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REGULAR ARTICLE

Simultaneous berth allocation and yard planning at tactical level

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Abstract We present a simultaneous berth allocation and yard planning problem at tactical level, since the berth allocation has a great impact on the yard planning and vice versa. This problem is solved by means of an alternating berth and yard planning heuristic approach. The alternating heuristic quickly converges to a local minimum which heavily depends on the starting point. Therefore, we formulate another optimization problem for generating a suitable starting point. A real size case study provided by PSA Antwerp shows that our approach to simultaneously solve both problems might reduce the total straddle carrier travel distance considerably as compared with a representative allocation.

Keywords Berth allocation · Yard planning · Linear programming · Alternating optimization

1 Introduction

We consider a terminal operator who provides a number of shipping companies with the facility logistics of discharging, loading, transporting and storing containers. Container shipping lines define so-called loops. A loop is a set of vessels visiting a sequence of terminals according to a more or less fixed schedule, similar to a time table for trains. For a terminal this results in a cyclic schedule of calling vessels, with typically a period of 7, 10 or 14 days.

The terminal operator faces the problem of determining berth positions for each of the vessel lines such that resources can be used as efficiently as possible. One of these resources is the quay, another resource is the yard. After being discharged containers are moved to an area in the yard, depending on the container type, to either wait for

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the arrival of a connecting vessel (transshipment containers) or to wait for movement to an exchange zone (inbound containers) from where they leave the terminal by truck, train or barge. Furthermore, export containers brought into the terminal by truck, train, or barge, are waiting in the yard for their vessel (outbound containers).

This problem can be considered at three different levels: strategic, tactical and operational. The time table itself, i.e. the arrival and departure times of the various vessels, is fixed for a longer period of time, usually in the order of years. It is subject to change only when contracts come up for renewal, or when new contracts are negotiated. At this strategic level, it is of paramount importance to determine the most suitable schedule for a terminal operator, and try to agree with the vessel operators upon the most favorable schedule. Given such a time table of arrivals and departures at a terminal, we can consider the same problem at a *tactical level*, which is what we do in this paper. This results in (optimal, but still proforma) schedules for weekly quay crane allocations and yard reservations. Decisions made at the tactical level may later need to be modified when vessels actually arrive at the port due to delays, equipment breakdown, etc. Then an *operational* problem must be solved.

Since both the yard planning and the berth allocation heavily depend on each other, we consider in this paper a *simultaneous* berth and yard planning problem. We assume that straddle carriers are used for transporting containers between vessels and yard, so our main objective is to reduce the total straddle carrier travel distance. Nevertheless, our approach can be applied for a PM operation (Prime Movers) as well. Since we simultaneously consider both berth allocation and yard planning at a tactical level, the yard planning as formulated in this paper is inconclusive. Therefore, after solving the problem presented in the paper, the resulting berth allocation should serve as an input to a more detailed yard planning (at a tactical level).

For an overview on applications and optimization models in this field, see the survey papers (Stahlbock and Voß 2008; Bierwirth and Meisel 2010) and references therein.

Many papers consider berth planning problems at an operational level (Cordeau et al. 2005; Imai et al. 2005, 2007; Kim and Moon 2003; Monaco and Sammarra 2007; Nishimura et al. 2001; Wang and Lim 2007) Also several papers study the berth planning problem integrated with crane planning, see Bierwirth and Meisel (2010, Table 6).

However, only a few papers consider tactical berth planning problems. Furthermore, a link with yard planning has not often been made. Often the berth allocation serves as an input for the yard planning problem (Kim and Park 2003a,b; Moccia et al. 2009; Zhang et al. 2003), and both problems are solved separately. One of the first papers on tactical berth planning while considering its effect on yard planning is (Moorthy and Teo 2006).

Moorthy and Teo (2006) consider the allocation of home berth (preferred berthing location) to a set of vessels scheduled to call at the terminal on a weekly basis. This home berth template serves as an input to create a yard template. They aim for a robust home berth template considering a trade-off between service level-waiting time and operational cost-connectivity. The movement cost for containers from vessel v to vessel w is defined as the number of containers moved from vessel v to vessel w multiplied by the distance between the berthing locations of vessels v and w. In this paper we do not derive a berth planning which serves as an input to the yard planning problem, but we consider both problems *simultaneously*. Instead of using a distance function as in

Moorthy and Teo (2006), we explicitly model the transportation of containers to and from their storage block in the yard, and minimize the total straddle carrier driving distance. Additionally, since vessels call at the terminal on a weekly basis, we allow for transshipment not only from vessel v to vessel w, but also from vessel w to vessel v. The latter transshipment is usually to vessel v in the next period, but in case vessel v and vessel w berth simultaneously, transshipment might also happen simultaneously. Furthermore we also include the transportation costs of import and export containers.

Cordeau et al. (2007) consider the service allocation problem at a tactical level. They assign services (vessels) to berths along the quay so as to minimize travel distance of containers between different vessels. As distance measure they also use the absolute distance between the corresponding berths. In Cordeau et al. (2007), discrete berths are modeled and the effect of time is ignored, so the service allocation does not necessarily result in a feasible berth allocation.

