A spatial approach to control of platooning vehicles: separating path-following from tracking

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Where innovation starts

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Introduction

Vehicle platooning: longitudinal

- Cooperative Adaptive Cruise Control (CACC).
- Reduced inter vehicle distance reduces drag. Resulting in reduced emission, reduced fuel consumption.
- Several approaches for Longitudinal Control

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Vehicle platooning: longitudinal and lateral

- 2D problem: longitudinal and lateral control
- Consider tracking problem: corner cutting
- Only 5cm per vehicle causes problems for long platoons
- Extended look-ahead reduces problem, but does not eliminate it.

Problem

Description in path length

Kinematic model car with length L:

$$\dot{x}(t) = v(t)\cos\theta(t) \qquad \dot{y}(t) = v(t)\sin\theta(t) \qquad \dot{\theta}(t) = \frac{v(t)}{L}\tan\phi(t)$$

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Let $s(t)$ denote travelled distance along path, i.e., $v(t) = \frac{\mathrm{d}s}{\mathrm{d}t}$.

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$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}s}x(s(t)) &= \cos\theta(s(t)) \\
\frac{\mathrm{d}}{\mathrm{d}s}y(s(t)) &= \sin\theta(s(t)) \\
\frac{\mathrm{d}}{\mathrm{d}s}\theta(s(t)) &= \frac{1}{L}\tan\phi(s(t)) = \underbrace{\kappa(s(t))}_{\text{curvature}}
\end{aligned}$$

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Problem

Follow feasible reference

Dropping dependency of time, we have:

Leader:

$$\frac{\mathrm{d}}{\mathrm{d}s_l}x_l(s_l) = \cos\theta_l(s_l) \qquad \frac{\mathrm{d}}{\mathrm{d}s_l}y_l(s_l) = \sin\theta_l(s_l) \qquad \frac{\mathrm{d}}{\mathrm{d}s_l}\theta_l(s_l) = \kappa_l(s_l)$$

Follower:

$$\frac{\mathrm{d}}{\mathrm{d}s}x(s) = \cos\theta(s) \qquad \frac{\mathrm{d}}{\mathrm{d}s}y(s) = \sin\theta(s) \qquad \frac{\mathrm{d}}{\mathrm{d}s}\theta(s) = \kappa(s)$$

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Difficulty

Path of follower is different. In particular: different length.

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Problem

Lateral control problem (path following)

For bounded $\kappa_l(s_l)$, determine diffeomorphism $\alpha\colon \mathbb{R}^+ \to \mathbb{R}^+$, $s_l = \alpha(s)$ and control law $\kappa(s)$ such that for the closed-loop system

$$\lim_{s\to\infty} |x_l(\alpha(s)) - x(s)| + |y_l(\alpha(s)) - y(s)| + |\theta_l(\alpha(s)) - \theta(s)| = 0$$

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Longitudinal control problem (tracking)

Given leader trajectory and solution for lateral control problem, determine velocity profile $v(t)$ such that

$$\lim_{t\to\infty} \alpha^{-1}(s_l(t)) - s(t) - hv(t) - r = 0$$

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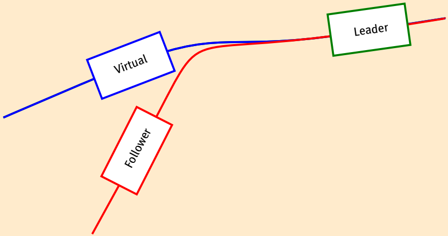
Lateral controller design

Path following: main idea

Let virtual vehicle drive along path of leader.



Virtual vehicle at $s_l = \alpha(s)$, when follower at s.

Path following: dynamics

Let $\bar{v}(s) = \frac{\mathrm{d}\alpha(s)}{\mathrm{d}s}$, $\bar{x}_l(s) = x_l(\alpha(s))$, $\bar{y}_l(s) = y_l(\alpha(s))$, $\bar{\theta}_l(s) = \theta_l(\alpha(s))$, $\bar{\kappa}_l(s) = \kappa_l(\alpha(s))$.

Lateral controller design

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$$
\begin{aligned}
\frac{\mathrm{d}x(s)}{\mathrm{d}s} &= \cos\theta(s) & \qquad \frac{\mathrm{d}\bar{x}_l(s)}{\mathrm{d}s} &= \bar{v}(s)\cos\bar{\theta}_l(s) \\
\frac{\mathrm{d}y(s)}{\mathrm{d}s} &= \sin\theta(s) & \qquad \frac{\mathrm{d}\bar{y}_l(s)}{\mathrm{d}s} &= \bar{v}(s)\sin\bar{\theta}_l(s) \\
\frac{\mathrm{d}\theta(s)}{\mathrm{d}s} &= \kappa(s) & \qquad \frac{\mathrm{d}\bar{\theta}_l(s)}{\mathrm{d}s} &= \bar{v}(s)\bar{\kappa}_l(s)
\end{aligned}
$$

