Globally bounded tracking controllers for rigid robot systems

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Outline

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- Problem formulation
- Composite controllers
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Rigid robot manipulator

A rigid robot manipulator with revolute joints can be described as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \qquad (1)$$

q	$n \times 1$ vector of joint displacements
τ	$n \times 1$ vector of applied torques
$M(q)$	$n \times n$ manipulator inertia matrix
$C(q,\dot{q})\dot{q}$	$n \times 1$ vector of centripetal and Coriolis torques
$G(q)$	$n \times 1$ vector of gravitational torques

Problem formulation

Design a controller for τ that globally steers the system

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

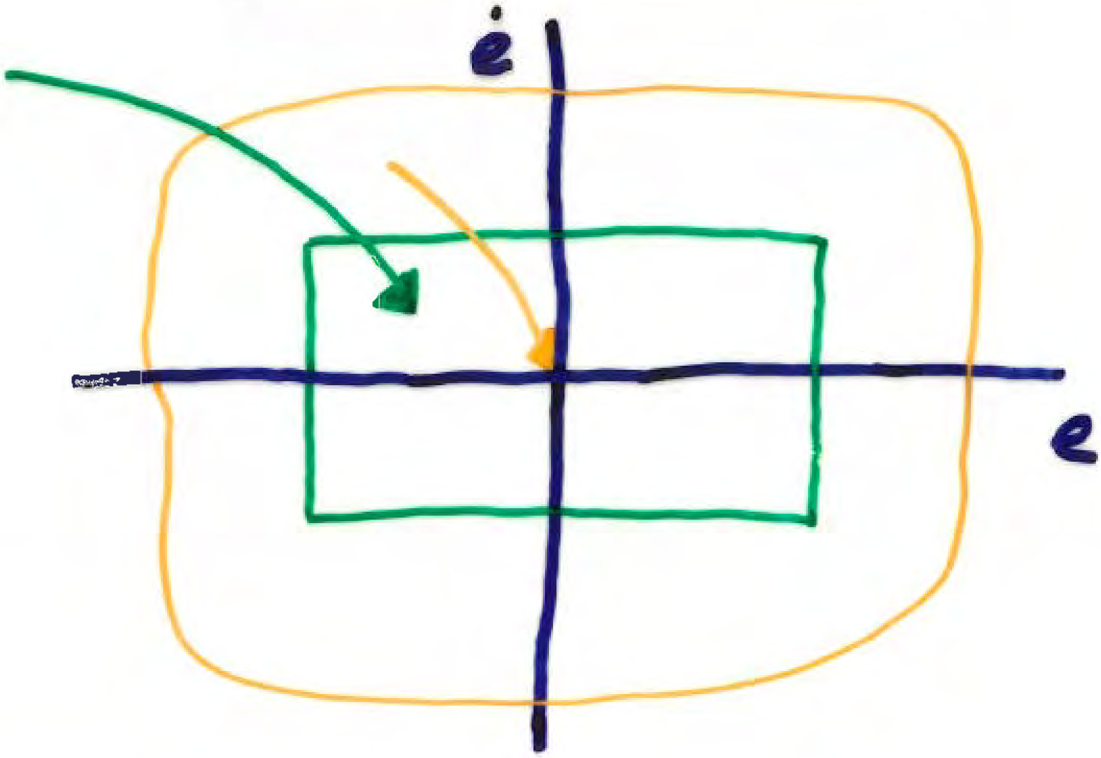
towards any desired trajectory $q_d(t) \in C^2$ under the input limitations

$$|\tau_i(t)| \leq \tau_{max,i}$$

provided q_d satisfies

$$||q_d(t)|| \leq B_0 \quad ||\dot{q}_d(t)|| \leq B_1 \quad ||\ddot{q}_d(t)|| \leq B_2$$

Composite controllers



Approach used:

- Find tracking controller whose region of attraction contains the set

$$\{(e, \dot{e}) \mid \|e\| \leq B_0 \wedge \|\dot{e}\| \leq B_1\}$$

  where $e \equiv q - q_d$.

- Find globally bounded regulating controller.

- Combine both controllers.

The classes $\mathcal{F}^n$ and $\mathcal{B}^n$

Definition of $\mathcal{F}^n$:

All continuous functions $f : I\!R^n \to I\!R^n$ for which there exists an $F : I\!R^n \to I\!R$ such that

$$f(x) = \frac{\partial F}{\partial x}(x) = \begin{bmatrix} \frac{\partial F}{\partial x_1}(x_1, \ldots, x_n) \\ \vdots \\ \frac{\partial F}{\partial x_n}(x_1, \ldots, x_n) \end{bmatrix}$$

and for which $x^T f(x)$ is positive definite.

