Adaptive Tracking Control of Nonholonomic Systems: An Example

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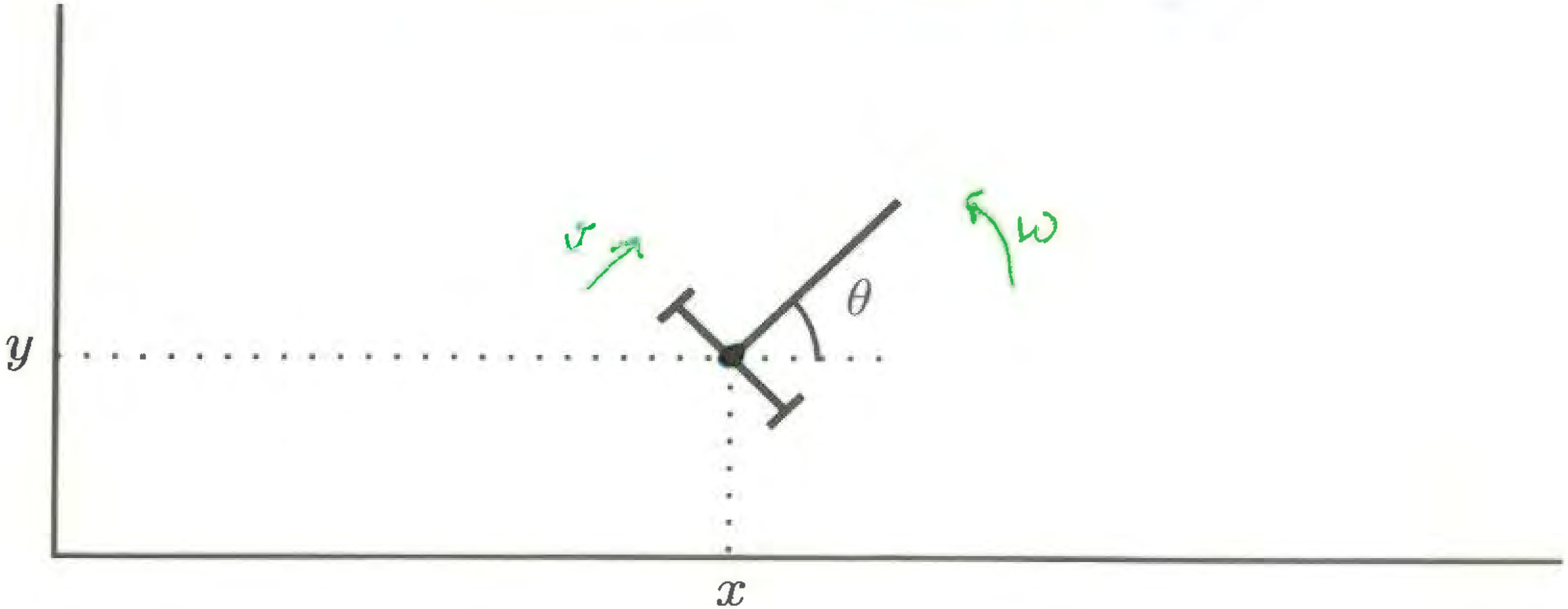
Overview

- Formulation of "ordinary" state tracking control problem
- How to formulate the adaptive state tracking control problem ?
- Solution to the simple academic example
- Conclusions

A simple kinematic model



$$
\begin{aligned}
\dot{x} &= v\cos\theta \\
\dot{y} &= v\sin\theta \\
\dot{\theta} &= \omega
\end{aligned}
$$

Formulating the tracking problem

Reference robot:

$$
\begin{aligned}
\dot{x}_r &= v_r \cos\theta_r \\
\dot{y}_r &= v_r \sin\theta_r \\
\dot{\theta}_r &= \omega_r
\end{aligned}
$$

Find control laws

$$
\begin{aligned}
v &\equiv v(x, y, \theta, x_r, y_r, \theta_r, v_r, \omega_r) \quad \dot{x}_r, \dot{y}_r, \dots \\
\omega &\equiv \omega(x, y, \theta, x_r, y_r, \theta_r, v_r, \omega_r)
\end{aligned}
$$

