TU/e technische universiteit eindhoven

Control of Manufacturing Systems

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Intel (afternoon)

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/department of mechanical engineering

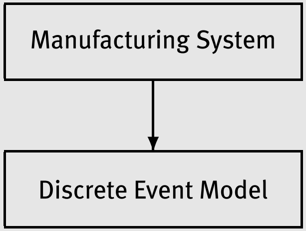
Outline

- Control Framework
- Case I
- Case II

Control Framework

Manufacturing System

Control Framework

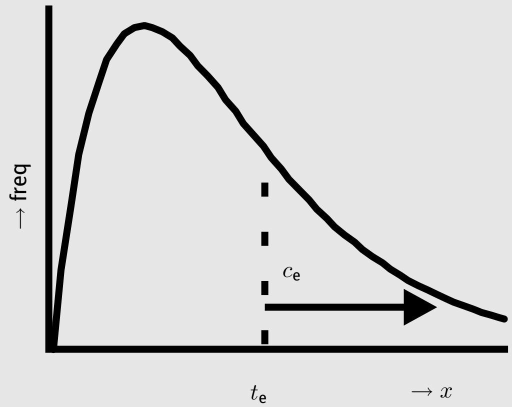


The effective process time method

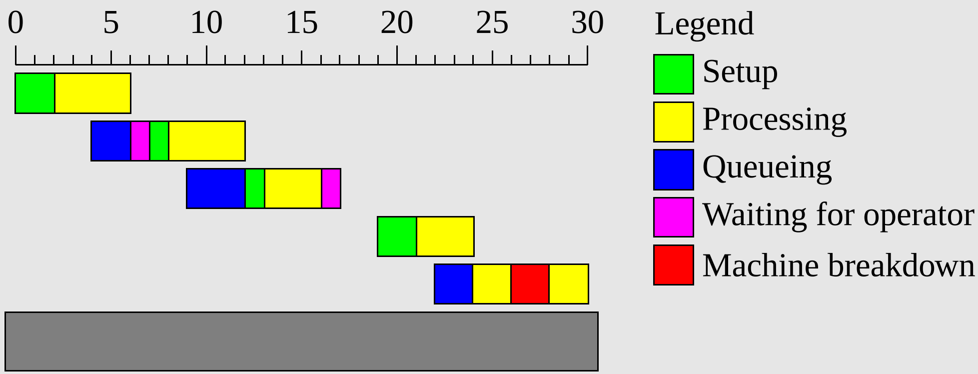
- raw process time t_0 and c_0
- setups t_s and c_s
- TBF t_f and c_f, TTR t_r and c_r
- operator delays
- rework
- . . . (!)

Idea:

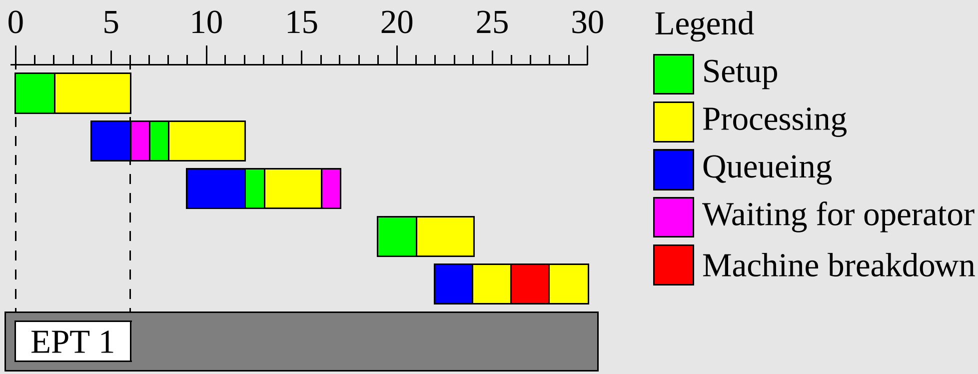
Combine all disturbances in one single EPT probability density function