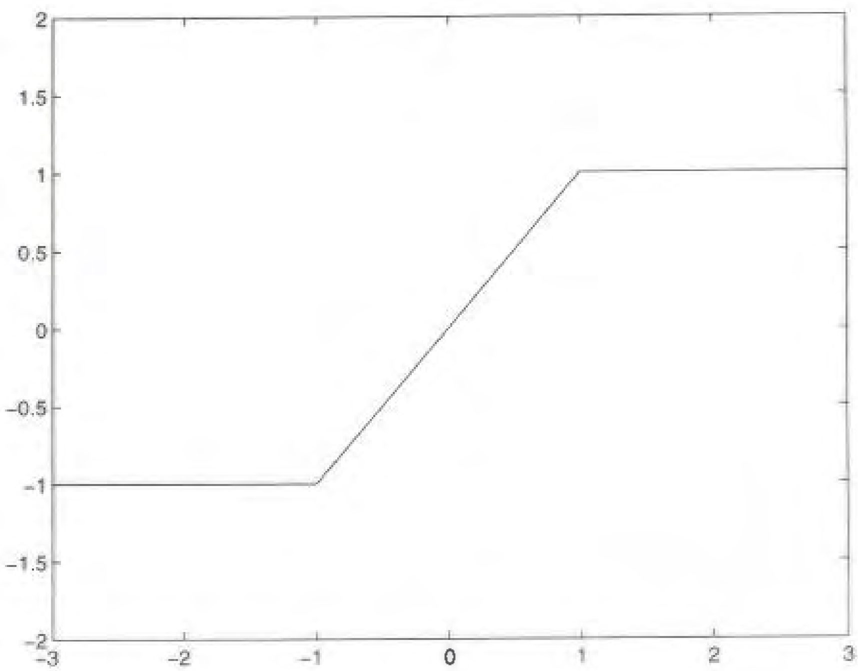
Definition of $\mathcal{B}^n$:

$$\mathcal{B}^n \triangleq \mathcal{F}^n \cap L_\infty$$

Examples:

- $f(x) = \Lambda[f_1(x), \ldots, f_n(x)]^T$ where f_i non-decreasing with $f_i(0) = 0$ and $\Lambda = \Lambda^T > 0$ diagonal.

- $f(x) = Kx$ where $K = K^T > 0$.



Globally tracking controllers

The control law

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + G(q) - K_d f_1(\dot{e}) - K_p f_2(e)$$

where $e \equiv q - q_d$ results in the closed-loop system

$$M(q)\ddot{e} + C(q,\dot{q})\dot{e} + K_d f_1(\dot{e}) + K_p f_2(e) = 0. \qquad (*)$$

Proposition If $f_1 \in C^1$, $f_2 \in C^2$, $\frac{\partial f_2}{\partial e}(0) > 0$ then $(*)$ is globally asymptotically stable.

Corollary If q_d is a fixed point then $(*)$ is globally asymptotically stable. $(f_1, f_2 \in C^0)$

Note that in case q_d is a fixed point the controller reduces to

$$\tau = G(q) - K_d f_1(\dot{e}) - K_p f_2(e)$$

which for $f_1, f_2 \in B^n$ is bounded.

Proof Consider the candidate Lyapunov function

$$V(t, e, \dot{e}) = \frac{1}{2}\dot{e}^T M(q)\dot{e} + F_2(e)$$

It's time-derivative along (1) becomes

$$\begin{aligned}
\dot{V}(t, e, \dot{e}) &= \dot{e}^T M(q)\ddot{e} + \frac{1}{2}\dot{e}^T \dot{M}(q)\dot{e} + \dot{e}^T f_2(e) \\
&= \dot{e}^T[-C(q, \dot{q})\dot{e} - f_1(\dot{e})] + \frac{1}{2}\dot{e}^T \dot{M}(q)\dot{e} \\
&= \dot{e}^T[\frac{1}{2}\dot{M}(q) - C(q, \dot{q})]\dot{e} - \dot{e}^T f_1(\dot{e}) \\
&= -\dot{e}^T f_1(\dot{e})
\end{aligned}$$

which is negative semi-definite in $(e, \dot{e})$.

Finish the proof of the proposition using Matrosov's Theorem, where

$$W(t, e, \dot{e}) = \ddot{V}(t, e, \dot{e})$$

Finish the proof of the corollary using LaSalle's invariance principle.

Composite controllers that solve the bounded globally tracking problem (I)

Consider the system (1). Then there exists a switching time $t_s \geq 0$ such that given any $\tilde{t}_s \geq t_s$ the composite controller

$$\tau = \begin{cases} G(q) - f_1(\dot{q}) - f_2(q) & t < \tilde{t}_s \\ \\ M(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + \\ G(q) - f_3(\dot{e}) - f_4(e) & t \geq \tilde{t}_s \end{cases} \tag{3}$$

results in a globally asymptotically stable closed-loop system.