Lateral controller design

Path following: error

Define errors in frame of follower

$$\begin{bmatrix} x_e(s) \\ y_e(s) \\ \theta_e(s) \end{bmatrix} = \begin{bmatrix} \cos\theta(s) & \sin\theta(s) & 0 \\ -\sin\theta(s) & \cos\theta(s) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_l(s) - x(s) \\ \bar{y}_l(s) - y(s) \\ \bar{\theta}_l(s) - \theta(s) \end{bmatrix}.$$

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Drop dependency on s. Use $'$ for $\frac{\mathrm{d}}{\mathrm{d}s}$. Resulting error-dynamics:

$$\begin{aligned} x_e' &= \kappa y_e + \bar{v}\cos\theta_e - 1, \\ y_e' &= -\kappa x_e + \bar{v}\sin\theta_e, \\ \theta_e' &= -\kappa + \bar{v}\bar{\kappa}_l. \end{aligned}$$

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Path following: Lyapunov (1 of 2)

Let $|\theta_e(0)| < \pi/2$. Recall error-dynamics:

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For $V_1 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 - \frac{1}{c_3}\log(\cos\theta_e)$ with $c_3 > 0$ we have

$$V_1' = x_e(\bar{v}\cos\theta_e - 1) + y_e(\bar{v}\sin\theta_e) + \frac{1}{c_3}\tan\theta_e(-\kappa + \bar{v}\bar{\kappa}_l).$$

Path following: Lyapunov (2 of 2)

For $|\theta_e(0)| < \pi/2$ and $V_1 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 - \frac{1}{c_3}\log(\cos\theta_e)$:

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Input

$$\begin{aligned} \bar{v} &= \frac{1 - c_1\sigma_1(x_e)}{\cos\theta_e} && 0 < c_1 < 1 \\ \kappa &= c_3 y_e(1 - c_1\sigma_1(x_e)) + \bar{v}\bar{\kappa}_l + c_2\sigma_2(\theta_e) && 0 < c_2 \end{aligned} \tag{1}$$

with $x\sigma(x) > 0$ for $x \neq 0$, $\sigma'(0) > 0$, $|\sigma(x)| \leq 1$, results in

$$V_1' = -c_1 x_e \sigma_1(x_e) - \frac{c_2}{c_3}\sigma_2(\theta_e)\tan\theta_e \leq 0$$

Path following: Lyapunov (2 of 2)

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Note that $\theta_e(s) < \pi/2$, and $\bar{v} > 0$. Diffeomorphism: $\frac{\mathrm{d}\alpha}{\mathrm{d}s} = \bar{v}$, $\alpha(0) = 0$.

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Lateral controller design

Path following: result

The controller (1) solves the lateral control problem, for all initial states satisfying $|\theta_e(0)| < \pi/2$, where the function α is given from $\frac{d\alpha}{ds} = \bar{v}$, $\alpha(0) = 0$. Furthermore, the resulting closed-loop system is uniformly globally asymptotically stable and locally exponentially stable in the distance driven.

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Proof

Use Nested Matrosov Theorem. Taking

$$V_2 = -x_e x_e' - \theta_e \theta_e'$$

results in

$$V_2' = \underbrace{Y(x_e, y_e, \theta_e)}_{=0 \text{ for } V_1' = 0} - (c_3 y_e + \bar{\kappa}_l)^2 y_e^2 - (c_3 y_e)^2$$

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Lateral controller design

Extension: bound on curvature

Let $\sup_s \kappa_l(s) = \kappa_l^{\text{max}}$. Additional requirement: $|\kappa(s)| \leq \kappa^{\text{max}} > \kappa_l^{\text{max}}$.

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\end{aligned}
\tag{2}
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Path following: result

For all initial states satisfying $|\theta_e(0)| < \arccos(\kappa_l^{\max}/\kappa^{\max})$, there exist $c_2 > 0$, $c_3 > 0$ such that (2) solves lateral control problem.

Longitudinal controller design

Longitudinal control problem

Given leader trajectory and solution for lateral control problem, determine velocity profile $v(t)$ such that

$$\lim_{t \to \infty} \alpha^{-1}(s_l(t)) - s(t) - hv(t) - r = 0 \tag{3}$$

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Observation

Reduced to 1D longitudinal control problem

- Use acceleration as input
- Apply input-output linearization (output: cf. (3))

Longitudinal controller design

Problem

Output/Error: $e(t) = \alpha^{-1}(s_l(t)) - s(t) - hv(t) - r$

Longitudinal controller design

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At $t = 0$: $s_l(t) = 0$, $s(t) = 0$, so $e(0) = -hv(0) - r \leq -r < 0$.

Even when far behind: follower initially slows down!

