Almost Global Tracking Control of a Quadrotor UAV on SE(3)

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Tracking control of drones

Three approaches for modeling dynamics and deriving controllers:

- Euler angles
- (Unit) quaternions
- SE(3)

Introduction

Tracking control of drones

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Euler angles

Singularities in representation (gimbal lock)

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Euler angles

Singularities in representation (gimbal lock)

(Unit) quaternions

Let both q and $\bar{q}$ represent same attitude.
Need same control actions: $u(q) = u(\bar{q})$, otherwise: ambiguity.

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Introduction

Remaining option: SE(3)

Both problems are overcome by considering dynamics on SE(3)

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Almost global result under assumption of *non-zero thrust follower*.

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Consequence: only local result.

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Almost global result under assumption of *non-zero thrust follower*.
Consequence: only local result.

Contribution

Almost global result under assumption of *non-zero thrust reference*.

Introduction

Comparable result

M.-D. Hua, T. Hamel, P. Morin, and C. Samson, "A control approach for thrust-propelled underactuated vehicles and its application to VTOL drones," IEEE Transactions on Automatic Control, vol. 54, no. 8, pp. 1837–1853, 2009.

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Major differences

In this paper

- torques as input (vs. velocities)
- uniform almost global asymptotic stability

Problem

Drone dynamics

$$
\begin{aligned}
\dot{\rho} &= R\nu \\
\dot{\nu} &= -S(\omega)\nu + gR^T e_3 - \frac{f}{m} e_3 \\
\dot{R} &= RS(\omega) \\
J\dot{\omega} &= S(J\omega)\omega + \tau,
\end{aligned}
$$

Problem

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\begin{aligned}
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\dot{R} &= RS(\omega) \\
J\dot{\omega} &= S(J\omega)\omega + \tau,
\end{aligned}
$$

Reference dynamics

$$
\begin{aligned}
\dot{\rho}_r &= R_r \nu_r \\
\dot{\nu}_r &= -S(\omega_r)\nu_r + gR_r^T e_3 - \frac{f_r}{m} e_3 \\
\dot{R}_r &= R_r S(\omega_r) \\
J\dot{\omega}_r &= S(J\omega_r)\omega_r + \tau_r
\end{aligned}
$$

Problem

Drone dynamics

$$\dot{\rho} = R\nu$$
$$\dot{\nu} = -S(\omega)\nu + gR^T e_3 - \frac{f}{m} e_3$$
$$\dot{R} = RS(\omega)$$
$$J\dot{\omega} = S(J\omega)\omega + \tau,$$

Reference dynamics

$$\dot{\rho}_r = R_r \nu_r$$
$$\dot{\nu}_r = -S(\omega_r)\nu_r + gR_r^T e_3 - \frac{f_r}{m} e_3$$
$$\dot{R}_r = R_r S(\omega_r)$$
$$J\dot{\omega}_r = S(J\omega_r)\omega_r + \tau_r$$

Feasible reference trajectory

Trajectory $(\rho_r, R_r, \nu_r, \omega_r, f_r, \tau_r)$ satisfying reference dynamics, with $0 < f_r^{\min} \leq f_r(t)$ and $\omega_r(t)$ bounded.

Problem

Error coordinates

$$\tilde{\rho} = R_r^T(\rho - \rho_r) \qquad \tilde{R} = R_r^T R$$

$$\tilde{\nu} = -\tilde{R}^T S(\omega_r)\tilde{\rho} + \nu - \tilde{R}^T \nu_r \qquad \tilde{\omega} = \omega - \tilde{R}^T \omega_r$$

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Error measure

$$\varepsilon(\tilde{\rho}, \tilde{R}, \tilde{\nu}, \tilde{\omega}) = \|\tilde{\rho}\| + \|\log \tilde{R}\| + \|\tilde{\nu}\| + \|\tilde{\omega}\|$$

Problem

Error coordinates

$$\tilde{\rho} = R_r^T(\rho - \rho_r) \qquad \tilde{R} = R_r^T R$$

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Error measure

$$\varepsilon(\tilde{\rho}, \tilde{R}, \tilde{\nu}, \tilde{\omega}) = \|\tilde{\rho}\| + \|\log \tilde{R}\| + \|\tilde{\nu}\| + \|\tilde{\omega}\|$$