Furthermore, if $f_1, f_2 \in \mathcal{B}^n$ we can determine a τ_{max} such that the controller (3) satisfies

$$||\tau(t)|| \leq \tau_{max} \quad \forall t \geq 0.$$

Composite controllers that solve the bounded globally tracking problem (II)

Consider the system (1). Then there exists a switching time $t_s > 0$ and a $\Delta > 0$ such that the controller

$$\tau = [1 - s_\Delta(t - t_s)]\tau_1 + s_\Delta(t - t_s)\tau_2$$

where

$$\tau_1 = G(q) - \text{[redacted]} f_1(\dot{q}) - \text{[redacted]} f_2(q)$$
$$\tau_2 = M(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + G(q) - \text{[redacted]} f_3(\dot{e}) - \text{[redacted]} f_4(e)$$

results in a globally asymptotically stable closed-loop system.

Here $s_\Delta : I\!R \to I\!R$ is a smooth nondecreasing function such that

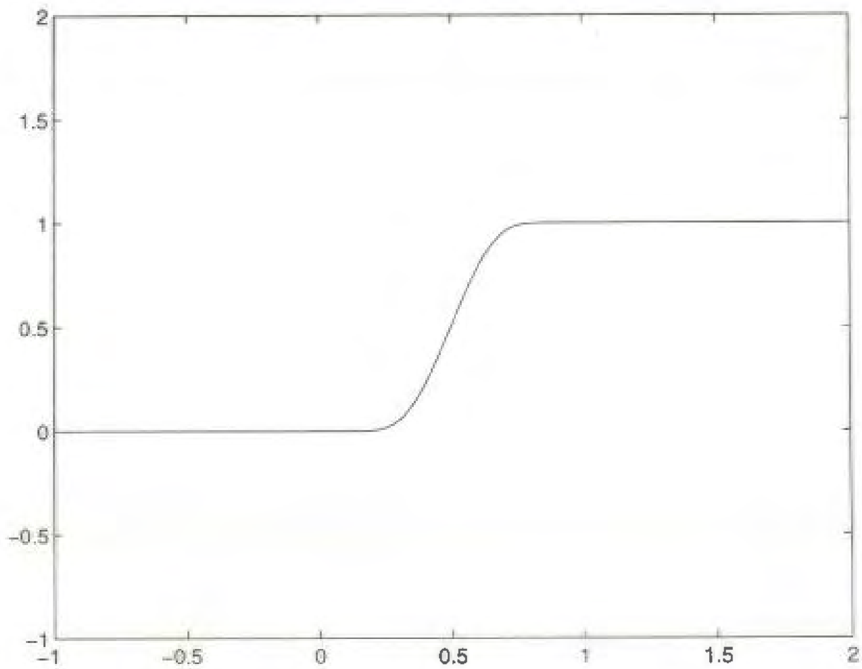
$$s_\Delta(x) = \begin{cases} 0 & x \leq 0 \\ \ldots & 0 < x < \Delta \\ 1 & \Delta \leq x \end{cases}$$