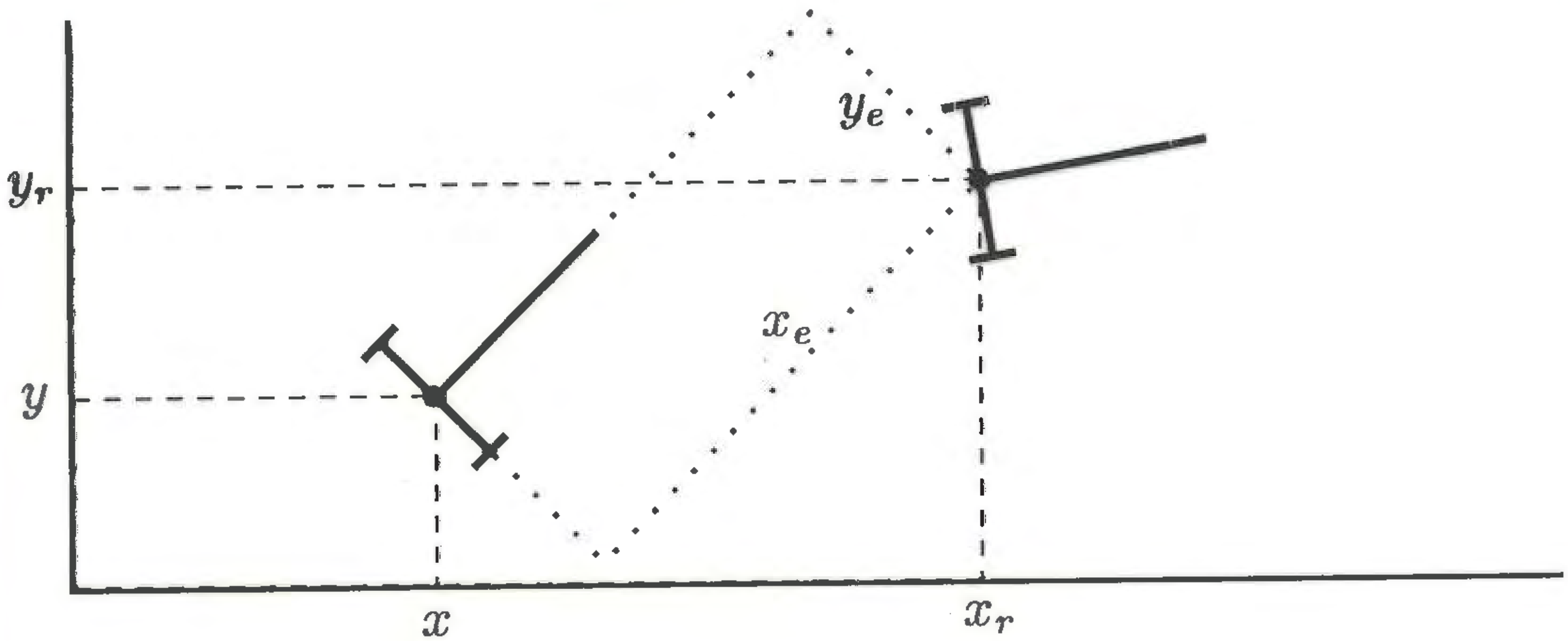
that yield

$$
\lim_{t \to \infty} |x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| = 0
$$

Remark

Why <u>state</u> tracking, not tracking of output (x, y)?

- Given $x_r(t)$, $y_r(t)$ the reference is not uniquely specified
- Tracking of $x_r(t)$ and $y_r(t)$ does not guarantee tracking of $\theta_r(t)$.
  (typically: car rotates 180° and follows reference backwards. However, "stranger things" can happen)



Define new coordinates (Kanayama et al, 1990)

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}$$

Error-dynamics

$$
\begin{aligned}
\dot{x}_e &= \omega y_e - v + v_r \cos\theta_e \\
\dot{y}_e &= -\omega x_e + v_r \sin\theta_e \\
\dot{\theta}_e &= \omega_r - \omega
\end{aligned}
$$

Micaelli Samson ('93)

Several solutions have been found, e.g Jiang, Nijmeijer ('97)