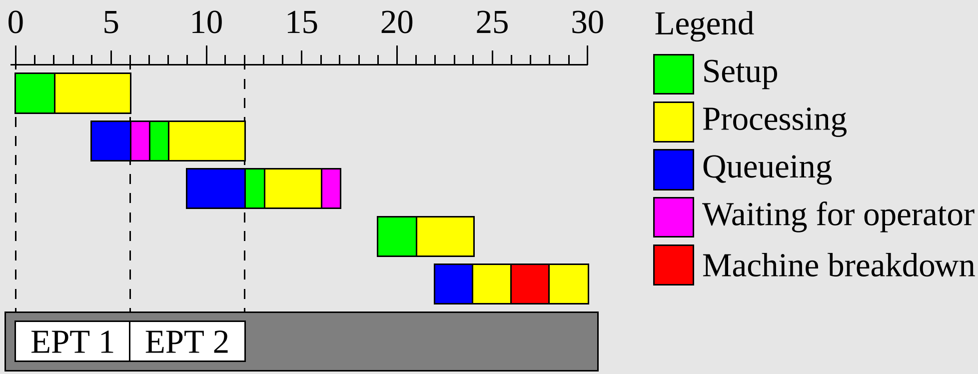
Effective Processing Times



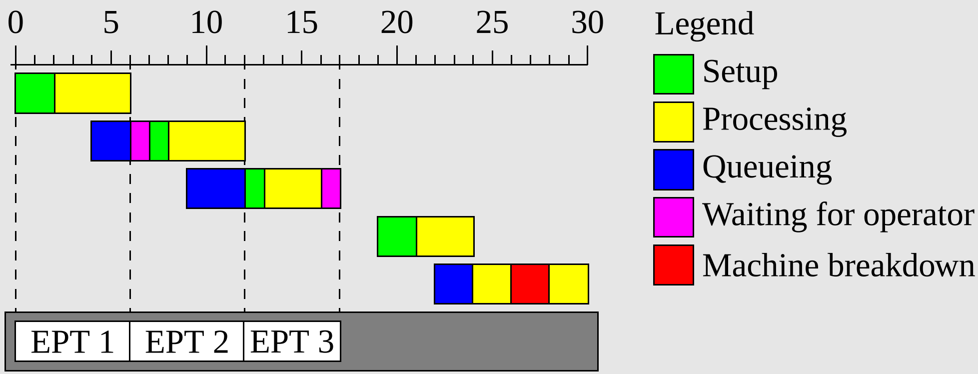
Effective Processing Times



Effective Processing Times



Effective Processing Times



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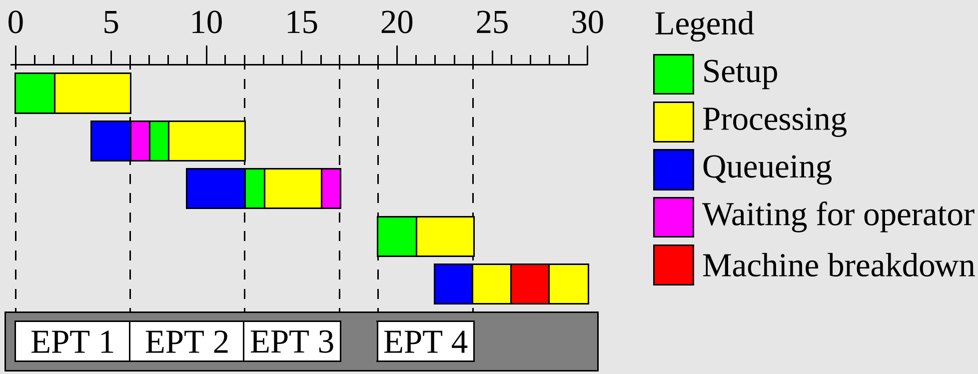

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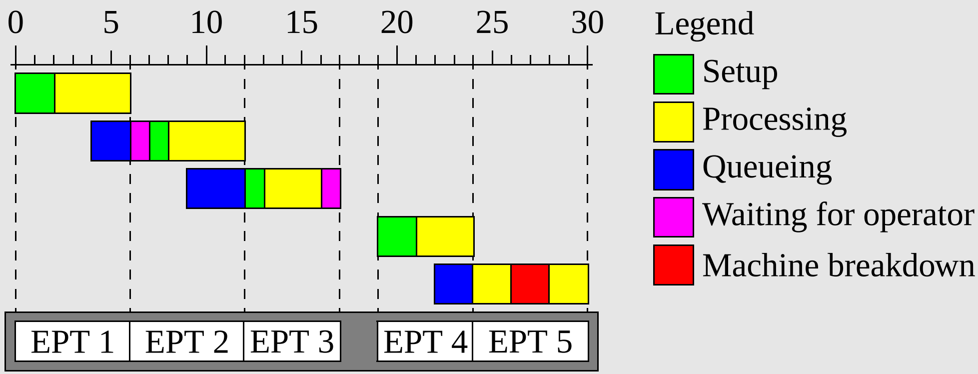
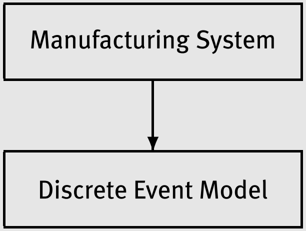
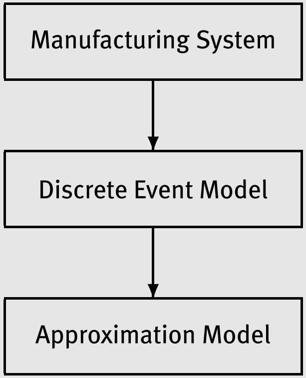


Control Framework



Control Framework



Approximation model

$$
\begin{tikzcd}
 & x_1 & u_1 & x_2 & u_2 & x_3 & u_3 & \\
{} \arrow[r, "u_0"] & B_1 \arrow[r] & M_1 \arrow[r] & B_2 \arrow[r] & M_2 \arrow[r] & B_3 \arrow[r] & M_3 \arrow[r] & {}
\end{tikzcd}
$$