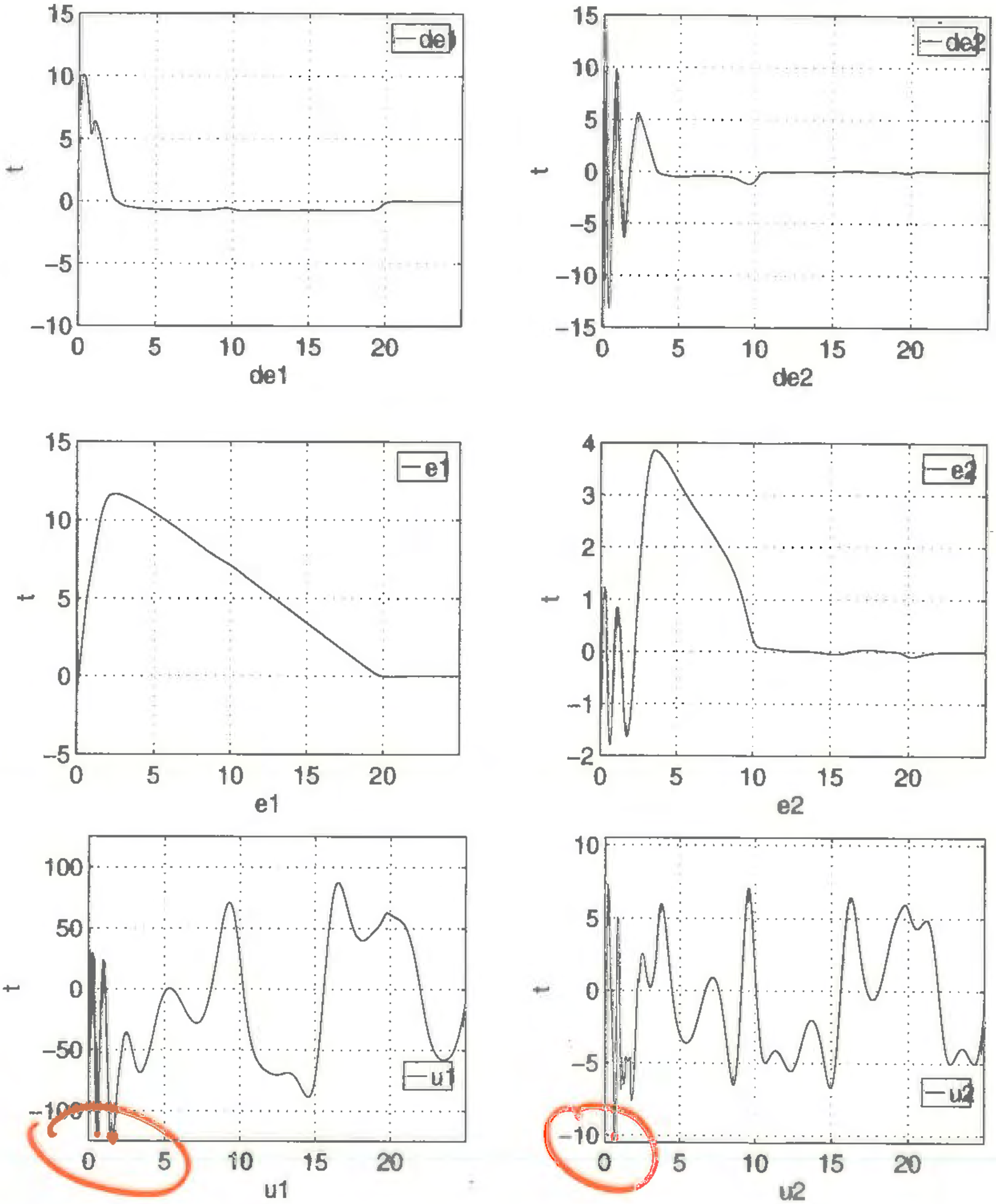
Examples of such $s_\Delta(x) \in C^\infty$ are given by:

$$\begin{cases} 0 & x \le 0 \\ \frac{1}{2}\exp\left[\frac{a}{x(x-b)} - \frac{a}{\frac{1}{2}\Delta(\frac{1}{2}\Delta-b)}\right] & 0<x\le\frac{1}{2}\Delta \\ 1-\frac{1}{2}\exp\left[\frac{a}{(x-\Delta)(x-\Delta+b)} - \frac{a}{\frac{1}{2}\Delta(\frac{1}{2}\Delta-b)}\right] & \frac{1}{2}\Delta<x<\Delta \\ 1 & \Delta \le x \end{cases}$$

where $a > 0$, $b \ge \Delta > 0$.