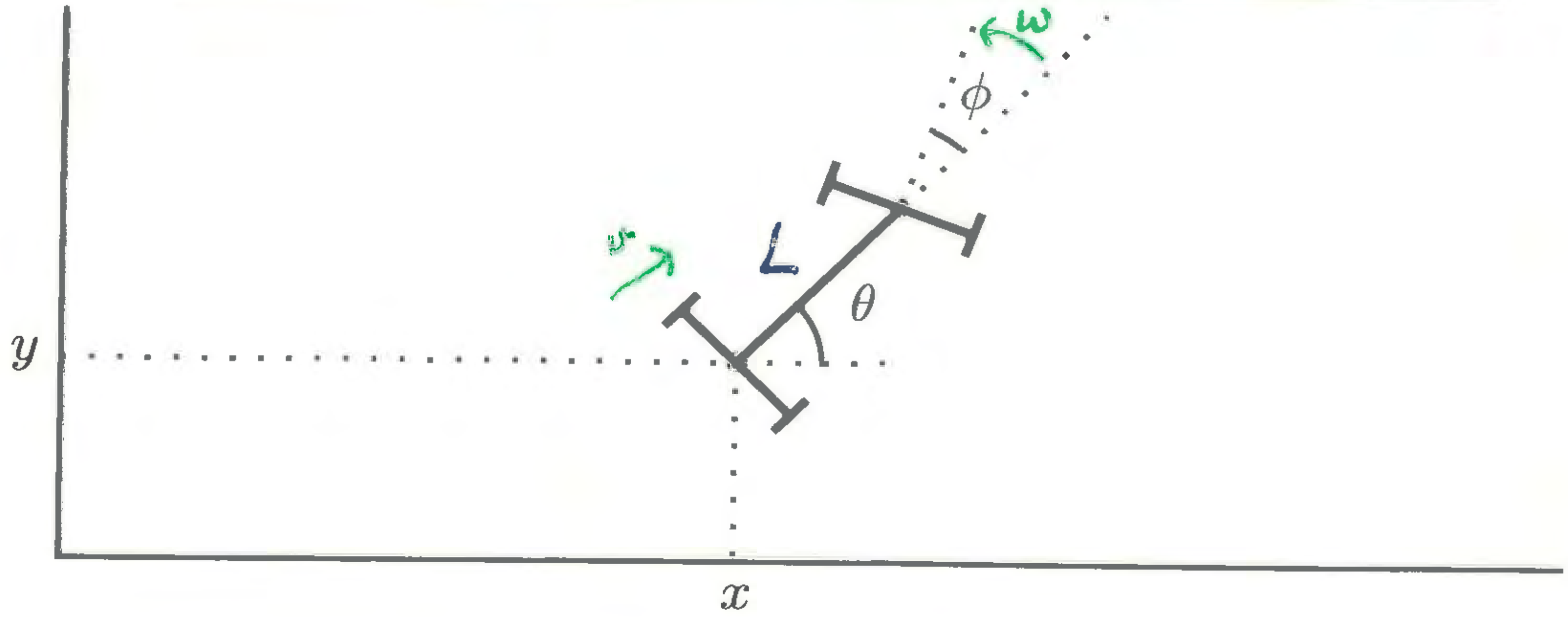
$$
\begin{aligned}
v &= v_r \cos\theta_e + c_x x_e + \gamma y_e v_r \underbrace{\int_0^1 \cos(s\theta_e)ds}_{=\frac{\sin\theta_e}{\theta_e}} \qquad \gamma, c_x > 0 \\
\omega &= \omega_r + c_\theta \theta_e \qquad c_\theta > 0
\end{aligned}
$$

yields

$$
\lim_{t\to\infty} |x_e| + |y_e| + |\theta_e| = 0
$$

provided either $v_r \not\to 0$ or $\omega_r \not\to 0$. !

A simple kinematic model containing a parameter



$$
\begin{aligned}
\dot{x} &= v\cos\theta \\
\dot{y} &= v\sin\theta \\
\dot{\theta} &= \frac{v}{L}\tan\phi \\
\dot{\phi} &= \omega
\end{aligned}
$$

Formulating the tracking problem

Reference robot:

$$
\begin{aligned}
\dot{x}_r &= v_r \cos\theta_r \\
\dot{y}_r &= v_r \sin\theta_r \\
\dot{\theta}_r &= \frac{v_r}{L} \tan\phi_r \\
\dot{\phi}_r &= \omega_r
\end{aligned}
$$

Find control laws

$$
\begin{aligned}
v &\equiv v(x, y, \theta, \phi, x_r, y_r, \theta_r, \phi_r, v_r, \omega_r) \\
\omega &\equiv \omega(x, y, \theta, \phi, x_r, y_r, \theta_r, \phi_r, v_r, \omega_r)
\end{aligned}
$$

$\dot{x}_r, \dot{y}_r, \ldots$

that yield

$$
\lim_{t\to\infty} |x^{(k)} - x_r^{(k)}| + |y^{(k)} - y_r^{(k)}| + |\theta^{(k)} - \theta_r^{(k)}| + |\phi^{(k)} - \phi_r^{(k)}| = 0
$$

Formulating the adaptive state tracking problem

- No problem for stabilization $(x_r \equiv 0)$
- No problem for rigid robot manipulator:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

any $q_r(t) \in C^2$ is feasible

Also <u>output</u> tracking usually no problem

e.g
$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) \\ \dot{x}_2 = x_3 + f_2(x_1, x_2) \\ \vdots \\ \dot{x}_n = u + f_n(x) \\ y = x_1 \end{cases}$$

any y_r is feasible

How to formulate the *adaptive* state tracking problem?