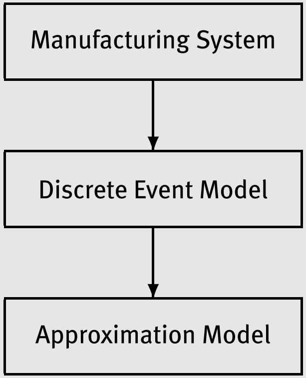
$$
\begin{aligned}
x_1(k+1) &= x_1(k) + u_0(k) - u_1(k)\\
x_2(k+1) &= x_2(k) + u_1(k) - u_2(k)\\
x_3(k+1) &= x_3(k) + u_2(k) - u_3(k)
\end{aligned}
$$

or

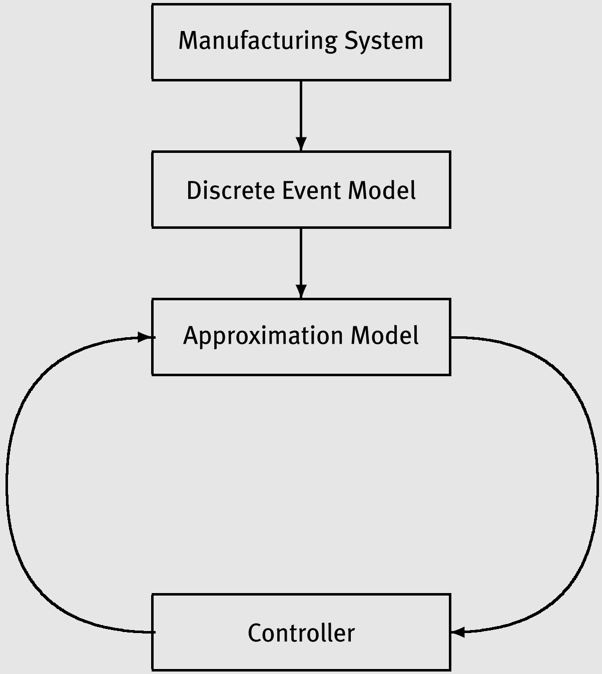
$$\frac{\partial \rho}{\partial t}(x,t) + \frac{\partial u}{\partial x}(x,t) = 0$$

$$u(x,t) = S(\rho(x,t))$$

Control Framework



Control Framework



Model Predictive Control (MPC)

Model:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned}$$

Reference: $y_{\text{ref}}(k) \qquad k = 1, 2, 3, \ldots$

Control horizon:

$$u(k), u(k+1), u(k+2), \ldots, u(k+c)$$

Prediction horizon:

$$y(k|k), y(k+1|k), y(k+2|k), \ldots y(k+c|k), \ldots y(k+p|k)$$

assuming $u(k+c+1) = \cdots = u(k+p)$

Model Predictive Control (MPC)

Cost function:

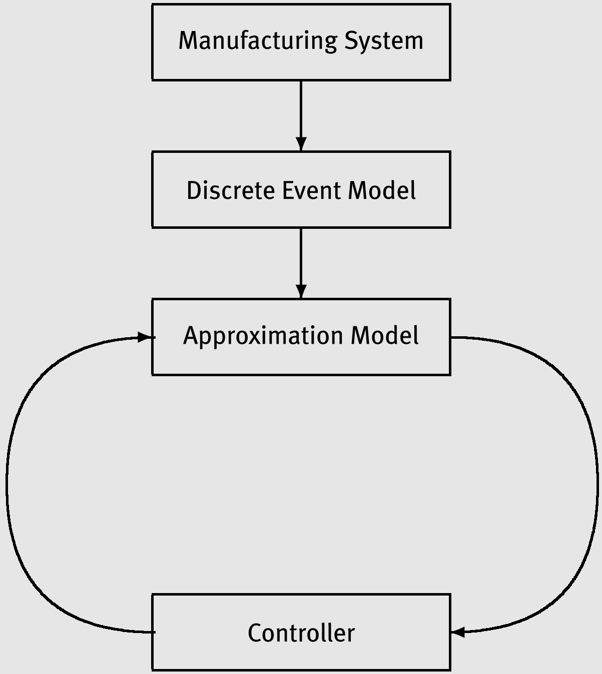
$$J(u) = \sum_{i=0}^{p} [y(k+i|k) - y_{\text{ref}}(k)]^T Q [y(k+i|k) - y_{\text{ref}}(k)]$$
$$+ \sum_{i=0}^{c} u(k+i)^T R u(k+i)$$

At each time instant k, solve

$$\min_{u(k),\ldots u(k+c)} J(u)$$

Implement $u(k)$
At next time instant, $k+1$, start all over again, i.e. $\min_{u(k+1),\ldots u(k+c+1)} J(u)$ and implement $u(k+1)$, etc.