$\Delta = 1$
$a = 3$
$b = \frac{1}{2}$

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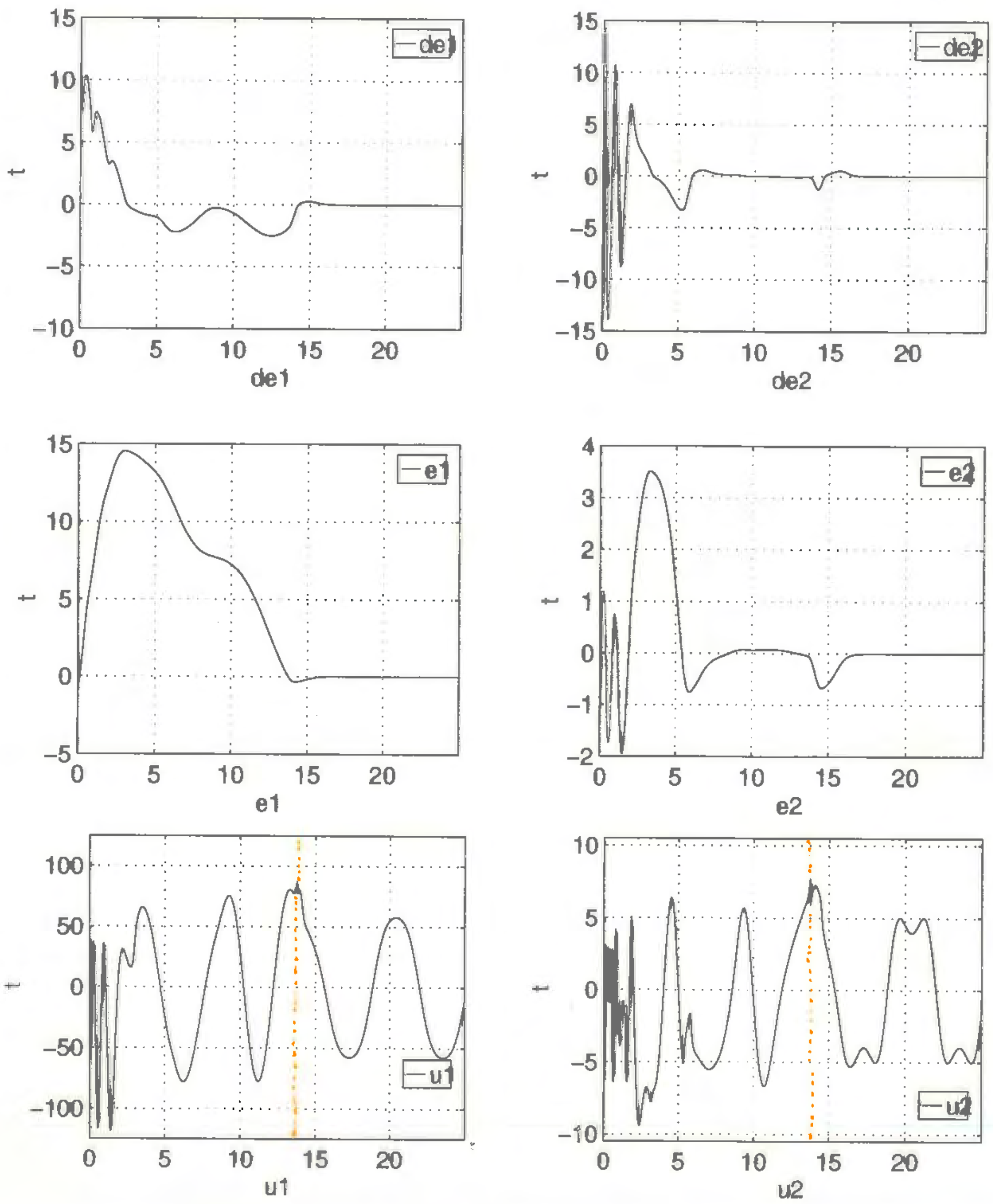
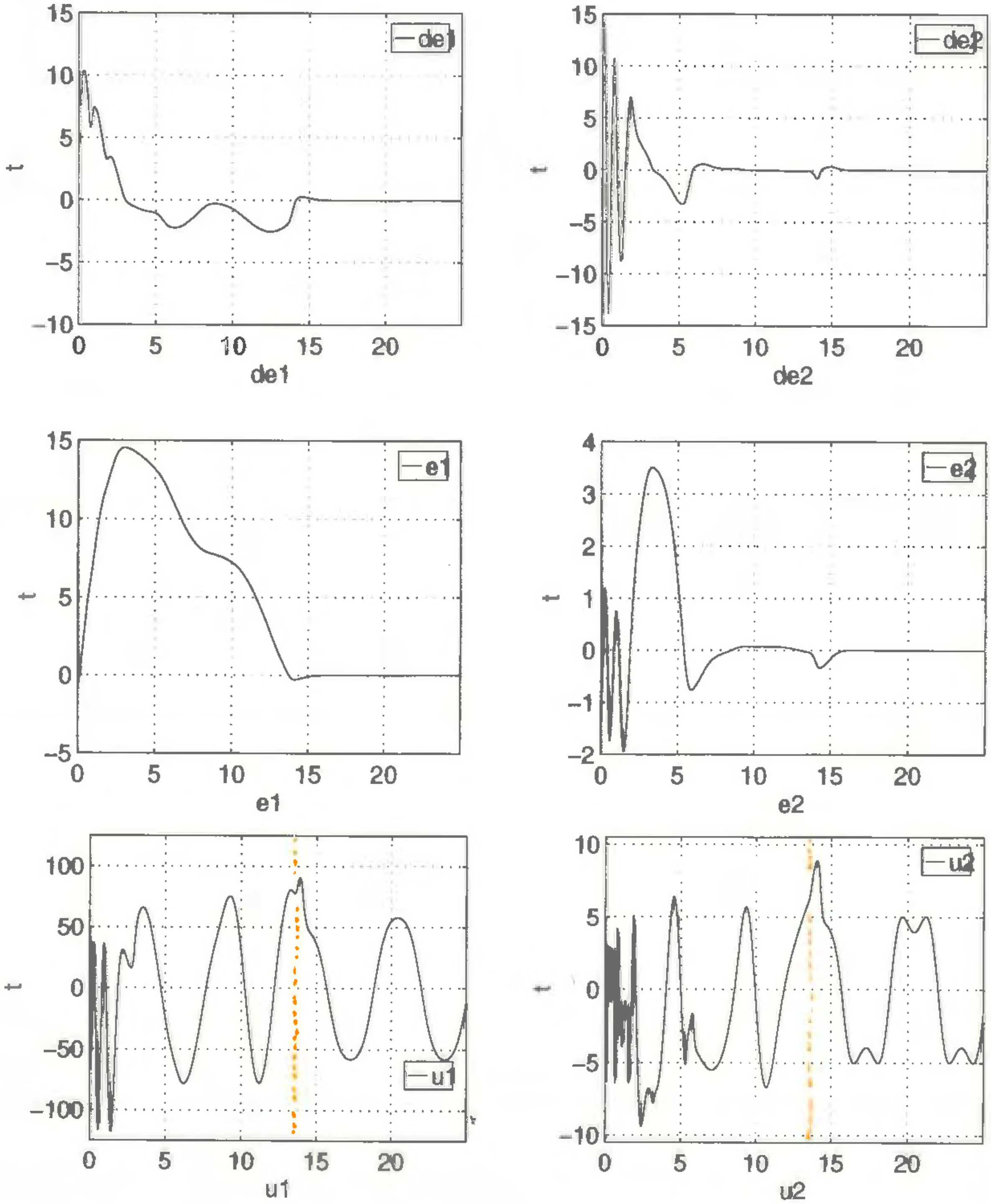