An extension of the service allocation problem dealt with in Cordeau et al. (2007) is presented in Giallombardo et al. (2010), where a tactical berth allocation problem is studied. Compared with Cordeau et al. (2007), in Giallombardo et al. (2010) time is included, as well as the usage of quay cranes. A difference between Giallombardo et al. (2010) and our work, is that we use a continuous berth location, and take into account the periodic nature of the schedule.

Compared with both Moorthy and Teo (2006) and Giallombardo et al. (2010), in this paper we explicitly model the transportation of containers to and from their storage block in the yard and try to determine the best block for a container to be temporarily stored. Additionally, we distinguish between container types (reefer, dangerous goods, empty containers, full containers).

As mentioned above, we consider the simultaneous berth allocation and yard planning problem at *tactical* level for a terminal operator who provides facility logistics for a number of shipping lines. This implies that certain decisions have already been made at a strategic level. We assume that an optimal layout of the yard has been determined see e.g. Kim et al. (2008) Following along the lines of Hendriks et al. (2012) we therefore also assume that the assignment of vessels to terminals has been made, that the terminal operator has decided on the berthing and departure time of each cyclically calling vessel, taking into account the expected amount of containers to be handled and the necessary quay and crane capacity to do so. As presented in Hendriks et al. (2010), this planning at a strategic level takes into account the fact that container terminal operator and shipping line have agreed that if a vessel arrives within an arrival window, then a certain vessel productivity and hence departure time is guaranteed.

In this paper we formulate a simultaneous berth allocation and yard planning problem, which looks like a (Multi-commodity) Capacitated Multi-source Weber Problem, (M)CMWP, suitably tailored to take into account both the time dimension, over a discretized periodic planning horizon, and the fact that a container can be stacked in several yard blocks. The main difficulty in solving this problem arises from the non-convexity of the objective function and the existence of multiple local minima (Brimberg et al. 2008).

The resulting Mixed Integer Quadratic Program (MIQP) is solved by adapting the classical Alternating Transportation - Location (ATL) heuristic approach for CMWP proposed by Cooper (1964), which converges to a local minimum after only a few iterations. A multi-start version from random starting points shows that the resulting local minimum heavily depends on the starting point. Therefore, a Mixed Integer Linear Program (MILP) is formulated to generate a good initial condition for this heuristic.

A real size case study provided by PSA Antwerp shows that our approach might result in a reduction of the total straddle carrier travel considerably compared with a representative allocation.

The outline of this paper is as follows: In Sect. 2 the problem is formally phrased as an MIQP. In Sect. 3 we present the alternating heuristic for quickly arriving at a local optimum, as well as an MILP for arriving at a suitable starting point for the alternating heuristic. In Sect. 4 we present a case study based on representative data from PSA Antwerp. Section 5 concludes the paper.

2 Model

As mentioned in the introduction, we consider a simultaneous berth and yard planning problem at *tactical level* for a terminal operator, who provides facility logistics for a number of shipping companies.

In particular this implies that we assume that several issues have been resolved at the strategic level. We assume that an optimal layout of the yard has been determined (see e.g. Kim et al. 2008), i.e. the number of storage blocks as well as their sizes and locations are known. Also the location of exchange zones is assumed to be given.

Furthermore, we assume that at strategic level a terminal assignment and cyclic berth planning has been determined along the lines of Hendriks et al. (2010, 2012). In particular this implies that arrival times and departure times of vessels are given, and that these are such that sufficient quay length is available. Additionally, for each time interval the total quay crane capacity devoted to each vessel is given, i.e., the amount of containers entering and leaving the yard has been determined and is such that containers can only be loaded or discharged when the vessel is berthing in the terminal. These amounts are real valued (which suffices at a tactical level) and also such that conservation of containers is satisfied, i.e. everything brought into a terminal by vessel or from the hinterland to be loaded onto a vessel will also leave by either a vessel or to the hinterland. Since we assume that at a strategic level a robust planning has been made as presented by Hendriks et al. (2010), the system has some slack for dealing with disturbances. Therefore, the departure time as agreed between terminal operator and shipping line can be met. As a result, the goal of this study is to not to reduce the berthing time of vessels, but to optimally use the resources on the quay. For instance using less straddle carriers if possible.

Note that at this tactical level we focus on the large vessels and ignore smaller barges for which we assume the left-over capacity of both quay and quay cranes will be used.

2.1 Model data (parameters)

The straddle carrier activity of moving between vessels and storage blocks is illustrated in Fig. 1.