Problem

For $(\rho_r, R_r, \nu_r, \omega_r, f_r, \tau_r)$ being a given feasible reference trajectory, find appropriate control laws

$$f = f(\rho, R, \nu, \omega, \rho_r, R_r, \nu_r, \omega_r) > 0, \qquad \tau = \tau(\rho, R, \nu, \omega, \rho_r, R_r, \nu_r, \omega_r)$$

such that for the resulting closed-loop system

$$\lim_{t\to\infty} \varepsilon\big(\tilde{\rho}(t), \tilde{R}(t), \tilde{\nu}(t), \tilde{\omega}(t)\big) = 0.$$

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Approach

Controller design

Two steps:

- Position tracking (body-fixed accelerations as *virtual input*)
- Attitude control (using actual inputs)

Show stability result using cascade analysis

Position tracking

Error definition

Express tracking error in body fixed frame of the reference:

$$\begin{bmatrix}\rho_e \\ \nu_e\end{bmatrix} = \begin{bmatrix}R_r^T(\rho_r - \rho) \\ \nu_r - R_r^T R \nu\end{bmatrix}$$

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Position tracking

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Tracking error dynamics

$$\begin{aligned} \dot{\rho}_e &= -S(\omega_r)\rho_e + \nu_e \\ \dot{\nu}_e &= -S(\omega_r)\nu_e + \underbrace{\tfrac{f}{m} R_r^T R e_3 - \tfrac{f_r}{m} e_3}_{u} \end{aligned}$$

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Position tracking

Error definition

Express tracking error in body fixed frame of the reference:

$$\begin{bmatrix} \rho_e \\ \nu_e \end{bmatrix} = \begin{bmatrix} R_r^T(\rho_r - \rho) \\ \nu_r - R_r^T R \nu \end{bmatrix}$$

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$$\begin{aligned} \dot{\rho}_e &= -S(\omega_r)\rho_e + \nu_e \\ \dot{\nu}_e &= -S(\omega_r)\nu_e + \underbrace{\frac{f}{m} R_r^T R e_3 - \frac{f_r}{m} e_3}_{u} \end{aligned}$$

Failing alternative

Express tracking error in body fixed frame of the drone:

$$\begin{bmatrix} \rho_e \\ \nu_e \end{bmatrix} = \begin{bmatrix} R^T(\rho - \rho_r) \\ \nu - R^T R_r \nu_r \end{bmatrix}$$

Tracking error dynamics

$$\begin{aligned} \dot{\rho}_e &= -S(\omega)\rho_e + \nu_e \\ \dot{\nu}_e &= -S(\omega)\nu_e + \underbrace{\frac{f_r}{m} R^T R_r e_3 - \frac{f}{m} e_3}_{\text{no full control}} \end{aligned}$$

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Result

Consider the dynamics

$$\dot{\rho}_e = -S(\omega_r)\rho_e + \nu_e$$
$$\dot{\nu}_e = -S(\omega_r)\nu_e + u$$

in closed loop with the dynamic state feedback

$$\begin{aligned} u &= R_r^T(K_P P_e + K_p p_e) - R_r^T K_P R_r \sigma_1(\rho_e + R_r^T P_e) \\ &\quad - k_\rho \sigma_2(\rho_e + R_r^T P_e) - K_\nu \sigma_3(\nu_e + R_r^T p_e) \\ \dot{P}_e &= p_e \\ \dot{p}_e &= -K_P P_e - K_p p_e + K_P R_r \sigma_1(\rho_e + R_r^T P_e) \end{aligned}$$

where $K_P = K_P^T > 0$, $K_p = K_p^T > 0$, $K_\nu = K_\nu^T > 0$, and $k_\rho > 0$.

The origin of the closed-loop system is UGAS.

