Bounded Tracking Control of a Wheeled Mobile Robot

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Outline

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  - Problem formulation
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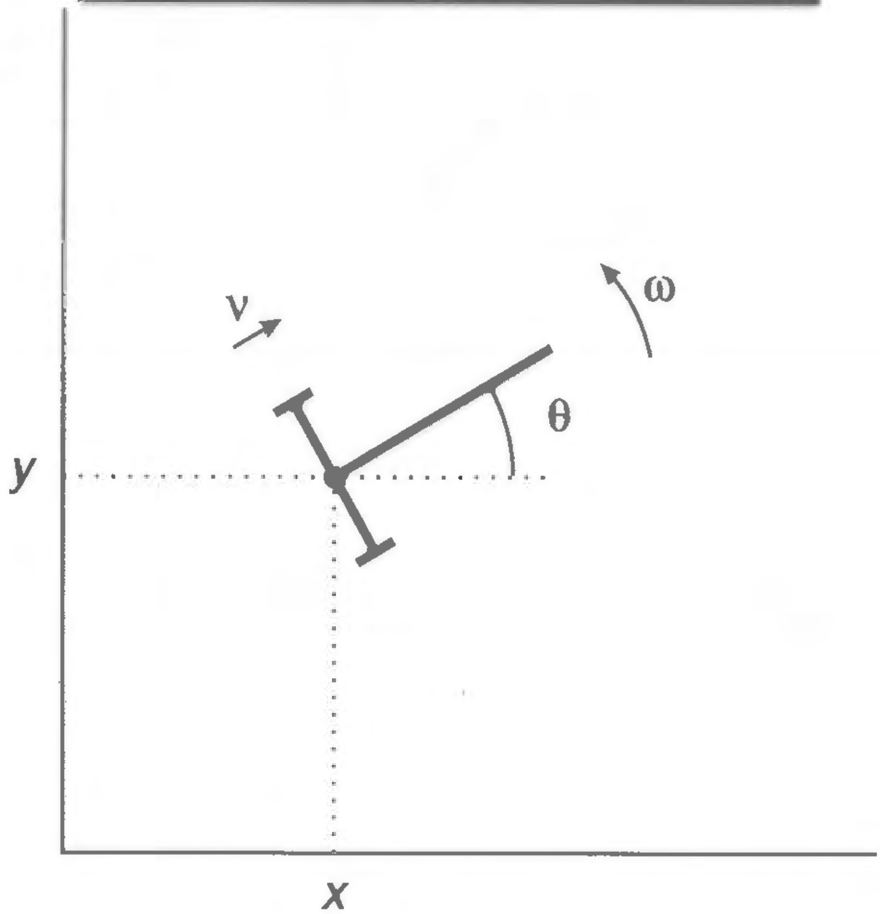
- Conclusions

A wheeled mobile robot



The dynamics is described by:

$$\begin{aligned} \dot{x} &= \nu \cos(\theta) \\ \dot{y} &= \nu \sin(\theta) \\ \dot{\theta} &= \omega \end{aligned}$$