Control Framework



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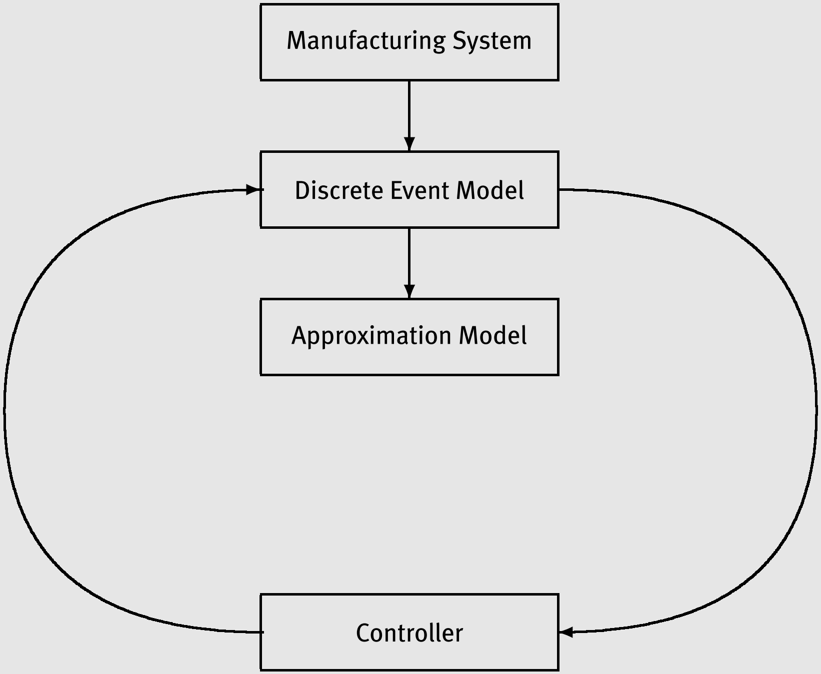
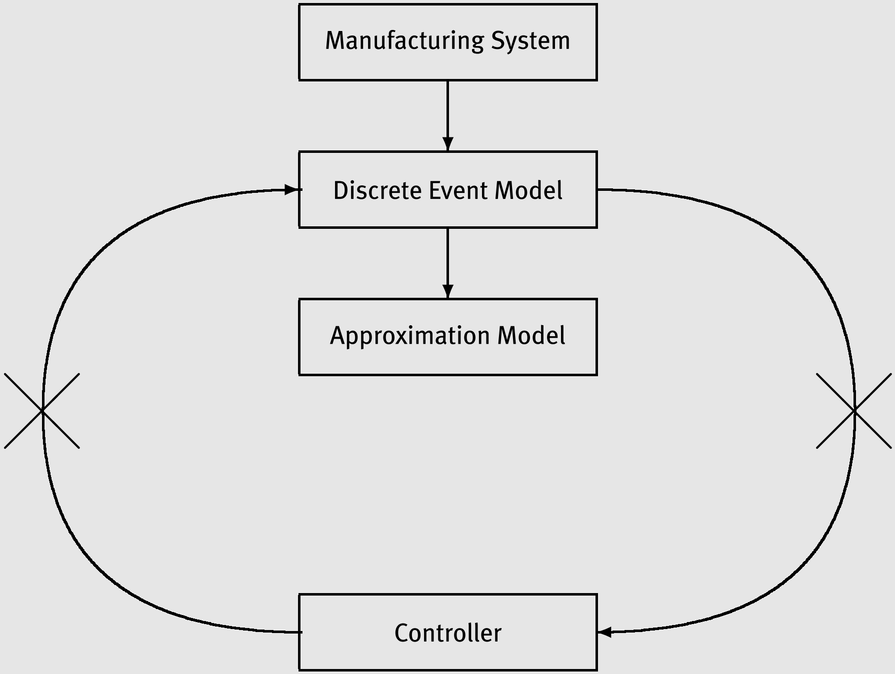
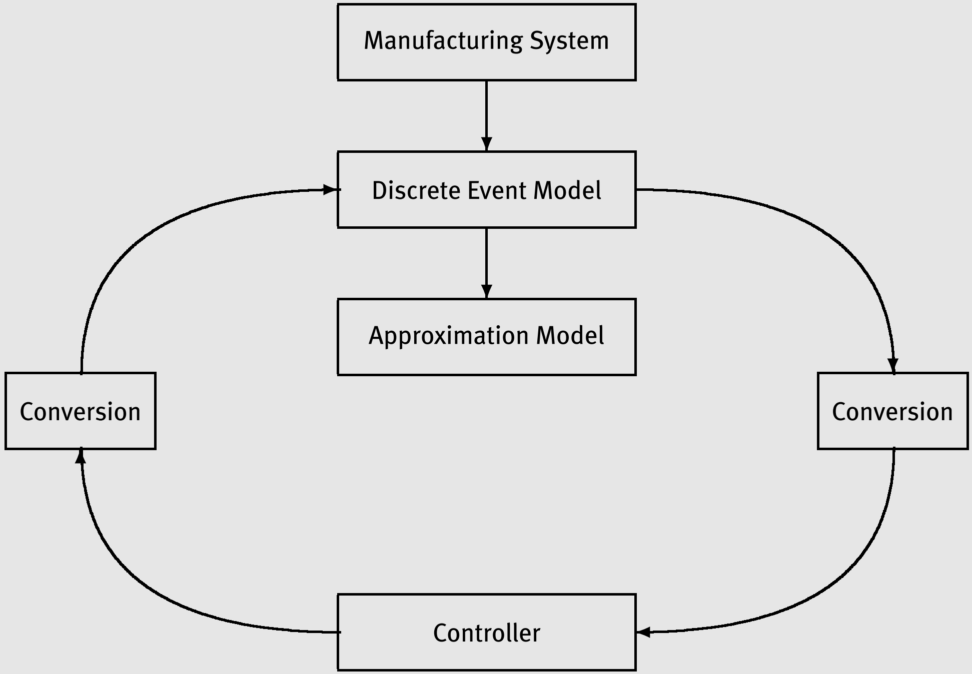