Proof

Define $\bar{\rho}_e = \rho_e + R_r^T P_e$, $\bar{\nu}_e = \nu_e + R_r^T p_e$. Then

$$\begin{aligned}
\dot{P}_e &= p_e \\
\dot{p}_e &= -K_P P_e - K_p p_e + K_P R_r \sigma_1(\bar{\rho}_e) \\
\dot{\bar{\rho}}_e &= -S(\omega_r)\bar{\rho}_e + \bar{\nu}_e \\
\dot{\bar{\nu}}_e &= -S(\omega_r)\bar{\nu}_e - k_\rho \sigma_2(\bar{\rho}_e) - K_\nu \sigma_3(\bar{\nu}_e)
\end{aligned}$$

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\dot{\bar{\nu}}_e &= -S(\omega_r)\bar{\nu}_e - k_\rho \sigma_2(\bar{\rho}_e) - K_\nu \sigma_3(\bar{\nu}_e)
\end{aligned}$$

Differentiating $V_1(\bar{\rho}_e, \bar{\nu}_e) = k_\rho V_{\sigma_2}(\bar{\rho}_e) + \frac{1}{2}\bar{\nu}_e^T \bar{\nu}_e$ yields

$$\dot{V}_1(\bar{\rho}_e, \bar{\nu}_e) = -\bar{\nu}_e^T K_\nu \sigma_3(\bar{\nu}_e) = Y_1(\bar{\nu}_e) \leq 0$$

Proof

Define $\bar{\rho}_e = \rho_e + R_r^T P_e$, $\bar{\nu}_e = \nu_e + R_r^T p_e$. Then

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Differentiating $V_2(\bar{\rho}_e, \bar{\nu}_e) = \bar{\nu}_e^T \bar{\rho}_e$ yields

$$\dot{V}_2(\bar{\rho}_e, \bar{\nu}_e) = \bar{\nu}_e^T \bar{\nu}_e - \bar{\rho}_e^T k_\nu \sigma_3(\bar{\nu}_e) - \bar{\rho}_e^T k_\rho \sigma_2(\bar{\rho}_e) = Y_2(\bar{\rho}_e, \bar{\nu}_e).$$

Position tracking

Proof

Define $\bar{\rho}_e = \rho_e + R_r^T P_e$, $\bar{\nu}_e = \nu_e + R_r^T p_e$. Then

$$\begin{aligned}
\dot{P}_e &= p_e \\
\dot{p}_e &= -K_P P_e - K_p p_e + K_P R_r \sigma_1(\bar{\rho}_e) \\
\dot{\bar{\rho}}_e &= -S(\omega_r)\bar{\rho}_e + \bar{\nu}_e \\
\dot{\bar{\nu}}_e &= -S(\omega_r)\bar{\nu}_e - k_\rho \sigma_2(\bar{\rho}_e) - K_\nu \sigma_3(\bar{\nu}_e)
\end{aligned}$$

Differentiating $V_1(\bar{\rho}_e, \bar{\nu}_e) = k_\rho V_{\sigma_2}(\bar{\rho}_e) + \frac{1}{2}\bar{\nu}_e^T \bar{\nu}_e$ yields

$$\dot{V}_1(\bar{\rho}_e, \bar{\nu}_e) = -\bar{\nu}_e^T K_\nu \sigma_3(\bar{\nu}_e) = Y_1(\bar{\nu}_e) \leq 0$$

Differentiating $V_2(\bar{\rho}_e, \bar{\nu}_e) = \bar{\nu}_e^T \bar{\rho}_e$ yields

$$\dot{V}_2(\bar{\rho}_e, \bar{\nu}_e) = \bar{\nu}_e^T \bar{\nu}_e - \bar{\rho}_e^T k_\nu \sigma_3(\bar{\nu}_e) - \bar{\rho}_e^T k_\rho \sigma_2(\bar{\rho}_e) = Y_2(\bar{\rho}_e, \bar{\nu}_e).$$

Use a nested Matrosov result and a cascaded result to show UGAS

Desired thrust and orientation

From position tracking we have desired virtual input $u = \frac{f}{m} R_r^T R e_3 - \frac{f_r}{m} e_3$.

Then $f = \|mu + f_r e_3\|$ and $R_r^T R e_3 = \frac{mu+f_r e_3}{\|mu+f_r e_3\|} = f_d = \begin{bmatrix} f_{d1} & f_{d2} & f_{d3} \end{bmatrix}^T$.

