TU/e technische universiteit eindhoven

Modeling Manufacturing Systems for Control

Erjen Lefeber

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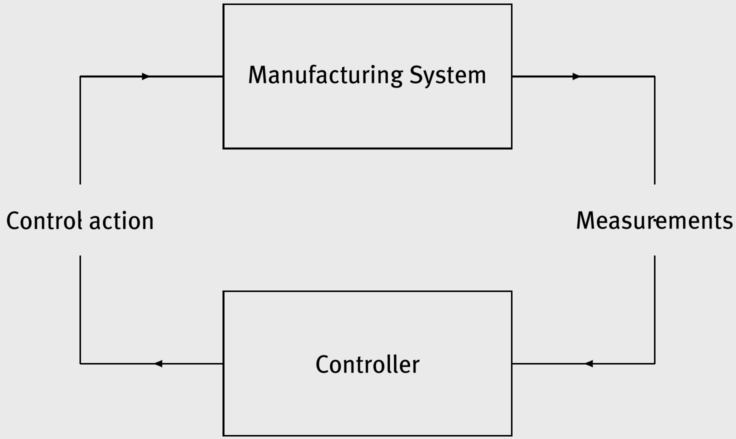
/department of mechanical engineering

Outline

- Modeling problem
- Available models
  - Discrete event, Queuing theory, Fluid models
  - Extensions to fluid models (ramp up)
  - Control framework
- Alternative model
  - Traffic flow
  - Flow model
- Properties for model
- Conclusions

Modeling problem (I)

Modeling for control (supply chain/mass production).



- Like to understand dynamics of factories
- Throughput, cycle time, variance of cycle time
- Answer questions like: How to ramp up a factory?

Modeling problem (II)

Some observations from practice:

- Quick answers (“What if . . . ”).
- A factory is (almost) never in steady state
- Throughput and cycle time are related

We look for a model that

- is computationally feasible,
- describes dynamics, and
- incorporates both throughput and cycle time

Available models (I)

Discrete Event

- Advantages
  - Include dynamics
  - Throughput and cycle time related
- Disadvantage
  - Not computationally feasible for real life fab

Available models (II)

Theory

- Advantages
  - Throughput and cycle time related
  - Computationally feasible (approximations)
- Disadvantage
  - Only steady state, no dynamics

Available models (III)

Fluid models

- Kimemia and Gershwin: Flow model
- Queuing theorists: Fluid models/Fluid queues

$$\xrightarrow{u_0} \overset{y_1}{B_1} \xrightarrow{u_1} M_1 \xrightarrow{u_1} \overset{y_2}{B_2} \xrightarrow{u_2} M_2 \xrightarrow{u_2} \overset{y_3}{B_3}$$

$$\begin{aligned}
\dot{y}_1 &= u_0 - u_1 \\
\dot{y}_2 &= u_1 - u_2 \\
\dot{y}_3 &= u_2
\end{aligned}$$

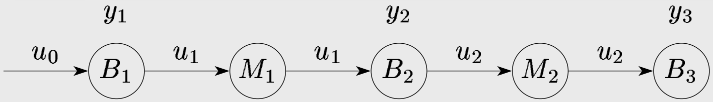
- Cassandras: Stochastic Fluid Model

Available models (III cont'd.)

Fluid models

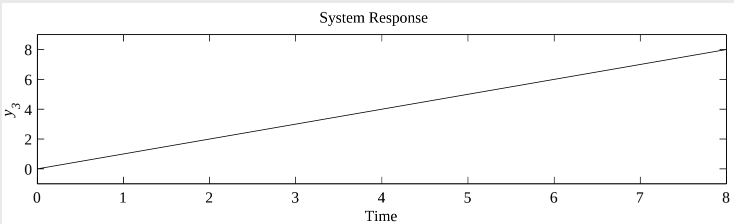
- Advantages
  - Dynamical model
  - Computationally feasible
- Disadvantage
  - Only throughput incorporated in model, no cycle time
  - And more . . .

