}_r = v_r \cos\theta_r \qquad \dot{\theta}_r = \frac{v_r}{L}\tan\phi_r$$

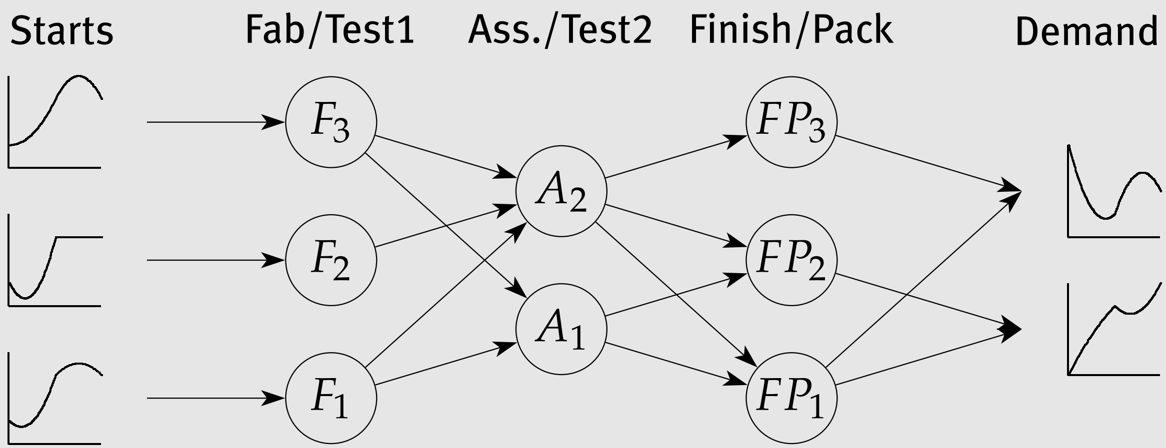
$$\dot{y}_r = v_r \sin\theta_r \qquad \dot{\phi}_r = \omega_r$$

Controller design

Find $v = v(t, x, y, \theta, \phi)$ and $\omega = \omega(t, x, y, \theta, \phi)$ such that for the resulting closed-loop system

$$\lim_{t\to\infty} |x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| + |\phi(t) - \phi_r(t)| = 0$$

Motivation



Modeling for control (supply chain/mass production).

- Like to understand dynamics of factories.
- Throughput, throughput time, variance of throughput time.
- Answer questions like: How to perform ramp up?

Modeling problem

Some observations from practice:

- Quick answers ("What if . . . ").
- A factory is (almost) never in steady state.
- Throughput and throughput time are related.

We look for a model that

- is computationally feasible,
- describes dynamics, and
- incorporates both throughput and throughput time.

First step

Model each manufacturing system as a network of queues.

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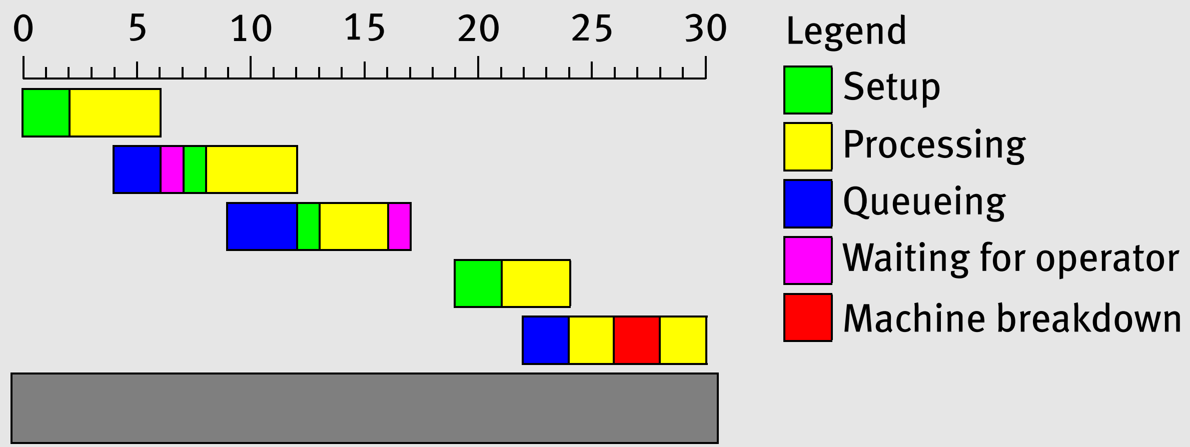
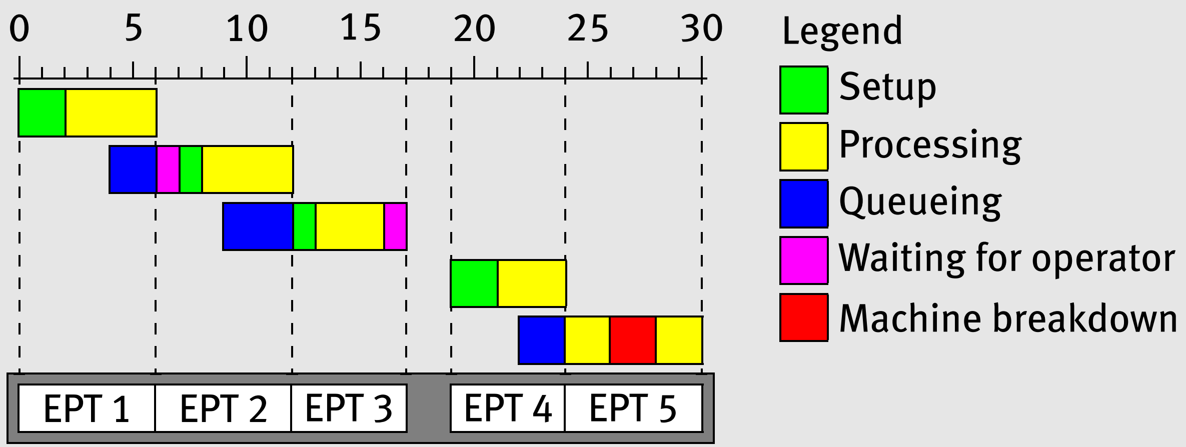
Some data