$t_s = 13.7755$



$t_s = 13.7755$

$\Delta = 1 \quad (a=3,\ b=\frac{1}{2})$

Controller used:

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + G(q) - K_p e - K_d \dot{e}$$

where

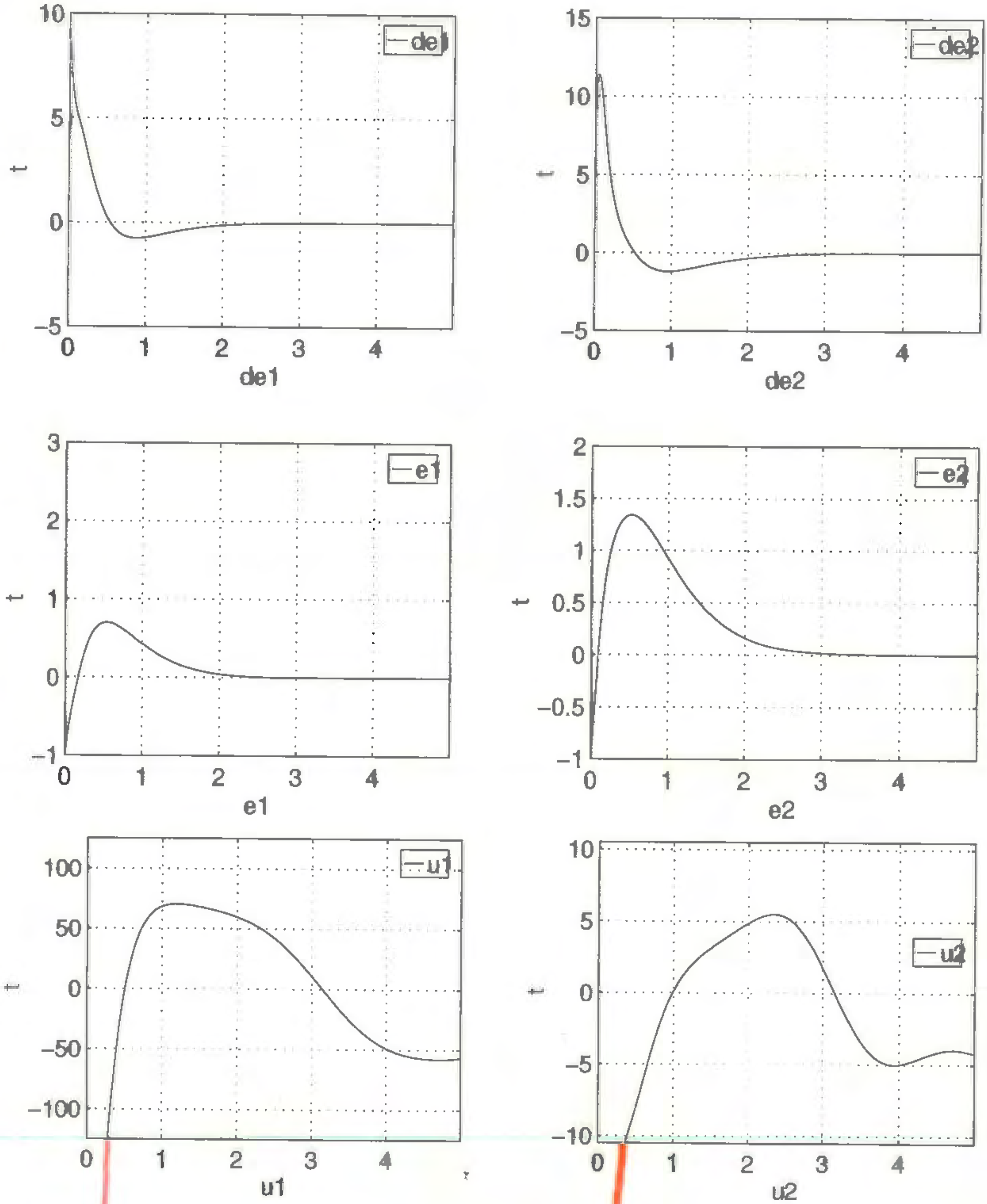
$$K_p = \begin{bmatrix} 76 & 0 \\ 0 & 6.0 \end{bmatrix} \qquad K_d = \begin{bmatrix} 50 & 0 \\ 0 & 5.1 \end{bmatrix}$$