Ramp up of fluid model



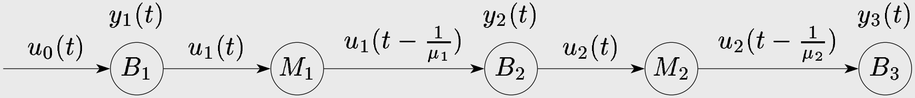
- Initially empty fab, $u_0 = 1$, $\mu_1 = \mu_2 = 1$.
- Machine produces whenever possible:

$$u_i = \begin{cases} 1 & \text{if } y_i > 0 \\ 0 & \text{otherwise} \end{cases} \qquad i \in \{1, 2\}.$$



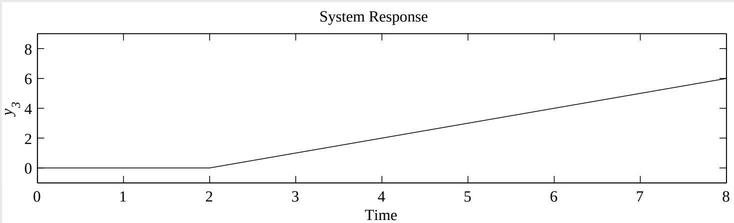
Extension to fluid model (I)

Possible solution: Add delay



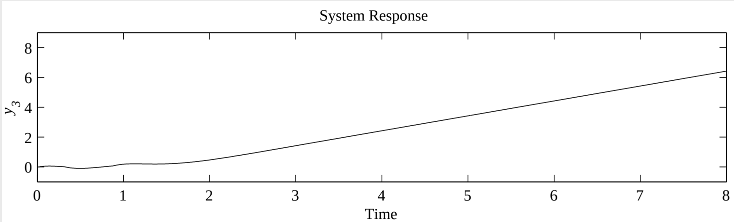
$$\dot{y}_1(t) = u_0(t) - u_1(t)$$
$$\dot{y}_2(t) = u_1(t - \frac{1}{\mu_1}) - u_2(t)$$
$$\dot{y}_3(t) = u_2(t - \frac{1}{\mu_2})$$

Result:



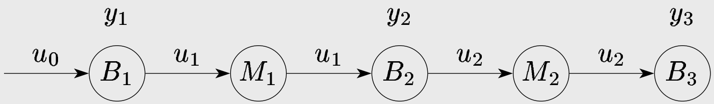
Extension to fluid model (II)

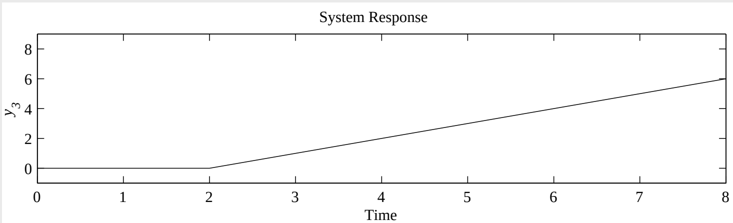
Padé approximations (2nd order):



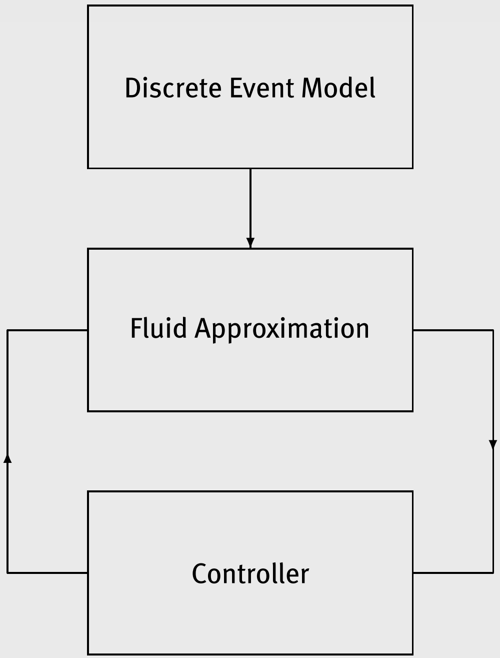
Extension to fluid model (III)

Hybrid model:

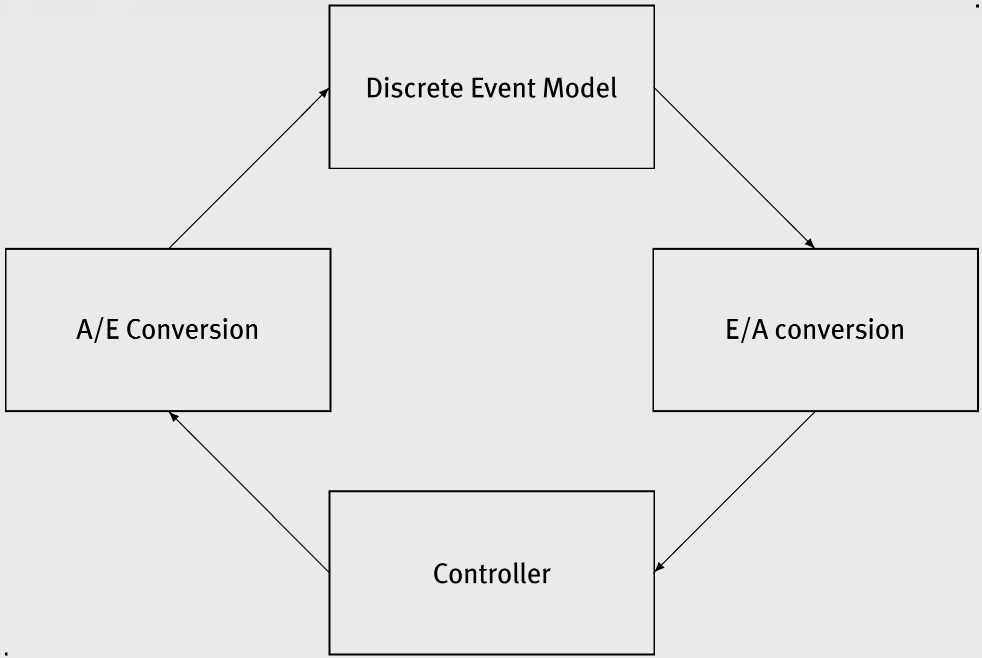




Control framework (I)



Control framework (II)



Available models (conclusion)

- Discrete Event: Not computationally feasible
  Queuing Theory: No dynamics
  Fluid models: No cycle time
- Need something else!
- Discrete event models (and queuing theory) have proved themselves. Can be used for verification!

Traffic flow: LWR model

Lighthill, Whitham ('55), and Richards ('56)

Traffic behavior on one-way road:

- density $\rho(x,t)$,
- speed $v(x,t)$,
- flow $u(x,t) = \rho(x,t)v(x,t)$.

Conservation of mass:

$$\frac{\partial \rho}{\partial t}(x,t) + \frac{\partial u}{\partial x}(x,t) = 0.$$

Static relation between flow and density:

$$u(x,t) = S(\rho(x,t)).$$

Modeling manufacturing flow

- density $\rho(x,t)$,
- speed $v(x,t)$,
- flow $u(x,t)=\rho(x,t)v(x,t)$,
- Conservation of mass: $\frac{\partial\rho}{\partial t}(x,t)+\frac{\partial u}{\partial x}(x,t)=0.$

Static relation between flow and density:

- single queue: $\frac{1}{v(x,t)}=\frac{1}{\mu}(1+\int_0^1\rho(s,t)\,\mathrm{d}s)$,
- re-entrant line: $v(x,t)=v_0\left(1-\frac{\int_0^1\rho(s,t)\,\mathrm{d}s}{W_{\text{max}}}\right)$,
- Many identical M/M/1 queues: $u(x,t)=\frac{\mu\rho(x,t)}{1+\rho(x,t)}$.

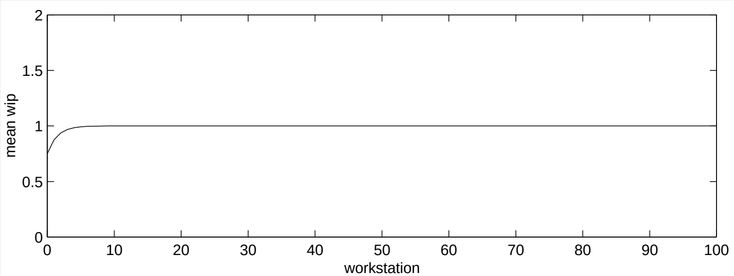
Properties (I)

- No backward-flow allowed (cf. Daganzo '95)
- No negative density
- Stable steady states
  - constant feed rate $\longrightarrow$ equilibrium
  - equilibrium meets relations queuing theory