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Attitude control

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Since $0 < f_r^{\min} \leq f_r(t)$, we can make $\|u\| \leq \frac{f_r^{\min} - \epsilon}{m}$, so $f_{d3} > 0$.

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Since $0 < f_r^{\min} \leq f_r(t)$, we can make $\|u\| \leq \frac{f_r^{\min} - \epsilon}{m}$, so $f_{d3} > 0$. Define

$$R_d = \begin{bmatrix} 1 - \frac{f_{d1}^2}{1+f_{d3}} & -\frac{f_{d1} f_{d2}}{1+f_{d3}} & f_{d1} \\ -\frac{f_{d1} f_{d2}}{1+f_{d3}} & 1 - \frac{f_{d2}^2}{1+f_{d3}} & f_{d2} \\ -f_{d1} & -f_{d2} & f_{d3} \end{bmatrix} \in SO(3) \qquad \omega_d = \begin{bmatrix} -\dot{f}_{d2} + \frac{f_{d2} \dot{f}_{d3}}{1+f_{d3}} \\ \dot{f}_{d1} - \frac{f_{d1} \dot{f}_{d3}}{1+f_{d3}} \\ \frac{f_{d2} \dot{f}_{d1} - f_{d1} \dot{f}_{d2}}{1+f_{d3}} \end{bmatrix}$$

Then $R_d e_3 = f_d$. Note: R_d rotates from e_3 to f_d in spanned plane.

Attitude control

Desired thrust and orientation

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Then $f = \|mu + f_r e_3\|$ and $R_r^T R e_3 = \frac{mu + f_r e_3}{\|mu + f_r e_3\|} = f_d = \begin{bmatrix} f_{d1} & f_{d2} & f_{d3} \end{bmatrix}^T$.

Since $0 < f_r^{\min} \leq f_r(t)$, we can make $\|u\| \leq \frac{f_r^{\min} - \epsilon}{m}$, so $f_{d3} > 0$. Define

$$R_d = \begin{bmatrix} 1 - \frac{f_{d1}^2}{1+f_{d3}} & -\frac{f_{d1} f_{d2}}{1+f_{d3}} & f_{d1} \\ -\frac{f_{d1} f_{d2}}{1+f_{d3}} & 1 - \frac{f_{d2}^2}{1+f_{d3}} & f_{d2} \\ -f_{d1} & -f_{d2} & f_{d3} \end{bmatrix} \in SO(3) \qquad \omega_d = \begin{bmatrix} -\dot{f}_{d2} + \frac{f_{d2} \dot{f}_{d3}}{1+f_{d3}} \\ \dot{f}_{d1} - \frac{f_{d1} \dot{f}_{d3}}{1+f_{d3}} \\ \frac{f_{d2} \dot{f}_{d1} - f_{d1} \dot{f}_{d2}}{1+f_{d3}} \end{bmatrix}$$

Then $R_d e_3 = f_d$. Note: R_d rotates from e_3 to f_d in spanned plane.

Define attitude errors:

$$R_e = R_d^T (R_r^T R) \qquad \omega_e = \omega - R^T R_r \omega_r - R_e^T \omega_d$$

Attitude error dynamics

$$\begin{aligned}\dot{R}_e &= R_e S(\omega_e) \\ J\dot{\omega}_e &= \tau - JR^T R_r J^{-1}[S(J\omega_r)\omega_r + \tau_r] + S(J\omega)\omega \\ &\quad + JS(\omega_e)[\omega - \omega_e] + JR_e^T[S(\omega_d)R_d^T\omega_r - \dot{\omega}_d].\end{aligned}$$

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Attitude control

Attitude error dynamics

$$\dot{R}_e = R_e S(\omega_e)$$

$$J\dot{\omega}_e = \tau - JR^T R_r J^{-1}[S(J\omega_r)\omega_r + \tau_r] + S(J\omega)\omega$$
$$+ JS(\omega_e)[\omega - \omega_e] + JR_e^T[S(\omega_d)R_d^T\omega_r - \dot{\omega}_d].$$

Controller (standard)

$$\tau = -K_\omega\omega_e + K_R\sum_{i=1}^{3} k_i(e_i \times R_e^T e_i) + JR^T R_r J^{-1}[S(J\omega_r)\omega_r + \tau_r]$$
$$- S(J\omega)\omega - JS(\omega_e)[\omega - \omega_e] - JR_e^T[S(\omega_d)R_d^T\omega_r - \dot{\omega}_d]$$

with distinct $k_i > 0$ and $K_\omega = K_\omega^T > 0$, $K_R = K_R^T > 0$.