First step

Model each manufacturing system as a network of queues.

Identification: EPT's



Available models: Discrete Event

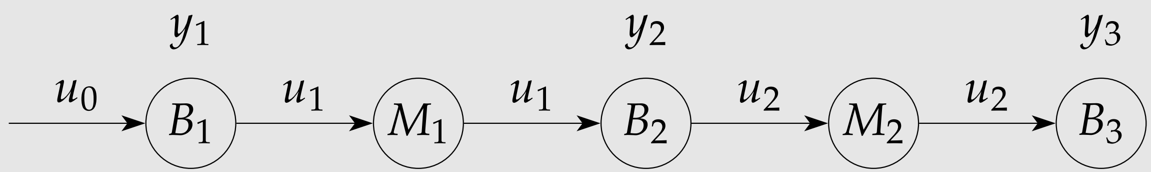
- Advantages
  - Include dynamics
  - Throughput and throughput time related
- Disadvantage
  - Not computationally feasible for real life fab

Available models: Queuing Theory

- Advantages
  - Throughput and throughput time related
  - Computationally feasible (approximations)
- Disadvantage
  - Mainly steady state, almost no dynamics

Available models: Fluid models

- Kimemia and Gershwin: Flow model
- Queuing theorists: Fluid models/Fluid queues



$$\dot{y}_1 = u_0 - u_1$$

$$\dot{y}_2 = u_1 - u_2$$

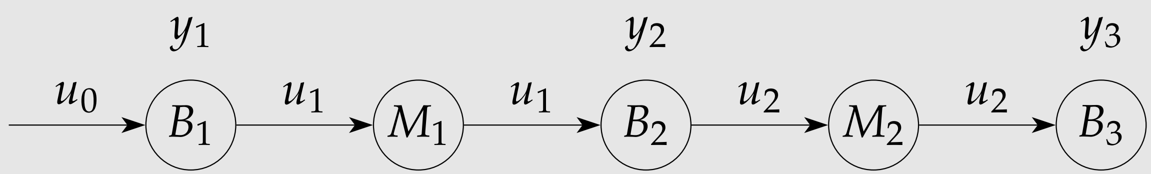
$$\dot{y}_3 = u_2$$

- Cassandras: Stochastic Fluid Model

Available models: Fluid models

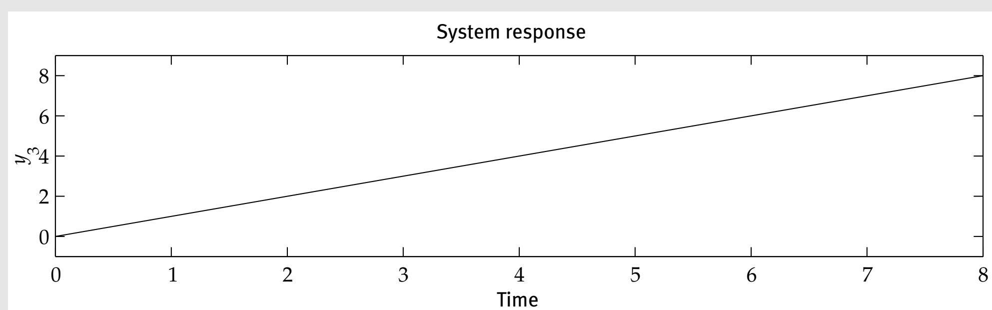
- Advantages
  - Dynamical model
  - Computationally feasible
- Disadvantage
  - Only throughput incorporated in model, no throughput time
  - And more ...