Control Framework



Control Framework



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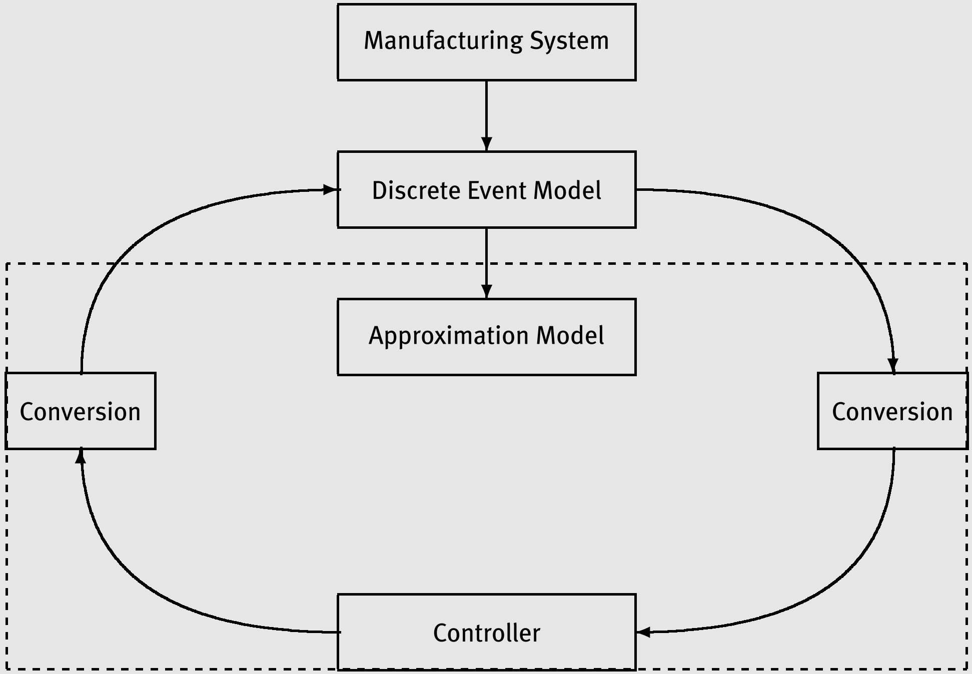

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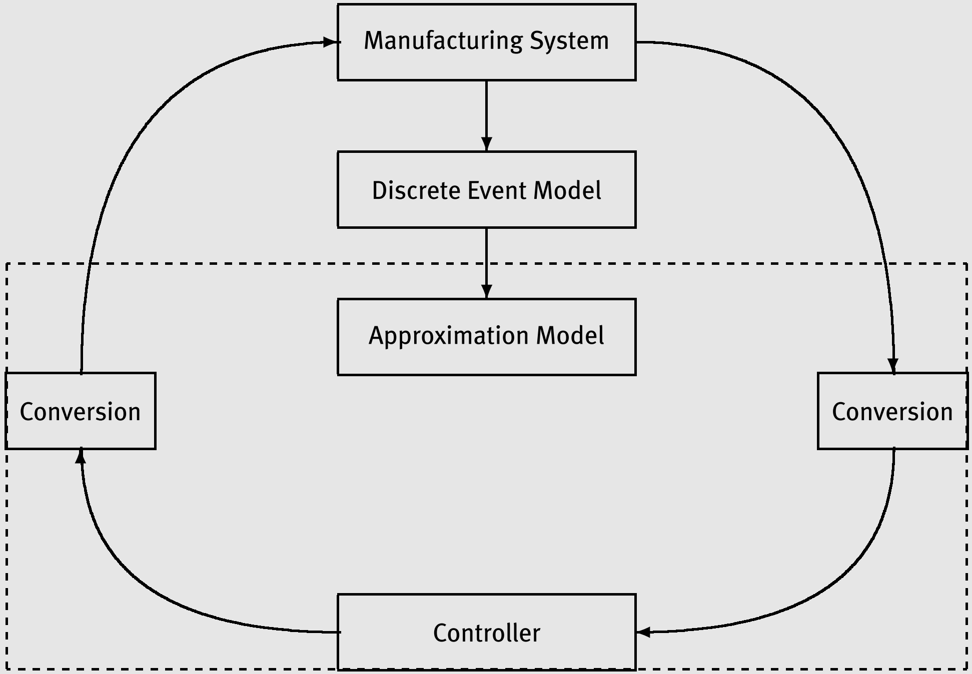

Outline

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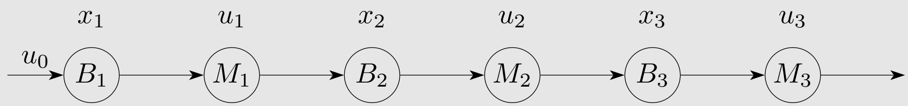
- Case II

Case I



- Three machine flowline
- Eight product types (equal demand)
- Infinite buffers
- No setup, no breakdown, no re-entrance
- Exponential processing times (mean 1.0)

MPC formulation

Model:

$$
\begin{aligned}
x_1(k+1) &= x_1(k) + u_0(k) - u_1(k)\\
x_2(k+1) &= x_2(k) + u_1(k) - u_2(k)\\
x_3(k+1) &= x_3(k) + u_2(k) - u_3(k)
\end{aligned}
$$