ν: linear velocity of mobile robot
ω: angular velocity of mobile robot

Problem

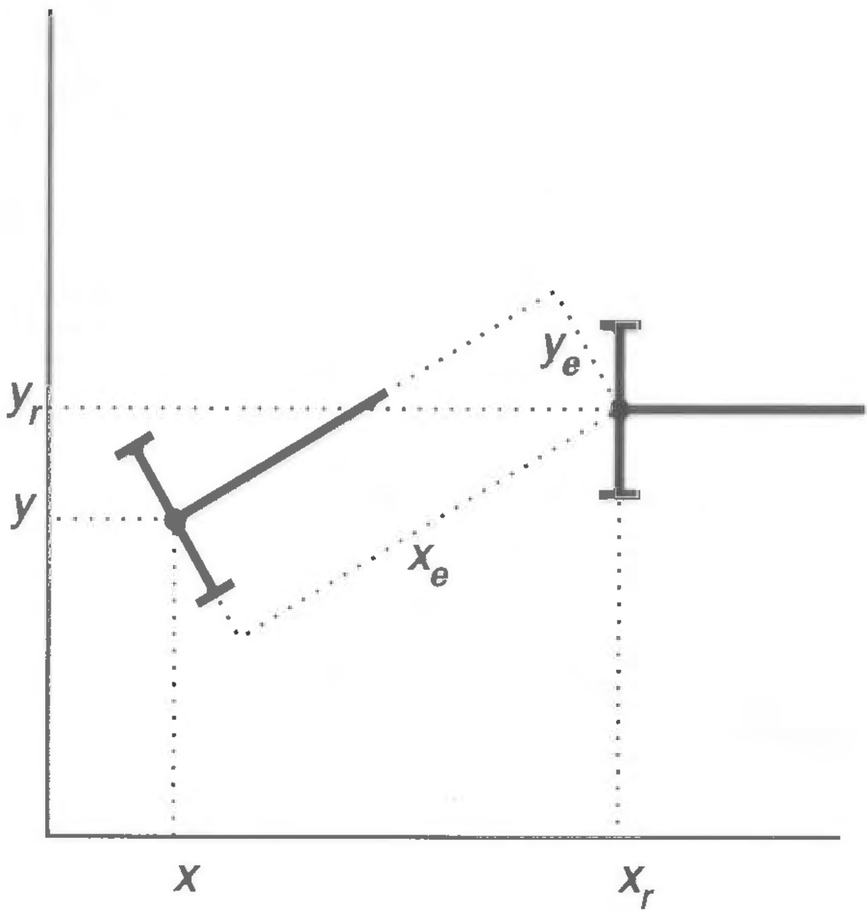
Find control laws for ν and ω such that the robot follows a *reference robot*, with position $(x_r, y_r, \theta_r)^T$ and inputs ν_r and ω_r, where the inputs are constrained to

$$|\nu(t)| \leq \nu^{max}, \;\; |\omega(t)| \leq \omega^{max} \qquad \forall t \geq 0$$

Assumption:

$$\begin{aligned} \nu^{max} &> \max_{t \geq 0} \; \nu_r(t) \\ \omega^{max} &> \max_{t \geq 0} \; \omega_r(t) \end{aligned}$$

A solution of the problem without the input constraints can be found in:
Z.-P. Jiang and H. Nijmeijer *Tracking Control of Mobile Robots: A Case Study in Backstepping*, to appear in Automatica.



Denote error coordinates as

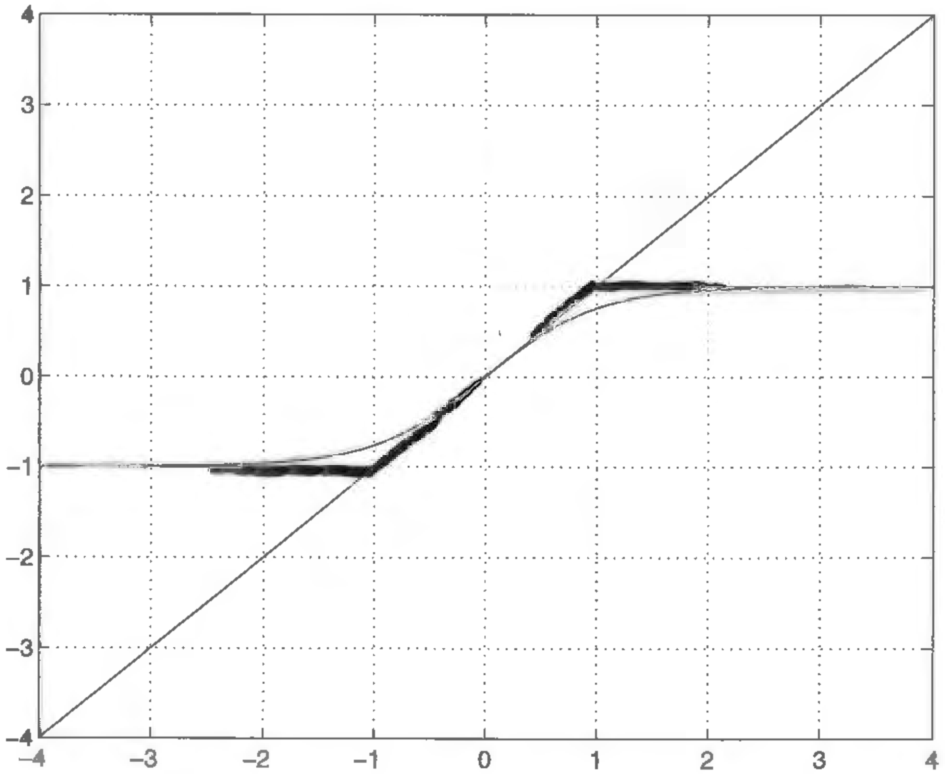
$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}$$

