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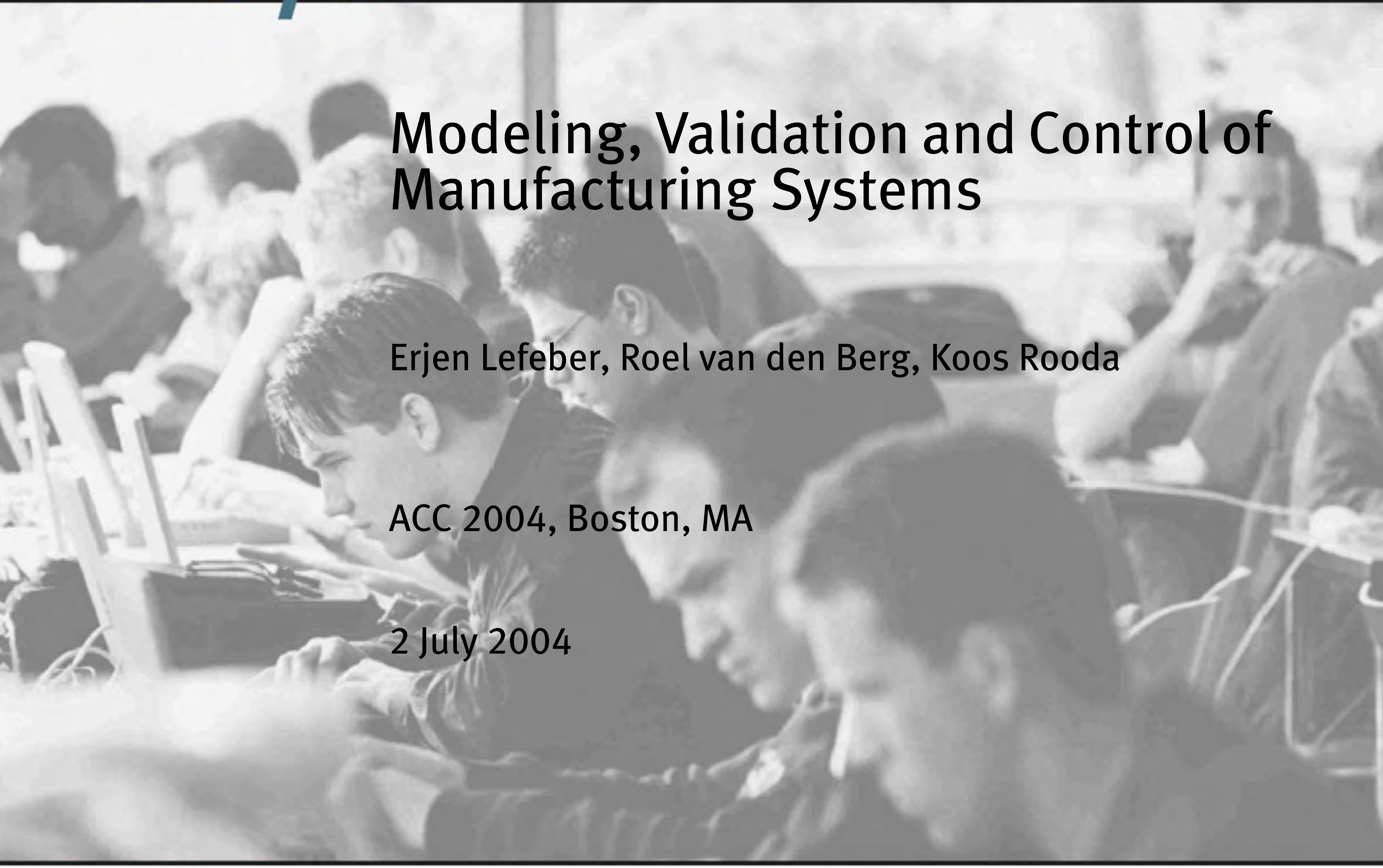