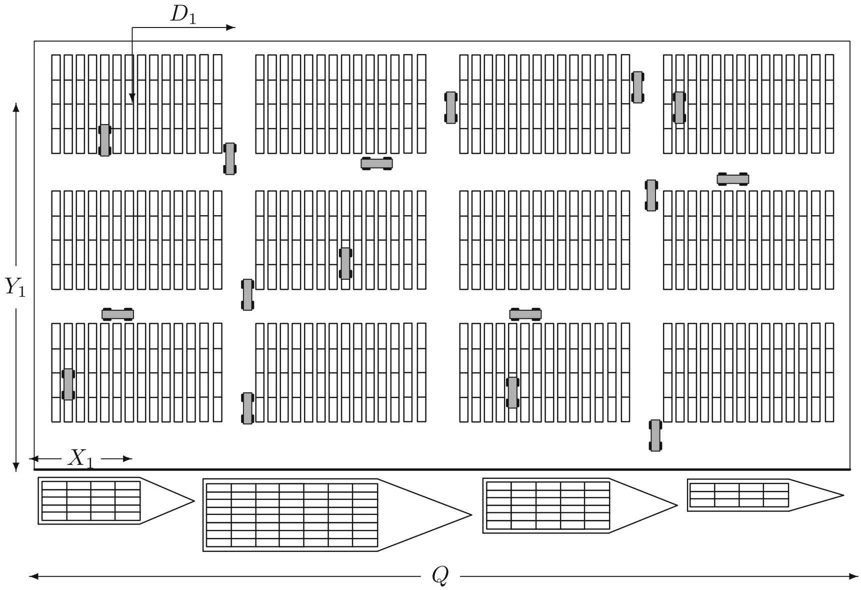


Fig. 1 Straddle carriers operating between vessels and storage blocks

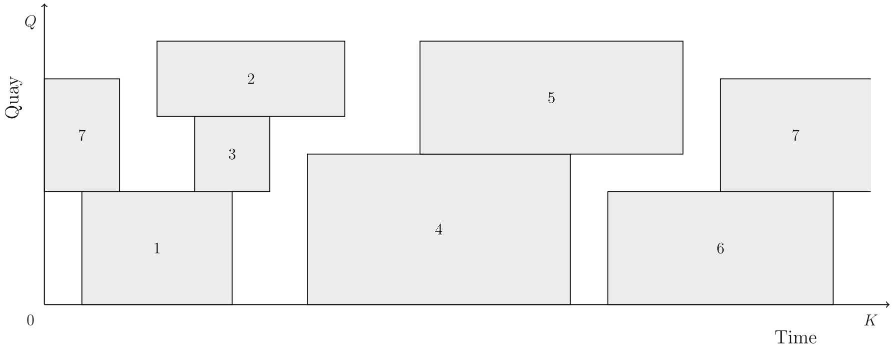


Fig. 2 An example of a periodic time table. Note that vessel 7 arrives at the end of the cycle and departs in the next one

We define the terminal quay length to be Q meter with a yard width of W meter. The yard is divided into N storage blocks, each with a capacity of C_n TEU (twenty foot equivalent units). The location of storage block n is determined by its center (X_n, Y_n), and its Manhattan distance to the nearest exchange zone is D_n. We consider T container types (e.g. reefer, dangerous goods, 20, 40 foot, empty, full containers, etc.), each with a size of M^t TEU. A number of V vessels berth at the quay cyclically, each with a length L_v meter, including the safety distance between vessels. Since arrival and departure time of the vessels are given, the set of vessels which berth simultaneously is also given. We define the set $\mathcal{S}$ as the set of vessel pairs $(v, w) \in \mathcal{S}$ that berth simultaneously. Note that if $(v, w) \in \mathcal{S}$, then also $(w, v) \in \mathcal{S}$. As an example, consider the time table depicted in Fig. 2. For this example the set of vessel pairs is given by

Table 1 Model parameters

Parameter	Definition
Q	Quay length [m]
W	Yard width [m]
N	Number of storage blocks in the yard
C_n	Capacity of storage block $n \in \{1, \ldots, N\}$ [TEU]
X_n	x-Position of the center of storage block $n \in \{1, \ldots, N\}$ [m]
Y_n	y-Position of the center of storage block $n \in \{1, \ldots, N\}$ [m]
D_n	Manhattan distance of the center of storage block $n \in \{1, \ldots, N\}$ to the nearest exchange zone [m]
T	Number of container types
M^t	Size of container of type $t \in \{1, \ldots, T\}$ [TEU]
V	Number of vessels
L_v	Length of vessel $v \in \{1, \ldots, V\}$, including safety distance [m]
$\mathcal{S}$	Set of pairs of vessels $(v, w) \in \{1, \ldots, V\}^2$ that berth simultaneously
K	Number of discrete time slots in the cycle
$I^t_{v,w}(k)$	Amount of containers of type $t \in \{1, \ldots, T\}$ entering the yard with source vessel $v \in \{0, 1, \ldots, V\}$ and destination vessel $w \in \{0, 1, \ldots, V\}$ during time slot k, for $k \in \{1, \ldots, K\}$
$O^t_{v,w}(k)$	Amount of containers of type $t \in \{1, \ldots, T\}$ leaving the yard with source vessel $v \in \{0, 1, \ldots, V\}$ and destination vessel $w \in \{0, 1, \ldots, V\}$ during time slot k, for $k \in \{1, \ldots, K\}$

$$\mathcal{S} = \{(1, 2), (1, 3), (1, 7), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 2), (4, 5), (5, 4), (5, 6), (6, 5), (6, 7), (7, 1), (7, 6)\}$$