resulting in

$$\begin{aligned} \dot{x}_e &= \omega y_e - \nu + \nu_r \cos\theta_e \\ \dot{y}_e &= -\omega x_e + \nu_r \sin\theta_e \\ \dot{\theta}_e &= \omega_r - \omega \end{aligned}$$

Let $\mathcal{F}$ denote the set of all nondecreasing C^1 functions satisfying $f(0) = 0, f'(0) > 0$.

Examples



$f(x) = ax$, $f(x) = a\tanh(bx)$, $f(x) = a\text{sat}(bx)$ $(a, b > 0)$.

$\Rightarrow$ $x f(x)$ positive definite

Solution (sketch)

Recall the error dynamics

$$\begin{aligned}
\dot{x}_e &= \omega y_e - \nu + \nu_r \cos\theta_e \\
\dot{y}_e &= -\omega x_e + \nu_r \sin\theta_e \\
\dot{\theta}_e &= \omega_r - \omega
\end{aligned}$$

Since $x_e = f_1(\omega) f_2(y_e)$ $(f_1, f_2 \in \mathcal{F})$ and $\theta_e = 0$ are stabilising functions for the y_e system, we can use the idea of integrator backstepping and define the new variable

$$\bar{x}_e = x_e - f_1(\omega) f_2(y_e).$$

Consider the positive definite function

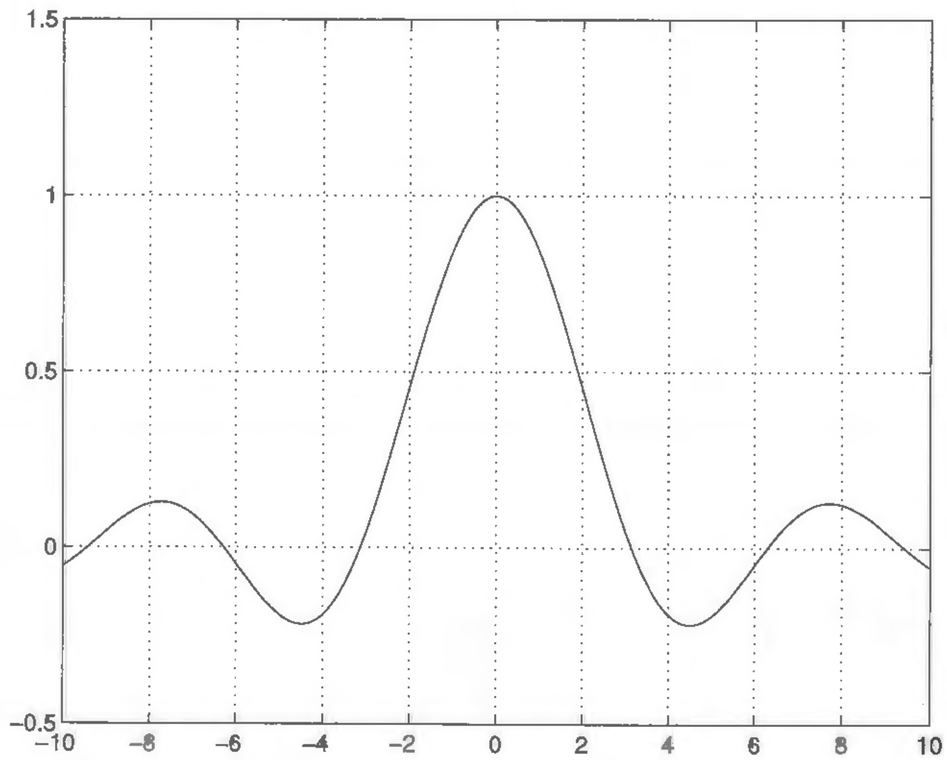
$$V(t, x_e, y_e, \theta_e) = \frac{1}{2}\bar{x}_e^2 + \frac{1}{2}{y_e}^2 + \frac{1}{2\gamma}{\theta_e}^2 \qquad (\gamma > 0)$$