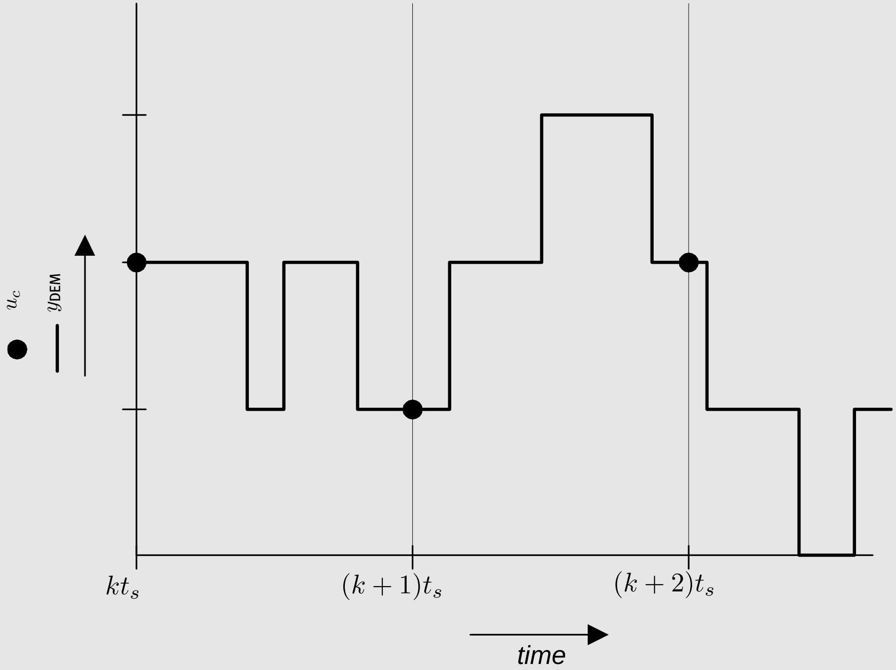
Cost function:

$$
\min_u \sum_{i=1}^{p} ||x(k+i|k) - x_{\mathsf{ref}}(k+i)||_Q^2 + \sum_{i=0}^{H_u-1} ||\Delta u(k)||_R^2
$$

with

$$
\Delta u(k) = u(k) - u(k-1)
$$

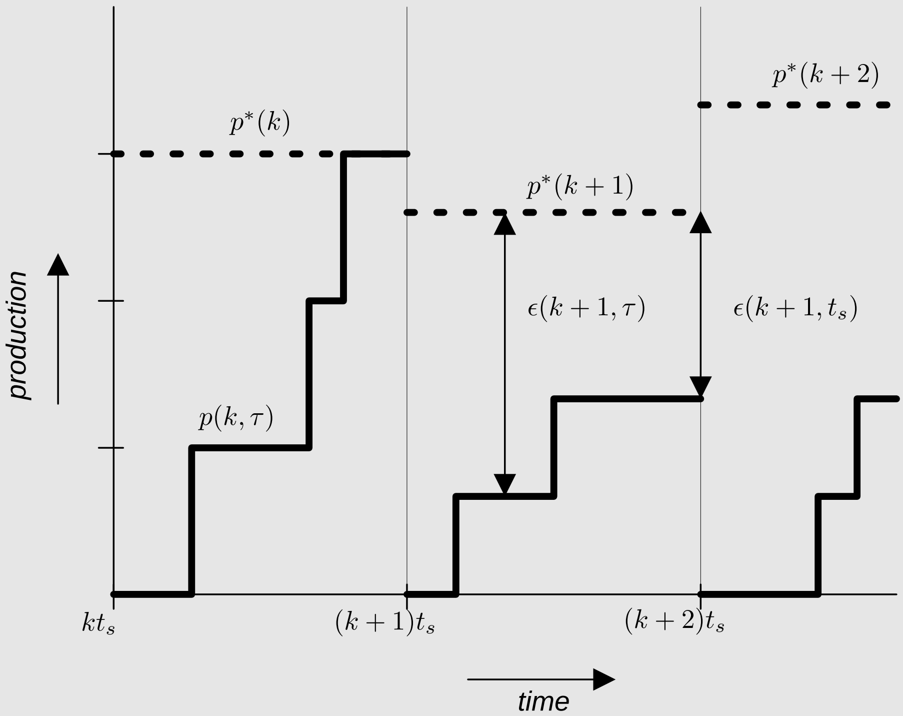
Conversion E/D



Conversion D/E

- Process until target reached (provided lots available)
- Define backlog: $\epsilon(k,\tau) = p^*(k) - p(k,\tau)$ where p^* denotes target, k denotes period, τ denotes time
- Take product type with largest backlog
- If $\epsilon <$ bound: target assumed to be met
- New target: $p^*(k) = u(k) \cdot t_s + \alpha \cdot \epsilon(k-1, t_s)$