Modeling, Validation and Control of Manufacturing Systems

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ACC 2004, Boston, MA

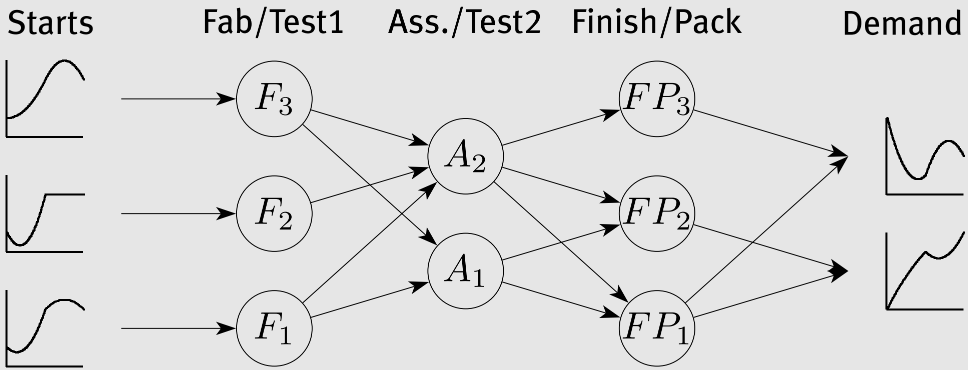
2 July 2004

/department of mechanical engineering

Outline

- Modeling manufacturing systems
- Effective Processing Times
- Overview of available models
- Introduction of PDE models
- Validation of PDE models
- Control using PDE models
- Conclusions

Modeling problem

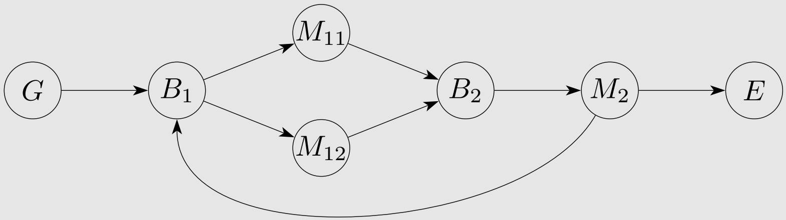


Modeling for control (supply chain/mass production).

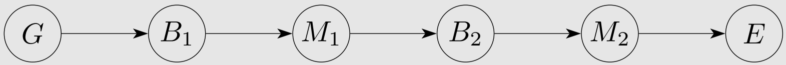
- Like to understand dynamics of factories
- Throughput, flow time, variance of flow time
- Answer questions like: How to perform ramp up?