When we use

$$\begin{aligned}\nu &= \nu_r \cos\theta_e - \dot{\omega} f_1'(\omega) f_2(y_e) - \\ &\quad - f_1(\omega) f_2'(y_e)\dot{y}_e + f_3(\bar{x}_e) \\ \omega &= \omega_r + \gamma y_e \nu_r \frac{\sin\theta_e}{\theta_e} + f_4(\theta_e)\end{aligned}$$

where $f_3, f_4 \in \mathcal{F}$, we obtain

$$\begin{aligned}\dot{V}(t, x_e, y_e, \theta_e) &= -\bar{x}_e f_3(\bar{x}_e) - \omega f_1(\omega) y_e f_2(y_e) - \\ &\quad -\theta_e f_4(\theta_e)\end{aligned}$$



Theorem: Assume that ν_r, $\dot{\nu}_r$, ω_r and $\dot{\omega}_r$ are bounded on $[0, \infty)$. Then, all trajectories of the resulting system composed of

$$\begin{aligned} \dot{x}_e &= \omega y_e - \nu + \nu_r \cos\theta_e \\ \dot{y}_e &= -\omega x_e + \nu_r \sin\theta_e \\ \dot{\theta}_e &= \omega_r - \omega \end{aligned} \qquad (1)$$

and

$$\begin{aligned} \nu &= \nu_r \cos\theta_e - \dot{\omega} f_1'(\omega) f_2(y_e) - \\ &\quad - f_1(\omega) f_2'(y_e)\dot{y}_e + f_3(\bar{x}_e) \\ \omega &= \omega_r + \gamma y_e \nu_r \frac{\sin\theta_e}{\theta_e} + f_4(\theta_e) \end{aligned} \qquad (2)$$

$(f_1, f_2, f_3, f_4 \in \mathcal{F})$ are globally uniformly bounded. Furthermore, if $\nu_r(t)$ does not converge to zero, or if $\nu_r(t)$ tends to zero but $\omega_r(t)$ does not converge to zero, then the closed-loop solutions converge to zero, i.e.

$$\lim_{t\to\infty} (|x_e(t)| + |y_e(t)| + |\theta_e(t)|) = 0$$

Remark: By taking $f_1(\omega) = c_3\omega$, $f_2(y_e) = y_e$, $f_3(\bar{x}_e) = c_4\bar{x}_e$ and $f_4(\theta_e) = c_5\gamma\theta_e$ the result reduces to that of Jiang and Nijmeijer.

Theorem: If additionally $\liminf_{t\to\infty} |\omega_r(t)| > 0$, the zero equilibrium is exponentially stable for small initial errors. In other words, all the closed-loop trajectories go to zero at an exponential rate after a considerable period of time.

Bounded tracking

Theorem: Consider the system (1) together with (2). And suppose we have to deal with input bounds, i.e.

$$|\nu(t)| \le \nu^{max}, \quad |\omega(t)| \le \omega^{max} \qquad \forall t \ge 0$$

where $\nu^{max} > \max\limits_{t\ge 0} \nu_r(t)$ and $\omega^{max} > \max\limits_{t\ge 0} \omega_r(t)$.

Then for any initial condition $(x_e(0), y_e(0), \theta_e(0))$ there exist $f_1, f_2, f_3, f_4 \in \mathcal{F}$ such that

$$|\nu(t)| \le \nu^{max}, \quad |\omega(t)| \le \omega^{max} \qquad \forall t \ge 0$$

Simulations

Initial conditions:
$(x_e(0), y_e(0), \theta_e(0)) = (16.6, 1.5, -1)$

Reference trajectory: $\nu_r(t) = 1$, $\omega_r(t) = 0$, i.e. straight line.