We discretize time such that the cycle consists of K discrete time slots. At a strategic level the assignment of quay crane capacity to vessels has been determined for each time slot. Therefore, for each time slot k we know the amount of containers of type t entering the yard being discharged from vessel v with destination vessel w: $I^t_{v,w}(k)$. We also know the amount of containers of type t leaving the yard being loaded onto vessel w which originated from vessel v: $O^t_{v,w}(k)$. For the ease of notation in the definition of $I^t_{v,w}(k)$ and $O^t_{v,w}(k)$ we allow for vessel 0 which represents the hinterland. So $I^1_{2,0}(3)$ denotes the amount of import containers of type 1 being discharged from vessel 2 during the third time slot. Similarly, $O^1_{2,0}(3)$ denotes the amount of import containers of type 1 from vessel 2 leaving the yard to the exchange zone during the third time slot, and $I^1_{0,4}(5)$ denotes the amount of export containers of type 1 with destination vessel 4 entering the yard from the exchange zone during the fifth time slot. The import and export containers are assumed to respectively leave and enter the yard according to empirical dwell time distributions. The model parameters together with their definitions are summarized in Table 1.

2.2 Model formulation

The objective of this study is to minimize the overall straddle carrier travel distance. We take into account the straddle carrier transportation between quay and yard, and between yard and hinterland (exchange zones). For the travel distance we use the Manhattan distance between the centers of storage blocks and vessels, i.e. an L_1-norm.

The problem is to both find berth locations for vessels and assign storage blocks to containers such that the total carrier distance is minimized, under the constraints that vessels do not overlap and for each block storage capacity is not exceeded. To that end, we define the following (real-valued) decision variables:

q_v	berth location of center of vessel $v \in \{1, \ldots, V\}$ along the quay
$i^t_{n,v,w}(k)$	amount of containers of type $t \in \{1, \ldots, T\}$ entering storage block $n \in \{1, \ldots, N\}$ with source vessel $v \in \{0, 1, \ldots, V\}$ and destination vessel $w \in \{0, 1, \ldots, V\}$ during time slot k, for $k \in \{1, \ldots, K\}$
$o^t_{n,v,w}(k)$	amount of containers of type $t \in \{1, \ldots, T\}$ leaving storage block $n \in \{1, \ldots, N\}$ with source vessel $v \in \{0, 1, \ldots, V\}$ and destination vessel $w \in \{0, 1, \ldots, V\}$ during time slot k, for $k \in \{1, \ldots, K\}$

As mentioned before, we allow for vessel 0, which represents the hinterland. In addition, we define the following (real-valued) auxiliary variables:

$d_{n,v}$	Manhattan distance between the center of storage block $n \in \{1, \ldots, N\}$ and the center of vessel $v \in \{1, \ldots, V\}$, respectively nearest exchange zone $(v = 0)$
$c^t_{n,w}(k)$	amount of containers of type $t \in \{1, \ldots, T\}$ in storage block $n \in \{1, \ldots, N\}$ for destination vessel $w \in \{0, 1, \ldots, V\}$ during time slot k, for $k \in \{1, \ldots, K\}$,

as well as the (binary) auxiliary variables:

$$e_{v,w} \begin{cases} 0 & \text{if } q_v > q_w \\ 1 & \text{if } q_v < q_w \end{cases} \quad \text{for all} \quad (v, w) \in \mathcal{S}.$$

The total straddle carrier travel distance is given by the product of the amount of containers transported and their distance of transportation. Therefore, the objective becomes:

$$\min_{\substack{q_v,\, d_{n,v},\, e_{v,w},\\ i^t_{n,v,w}(k),\, o^t_{n,v,w}(k)}} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{v=0}^{V} \sum_{w=0}^{V} \left(d_{n,v} \cdot i^t_{n,v,w}(k) + d_{n,w} \cdot o^t_{n,v,w}(k)\right) \quad (1)$$

This minimization is subject to constraints. With respect to the berth allocation problem, the following constraints are valid:

$$\frac{1}{2}L_v \le q_v \le Q - \frac{1}{2}L_v \quad v \in \{1, \ldots, V\} \quad (2a)$$

$$q_v - q_w \geq \frac{1}{2}L_v + \frac{1}{2}L_w - e_{v,w}Q \quad (v, w) \in \mathcal{S} \tag{2b}$$

$$e_{v,w} + e_{w,v} = 1 \quad (v, w) \in \mathcal{S} \tag{2c}$$

$$d_{n,v} \geq q_v - X_n + Y_n \quad n \in \{1, \ldots, N\}, v \in \{1, \ldots, V\} \tag{2d}$$

$$d_{n,v} \geq -q_v + X_n + Y_n \quad n \in \{1, \ldots, N\}, v \in \{1, \ldots, V\} \tag{2e}$$

$$d_{n,0} = D_n \quad n \in \{1, \ldots, N\} \tag{2f}$$

The constraint (2a) guarantees that vessel v is positioned within the available quay space. Constraints (2b) and (2c) ensure non-overlapping of vessels that berth simultaneously. Constraints (2d) and (2e) ensure that $d_{n,v}$ becomes at least the Manhattan distance between storage block n and vessel v. Note that equality follows from minimizing the objective. Finally, (2f) makes that $d_{n,0}$ equals the Manhattan distance between storage block n and the nearest exchange zone.