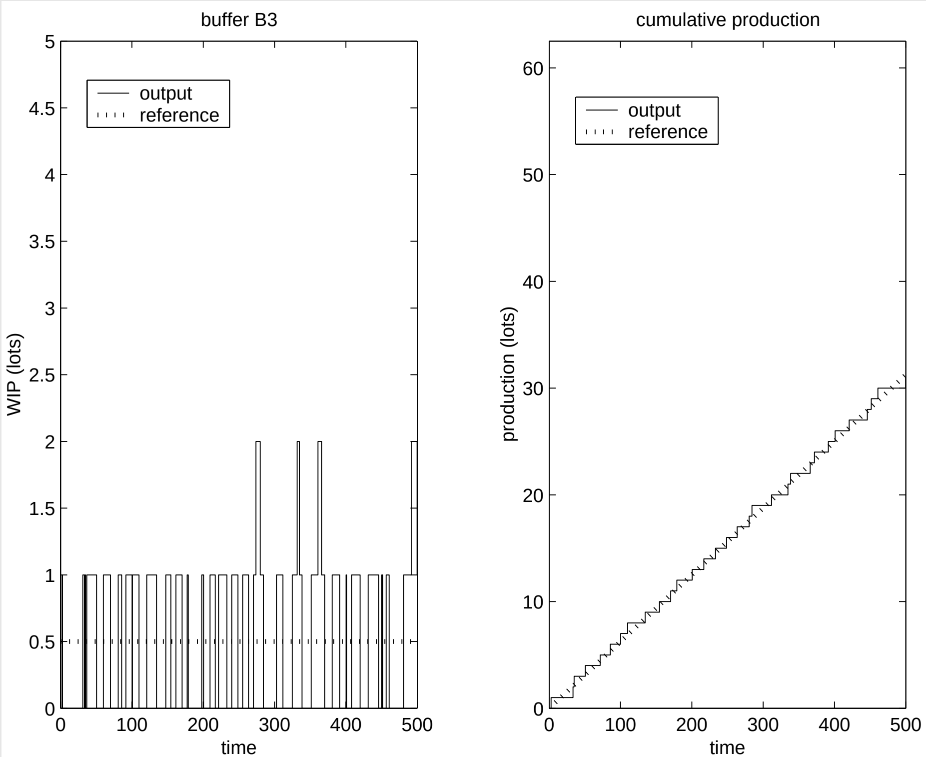
Conversion D/E



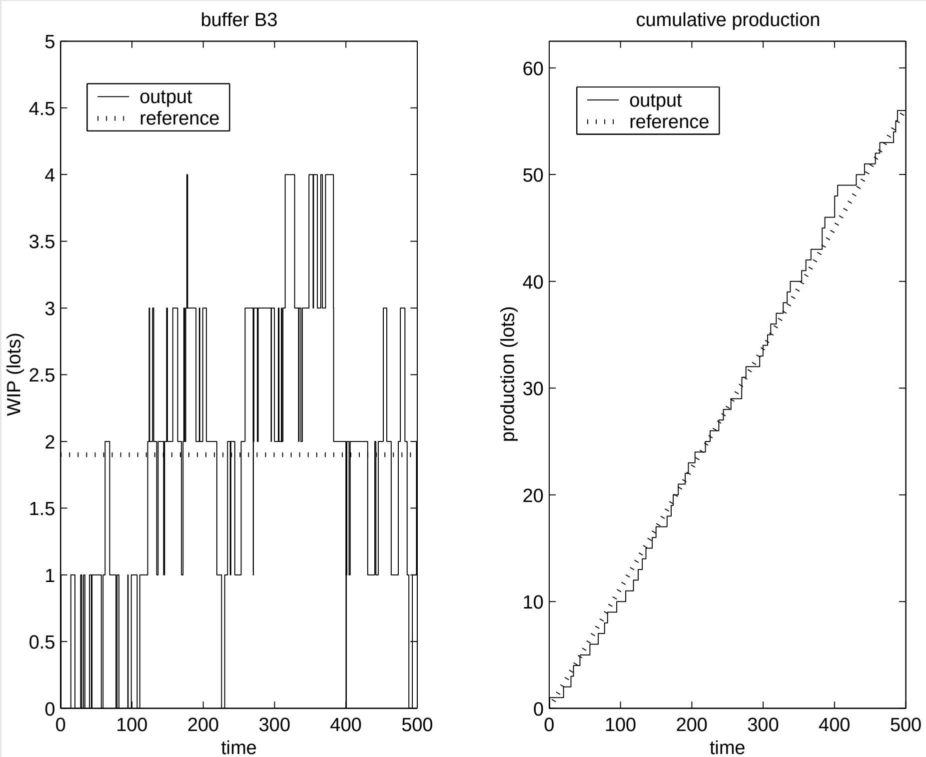
Experiments

H_u	control horizon	3 (samples)
H_p	prediction horizon	3 (samples)
H_w	penalty start	1 (sample)
$Q : R$	weighing ratio	1:8
t_s	sample time	10 (units time)
α	filtering factor for conversion D/E	0.3
ϵ^t	production threshold	0.3

Experiment: $u = 0.5$



Experiment: $u = 0.9$



Case II

- Number of machines $n = 10$
- Mean processing time: 0.5h
- Desired $u = 0.75$ (1.5 lot per h)
- Initial WIP $x_i(0) = 0$