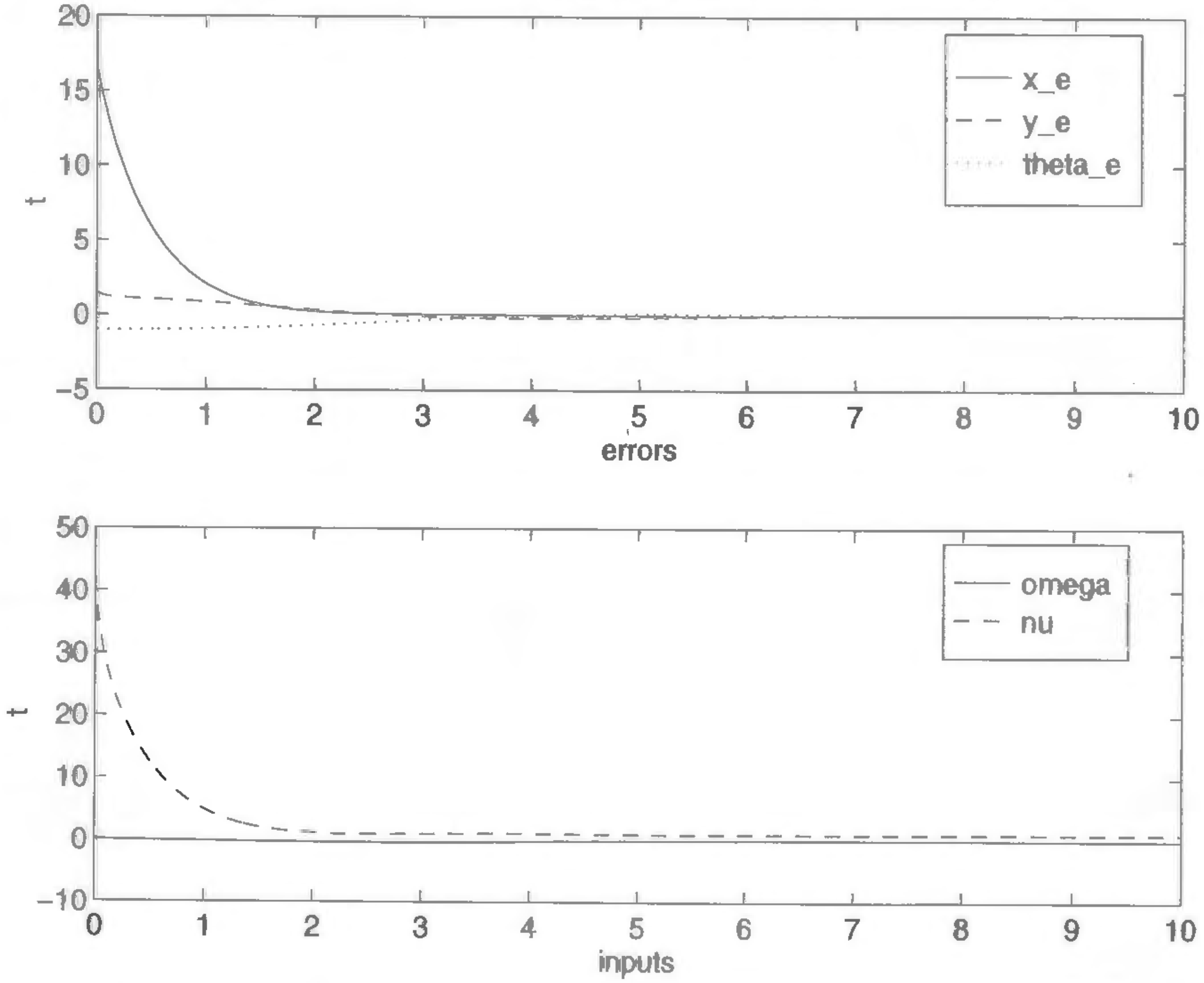
Controller Jiang& Nijmeijer:
$f_1(\omega) = \omega$, $f_2(y_e) = y_e$,
$f_3(\bar{x}_e) = 2\bar{x}_e$, $f_4(\theta_e) = \theta_e$,
$\gamma = 1$.