Two main structures

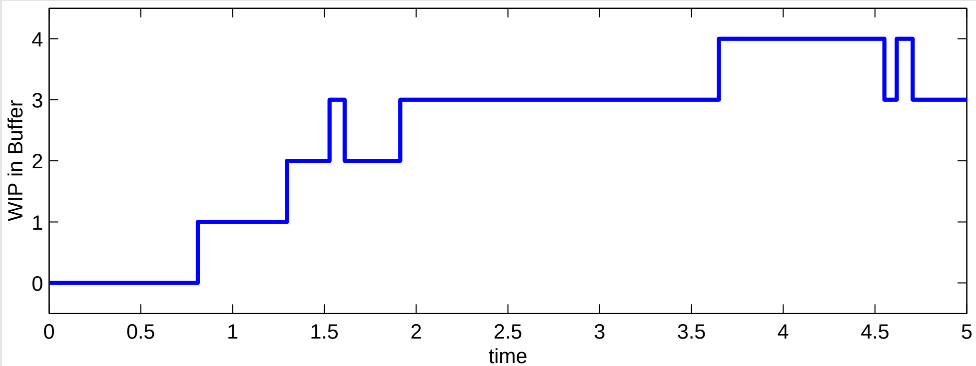
- Re-entrant line

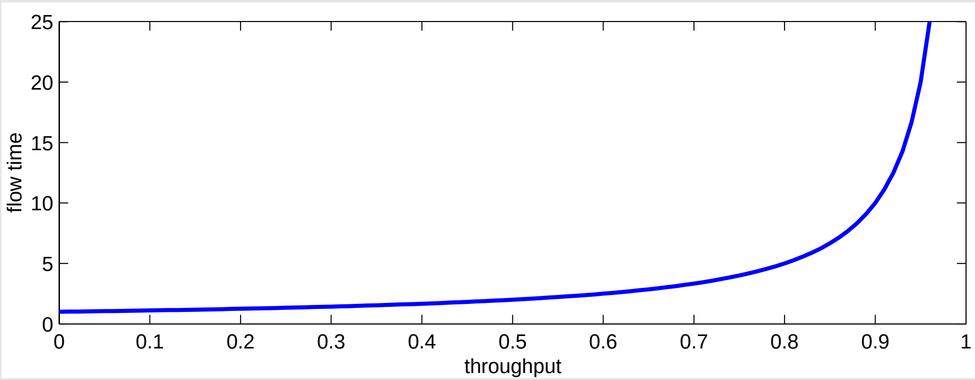


- Acyclic line



Typical signal + Nonlinear relation





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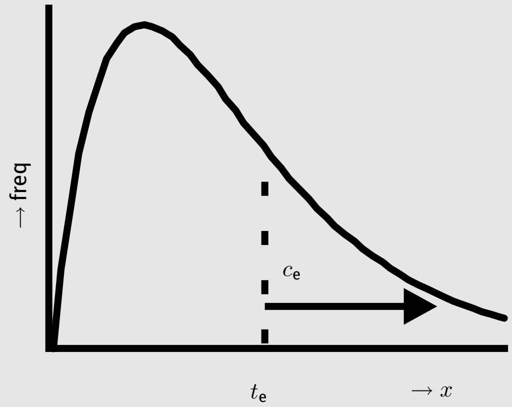
Effective Processing Times

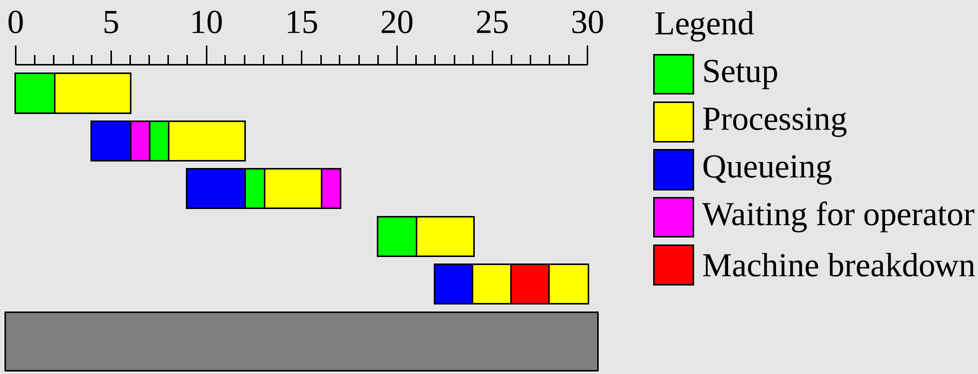
- raw process time t_0 and c_0
- setups t_s and c_s
- TBF t_f and c_f, TTR t_r and c_r
- operator delays
- rework
- …(!)

Idea:

Combine all disturbances in one single EPT probability density function

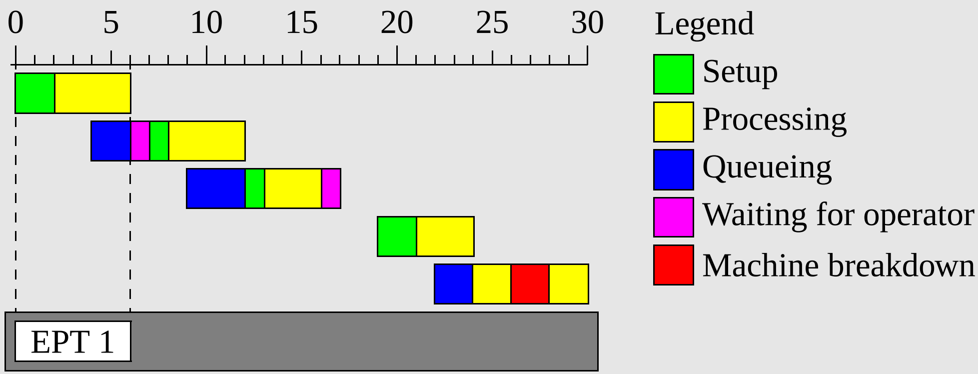


Effective Processing Times



- Time a lot experiences (from a logistic point of view)
- Time a lot either was or could have been processed

Effective Processing Times



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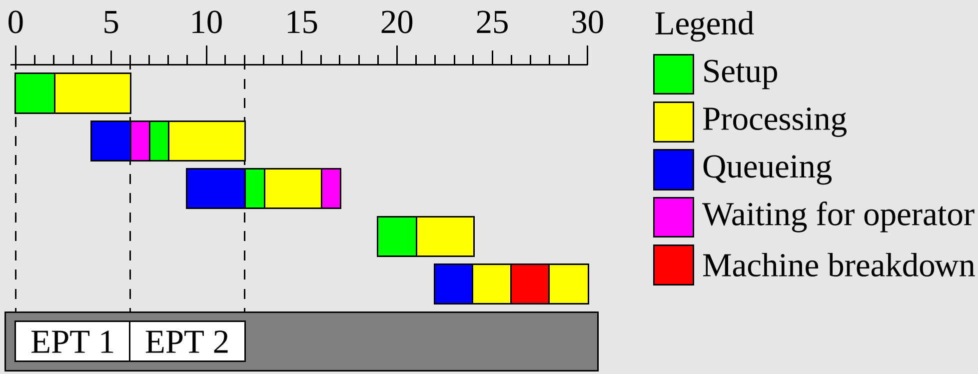

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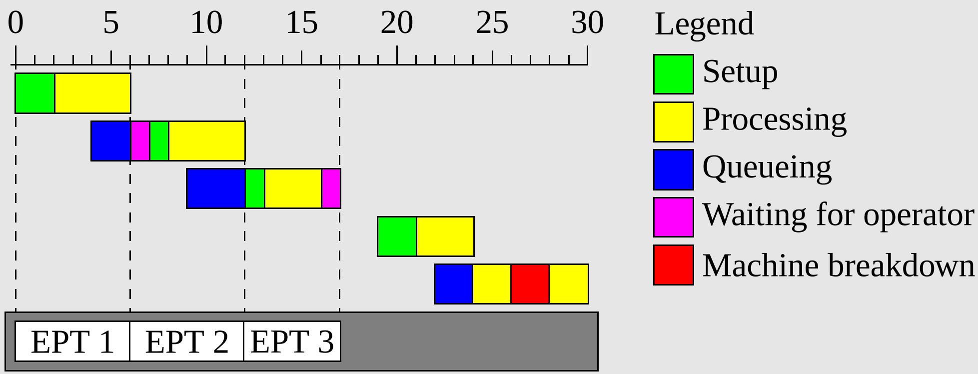

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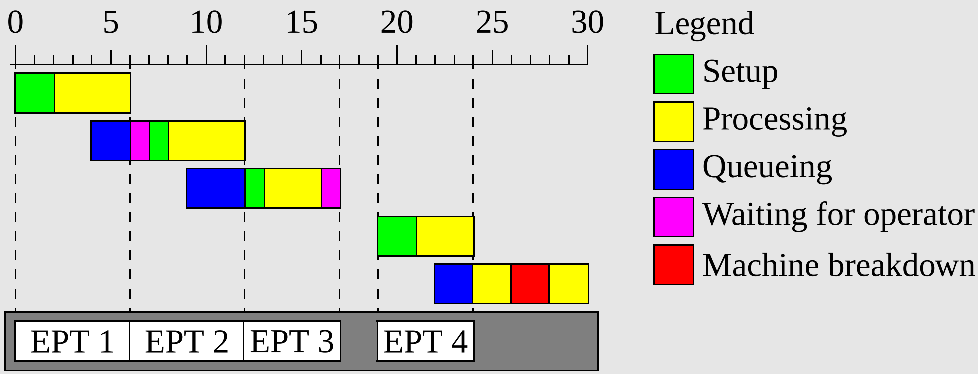