Example: ramp up of fluid model



- Initially empty fab, $u_0 = \lambda = 1$, $\mu_1 = \mu_2 = 1$.
- Machine produces whenever possible:

$$u_i = \begin{cases} \mu_i & \text{if } y_i > 0 \\ 0 & \text{otherwise} \end{cases} \qquad i \in \{1, 2\}.$$



Available models (conclusion)

- Discrete Event: Not computationally feasible

  Queuing Theory: No dynamics

  Fluid models: No throughput time
- Need something else!
- Discrete event models (and queuing theory) have proved themselves. Can be used for verification!

Traffic flow: LWR model

Lighthill, Whitham ('55), and Richards ('56)

Traffic behavior on one-way road:

- density $\rho(x, t)$,
- speed $v(x, t)$,
- flow $u(x, t) = \rho(x, t)v(x, t)$.

Conservation of mass:

$$\frac{\partial \rho}{\partial t}(x, t) + \frac{\partial u}{\partial x}(x, t) = 0.$$

Static relation between flow and density:

$$u(x, t) = S(\rho(x, t)).$$

Modeling manufacturing flow

- density $\rho(x,t)$,
- speed $v(x,t)$,
- flow $u(x,t) = \rho(x,t)v(x,t)$,
- conservation of mass: $\frac{\partial \rho}{\partial t}(x,t) + \frac{\partial \rho v}{\partial x}(x,t) = 0$.
- Boundary condition: $u(0,t) = \lambda(t)$

Modeling manufacturing flow

Armbruster, Marthaler, Ringhofer (2002):

- Single queue: $\frac{1}{v(x,t)} = \frac{1}{\mu}(1 + \int_0^1 \rho(s,t)\,\mathrm{d}s)$
- Single queue: $\frac{\partial \rho v}{\partial t}(x,t) + \frac{\partial \rho v^2}{\partial x}(x,t) = 0$

$$\rho v^2(0,t) = \frac{\mu \cdot \rho v(0,t)}{1+\int_0^1 \rho(s,t)\,\mathrm{d}s}$$

- Re-entrant: $v(x,t) = v_0 \left(1 - \frac{\int_0^1 \rho(s,t)\,\mathrm{d}s}{W_{\max}}\right)$
- Re-entrant: $\frac{\partial \rho v}{\partial t}(x,t) + \frac{\partial \rho v^2}{\partial x}(x,t) = 0$

$$\rho v^2(0,t) = \rho v(0,t) \cdot v_0 \left(1 - \frac{\int_0^1 \rho(s,t)\,\mathrm{d}s}{W_{\max}}\right)$$

Lefeber (2003):

- Line of m identical queues: $v(x,t) = \frac{\mu}{m+\rho(x,t)}$

Validation

- Line of 15 identical machines.
- Infinite queues.
- FIFO-policy.
- Exponential Effective Processing Times.
- Step-response (initially empty, start rate λ).
- Model 1, 2, 5 versus averaged discrete event.

Show movie

MPC based controller design

Approximation model (nonlinear)

$$x_1(k+1) = x_1(k) - \frac{\mu x_1(k)}{m + x_1(k)} + \lambda_{\text{in}}(k)$$

$$x_2(k+1) = x_2(k) - \frac{\mu x_2(k)}{m + x_2(k)} + \frac{\mu x_1(k)}{m + x_1(k)}$$