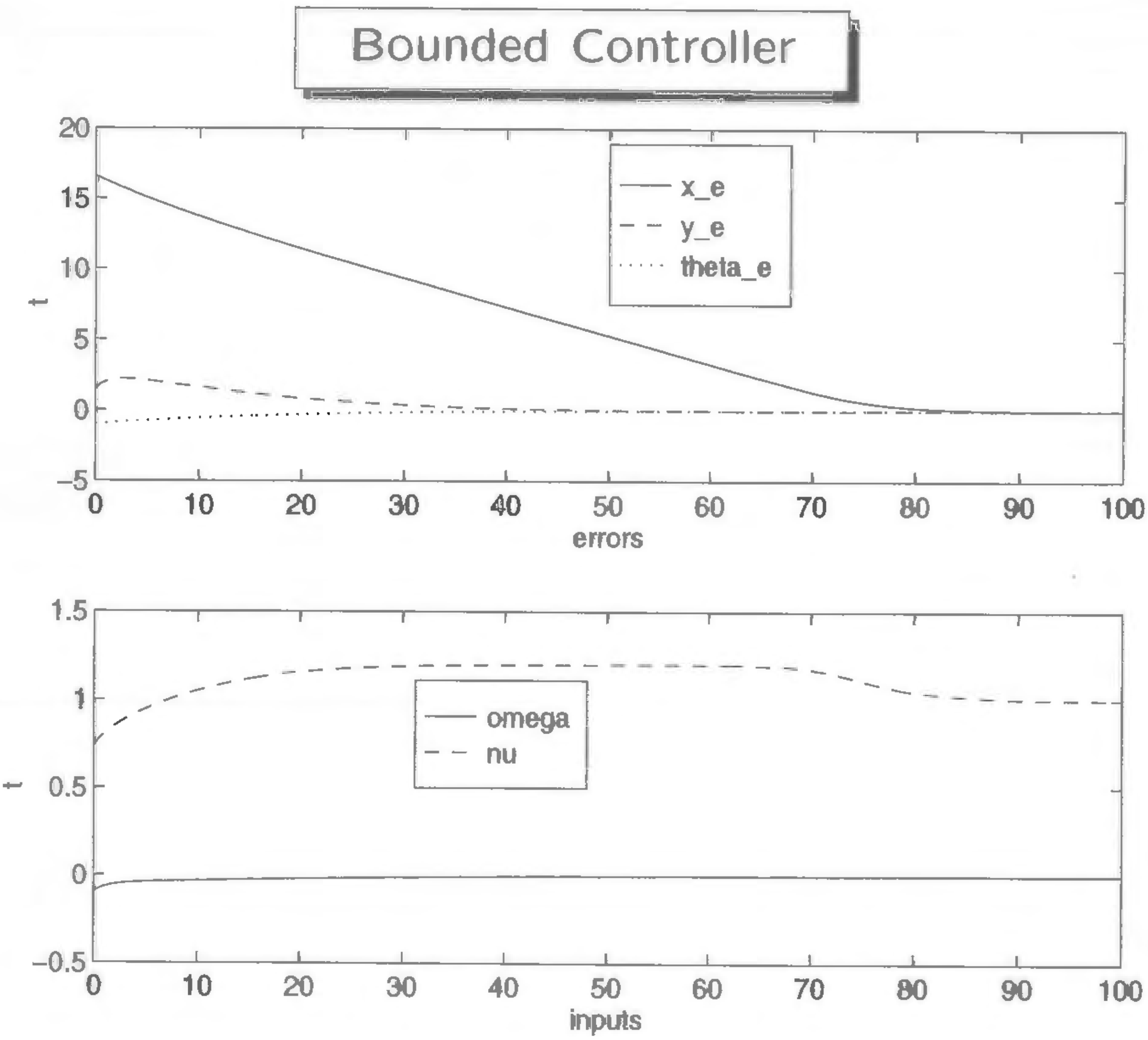
Bounded controller:
$f_1(\omega) = 0.2\tanh(\omega)$, $f_2(y_e) = 0.2\tanh(y_e)$,
$f_3(\bar{x}_e) = 0.2\tanh(\bar{x}_e)$, $f_4(\theta_e) = 0.2\tanh(\theta_e)$,
$\gamma = 0.045$.