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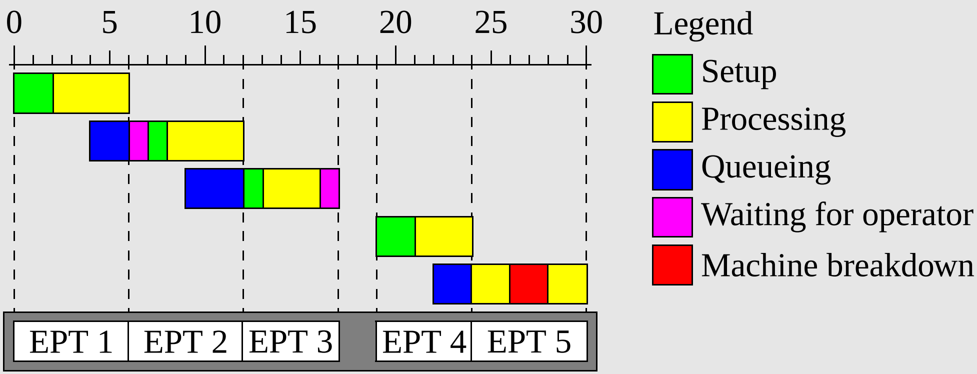


- Time a lot experiences (from a logistic point of view)
- Time a lot either was or could have been processed

Modeling problem

Some observations from practice:

- Quick answers (“What if . . . ”).
- A factory is (almost) never in steady state
- Throughput and flow time are related

We look for a model that

- is computationally feasible,
- describes dynamics, and
- incorporates both throughput and flow time

Available models

Discrete Event

- Advantages
  - Include dynamics
  - Throughput and flow time related
- Disadvantage
  - Clearly infeasible for entire supply chain

Available models

Queueing Theory

- Advantages
  - Throughput and flow time related
  - Computationally feasible (approximations)
- Disadvantage
  - Only steady state, no dynamics

Available models

Fluid models

- Kimemia and Gershwin: Flow model
- Queuing theorists: Fluid models/Fluid queues

$$\xrightarrow{u_0(t)} \overset{x_1(t)}{B_1} \xrightarrow{u_1(t)} M_1 \xrightarrow{u_1(t)} \overset{x_2(t)}{B_2} \xrightarrow{u_2(t)} M_2 \xrightarrow{u_2(t)} \overset{x_3(t)}{B_3}$$

$$\begin{aligned} \dot{x}_1 &= u_0 - u_1 \\ \dot{x}_2 &= u_1 - u_2 \\ \dot{x}_3 &= u_2 \end{aligned} \quad \text{or} \quad \begin{aligned} x_1(k+1) &= x_1(k) + u_0(k) - u_1(k) \\ x_2(k+1) &= x_2(k) + u_1(k) - u_2(k) \\ x_3(k+1) &= x_3(k) + u_2(k) \end{aligned}$$

- Cassandras: Stochastic Fluid Model

Available models

Fluid models

- Advantages
  - Dynamical model
  - Computationally feasible
- Disadvantage
  - Only throughput incorporated in model, no flow time
  - Possible to run factory with no WIP

Available models (conclusion)

- Discrete Event: Not computationally feasible
  Queuing Theory: No dynamics
  Fluid models: No flow time
- Need something else!
- Discrete event models (and queuing theory) have proved themselves. Can be used for verification!

Traffic flow: LWR model

Lighthill, Whitham ('55), and Richards ('56)

Traffic behavior on one-way road:

- density $\rho(x,t)$,
- speed $v(x,t)$,
- flow $u(x,t) = \rho(x,t)v(x,t)$.