With respect to the yard planning problem, the following constraints are valid:

$$\sum_{n=1}^{N} i^t_{n,v,w}(k) = I^t_{v,w}(k) \quad \forall k, t, v, w \tag{3a}$$

$$\sum_{n=1}^{N} o^t_{n,v,w}(k) = O^t_{v,w}(k) \quad \forall k, t, v, w \tag{3b}$$

$$c^t_{n,w}(k+1) = c^t_{n,w}(k) + \sum_{v=0}^{V}(i^t_{n,v,w}(k) - o^t_{n,v,w}(k)) \quad \forall k, n, t, w \tag{3c}$$

$$\sum_{t=1}^{T} M^t \cdot \sum_{w=0}^{V} \left(c^t_{n,w}(k) + \sum_{v=0}^{V} i^t_{n,v,w}(k) \right) \leq C_n \quad \forall k, n \tag{3d}$$

$$c^t_{n,w}(k) - \sum_{v=0}^{V} o^t_{n,v,w}(k) \geq 0 \quad \forall k, n, t, w \tag{3e}$$

The constraints (3a) and (3b) represent conservation laws: adding the amount of containers over all blocks yields the total amount of containers. Constraint (3c) describes the dynamics of the amount of containers in each storage block. Due to the cyclic nature we should understand this relation modulo K, i.e. $c^t_{n,w}(K+1) = c^t_{n,w}(1)$. The constraint (3d) ensures that the storage capacity of a block is never exceeded, even if during a time slot all containers first enter the yard and only then the containers to be loaded leave again. Similarly, (3e) ensures a non-negative amount of containers of each type in each block, even if during a time slot first all containers leave the yard and then all containers enter. Notice that we assume that a straddle carrier can only handle one container at a time, no matter what type t. So each container induces an individual driving distance. The size M^t (20 or 40 ft) is only needed to account for the capacity consumed by containers in the storage blocks.

In addition to the constraints (2) and (3) we also require that all variables are nonnegative.

Note that for each of the special types of reefer, dangerous goods, and empty containers usually specific blocks are designated. Hence, for container types having one or more designated storage blocks, the variables $i^t_{n,v,w}(k)$ and $o^t_{n,v,w}(k)$ must all be set to zero for the remainder of the blocks. Similarly for $v = w$ the variables $i^t_{n,v,w}(k)$ and $o^t_{n,v,w}(k)$ can be set to zero. Furthermore, notice that all variables are real-valued, except $e_{v,w} \in \{0, 1\}$ for $(v, w) \in \mathcal{S}$. Therefore, the resulting optimization problem is a Mixed Integer Quadratic Program (MIQP).

We conclude this section by remarking that the formulation presented above is typical for an SC operation as we consider in this paper. For a PM operation one would prefer to have no more than say 15 containers per block with the same destination due to the limited throughput of a stacking crane on a block. This can be incorporated easily along the lines of the constraints (3c)–(3e). Also yard-to-yard movements of transshipment containers and congestion effects, cf. (Lee et al. 2006; Han et al. 2008), can be incorporated straightforwardly in the yard planning problem.

3 Alternating BAP–YAP heuristic

The MIQP as formulated in the previous section looks like an MCMWP. The main difficulty in solving this problem arises from the non-convexity of the objective function and the existence of multiple local minima Brimberg et al. (2008). Our approach to find a local minimum closely resembles the classical ATL heuristic approach for CMWP as originally proposed by Cooper (1964).

Notice that the optimization problem (1)–(3) consists of two components: the Berth Allocation Problem (BAP) and the Yard Allocation Problem (YAP), which both are easy to solve in isolation. That is, given the Berth Allocation [(i.e. q_v with corresponding $e_{v,w}$ and $d_{n,v}$ satisfying (2)], the problem reduces to the Linear Program (1), (3). And once the container flows $i^t_{n,v,w}(k)$ and $o^t_{n,v,w}(k)$ are given with corresponding $c^t_{n,w}(k)$ satisfying (3), the problem reduces to the MILP (1), (2) which is easy to solve since the number of binary variables $e_{v,w}$ is relatively small.

The heuristic alternates between BAP and YAP until no further improvement is possible. This is usually the case after a small number of BAP–YAP iterations. In this way we quickly obtain a local minimum.

Since the problem (1)–(3) has many local minima, the resulting minimum heavily depends on the starting point used. In this paper we consider two options for arriving at a "good" local minimum: a multi-start version of the heuristic, and determining a suitable starting point by solving an additional MILP.

3.1 Multi start

A generally applicable and rather straightforward way to arrive at a "good" local minimum is to consider a multi-start version of the alternating BAP–YAP heuristic. We repeat the heuristic from random starting points and retain the best local minimum as the final solution. Since typically first the BAP is solved, and used as an input for the

YAP, one of the starting points should also be the solution to the YAP corresponding with the berth allocation determined at a strategic level.