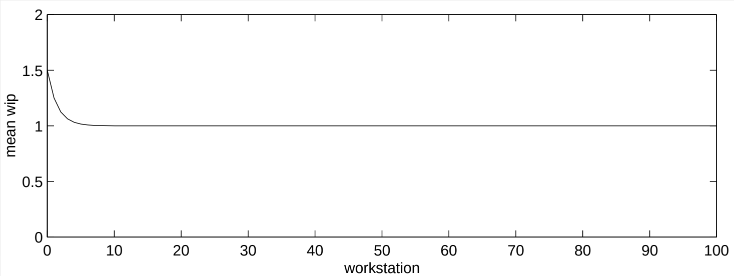
Properties (II)

100 machines, $\mu = 1$, exponential. Utilization: 50%.

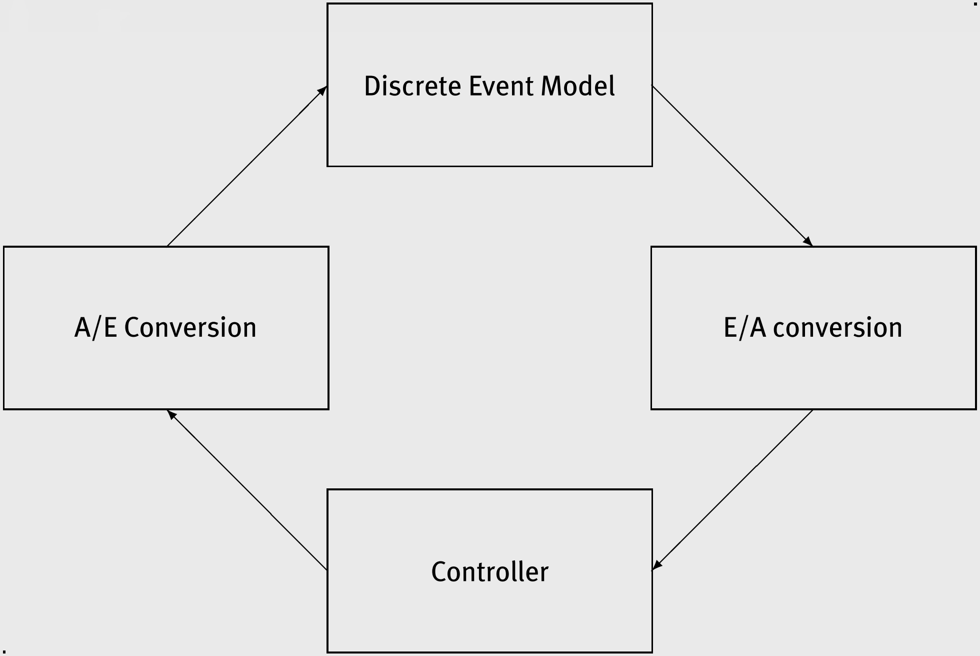
- Regular arrivals: $c_a^2 = 0$



- Irregular arrivals: $c_a^2 = 3$



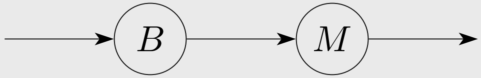
Recall: Control framework



Properties (III)

Variability needs to be included. However, ...

1 machine, $\mu = 1$, exponential



- Push control: exponential arrivals. Utilization 50%
  - Throughput: 0.5 lots per unit time
  - Cycle time: 2 hours
  - Mean wip: 1 lot
- CONWIP control: WIP=1
  - Throughput: 1 lots per unit time
  - Cycle time: 1 hours
  - Mean wip: 1 lot

Conclusions

Computationally feasible dynamical model incorporating both throughput and cycle time.

- NOT: Discrete event, Queuing theory, Fluid models
- Extensions to fluid model
- Possible: flow model (PDE)
  - Variability not dealt with by LWR-like models
  - Push $\leftrightarrow$ CONWIP: Correlation determines steady state
  - Queuing theory, discrete event models can be used for verification

Future Research

- Derive ‘good’ (PDE)models (throughput, (variance) cycle time)
- PDE-based controller design
- PDE-based observer design
- E/A conversion
- A/E conversion