Reference robot:

$$
\begin{aligned}
\dot{x}_r &= v_r \cos\theta_r \\
\dot{y}_r &= v_r \sin\theta_r \\
\dot{\theta}_r &= \frac{v_r}{L}\tan\phi_r \\
\dot{\phi}_r &= \omega_r
\end{aligned}
$$

Find control laws for v and ω that yield

$$
\lim_{t\to\infty} |x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| + |\phi(t) - \phi_r(t)| = 0
$$

where L is an unknown parameter.

Problem

How to specify the dynamics of the reference robot:

$$\dot{x}_r = v_r \cos\theta_r \quad (1)$$

$$\dot{y}_r = v_r \sin\theta_r \quad (2)$$

$$\dot{\theta}_r = \frac{v_r}{L}\tan\phi_r \quad (3)$$

$$\dot{\phi}_r = \omega_r \quad (4)$$

Not by specifying $x_r(t)$, $y_r(t)$, $\theta_r(t)$, $\phi_r(t)$! Since:

$$v_r = \dot{x}_r \cos\theta_r + \dot{y}_r \sin\theta_r$$

$$L = \frac{v_r}{\dot{\theta}_r}\tan\phi_r$$

A first attempt

We know that $[x_r(t), y_r(t)]$ is flat output, i.e.

$$[x_r, y_r, \theta_r, \phi_r, v_r, \omega_r] = f(x_r, \dot{x}_r, \ddot{x}_r, x_r^{(3)}, y_r, \dot{y}_r, \ddot{y}_r, y_r^{(3)})$$

This can be seen as follows:

$$\begin{aligned} \dot{x}_r &= v_r \cos\theta_r & \qquad \theta_r &= \arctan(\dot{y}_r/\dot{x}_r) \\ \dot{y}_r &= v_r \sin\theta_r & v_r &= \sqrt{\dot{x}_r^2 + \dot{y}_r^2} \\ \dot{\theta}_r &= \frac{v_r}{L}\tan\phi_r & \phi_r &= \arctan(L\dot{\theta}_r/v_r) \\ \dot{\phi}_r &= \omega_r & \omega_r &= \dot{\phi}_r \end{aligned}$$

So by specifying $[x_r(t), y_r(t)]$ we can recover the entire state.

What signals can we use in control law?

If we specify $x_r(t), y_r(t)$ we obtain

$$
\begin{aligned}
\theta_r &= \arctan(\dot{y}_r/\dot{x}_r) \\
v_r &= \sqrt{\dot{x}_r^2 + \dot{y}_r^2} \\
\phi_r &= \arctan(L\dot{\theta}_r/v_r) \\
\omega_r &= \dot{\phi}_r
\end{aligned}
$$

What signals can we 'use' if L is unknown?

- $x_r(t)$, $y_r(t)$, $\theta_r(t)$, $v_r(t)$ are independent of L
- $\phi_r(t)$, $\omega_r(t)$ dependent of L.

A way to formulate the adaptive tracking problem

By specifying $x_r(t), y_r(t)$ we specify the entire reference dynamics:

- $x_r(t)$, $y_r(t)$, $\theta_r(t)$, $v_r(t)$ independent of L
- $\phi_r(t)$, $\omega_r(t)$ dependent of L.

Find control laws

$$
\begin{aligned}
v &\equiv v(x, y, \theta, \phi, x_r, y_r, \theta_r, v_r) \\
\omega &\equiv \omega(x, y, \theta, \phi, x_r, y_r, \theta_r, v_r)
\end{aligned}
$$

$\dot{x}_r, \dot{y}_r, \dots$

NOT $\phi_r, \omega_r, \dot{\phi}_r, \dots$!

that yield

$$\lim_{t\to\infty} |x(t) - x_r(t)| + |y(t) - y_r(t)| + |\theta(t) - \theta_r(t)| + |\phi(t) - \phi_r(t)| = 0$$

↳ $\phi \to \phi_r$ even though we don't know ϕ_r (since it "depends" on the unknown L)

State Feedback (known L)

Assumptions
- Car is always moving forward, i.e. $v_r(t) \geq v_r^{min} > 0$
- Forward velocity v_r and angular velocity $\dot{\theta}_r$ are bounded
- Accelerations $\dot{v}_r$ and $\ddot{\theta}_r$ are bounded.