3.2 Suitable starting point

Instead of using a multi-start approach, we can also carefully select the initial yard allocation by formulating an additional MILP to properly select initial values for $i^t_{n,v,w}(k)$ and $o^t_{n,v,w}(k)$.

To that end, we assume that containers of type t originating from vessel v with destination vessel w are always assigned to one and the same storage block, whereas in the original MIQP formulation the assigned storage block is allowed to vary over time and need not be the same. We neglect the constraint on maximum storage block capacity, since such a starting point still results in a feasible BAP, and after solving the BAP, solving the YAP yields a feasible solution (taking into account the storage constraints).

We therefore use the additional decision variable:

$$a^t_{n,v,w} = \begin{cases} 0 & \text{if containers of type } t \in \{1,\ldots,T\} \text{ with source vessel } v \in \{0,1,\ldots,V\} \\ & \text{and destination vessel } w \in \{0,1,\ldots,V\} \text{ are not assigned} \\ & \text{to storage block } n \in \{1,\ldots,N\} \\ 1 & \text{if containers of type } t \in \{1,\ldots,T\} \text{ with source vessel } v \in \{0,1,\ldots,V\} \\ & \text{and destination vessel } w \in \{0,1,\ldots,V\} \text{ are assigned to} \\ & \text{storage block } n \in \{1,\ldots,N\} \end{cases}$$

Then we obtain $i^t_{n,v,w}(k) = I^t_{v,w}(k) \cdot a^t_{n,v,w}$, and $o^t_{n,v,w}(k) = O^t_{v,w}(k) \cdot a^t_{n,v,w}$. Hence, the objective becomes

$$\min_{\substack{q_v,\, d_{n,v},\, e_{v,w},\\ a^t_{n,v,w}}} \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{v=0}^{V}\sum_{w=0}^{V}\left(I^t_{v,w}(k)\cdot d_{n,v}\cdot a^t_{n,v,w} + O^t_{v,w}(k)\cdot d_{n,w}\cdot a^t_{n,v,w}\right)$$

Since we do not have a product of two continuous design variables anymore, but a product of a binary variable and a bounded continuous variable, we can modify the formulation so that the objective becomes a linear expression. For that, we need to introduce additional continuous variable $\delta^t_{n,v,w} = d_{n,v} \cdot a^t_{n,v,w}$ and $\epsilon^t_{n,v,w} = d_{n,w} \cdot a^t_{n,v,w}$ which result from the following linearizing constraints:

$$\begin{aligned} &0 \le \delta^t_{n,v,w} \le (Q+W)a^t_{n,v,w} \quad d_{n,v} - (Q+W)(1-a^t_{n,v,w}) \le \delta^t_{n,v,w} \le d_{n,v} \\ &0 \le \epsilon^t_{n,v,w} \le (Q+W)a^t_{n,v,w} \quad d_{n,w} - (Q+W)(1-a^t_{n,v,w}) \le \epsilon^t_{n,v,w} \le d_{n,w} \end{aligned} \tag{4}$$

which hold for all t, n, v, w. Since $0 \le d_{n,v} \le Q + W$ it follows that due to these constraints $\delta^t_{n,v,w} = d_{n,v} \cdot a^t_{n,v,w}$, and similarly that $\epsilon^t_{n,v,w} = d_{n,w} \cdot a^t_{n,v,w}$. As a result we obtain the following MILP for determining a suitable starting point:

$$\min_{\substack{q_v,\, d_{n,v},\, e_{v,w},\\ a^t_{n,v,w},\, \delta^t_{n,v,w},\, \epsilon^t_{n,v,w}}} \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{v=0}^{V}\sum_{w=0}^{V}(I^t_{v,w}(k)\cdot\delta^t_{n,v,w} + O^t_{v,w}(k)\cdot\epsilon^t_{n,v,w}) \tag{5}$$

subject to the BAP constraints (2), the YAP constraints

$$\sum_{n=1}^{N} a^t_{n,v,w} = 1 \quad \forall t, v, w \tag{6a}$$

$$c^t_{n,w}(k+1) = c^t_{n,w}(k) + \sum_{v=0}^{V}(I^t_{v,w}(k) - O^t_{v,w}(k))a^t_{n,v,w} \quad \forall k, n, t, w \tag{6b}$$

$$C_n \geq \sum_{t=1}^{T} M^t \cdot \sum_{w=0}^{V}\left(c^t_{n,w}(k) + \sum_{v=0}^{V} I^t_{v,w}(k)a^t_{n,v,w}\right) \quad \forall k, n \tag{6c}$$

$$0 \leq c^t_{n,w}(k) - \sum_{v=0}^{V} O^t_{v,w}(k)a^t_{n,v,w} \quad \forall k, n, t, w \tag{6d}$$

and the linearizing constraints (4) for all t, n, v, w.