Composite controller:

$$\tau = \mathrm{Sat}(G(q) - K_p e - K_d \dot{e})$$

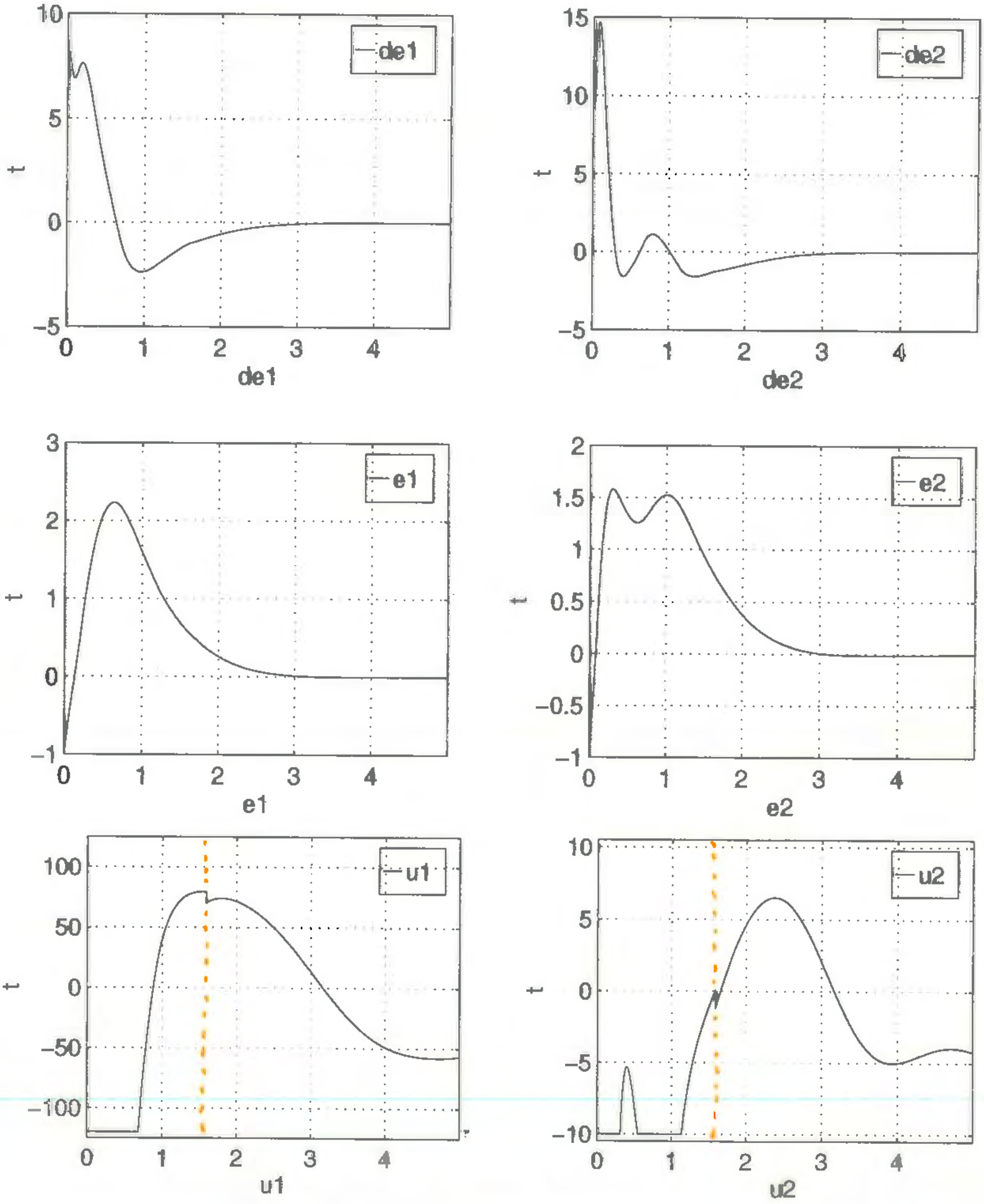
until $V_2(t, e, \dot{e}) \le 20$. From then on:

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + G(q) - K_p e - K_d \dot{e}$$

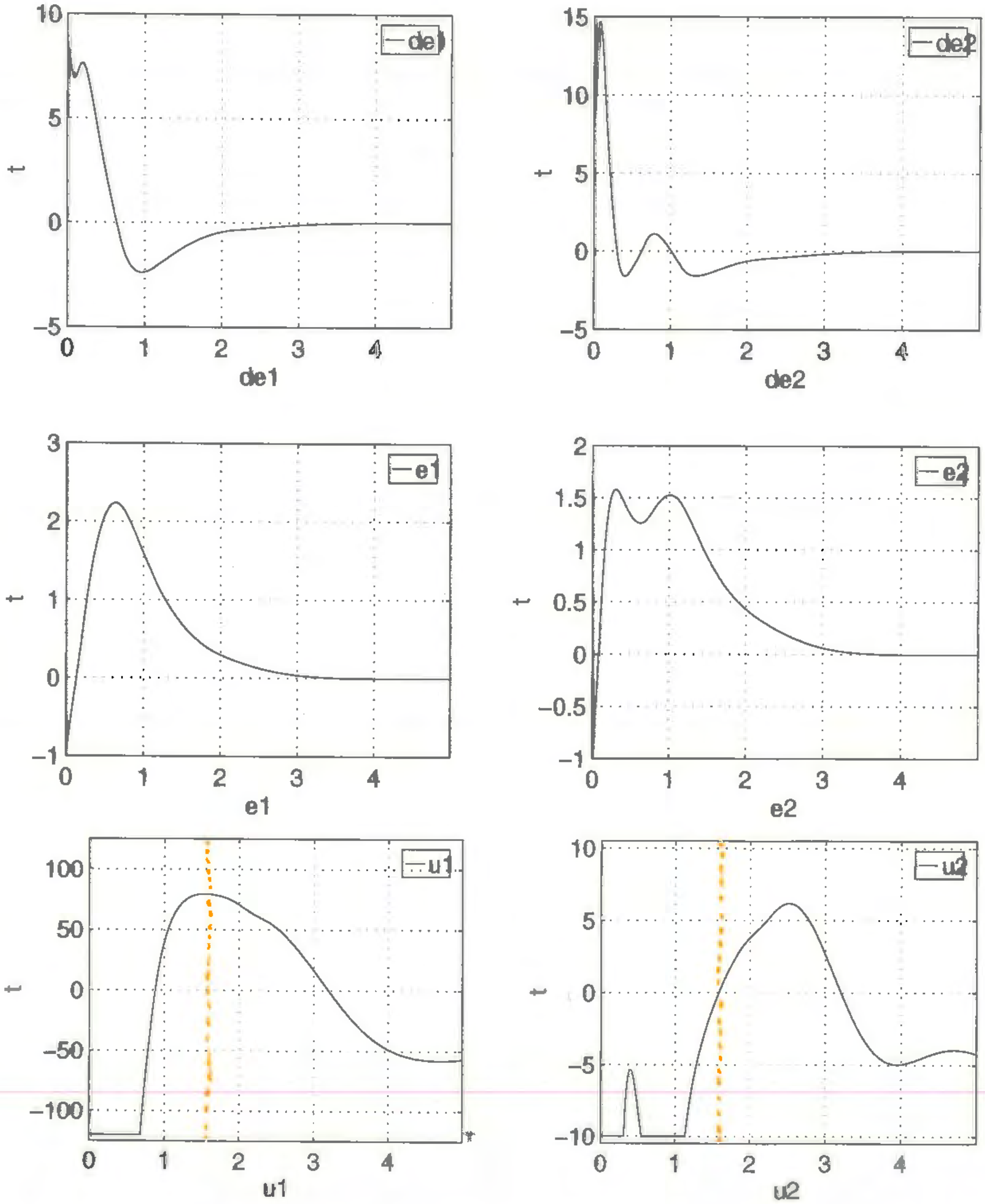


-430

-50



$t_s = 1.5902$



$t_s = 1.5902$

$\theta = 1 \quad (a = 3, b = \frac{1}{2})$

Conclusions

- Composite controllers can solve the globally bounded tracking control problem, not only for rigid robot systems.

- A drawback still exists in the time-varying nature of the composite controller.

- How to obtain the best performance possible within prespecified bounds on the input, is an interesting question for further research.