Result: $(R_e, \omega_e) = (I, 0)$ ULES and UaGAS

Combined result

Cascaded system

$$
\begin{aligned}
\dot{P}_e &= p_e \\
\dot{p}_e &= -K_P P_e - K_p p_e + K_P R_r \sigma_1(\bar{\rho}_e) \\
\dot{\bar{\rho}}_e &= -S(\omega_r)\bar{\rho}_e + \bar{\nu}_e \\
\dot{\bar{\nu}}_e &= -S(\omega_r)\bar{\nu}_e - k_\rho \sigma_2(\bar{\rho}_e) - K_\nu \sigma_3(\bar{\nu}_e) + \frac{f}{m} R_r^T R(I - R_e^T) e_3 \\
\dot{R}_e &= R_e S(\omega_e) \\
J\dot{\omega}_e &= -K_\omega \omega_e + K_R \sum_{i=1}^{3} k_i (e_i \times R_e^T e_i)
\end{aligned}
$$

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\begin{aligned}
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\dot{p}_e &= -K_P P_e - K_p p_e + K_P R_r \sigma_1(\bar{\rho}_e) \\
\dot{\bar{\rho}}_e &= -S(\omega_r)\bar{\rho}_e + \bar{\nu}_e \\
\dot{\bar{\nu}}_e &= -S(\omega_r)\bar{\nu}_e - k_\rho \sigma_2(\bar{\rho}_e) - K_\nu \sigma_3(\bar{\nu}_e) + \frac{f}{m} R_r^T R(I - R_e^T) e_3 \\
\dot{R}_e &= R_e S(\omega_e) \\
J\dot{\omega}_e &= -K_\omega \omega_e + K_R \sum_{i=1}^{3} k_i (e_i \times R_e^T e_i)
\end{aligned}
$$

Result

If the functions σ_1, σ_2, σ_3, and $P_e(t_0)$ and $p_e(t_0)$ are properly chosen guaranteeing that $\|u\| \le \frac{f_r^{\min} - \epsilon}{m}$ for some $0 < \epsilon < f_r^{\min}$, then the origin $(P_e, p_e, \bar{\rho}, \bar{\nu}, R_e, \omega_e) = (0, 0, 0, 0, I, 0)$ is ULES and UaGAS.

Final result

Problem: we have solved a different problem

We have shown convergence of $(P_e, p_e, \bar{\rho}, \bar{\nu}, R_e, \omega_e)$ to $(0, 0, 0, 0, I, 0)$.
However, we need to show that $\varepsilon(\tilde{\rho}, \tilde{R}, \tilde{\nu}, \tilde{\omega})$ converges to 0.

Final result

Problem: we have solved a different problem

We have shown convergence of $(P_e, p_e, \bar{\rho}, \bar{\nu}, R_e, \omega_e)$ to $(0, 0, 0, 0, I, 0)$.
However, we need to show that $\varepsilon(\tilde{\rho}, \tilde{R}, \tilde{\nu}, \tilde{\omega})$ converges to 0.

Corollary (final result)

The derived controller also makes $\varepsilon(\tilde{\rho}, \tilde{R}, \tilde{\nu}, \tilde{\omega})$ converge to 0, i.e., $(\tilde{\rho}, \tilde{R}, \tilde{\nu}, \tilde{\omega})$ converges to $(0, I, 0, 0)$.

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Simulations

Data

AR-Drone: $m=0.456$ [kg]
$J=\mathrm{diag}(0.0022, 0.0025, 0.0045)$ [kgm^2]
$g=9.81$ [m/s^2]

Simulations

Data

AR-Drone: m=0.456 [kg]
J=diag(0.0022,0.0025,0.0045) [kgm^2]
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Reference trajectory: $\rho_r(t) = \begin{bmatrix} \cos t & \sin t & 1.5 + \sin t \end{bmatrix}^T$

R_r defined (from $\ddot{\rho}_r$ as presented earlier).