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Solution

Assume information planning for distance Δ into the future.
Define

$$e(t) = \alpha^{-1}(s_l(t) + \Delta) - (s(t) + \Delta) - hv(t) - r$$

Controller derivation

Since $e(t) = \alpha^{-1}(s_l(t) + \Delta) - (s(t) + \Delta) - hv(t) - r$:

Longitudinal controller design

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Since $e(t) = \alpha^{-1}(s_l(t) + \Delta) - (s(t) + \Delta) - hv(t) - r$:

$$\dot{e}(t) = \frac{v_l(t)}{\bar{v}(\alpha^{-1}(s_l(t) + \Delta))} - v(t) - ha(t)$$

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Controller:

$$a(t) = \frac{1}{h}\left[\frac{v_l(t)}{\bar{v}(\alpha^{-1}(s_l(t) + \Delta))} - v(t) + k\sigma(e(t))\right] \qquad k > 0$$

Closed-loop:

$$\dot{e}(t) = -k\sigma(e(t))$$

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Globally asymptotically stable and locally exponentially stable.

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Simulations

Paths for platoon of 4 cars ($h = 0.3$[s], $r = 4.5$[m], $\Delta = 10$[m])

Leader: $(0, 0, 0)$, $v = 0$

$a = 2$[m/s^2] until $v = 33.3$[m/s].
$t = 20$[s]: half circle (radius 800[m])
100–105[s]: $a = -2$[m/s^2]
105–110[s]: $a = 2$[m/s^2]

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Veh 3: $(-10, -10, 1.5)$, $v = 0$

Veh 4: $(-20, -10, 0)$, $v = 0$

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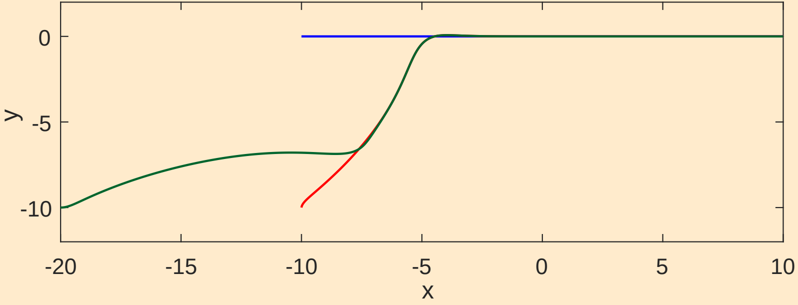
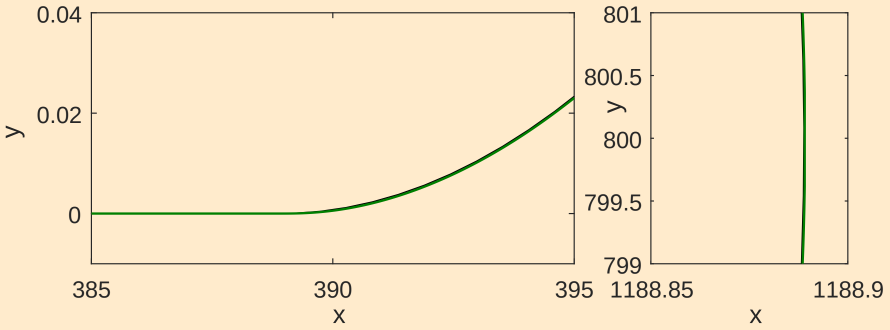


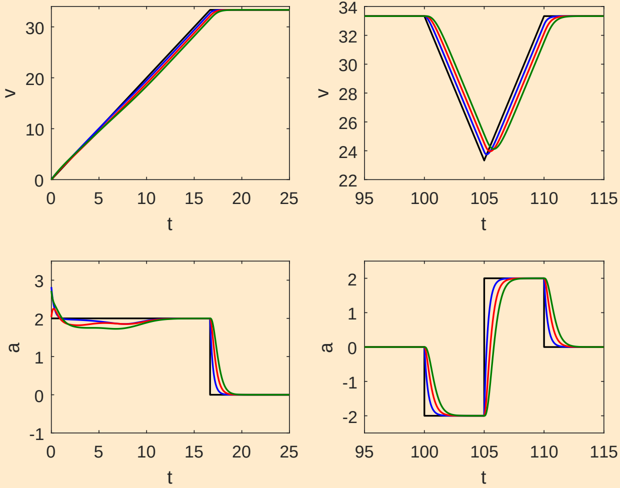
Simulations

No corner cutting



Simulations)

Response per vehicle



Conclusions and future work

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- Separate design for longitudinal and lateral controller
  - Lateral: virtual vehicle (generates mapping)
  - Longitudinal: standard 1D CACC controller (using mapping)
- Benefit: no corner cutting

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Future work

- Implement in experimental setup
  - Real time implementation
  - No full state information
- Improve lateral controller (cf. tracking for marine vessels)
- Definition of longitudinal error