$$\vdots$$

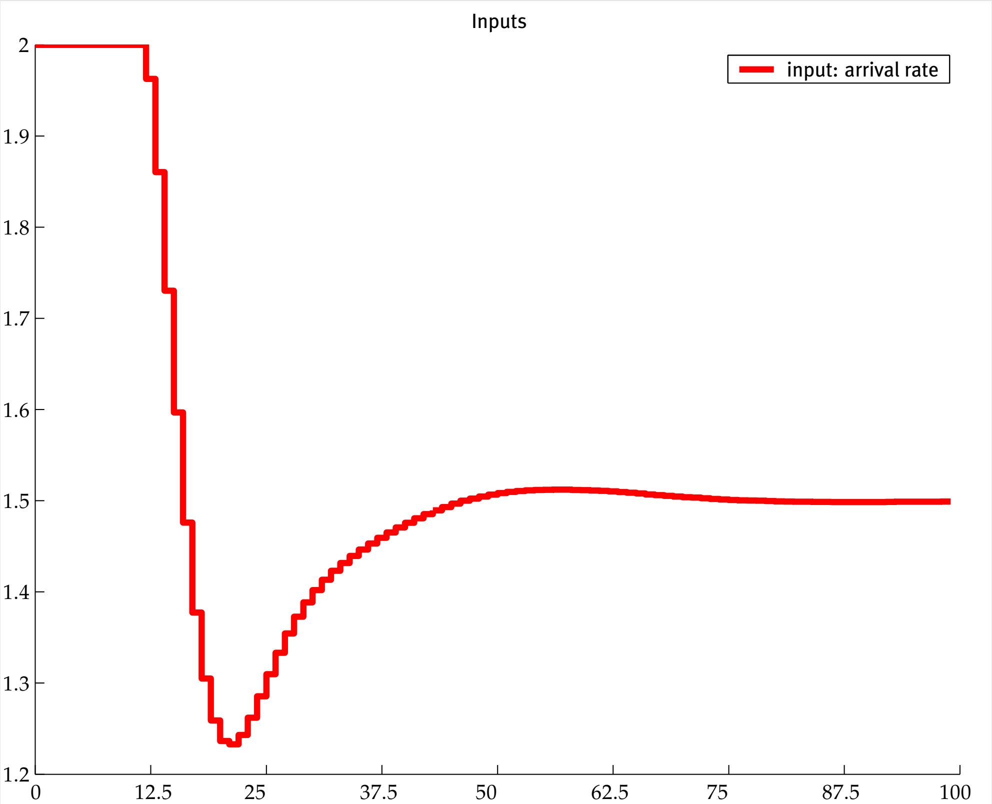
$$x_m(k+1) = x_m(k) - \frac{\mu x_m(k)}{m + x_m(k)} + \frac{\mu x_{m-1}(k)}{m + x_{m-1}(k)}$$

$$y(k) = \frac{\mu x_m(k)}{m + x_m(k)}$$

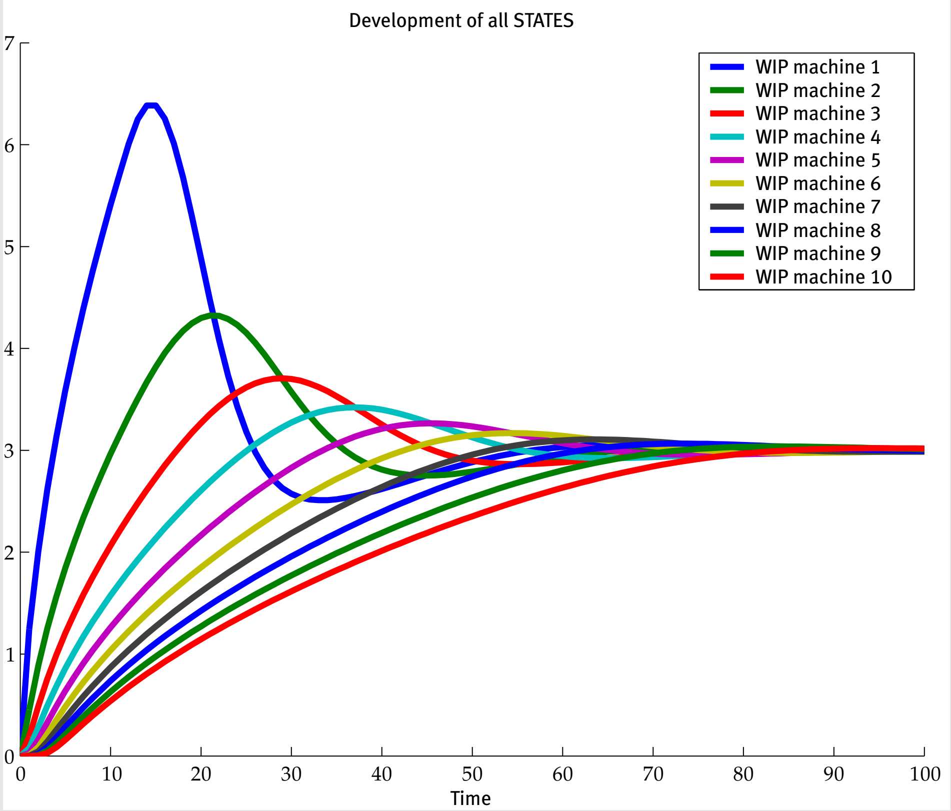
MPC based controller design

- Number of machines $m = 10$
- Mean processing time: 0.5h
- Desired $u = 0.75$ (1.5 lot per h)
- Initial WIP $x_i(0) = 0$
- Prediction horizon $p = 100$h
- Control horizon $c = 5$h
- Control constant over periods of 1h
- Time sampling: 40 steps per 1h

MPC based controller design



MPC based controller design



Conclusions

Need for computationally feasible dynamical models incorporating both throughput and throughput time.

- NOT: Discrete event, Queueing theory, Fluid models
- Possible: PDE-models
  - Correct steady state behavior
  - Better description transient needed
  - Queueing theory, discrete event models can be used for validation of PDE models
- PDE-based controller design (boundary control)