Where, as explained before, in case of infeasibility the constraint (6c) can be relaxed (or even omitted). Furthermore, when there is no flow from vessel v to vessel w of containers of type t, there is no need to assign a storage block and the associated variable $a^t_{n,v,w}$ can be omitted from the problem formulation, as well as the associated $\delta^t_{n,v,w}$ and $\epsilon^t_{n,v,w}$.

Nevertheless, notice that for real life problem instances, the number of additional (binary) decision variables $a^t_{n,v,w}$ becomes really large, making it impossible to solve the MILP (2), (4)–(6) to optimality within a reasonable amount of time. For a real life problem usually a reasonable allocation is available, e.g. the one currently in use. Then one can fix most of the additional decision variables in accordance with this allocation and keep only those additional decision variables that contribute most to the total straddle carrier driving distance, in order to obtain a computationally feasible MILP for generating a suitable starting point, see also Sect. 4.2.

4 Case study

The terminal operator PSA Antwerp provided us with a representative data set consisting of a cyclic timetable, vessels' load compositions and yard layout. Since the vessel lines used different cycles, we considered a cycle of four weeks which was the lowest common multiple of all cycle durations. For the vessel lines that have multiple vessels calling in this general cycle, we used only one decision variable with respect to the vessels position, such that all vessels of the same loop berth at the same position (as preferred in practice). The resulting cycle contained 103 vessels ($V = 103$).

The quay has a length of $Q = 2{,}000$ m and a yard width of $W = 400$ m. Two exchange zones are located at (550, 400) and (1,200,400) respectively, where the point

(0, 0) corresponds with the bottom left in Fig. 1. This figure is not representative for the yard, since the considered yard consists of $N = 20$ storage blocks. We distinguished eight different container types ($T = 8$): full, reefer, dangerous goods and empty containers, each of them in two sizes (1 and 2 TEU). We discretized time into $K = 168$ time slots (of 4 h).

A a result, the BAP was an MILP consisting of about 800 variables and 1,500 constraints. The YAP was an LP consisting of almost 2,000,000 variables and 550,000 constraints. Convergence to a local minimum was achieved after a few iterations. For computation times, see Sect. 4.3

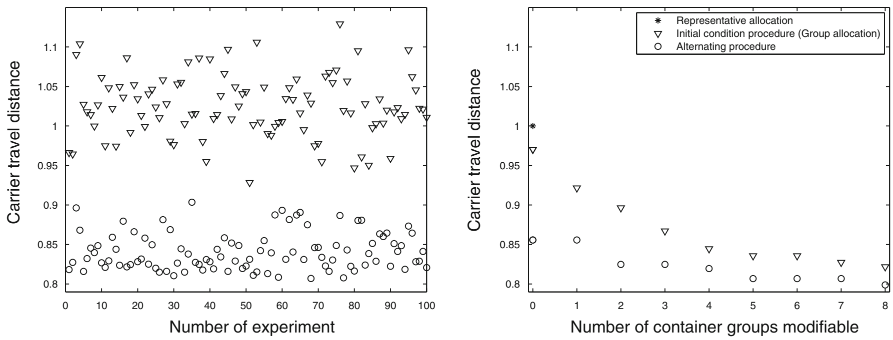
For the representative data set, the corresponding total straddle carrier driving distance can be determined, which serves as a reference point. All results we present in this section have been scaled with respect to this total straddle carrier driving distance. We first present the results from the multi-start heuristic, then the results obtained from first determining a suitable starting point.

4.1 Multi-start heuristic with random starting points

We solved the proposed MIQP heuristically by alternatingly solving the BAP-MILP and the YAP-LP. We did this 100 times using different starting points that we selected randomly. To generate a starting point we assigned all containers of a certain type t, originating from vessel v with destination vessel w to a storage block which we picked randomly (using a uniform distribution), ignoring the capacity constraint (6c). This is no problem, as after one iteration we always have a feasible assignment.

Results from these 100 experiments are depicted in Fig. 3a, where a triangle represents the total straddle carrier travel distance for the random starting point, and the circle straight below it represents the total straddle carrier travel distance after convergence of the alternating procedure for that particular starting point. All distances are scaled with respect to the total straddle carrier driving distance obtained from the representative data set provided by PSA Antwerp.

From this figure we learn the following:



(a) Random starting points **(b)** Suitable starting points

Fig. 3 Results of the alternating procedure

- In each experiment, the alternating procedure yields significant reductions in the total straddle carrier driving distance compared with the initial condition.
- A good, randomly selected, starting point does not necessarily provide a good end point.
- The majority (about 80 %) of the randomly selected starting points shows a larger total straddle carrier driving distance than in the representative allocation.
- For each of the 100 experiments, the alternating procedure yields a solution which is at least 10 % better than the representative allocation.
- The best solution outperforms the representative allocation by almost 20 %.

4.2 Suitable starting point

In this section we apply the alternating optimization for the starting point generated by the MILP presented in Sect. 3.2. Since the MILP consists of a relatively large number of variables, it can not be solved to optimality within a reasonable amount of time.