Nonlinear model

$$x_1(k+1) = x_1(k) - \frac{\mu x_1(k)}{1+x_1(k)} + u(k)$$

$$x_2(k+1) = x_2(k) - \frac{\mu x_2(k)}{1+x_2(k)} + \frac{\mu x_1(k)}{1+x_1(k)}$$

$$\vdots$$

$$x_n(k+1) = x_n(k) - \frac{\mu x_n(k)}{1+x_n(k)} + \frac{\mu x_{n-1}(k)}{1+x_{n-1}(k)}$$

$$y(k) = \frac{\mu x_n(k)}{1+x_n(k)}$$

MPC controller design

- Prediction horizon $p = 100$h
- Control horizon $p = 5$h
- Control constant over periods of 1h
- Time sampling: 40 steps per 1h

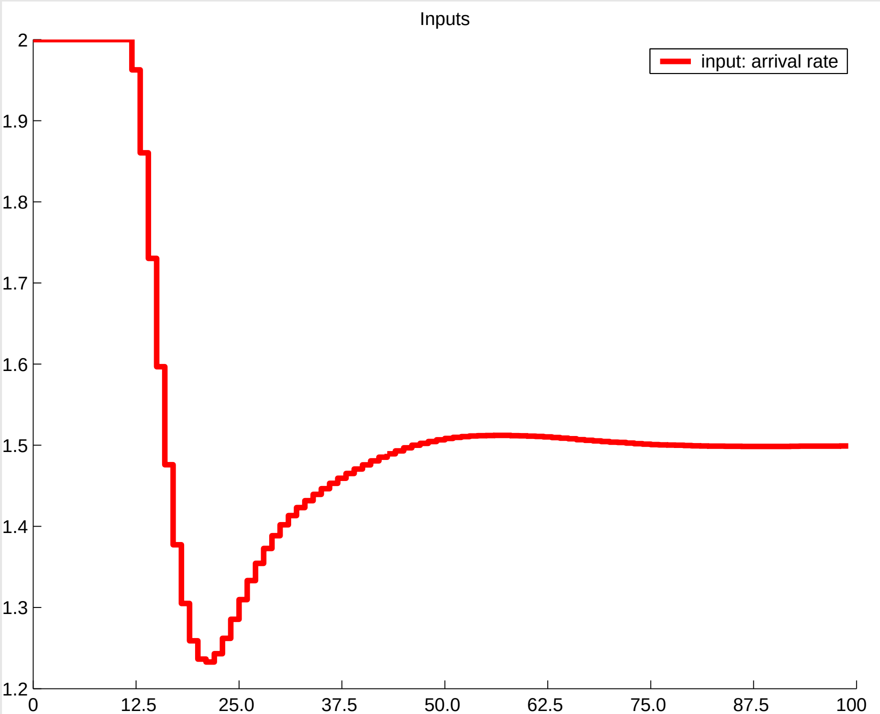
Cost function:

$$\min_u \sum_{i=0}^{p} ||y(k+i|k) - y_{\text{des}}||_Q^2$$

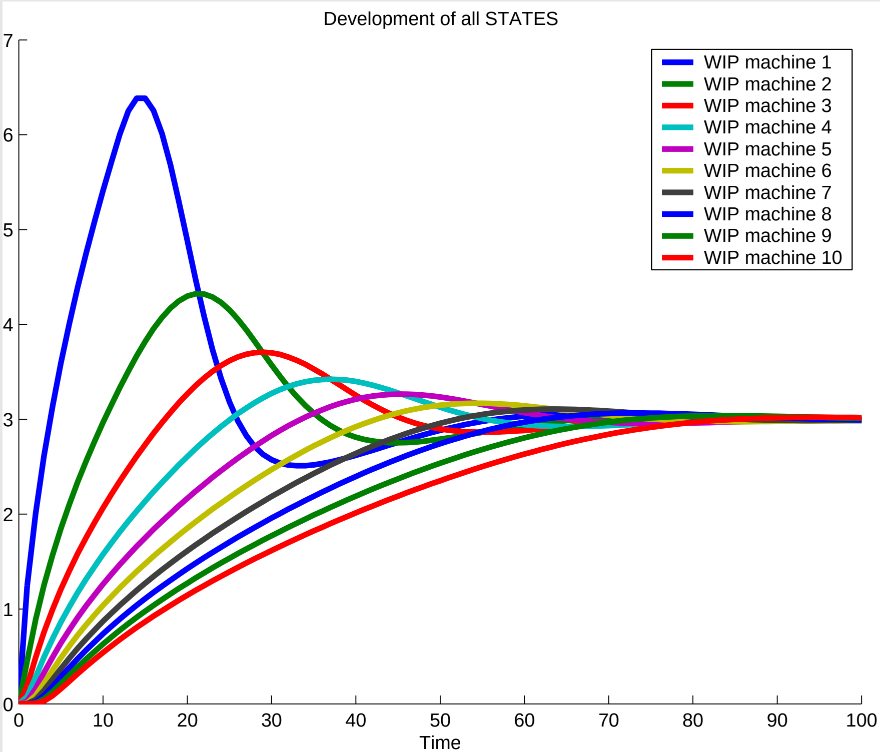
Constraints:

$$0 \leq u(k) \leq 2 \qquad 0 \leq \frac{\mu x_i(k)}{1 + x_i(k)} \leq 2$$

MPC based controller design



MPC based controller design



Conclusions

- Introduced effective processing times for obtaining simpler meta-models
- Control framework enables use of “standard control techniques” for controlling manufacturing system
- Case studies shows proof of concept (controlling discrete event model)