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Simulations

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Reference trajectory: $\rho_r(t) = \begin{bmatrix} \cos t & \sin t & 1.5 + \sin t \end{bmatrix}^T$

R_r defined (from $\ddot{\rho}_r$ as presented earlier).

Initial conditions:

$$\rho(t_0) = \begin{bmatrix} -1 \\ 0.7 \\ 4 \end{bmatrix} \qquad R(t_0) = \begin{bmatrix} -0.25 & -0.433 & 0.866 \\ 0.533 & -0.808 & -0.25 \\ 0.808 & 0.34 & 0.433 \end{bmatrix}$$

$$\nu(t_0) = \begin{bmatrix} 0.1 \\ -0.8 \\ 0.7 \end{bmatrix} \qquad \omega(t_0) = \begin{bmatrix} -1 \\ 0.3 \\ -2 \end{bmatrix}.$$

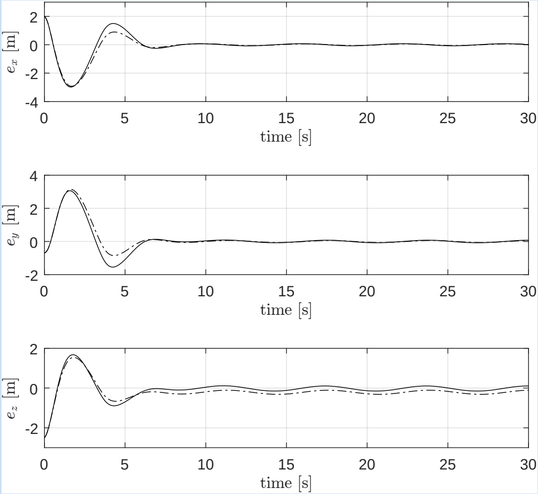
Simulations

Disturbances (more realistic)

- Mass discrepancy: $m = 0.456$[kg], $m_r = 0.48$[kg].
- Add model of sensors and actuators
  - sampling
  - delays
  - noisy measurements
- Use filtered measurements

Simulations

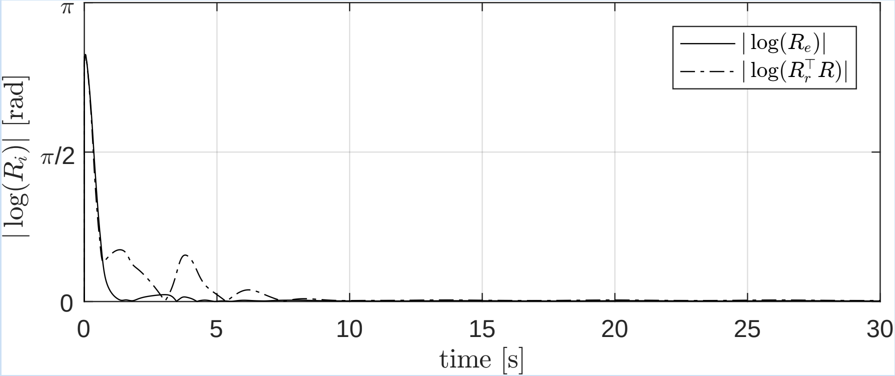
Results: Filtered errors $e = \rho_r(t) - \rho(t)$ in the inertial frame



- without integral control (dashed)
- with integral control (solid)

Simulations

Results: Attitude errors (metric: angle of rotation)



- with respect to the desired attitude, R_e, (solid),
- with respect to the reference attitude, $R_r^T R$, (dashed)

Experiments

[video]

Experiments

[video]

Conclusions

- Avoid singularities of Euler angles, ambiguity of quaternions, allows for large angular maneuvers.

Conclusions

- Avoid singularities of Euler angles, ambiguity of quaternions, allows for large angular maneuvers.
- Explicitly took into account the constraint of non-zero total thrust in our controller design

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Future work

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