Step 1: Take $\boxed{v = v_r + sat_\varepsilon(x_e)}$ where $\varepsilon < v_r^{min}$

where $sat_\varepsilon(\cdot)$ is C^1 and satisfies

$$x \cdot sat_\varepsilon(x) > 0 \quad \forall x \neq 0$$

$$|sat_\varepsilon(x)| \leq \varepsilon \quad \forall x$$

This assures that $v(t) > 0 \quad \forall t$

Step 2: Consider the tracking error dynamics (partially):

$$\dot{x}_e = y_e \cdot \frac{v}{L}\tan\varphi + v_r(\cos\theta_e - 1) - sat_2(x_e)$$

$$\dot{y}_e = -x_e \cdot \frac{v}{L}\tan\varphi + v_r \sin\theta_e$$

$$\dot{\theta}_e = \underbrace{\frac{v_r}{L}\tan\varphi_r - \frac{v}{L}\tan\varphi}_{\text{virtual control input}}$$

virtual control input (or actually $v_r \tan\varphi_r - v\tan\varphi$)

Taking $V = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{1}{2k_2}\theta_e^2$ yields

$$\dot{V} = -x_e\, sat_2(x_e) + \frac{1}{k_2}\theta_e\left[k_2 v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right) + \dot{\theta}_e\right]$$

Having $\frac{v_r}{L}\tan\varphi_r - \frac{v}{L}\tan\varphi = -k_1\theta_e - k_2 v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right)$ yields

$$\dot{V} = -x_e\, sat_1(x_e) - \frac{k_1}{k_2}\theta_e^2$$

Step 3: Define $z = v_r \tan\varphi_r - v\tan\varphi + c_1\theta_e + c_2 v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right)$

and backstepping gives control law for ω:

State Feedback (unknown L)

- 1^{st} step: $v = v_r + sat_2(x_e)$
- 2^{nd} step: Desired "input" $v_r \tan\varphi_r - v\tan\varphi = -c_1\theta_e - c_2 v_r\left(\frac{\cos\theta_e - 1}{\theta_e} x_e + \frac{\sin\theta_e}{\theta_e} y_e\right)$

Problem: φ_r is not available (since L is unknown)

"Fortunately" we "only" need $v_r \tan\varphi_r$ which happens to be $L \cdot \dot{\theta}_r$ and $\dot{\theta}_r$ is available!

Are we "sound" now?

What happens if we define

$$z = L\dot{\theta}_r - v\tan\varphi + c_1\theta_e + c_2 v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right)$$

Problem: We can not use z in feedback law for ω, since we don't know L

Define

$$\hat{z} = \hat{L}\dot{\theta}_r - v\tan\varphi + c_1\theta_e + c_2 v_r\left(\frac{\cos\theta_e - 1}{\theta_e}x_e + \frac{\sin\theta_e}{\theta_e}y_e\right)$$

Where $\hat{L}$ is an estimate for L.

When expressing reference signals in <u>known</u> signals (i.e. using $\dot{Q}_r, \ddot{Q}_r$) we run into the unknown parameter $\frac{1}{L}$.

If we additionally define $\hat{\rho}$ as estimate for the unknown $\rho = \frac{1}{L}$ we can find control law for w and update laws for $\hat{L}$ and $\hat{\rho}$ by means of backstepping.

Proposition: All trajectories are globally uniformly bounded. Furthermore

$$\lim_{t \to \infty} |\xi_e(t)| + |Q_e(t)| + |\hat{z}(t)| + |\phi(t) - \phi_r^d(t)| = 0$$

If in addition $\dot{Q}_r(t)$ is P.E. then also

$$\lim_{t \to \infty} |y_e(t)| + |\tilde{L}(t)| + |\tilde{\rho}(t)| = 0$$

Conclusions

- Formulation of the adaptive state tracking problem is non trivial since not knowing certain parameters can be in conflict with specifying the reference state.
- The adaptive state tracking problem has been formulated, such that in case of known parameters it reduces to the non-adaptive state tracking problem.
- For a simple academic example a solution was presented.