Conservation of mass:

$$\frac{\partial \rho}{\partial t}(x,t) + \frac{\partial u}{\partial x}(x,t) = 0.$$

Static relation between flow and density:

$$u(x,t) = S(\rho(x,t)).$$

Modeling manufacturing flow

- density $\rho(x,t)$,
- speed $v(x,t)$,
- flow $u(x,t) = \rho(x,t)v(x,t)$,
- Conservation of mass: $\frac{\partial \rho}{\partial t}(x,t) + \frac{\partial \rho v}{\partial x}(x,t) = 0.$
- Boundary condition: $u(0,t) = \lambda(t)$

Modeling manufacturing flow

Armbruster, Marthaler, Ringhofer (2002):

- Single queue: $\frac{1}{v(x,t)} = \frac{1}{\mu}(1 + \int_0^1 \rho(s,t)\,\mathrm{d}s)$
- Single queue: $\frac{\partial \rho v}{\partial t}(x,t) + \frac{\partial \rho v^2}{\partial x}(x,t) = 0$
  $\rho v^2(0,t) = \frac{\mu \cdot \rho v(0,t)}{1+\int_0^1 \rho(s,t)\,\mathrm{d}s}$
- Re-entrant: $v(x,t) = v_0 \left(1 - \frac{\int_0^1 \rho(s,t)\,\mathrm{d}s}{W_{\mathsf{max}}}\right)$
- Re-entrant: $\frac{\partial \rho v}{\partial t}(x,t) + \frac{\partial \rho v^2}{\partial x}(x,t) = 0$
  $\rho v^2(0,t) = \rho v(0,t) \cdot v_0 \left(1 - \frac{\int_0^1 \rho(s,t)\,\mathrm{d}s}{W_{\mathsf{max}}}\right)$

Lefeber (2003):

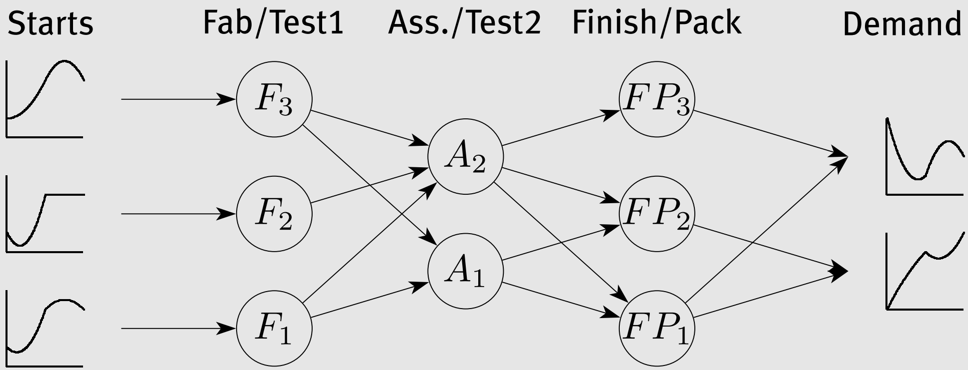
- Line of m identical queues: $v(x,t) = \frac{\mu}{m+\rho(x,t)}$

Validation

- Line of 15 identical machines
- Infinite queues
- FIFO-policy
- Exponential Effective Processing Times
- Step-response (initially empty, start rate λ)
- Model 1, 2, 5 versus averaged discrete event

Show movie

Control



- Lyapunov (boundary) controller design
- MPC on discretization of PDE
- Hamiltonian system (power conserving interconnection)
- Hybrid system

Conclusions

Need for computationally feasible dynamical models incorporating both throughput and flow time.

- NOT: Discrete event, Queueing theory, Fluid models
- Possible: PDE-models
  - Correct steady state behavior
  - Better description transient needed
  - Queueing theory, discrete event models can be used for validation of PDE models
- PDE-based controller design (boundary control)