The bounded controller was designed to satisfy the input contraints

$$|\nu(t)| \le 2, \quad |\omega(t)| \le 1 \quad \forall t \ge 0$$

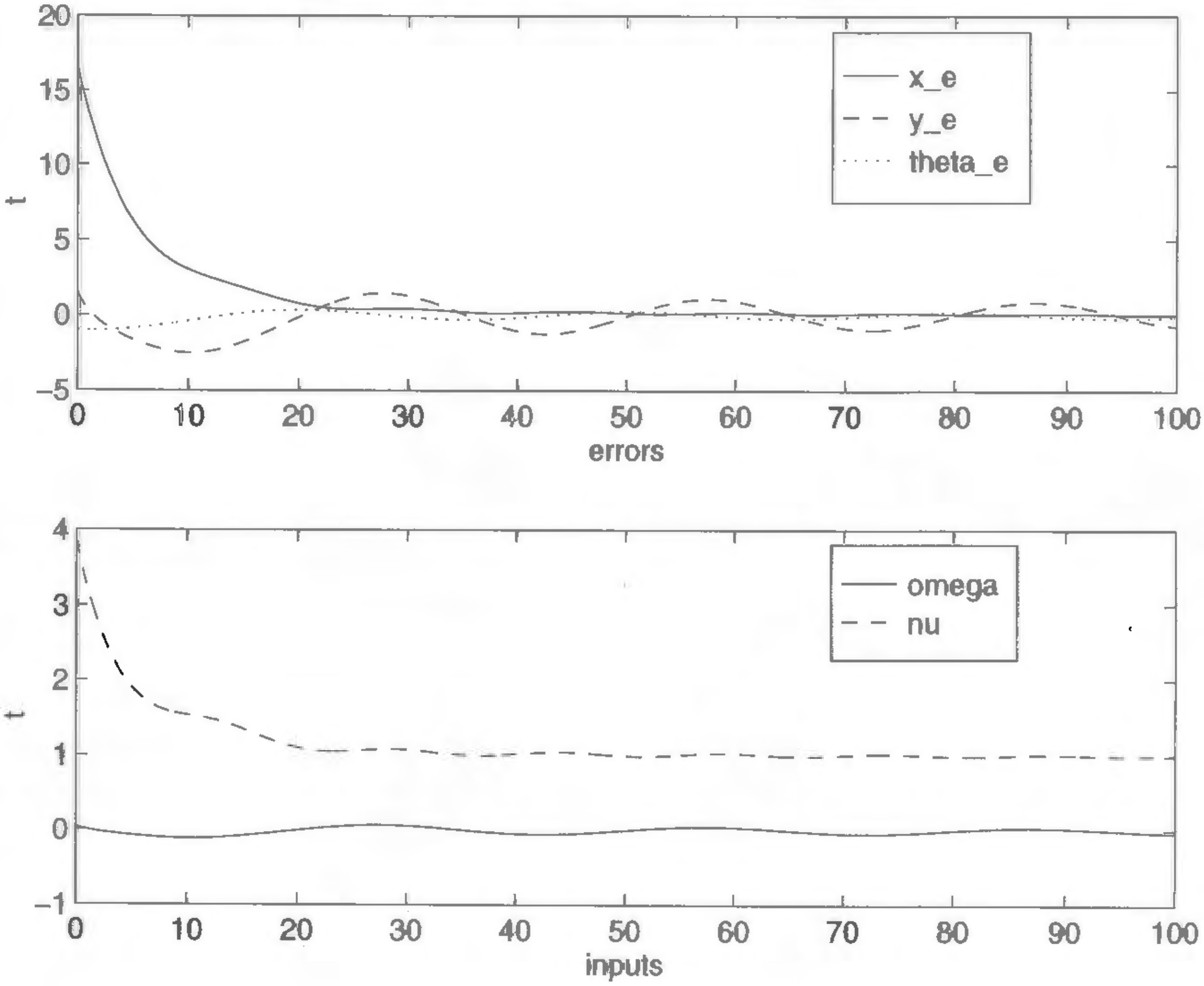
Controller Jiang& Nijmeijer





Bounded Controller
20
15
10
5
0
−5
t
x_e
y_e
theta_e
0
10
20
30
40
50
60
70
80
90
100
errors
1.5
1
0.5
0
−0.5
t
omega
nu
inputs

Jiang & Nijmeijer (II)



$f_1(\omega) = 0.2\omega$, $f_2(y_e) = 0.2y_e$,
$f_3(\bar{x}_e) = 0.2\bar{x}_e$, $f_4(\theta_e) = 0.2\theta_e$,
$\gamma = 0.045$.

Conclusions

- A wider class of (globally) tracking controllers for a wheeled mobile robot has been derived.

- Under input constraints, semi-globally tracking controllers have been found.

- There is a trade-off between convergence to the desired trajectory and the satisfying of input constraints.