Hence, we did the following. Starting from the representative allocation provided by PSA Antwerp we identified more than 200 groups of containers (v, w, t-combinations) varying in size between 1 and $\leq$1,500 containers. We assigned each group to the storage block to which the majority of the containers of that group had been assigned in the representative allocation. In this way we fixed the decision variables $a^t_{n,v,w}$. For this allocation the maximum storage block capacity of 2,500 was never exceeded. Next, we ordered the groups of containers according to their contributions in the total straddle carrier driving distance resulting from this assignment. From these groups, we took the G groups with the largest contributions to the total distance and set the corresponding values for $a^t_{n,v,w}$ to be variables again, while keeping the other $a^t_{n,v,w}$ fixed, and ran the MILP presented in Sect. 3.2. Subsequently, the resulting solution was used as a starting point for the alternating optimization procedure.

The outcomes are depicted in Fig. 3b, showing the resulting straddle carrier driving distance after the alternating procedure as a function of G. From this figure we conclude the following:

- In each experiment, the alternating procedure yields significant reductions in the carrier travel distance starting from the starting point. The reduction however decreases as the size of G decreases.
- Each generated starting point already outperforms the representative allocation.
- For $G = 0$ (no groups can be modified) the starting point already outperforms the representative allocation. Although the container block allocation is fixed and cannot be changed while generating this starting point, the berth positions of the vessels are variable. Apparently, a modification of only the vessels' berth positions already yields a reduction of about three percent in carrier travel distance.
- As G increases, the found objective value for the starting points (triangles) decreases. This makes sense since if more container groups are variable, no larger travel distance will result from the optimization.
- A better starting point (triangle) never yields a worse converged solution (circle). This disagrees with the observation made for Fig. 3a, where a better starting point not necessarily led to a better converged solution.

- The modification of only the eight largest contributions (and possibly all vessels' berth positions) already leads to a better solution than the best found solution for 100 random starting points. The corresponding reduction in travel distance with respect to the travel distance in the representative allocation is more than 20 %.

4.3 Discussion

Recall that the MIQP problem as presented in Sect. 2 can not be solved to optimality within a reasonable amount of time due to the fact that it is a non-convex problem with multiple local minima (the latter is also illustrated in Fig. 3a, since each circle corresponds to a local minimum of the MIQP). We therefore used an alternative BAP–YAP heuristic for arriving at a local minimum of the MIQP.

In order to arrive at a 'good' local minimum, we applied a standard multi-start heuristic with random starting points. As an alternative we solve a big MILP (with many equality constraints added compared to the formulation presented in Sect. 3.2) for generating a suitable starting point.

For our case study we used CPLEX 11 on a 3GHz Intel Xeon CPU. Convergence to a local minimum using BAP–YAP iterations was achieved after a few iterations, and typically took a few minutes (on average 0:01:13 with a standard deviation of 0:00:01 for the suitable starting points). The total computation time for generating the multi-start result presented in Fig. 3a was 5:03:54 (0:03:02 per random initial condition with a standard deviation of 0:02:42). For the alternative approach, solving a big MILP for generating a suitable starting point took 1:53:57 with a standard deviation of 0:11:26. Therefore, from a computation time point of view both methods are comparable, but the latter approach resulted in a better local minimum. From these results we conclude that for this case it pays off to generate a proper initial condition rather than executing an extensive number of experiments for random starting points.

5 Conclusions and recommendations

We considered a simultaneous berth allocation and yard planning problem at a tactical level. We assume that arrival and departure times of vessels have been determined at a strategic level, as well as the schedule of containers arriving to and leaving the yard in each time slot. We presented an MIQP formulation for solving both the berth allocation and yard planning problem simultaneously.

We presented an alternating berth and yard planning heuristic approach for solving this MIQP. The alternating heuristic converges to a local minimum after a few iterations. However, the obtained local minimum heavily depends on the starting point. Therefore, we formulate another optimization problem for generating a suitable starting point.

The main conclusion is that a real size case study provided by PSA Antwerp shows that our approach might reduce the total straddle carrier travel distance considerably compared with a representative allocation provided by PSA Antwerp.

The main goal in this study was to reduce the total straddle carrier driving distance. As a result, storage blocks closest to the berth positions of vessels were typically stacked up to three containers high (the largest possible stacking height when using straddle carriers), whereas other storage blocks remained relatively empty. In this way the total distance between vessel and containers indeed is minimized. A drawback of stacking containers on top of each other however is that it may require additional handling to retrieve a certain container. Namely, a container, which for instance is stacked at the bottom of a pile cannot be picked up directly, but first the top containers have to be shifted. Each individual container that has to be shifted to retrieve another one is called a shifter. The probability of a shifter increases as the stacking height increases. Since each shifter requires time and money, an interesting recommendation is to minimize the amount of shifters as a second (conflicting) objective. In a related case study, a relation between the stacking height and the expected amount of shifters has been derived and introduced in the model. Given the average time required for a shifter and the average speed of a straddle carrier, the costs of a shifter were translated into a "lost" straddle carrier distance. In this way, the two objectives could be traded off fairly, see also Hendriks (2009).

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